SMART DEVICES
Non-linear Resistive Sensors

Josué ALVAREZ, Cyril ANAK STELL, Aminata DIOP,
Cécile DUTHOIT, Linn MJELSTAD, Clovis OUEDRAOGO

December 2016

1 Introduction

This document answers an exercise about resistive sensors and how to choose the appropriate electrical assembly to power this sensor. It discusses non-linearity reduction of the measurement on non-linear resistive sensors, such as the one we made during our AIME internship. Five different types of potentiometric assemblies will be discussed and compared:

- Potentiometric assembly powered by a voltage or current source
- Quarter-bridge arm
- Half bridge
- Push-pull
- Active quarter-bridge arm linearization

2 Resistive Sensor

2.1 Question 5.1 - Deviation from linearity

The deviation from linearity is the most important deviation within the measurements between the real feature and its linear approximation. The extreme values pour m are here 0 and 2, which leads to R_c:

$$\delta R_c = max(R_c - R_{c,lin}), m \in [0;2]$$
$$\delta R_c = 0,19 \quad \Omega$$

2.2 Question 5.2 - Linearity error

The linearity error is the deviation from linearity divided by the difference between both extreme values of the sensor output, which is here its resistance:

$$\epsilon = \frac{\delta R_c}{max(R_c) - min(R_c)} = \frac{0,19}{121,20 - 100} = 0,9 \quad \%$$

2.3 Question 5.3 - Sensor sensibility

Using the linear approximation given at the beginning of the exercise, we got S_c, the sensor sensibility:

$$S_c = \frac{\Delta R_c}{\Delta m} = b' = 10,6 \quad \Omega/m's \quad unity$$

2.4 Question 5.4

It is better to choose a value at the middle of all the measurement values already done, in order to limit the non-linearity and the uncertainties. We can choose 1 here. By reading the table of results for $m_0 = 1$, we get:

$$R_c(m_0) = R_{c0} = 110,30 \quad \Omega$$

2.5 Question 5.5

We want to demonstrate that $\Delta R_c = A\Delta m^2 + B\Delta m$

$$\Delta R_c = R_c(m) - R_{c0}$$
$$\Delta R_c = R_c(m_0 + \Delta m) - R_{c0}$$
$$\Delta R_c = a(m_0 + \Delta m)^2 + b(m_0 + \Delta m) + c - (am_0^2 + bm_0 + c)$$
$$\Delta R_c = a\Delta m^2 + (b + 2am_0)\Delta m$$
$$\Delta R_c = A\Delta m^2 + B\Delta m$$

We get the expression we were looking for.

3 Potentiometer installation - Current supply

3.1 Question 5.6

Voltage divider:

$$V_{mes} = \frac{R_c}{R + R_c}V_g = \frac{R_{c,0} + \Delta R_c}{R + R_{c,0} + \Delta R_c}V_g \tag{1}$$

3.2 Question 5.7

$$V_{mes} = \Delta V_{mes} + V_{mes,0} \Rightarrow \Delta V_{mes} = V_{mes} - V_{mes,0} \tag{2}$$

$$\begin{aligned}
\Delta V_{mes} &= \frac{R_{c,0} + \Delta R_c}{R + R_{c,0} + \Delta R_c}V_g - \frac{R_{c,0}}{R + R_{c,0}}V_g \\
&= \frac{(R_{c,0} + \Delta R_c)(R + R_{c,0})}{(R + R_{c,0} + \Delta R_c)(R + R_{c,0}}V_g - \frac{R_{c,0}(R + R_{c,0} + \Delta R_c}{(R + R_{c,0})(R + R_{c,0} + \Delta R_c)}V_g \\
&= \frac{R_{c,0}R + R_{c,0}R_{c,0} + \Delta R_c R + \Delta R_c R_{c,0} - RR_{c,0} - R_{c,0}R_{c,0} - R_{c,0}\Delta R_c}{RR + RR_{c,0} + R\Delta R_c + RR_{c,0} + R_{c,0}R_{c,0} + R_{c,0}\Delta R_c}V_g \\
&= \frac{\Delta R_c R}{RR + 2RR_{c,0} + R_{c,0}R_{c,0} + \Delta R_c(R + R_{c,0})}V_g \\
&= \frac{\Delta R_c R}{(R + R_{c,0})^2 + \Delta R_c(R + R_{c,0})}V_g
\end{aligned} \tag{3}$$

3.3 Question 5.8

We are looking for the value of R that gives us $\frac{d\Delta V_{mes}}{dR} = 0$.

