Divide and Conquer

Binary Search

Quick Sort

Divide and Conquer

- **Divide and Conquer Methodology**
  - Given a problem, identify a small number of significantly smaller sub-problems of the same type
  - Solve each sub-problem recursively (the smallest possible size of a sub-problem is a base-case)
  - Combine these solutions into a solution for the main problem
- **Examples**
  - **Binary search**
  - **Merge Sort**

Binary Search

- Finds the position of a specified input value within a sorted array
- For binary search, the array should be arranged in ascending or descending order.

How binary search works ?

In each step, the algorithm compares the search value with the value of the middle element of the array.

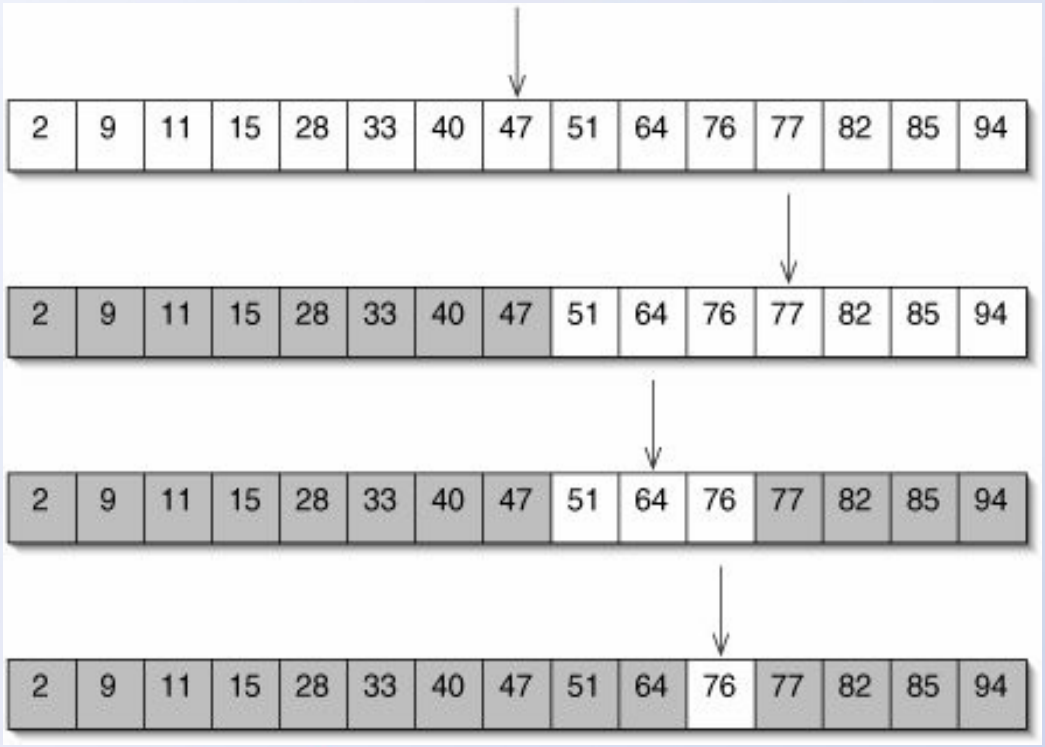
If the values match, then a match has been found and its index is returned.

If the search value is less than the middle element, then the algorithm repeats on the sub-array to the left of the middle element

If the search key is greater, then on the sub-array to the right

Binary Search

How it works ?



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Task

Try to implement finding a number in sorted array of numbers.

Keep two variables – left and right to point where to search.

Sorting Algorithms

Bubble Sort

Worst Case	O(n^2)
Average Case	O(n^2)
Best Case	O(n)

Selection Sort

Worst Case	O(n^2)
Average Case	O(n^2)
Best Case	O(n^2)

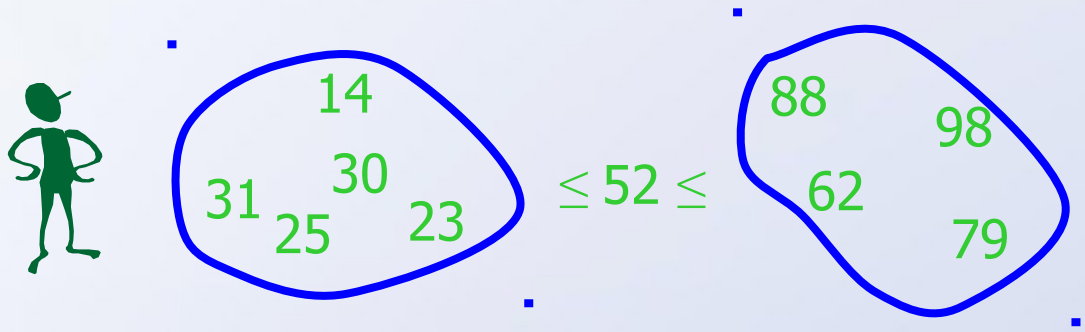
Counting Sort

Worst Case	O(n+k)
Average Case	O(n+k)
Best Case	O(n+k)

Quick Sort

Partition set into two using randomly chosen pivot

Quick Sort



sort the first half.

