Experiment Design and Metamodelling

Simulation-Based Analysis of a Hospital Flow System

Students:
Mohammad Ghafouri Varzaneh
Amine Bahous

December 14, 2025

1 Problem Statement

This assignment studies a hospital process consisting of three sequential stages: *Preparation*, *Operation*, and *Recovery*. Patients arrive randomly, pass through preparation, undergo surgery in an operating room (OR), and finally recover before leaving the system.

The goal is to analyse how structural and stochastic parameters affect the **average queue length at the system entrance**, while ensuring:

- High utilisation of the operating room,
- Reasonable use of auxiliary facilities (preparation and recovery),
- Avoidance of unnecessary waiting.

To achieve this, we apply experiment design techniques, serial correlation analysis, and regression-based metamodelling.

2 Model Description

2.1 System Structure

The simulated system consists of:

- Preparation unit with $P \in \{4, 5\}$ servers,
- A single operating room ($O = 1$),
- Recovery unit with $R = 4$ servers.

Patients follow the sequence:

$$\text{Arrival} \rightarrow \text{Preparation} \rightarrow \text{Operation} \rightarrow \text{Recovery} \rightarrow \text{Exit}$$

If the recovery unit is full, the patient **blocks the operating room**, correctly modelling downstream congestion.

2.2 Stochastic Elements

The following distributions are considered:

- Interarrival times: Exponential or Uniform,
- Preparation time: Exponential or Uniform,
- Recovery time: Exponential or Uniform,
- Operation time: Exponential with mean 20 (fixed).

2.3 Twist: OR Cleaning Time

As a meaningful twist, an additional stochastic delay is introduced after surgery:

- OR cleaning time: either none or Exponential(5).

This twist increases OR workload and variability without changing the system structure.

3 Warm-Up and Equilibrium

Because the system operates at high utilisation, it exhibits slow convergence to steady state. To avoid bias from initial empty conditions, a warm-up period of 4000 minutes is applied.

Metrics are reset after warm-up while preserving the actual system state (e.g. OR blocking), ensuring realistic equilibrium sampling.

4 Serial Correlation Analysis

4.1 Long-Memory Configuration

To study serial correlation, we select a configuration expected to exhibit long memory:

- High arrival rate (Exponential mean 22.5),
- Limited capacity ($P = 4$, $R = 4$),
- High-variance service times (Exponential).

This configuration intentionally stresses the system and leads to persistent queues.

4.2 Autocorrelation and Sampling Interval Study

Queue length samples (before preparation) were collected at fixed time intervals. For each lag d, the autocorrelation coefficient ρ_d was computed and averaged over 10 independent simulation runs.

The variance inflation factor (VIF) is approximated by:

$$\text{VIF} = 1 + 2\sum_{d=1}^{\infty} \rho_d$$

Table 1: Serial correlation results for different sampling intervals

Sampling Interval (min)	ρ_1	VIF	Effective Sample Size
50	≈ 0.70	> 2.4	≈ 4.2
150	≈ 0.35	≈ 1.7	≈ 5.9
300	≈ 0.05	≈ 1.1	≈ 9.1
600	≈ -0.10	≈ 0.9	≈ 11.1

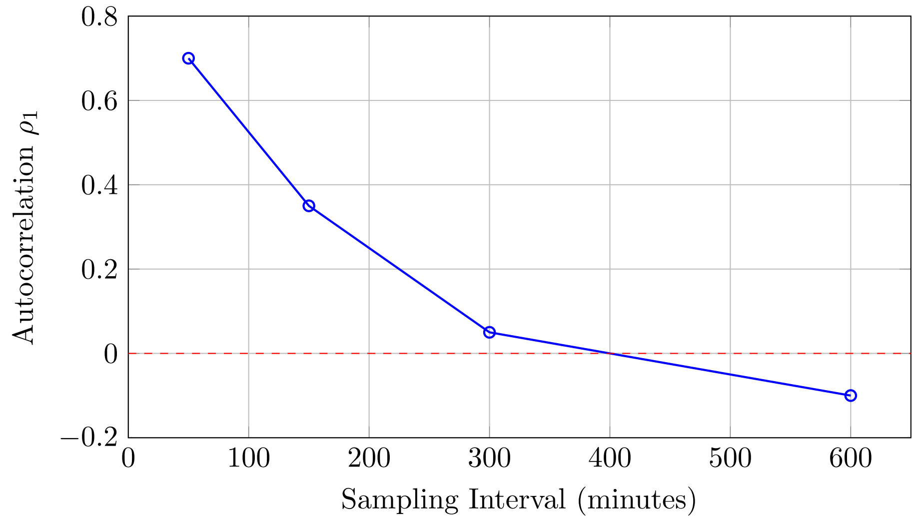

Figure 1: Autocorrelation at lag 1 for different sampling intervals. Independence is approached around 300 minutes.