Derived the expression becomes :

$$\frac{d\Delta V_{mes}}{dR} = \frac{R_c - R_{c,0}}{(R_{c,0} + R)^2(R_c + R)^2}V_g(R_cR_{c,0} - R^2) = 0 \tag{4}$$

R_c is evolving around $R_{c,0}$ and therefore we chose to use $R = R_{c,0}$. Which gives us:

$$\Delta V_{mes} = \frac{\Delta R_c}{4R_{c,0}(1 + \frac{\Delta R_c}{2R_{c,0}})}Vg = \frac{A\Delta m^2 + B\Delta m}{4R_{c,0}(1 + \frac{A\Delta m^2 + B\Delta m}{2R_{c,0}}}Vg \tag{5}$$

4 Montage potentionmétrique - Alimentation en courant

4.1 Question 5.12

Expression of Vmes=f(Rc and Ig) :

$$V_{mes} = R_cI_g \tag{6}$$

Let's calculate the variation ΔV_mes with Δm and ΔR_c evolution:

$$\begin{aligned}\Delta V_{mes} &= V_{mes} - V_{mes,0} \\ &= R_c I_g - R_{c,0} I_g \\ &= I_g \Delta R_c\end{aligned} \tag{7}$$

Where :

$$\begin{aligned}\Delta R_c &= R_c - R_{c,0} \\ &= a\Delta(m^2) + b\Delta m\end{aligned} \tag{8}$$

4.2 Question 5.13

Expression of $\Delta V_{mes,lin}$, the linear approximation of ΔV_{mes}:

$R_{c,lin} = b'm - c'$

$$\begin{aligned}\Delta V_{mes,lin} &= I_g \Delta R_{c,lin} \\ &= I_g(R_c - R_{c,0}) \\ &= I_g(bm + c' - b'm_0 - c') \\ &= I_g b'(m - m_0) = I_g b' \Delta m\end{aligned}$$

4.3 Question 5.14

Comparaison with "Alimentation en tension":

4.4 Question 5.15

Expression of error of linearity ϵ_2 :

$$\epsilon_2 = \Delta V_{mes} - \frac{\Delta V_{mes,lin}}{\Delta V_{mes}} \tag{10}$$

If

$$\Delta V_{mes} \simeq \Delta V_{mes,lin} \tag{11}$$

thus

$$\epsilon_2 = \Delta V_{mes,lin} - 1 \tag{12}$$

4.5 Question 5.16

The expression of V_{mes}:

$$V_{mes} = \left(\frac{R_c}{R_c + R_1} + \frac{R_3}{R_3 + R_2}\right)V_g = \frac{R_c R_2 - R_1 R_3}{(R_c + R_1)(R_3 + R_2)} V_g$$

4.6 Question 5.17

Balancing the bridge for the value m_0 of the measurand is equivalent to $R_{c0}R_2 - R_1 R_3 = 0$.

$$V_{mes} = \frac{R_c R_2 - R_1 R_3}{(R_c + R_1)(R_3 + R_2)} V_g = 0 \Rightarrow R_c R_2 - R_1 R_3 = 0$$

It is necessary to choose $R_1 = R_{c0}$ to have a better sensitivity in the potentiometric branch containing the sensor. This results is $R_2 = R_3$.In order to have the same power dissipated by the joule effect on each of the resistors (including the sensor) so as to balance the heating, $R_1 = R_2 = R_3 = R_{c0} = 110.30\Omega$. So the expression of V_{mes} become:

$$V_{mes} = \frac{R_c - R_{c0}}{R_c + R_{c0}} \frac{V_g}{2}$$

4.7 Question 5.18

The measurement voltage is written as:

$$V_{mes} = \Delta V_{mes} = \frac{\Delta R_c}{2R_{c0}(1 + \frac{\Delta R_c}{2R_{c0}})} \frac{V_g}{2} = \frac{B\Delta m(1 + \frac{A}{B}\Delta m)}{1 + \frac{A\Delta m^2 + B\Delta m}{2R_{c0}}} \frac{V_g}{4R_{c0}}$$