14,23,25,30,31

sort the second half.

62,79,98,88

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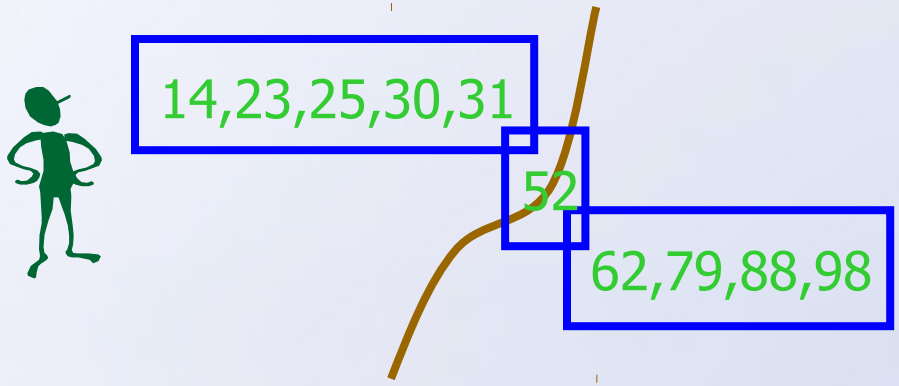
Quick Sort



Glue pieces together.
(No real work)

14,23,25,30,31,52,62,79,88,98

Quicksort

- **Quicksort advantages:**
  - Sorts in place – no additional array needed
  - Sorts $O(n \lg n)$ in the average case
  - Very efficient in practice

- **Quicksort disadvantages :**
  - Sorts $O(n^2)$ in the worst case
  - not stable
    - Does not preserve the relative order of elements with equal keys
    - Sorting algorithm is stable if 2 records with same key stay in original order
  - But in practice, it's quick
  - And the worst case doesn't happen often … sorted

Quicksort

- ***Divide***: $A[p \ldots r]$ is partitioned (rearranged) into two nonempty subarrays $A[p \ldots q-1]$ and $A[q+1 \ldots r]$ so that each element of $A[p \ldots q-1]$ is less than or equal to each element of $A[q+1 \ldots r]$. Index q is computed here, called pivot.

- ***Conquer***: two subarrays are sorted by recursive calls to quicksort.

- ***Combine***: no work needed since the subarrays are sorted in place already.

Quicksort

- **The basic algorithm to sort an array *A* consists of the following four easy steps:**
  - If the number in *A* is 0 or 1, then return
  - Pick any element ***v*** in *A*. This is called the ***pivot***
  - Partition *A*-{***v***} (the remaining elements in *A*) into two disjoint groups:
    - $A_1 = \{x \in A-\{v\} \mid x \leq v\}$, and
    - $A_2 = \{x \in A-\{v\} \mid x \geq v\}$
  - return
    - { quicksort(A_1) followed by ***v*** followed by quicksort(A_2)}

Quicksort

- **Small instance has $n \leq 1$**
  - Every small instance is a sorted instance
- **To sort a large instance:**
  - select a pivot element from out of the ***n*** elements
- **Partition the *n* elements into 3 groups left, middle and right**
  - The middle group contains only the pivot element
  - All elements in the left group are ≤ pivot
  - All elements in the right group are ≥ pivot
- **Sort left and right groups recursively**
- **Answer is sorted left group, followed by middle group followed by sorted right group**

Example

6	2	8	5	11	10	4	1	9	7	3

Use 6 as the pivot

2	5	4	1	3	6	7	9	10	11	8

Sort left and right groups recursively

Quicksort Code

```
Quicksort(A, p, r)
{
    if (p < r)
    {
        q = Partition(A, p, r)
        Quicksort(A, p  , q-1)
        Quicksort(A, q+1   , r)
    }
}
```

- Initial call is Quicksort(*A*, 1, *n*), where *n* in the length of *A*

Partition

- **Clearly, all the action takes place in the partition() function**
  - Rearranges the sub-array in place
  - End result:
    - Two sub-arrays
  - Returns the **index** of the “pivot” element separating the two sub-arrays

Task

Try to reorder elements of given array so that first k elements are lower than or equal to the last element of the array and next elements are greater than it.