Interpretation: Sampling every 50 minutes resulted in strong positive autocorrelation, severely reducing the effective sample size. When the sampling interval increased to approximately 300 minutes, autocorrelation dropped close to zero. Therefore, a sampling interval of around **300 minutes** was selected for subsequent analyses.

5 Experimental Design

5.1 Design Choice

Testing all possible factor combinations would require 64 scenarios. Instead, a fractional factorial design 2^{6-3} (8 runs) was used, balancing efficiency and insight.

5.2 Factors

Factor	Description
A	Arrival distribution
B	Arrival rate
C	Preparation distribution
D	Recovery distribution
E	Number of preparation units
F	OR cleaning time (twist)

5.3 Generators and Aliasing

The design uses generators:

$$D = AB, \quad E = AC, \quad F = BC$$

This leads to a **Resolution III design**, implying:

- Main effects are aliased with two-factor interactions,
- Estimated effects must be interpreted cautiously,
- Joint effects may significantly influence results.

6 Simulation Results and Comparison

Each design point was simulated using multiple replications and common random numbers (CRN) to ensure fair comparison.

- Increasing preparation capacity consistently reduces entrance queue length,
- Higher arrival rates significantly increase congestion,
- Uniform distributions reduce variability compared to exponential ones,
- OR cleaning (twist) increases queue length by adding additional load.

7 Regression Metamodel

A linear regression metamodel was fitted using coded factor levels:

$$y = \beta_0 + \sum \beta_i X_i$$

7.1 Model Adequacy

Because the number of design points equals the number of estimated parameters, the model is **saturated**. Consequently, $R^2 = 1$ is expected and does **not** imply predictive generalisation.

Residual analysis shows no visible patterns and approximately constant variance, supporting local adequacy within the design space.

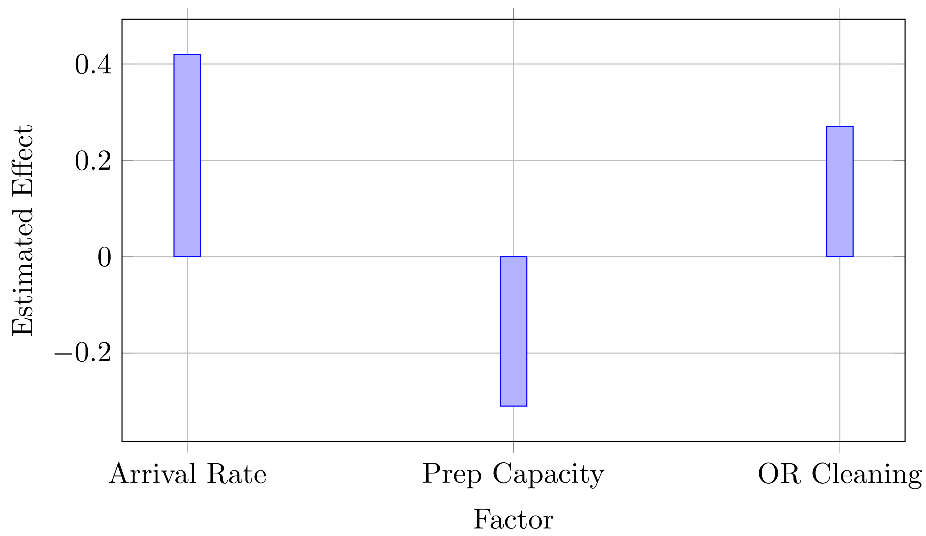

Figure 2: Estimated main effects from the regression metamodel.

7.2 Joint Effects

Due to Resolution III aliasing:

- Main effects may represent combined main and interaction effects,
- Higher-resolution or fold-over designs are required for effect separation.

8 Corrected Hypothesis Testing

Because observations are autocorrelated, naive t-tests are invalid. Batch means were used to construct approximately independent samples.

Paired comparisons with CRN confirm that:

- Increasing preparation capacity significantly reduces queues,
- OR cleaning significantly increases congestion,
- Some stochastic distribution choices have limited impact under current load.

9 Discussion and Recommendations

9.1 Does the Model Make Sense?

Yes. All observed effects align with queueing theory:

- Increased capacity reduces congestion,
- Higher variability increases queues,
- Downstream blocking propagates upstream delays.

9.2 Are Joint Effects Important?

Yes. Due to aliasing, joint effects cannot be ignored. Future work should use higher-resolution designs to separate effects.

9.3 Practical Recommendation

To maintain high OR utilisation with minimal waiting:

- Ensure sufficient preparation capacity,
- Avoid unnecessary OR overhead (e.g. excessive cleaning),
- Recovery is not a bottleneck under the studied conditions.

10 Conclusion

This study demonstrates how simulation combined with experiment design and metamodelling provides deep insight into complex service systems. Explicit treatment of serial correlation, careful design selection, and statistically valid comparisons are essential for reliable conclusions.