4.8 Question 5.19

The linear approximation $\Delta V_{mes,lin}$ of ΔV_{mes} is given by the first order expansion in Δm, that is:

$$\Delta V mes, lin = \frac{BV_g}{4R_{c0}}\Delta m$$

4.9 Question 5.20

The reduced sensitivity x of the measurement is deduced therefrom:

$$S_r = \frac{1}{V_g}\frac{\Delta V_{mes,lin}}{\Delta m} = \frac{B}{4R_{c0}}$$

S_r =24mV per unit of m/V

4.10 Question 5.21

The linearity error is calculated as in the case of potentiometric power supplied:

$$\epsilon_3 = \frac{(\frac{A}{B} - \frac{B}{2R_{c0}})\Delta_m - \frac{A}{2R_{c0}\Delta m^2}}{1 + \frac{A}{B}\Delta m}$$

The expansion to order 2 in Δm of the above expression gives:

$$\epsilon_3 = (\frac{A}{B} - \frac{B}{2R_{c0}})\Delta m - \frac{A^2}{B^2}\Delta m^2 = -1,97.10^{-2}\Delta m - 8,01.10^{-4}\Delta m^2$$

This term is maximum for: Δm= 1 and ϵ_3= -2,10%

5 Montage en demi-pont push-pull

In this part, we consider a second sensor identical to R_c that we substitute to R_1. R_c and R_1 are in push-pull mode.

5.1 Question 5.22

The balance of the bridge is still realised for the value m_0.

We have replaced R_1 with a sensor with the same characteristics as R_c. Because it is a push-pull mounting the value of all the four resistance will be the same at the beginning. We also know that $V_{mes,0} = 0$. This means that $R_2 = R_3 = R_{c0}$, we also know that $R_{1,0} = R_{c,0}$. We have $R_c = R_{c0} + \Delta R_c$, and $R_1 = R_{1,0} + \Delta R_1 = R_{c,0} + \Delta R_1$ We also know that $V_{mes,0} = 0$, which means that $V_{mes} = V_{mes,0} + \Delta V_{mes}$ becomes $V_{mes} = \Delta V_{mes}$ V_{mes} will be measured at the same point as earlier, and we therefore got:

$$\Delta V_{mes} = V_{mes} = \frac{R_c}{R_1 + R_c}V_g - \frac{R_3}{R_2 + R_3}V_g = \frac{R_{c,0} + \Delta R_c}{R_{c,0} + \Delta R_1 + R_{c,0} + \Delta R_c}V_g - \frac{R_{c,0}}{R_{c,0} + R_{c,0}}V_g \tag{13}$$

$$\Delta V_{mes} = (\frac{R_{c0} + \Delta R_c}{R_{c0} + \Delta R_c + R_{c0} + \Delta R_1} - \frac{R_{c0}}{2R_{c0}})V_g \tag{14}$$

$$\Delta V_{mes} = \frac{(V_g)(\Delta R_c - \Delta R_1)}{4(R_{c0} + \Delta R_1)} \tag{15}$$

5.2 Question 5.23

As mentioned earlier, the two sensors R_c and R_1 are identical, and in push-pull mode. So, we got :

$$\Delta R_c = R_c(m_0 + \Delta m) - R_c(m_0) = A\Delta m^2 + B\Delta m \tag{16}$$

$$\Delta R_c = R_c(m_0 - \Delta m) - R_c(m_0) = A\Delta m^2 - B\Delta m \tag{17}$$

5.3 Question 5.24

With the new expressions of ΔR_c and ΔR_1, ΔV_{mes} can be written as :

$$\Delta V_{mes} = \frac{V_g * B * \Delta m}{2(R_{c0} + A\Delta m^2)}$$

5.4 Question 5.25

In order to achieve the linear approximation, we must first put rewrite ΔV_{mes} :

$$\Delta V_{mes} = \frac{V_g * B * \Delta m}{2R_{c0}(1 + \frac{A\Delta m^2}{R_{c0}})}$$

