Experiment Design and Metamodelling

Simulation-Based Analysis of a Hospital Flow System

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1 Problem Statement

This assignment studies a hospital process consisting of three sequential stages: *Preparation*, *Operation*, and *Recovery*. Patients arrive randomly, pass through preparation, undergo surgery in an operating room (OR), and finally recover before leaving the system.

The goal is to analyse how structural and stochastic parameters affect the **average queue length at the system entrance**, while ensuring:

- High utilisation of the operating room,
- Reasonable use of auxiliary facilities (preparation and recovery),
- Avoidance of unnecessary waiting.

To achieve this, we apply experiment design techniques, serial correlation analysis, and regression-based metamodelling.

2 Model Description

2.1 System Structure

The simulated system consists of:

- Preparation unit with $P \in \{4, 5\}$ servers,
- A single operating room ($O = 1$),
- Recovery unit with $R = 4$ servers.

Patients follow the sequence:

$$\text{Arrival} \rightarrow \text{Preparation} \rightarrow \text{Operation} \rightarrow \text{Recovery} \rightarrow \text{Exit}$$

If the recovery unit is full, the patient **blocks the operating room**, correctly modelling downstream congestion.

2.2 Stochastic Elements

The following distributions are considered:

- Interarrival times: Exponential or Uniform,
- Preparation time: Exponential or Uniform,
- Recovery time: Exponential or Uniform,
- Operation time: Exponential with mean 20 (fixed).

2.3 Twist: OR Cleaning Time

As a meaningful twist, an additional stochastic delay is introduced after surgery:

- OR cleaning time: either none or Exponential(5).

This twist increases OR workload and variability without changing the system structure.

3 Warm-Up and Equilibrium

Because the system operates at high utilisation, it exhibits slow convergence to steady state. To avoid bias from initial empty conditions, a warm-up period of 4000 minutes is applied.

Metrics are reset after warm-up while preserving the actual system state (e.g. OR blocking), ensuring realistic equilibrium sampling.

4 Serial Correlation Analysis

4.1 Long-Memory Configuration

To study serial correlation, we select a configuration expected to have long memory:

- High arrival rate (Exponential mean 22.5),
- Limited capacity ($P = 4$, $R = 4$),
- High-variance service times (Exponential).

4.2 Autocorrelation Results

Queue length samples are collected at different time intervals. For each lag d, the autocorrelation coefficient ρ_d is computed and averaged across multiple runs.

The variance inflation factor is approximated by:

$$\text{VIF} = 1 + 2\sum_{d=1}^{\infty} \rho_d$$

Results show that:

- Short sampling intervals produce significant autocorrelation,
- Larger intervals reduce ρ_1 toward zero,
- Effective sample size is substantially smaller than nominal.

Conclusion: Samples should be spaced sufficiently far apart (hundreds of minutes) to reduce correlation, or batch means should be used.

5 Experimental Design

5.1 Design Choice

Testing all combinations would require 64 scenarios. Instead, a fractional factorial design 2^{6-3} (8 runs) is used, balancing efficiency and insight.

5.2 Factors

Factor	Description
A	Arrival distribution
B	Arrival rate
C	Preparation distribution
D	Recovery distribution
E	Number of prep units
F	OR cleaning time (twist)

5.3 Generators and Aliasing

The design uses generators:

$$D = AB, \quad E = AC, \quad F = BC$$

This leads to a **Resolution III design**, where:

- Main effects are aliased with two-factor interactions,
- Joint effects may influence estimated main effects.

6 Simulation Results and Comparison

Each design point is simulated with multiple replications using common random numbers (CRN) for fair comparison.

The response variable is the average entrance queue length.

6.1 Key Observations

- Increased preparation capacity consistently reduces entrance queue length,
- Higher arrival rates increase congestion,
- Uniform distributions generally reduce variability compared to exponential ones,
- OR cleaning (twist) increases queue length by adding load.

7 Regression Metamodel

A linear regression metamodel is fitted using coded factor levels:

$$y = \beta_0 + \sum \beta_i X_i$$

7.1 Model Adequacy

- High R^2 indicates strong explanatory power,
- Residuals show no strong patterns and approximate constant variance,
- Leave-one-out validation confirms reasonable predictive accuracy.

7.2 Joint Effects

Due to Resolution III aliasing:

- Significant coefficients may represent combined main and interaction effects,
- A higher-resolution or fold-over design would be required for separation.

8 Corrected Hypothesis Testing

Because observations are autocorrelated, naive t-tests are invalid. Batch means are used to obtain approximately independent samples.

Paired comparisons with CRN confirm that:

- Increased prep capacity significantly reduces queues,
- OR cleaning significantly increases congestion,
- Some factors show no statistically significant effect under current load.

9 Discussion and Recommendations

9.1 Does the Model Make Sense?

Yes. The direction and magnitude of effects align with queueing theory:

- More capacity reduces queues,
- Higher variability increases congestion,
- Downstream blocking propagates upstream delays.

9.2 Are Joint Effects Important?

Yes. Because of aliasing, joint effects cannot be ignored. Future studies should use higher-resolution designs.

9.3 Practical Recommendation

To ensure high OR utilisation with minimal waiting:

- Prioritise sufficient preparation capacity,
- Avoid unnecessary OR overhead (e.g. excessive cleaning),
- Recovery is not a bottleneck under studied conditions.

10 Conclusion

This study demonstrates how simulation combined with experiment design and metamodelling provides deep insight into complex service systems. Proper handling of serial correlation, careful design selection, and statistically valid comparisons are essential for reliable conclusions.