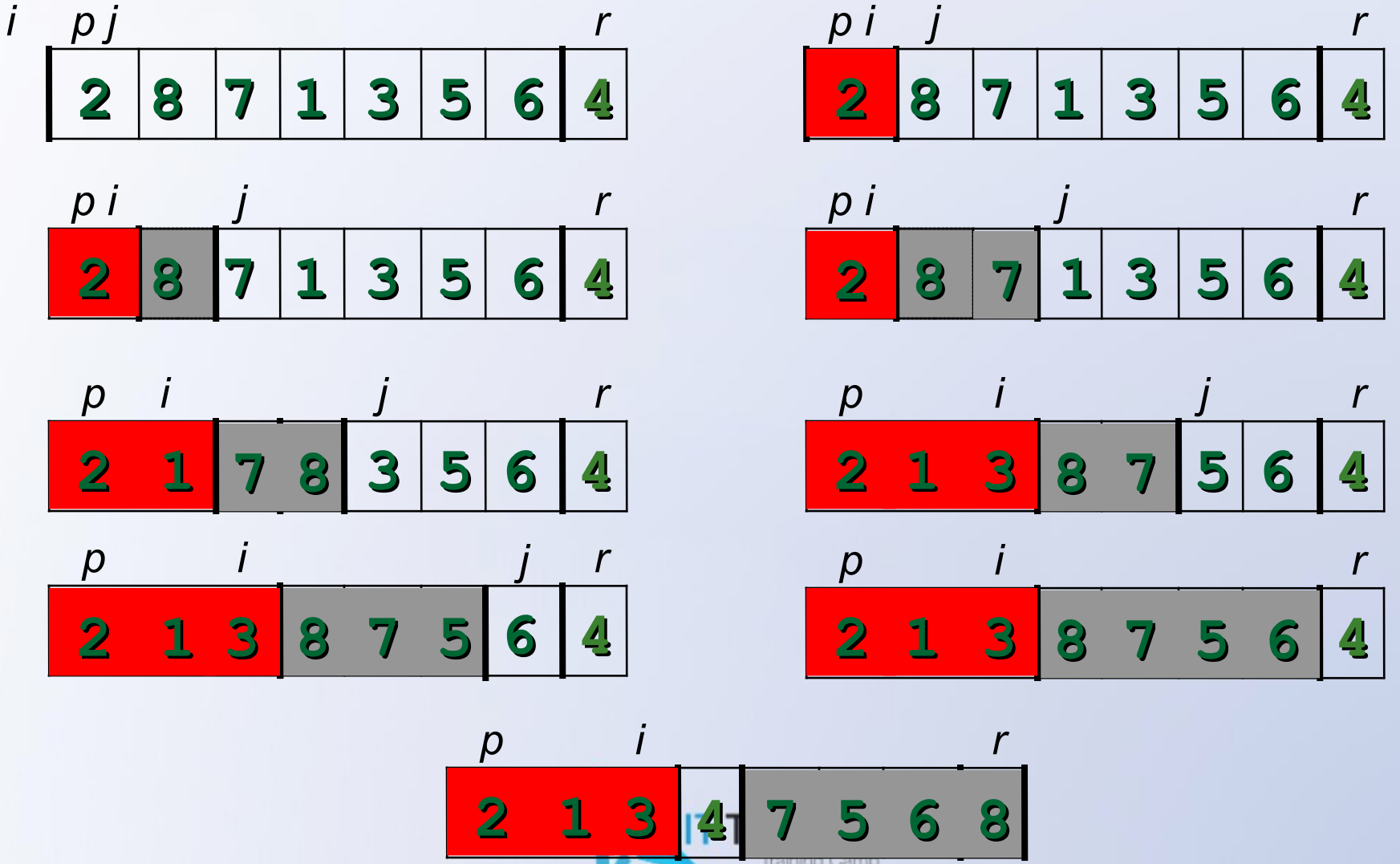
Your algorithm should work in O(N) time.

Partition Code

```
Partition(A, p, r)
{
   x = A[r];          // x is pivot
   i = p – 1;
     for(int j = p; j < r-1; j++)
     {
          if( A[j] <= x)
          {
            i = i + 1;
            exchange A[i] with A[j];
          }
     }
   exchange A[i+1] with A[r];
   return i+1;
}
```

`partition()` runs in O(n) time

Partition Example $A=\{2, 8$



Partition Example Explanation

- **Red shaded elements are in the first partition with values lower than the pivot.**
- **Gray shaded elements are in the second partition with values greater than the pivot.**
- **The unshaded elements have no yet been put in one of the first two partitions.**
- **The final white element is the pivot.**

Order Statistics

Task : given an array and number K, find K-th smallest element in the array.

1)Write the first solution that comes to your mind.

2)Next try to use partition function mentioned before to help you.