Because $R_{c0} >> A\ \Delta m^2$, we have :

$$\Delta V_{mes,lin} = \frac{V_g * B * \Delta m}{2R_{c0}}$$

5.5 Question 5.26

The reduced sensibility of the measure is given by :

$$S_r = \frac{1}{V_g}\frac{\Delta V_{mes,lin}}{\Delta m}$$

$$S_r = \frac{1}{V_g}\frac{B * V_g * \Delta m}{2R_{c0} * \Delta m}$$

$$S_r = \frac{B}{2R_{c0}}$$

6 Linéarisation amont - Montage en quart de pont actif

6.1 Question 5.28

Supposing that the operational amplifier is ideal, we have the following:

$$V_B = \frac{R_{c,0}}{2R_{c,0}}V_g = \frac{V_g}{2} \tag{18}$$

$$V_A = V_{mes} + \frac{R_c}{R_c + R_{c,0}}(V_g - V_{mes}) = \frac{R_c}{R_c + R_{c,0}}V_g + (1 - \frac{R_c}{R_c + R_{c,0}}) = \frac{R_c}{R_c + R_{c,0}}Vg + (\frac{R_c + R_{c,0}}{R_c + R_{c,0}} - \frac{R_c}{R_c + R_{c,0}})V_{mes} \tag{19}$$

$$V_A = \frac{R_c}{R_c + R_{c,0}}V_g + \frac{R_{c,0}}{R_c + R_{c,0}}V_{mes} \tag{20}$$

6.2 Question 5.29

As the operation amplifier is supposed to be ideal, we have the following relation: $V_A = V_B$ This will give us the following:

$$\frac{V_g}{2} = \frac{R_c}{R_c + R_{c,0}}V_g + \frac{R_{c,0}}{R_c + R_{c,0}}V_{mes} \tag{21}$$

$$(\frac{1}{2} - \frac{R_c}{R_c + R_{c,0}})V_g = \frac{R_{c,0}}{R_c + R_{c,0}}V_{mes} \tag{22}$$

$$(\frac{R_c + R_{c,0}}{2R_{c,0}} - \frac{R_c}{R_{c,0}})V_g = V_m es \tag{23}$$

$$V_{mes} = \frac{R_{c,0} - R_c}{2R_{c,0}}V_g \tag{24}$$

We know that $\Delta R_c = R_c - R_{c,0}$ and this will give us the following:

$$V_{mes} = \frac{-\Delta R_c V_g}{2R_{c,0}} \tag{25}$$

From the previous equations, we can deduce that:

$$V_{mes} = \Delta V_{mes} = \frac{-BV_g\Delta m}{2R_{c,0}}(1 + \frac{A}{B}\Delta m) \tag{26}$$

6.3 Question 5.30

The linear approximation of $\Delta V_{mes,lin}$ of V_{mes} is:

$$\Delta V_{mes,lin} = \frac{-BV_g\Delta m}{2R_{c,0}} \tag{27}$$

6.4 Question 5.31

We can now deduce the expression of the reduced sensitivity of the measure as the following:

$$S_r = \frac{\Delta V_{mes,lin}}{\Delta m V_g} \tag{28}$$

Then we will have:

$$S_r = \frac{-B}{2R_{c0}} \tag{29}$$

This will help us obtain the numerical value of the reduced sensibility which is -48mV/m/V.

6.5 Question 5.32

The linearity error is calculated like the following:

$$\epsilon_5 = \frac{\frac{-BV_g\Delta m}{2R_{c0}}(1+\frac{A\Delta m}{B}) + \frac{BV_g\Delta m}{2R_{c0}}}{\frac{-BV_g\Delta m}{2R_{c0}}(1+\frac{A\Delta m}{B})} = \frac{A\Delta m}{B+A\Delta m} \tag{30}$$

For the second order, we get the following:

$$\epsilon_5 \approx \frac{A\Delta m}{B}(1-\frac{A\Delta m}{B}) \tag{31}$$

When $\Delta m = -1$, this expression is at it's maximum and will give us:

$$\epsilon_5 = -2.91\% \tag{32}$$