

Lesson 10

Introduction to algorithms Sorting algorithms

Bubble sort

Selection sort

Counting sort



Introduction to algorithms

The goal of an algorithm –
to solve a particular problem as efficiently as possible



Introduction to algorithms

Algorithm Complexity –
a measure for the efficiency of an algorithm



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Time Complexity

how fast the algorithm is



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Space Complexity

how much memory the algorithm requires



Introduction to algorithms

Big O Notation (**simplified**)

$O(N)$ - the algorithm executes **approximately N simple operations**,
where N is the size of the input



Introduction to algorithms

Popular complexities for input of size N

<u>Number of operations</u>	<u>Big O</u>	<u>Short name</u>
1, 5, 10, 10 000	$O(1)$	constant
$\log N$, $\log N + 10$	$O(\log N)$	logarithmic
N , $2N$, $10N + 5$	$O(N)$	linear
$N \log N$, $N \log 2N$	$O(N \log N)$	linearithmic
N^2 , $3N^2 + N + 1$	$O(N^2)$	quadratic
N^3 , $N^3 + N^2$	$O(N^3)$	cubic
N^K , K – constant	$O(N^K)$	polynomial
2^N , 3^N , $2^N + N^2$	$O(2^N)$	exponential
$N!$	$O(N!)$	factorial



Introduction to algorithms

Complexity in the:

best case
average case
worst case



Sorting algorithms I

Sorting problem –
rearrange a sequence of items,
so that all the elements in the resulting sequence
are in increasing (decreasing) order



Sorting algorithms I

Sort an array of integers in increasing order



Mostly used in practice

- Usually using predefined function for sorting

```
public static void main(String[] args) {  
    int[] array = {12, 3, 53, 6, 9, 3, 43, -4, 0};  
  
    Arrays.sort(array);  
  
    for (int i = 0; i < array.length; i++) {  
        System.out.print(array[i] + " ");  
    }  
}
```



Bubble Sort

- Simple sorting algorithm
- Worst case scenario of N^2 operations
- Best case scenario of N operations
- Used for small arrays



Bubble Sort Philosophy

- Traverse elements
- Compare adjacent elements
- Swap adjacent elements if the next element is greater



Bubble Sort Pseudo Code

- Compare `array[0]` & `array[1]`
- If `array[0] > array[1]` swap it.
- Compare `array[1]` & `array[2]`
- If `array[1] > array[2]` swap it.
- ...
- Compare `array[n-1]` & `array[n]`
- if `array[n-1] > array[n]` then swap it.
- Repeat the same steps for `array[1]` to `array[n-1]`



Try it!
Also try to optimize the algorithm.



Bubble Sort

```
public static int[] bubbleSort(int[] intArray) {  
    int temp = 0;  
  
    for(int i=0; i < intArray.length; i++){  
        for(int j=1; j < (intArray.length - i); j++){  
  
            if(intArray[j-1] > intArray[j]){  
                //swap the elements!  
                temp = intArray[j-1];  
                intArray[j-1] = intArray[j];  
                intArray[j] = temp;  
            }  
        }  
    }  
    return intArray;  
}
```



Selection Sort

- Used only for small arrays
- Slow for bigger arrays
- Worst case scenario of N^2 operations
- Best case scenario of N^2 operations
- Find the next smallest element and move it to its final position
- Start by the smallest element
- Subset of sorted and unsorted elements



Selection Sort Algorithm

- Initially, the sorted sublist is empty and the unsorted sublist is the entire input list
- Start with first element of the unsorted subset
- Test with all elements after the assumed min element to find the smallest
- Swap the smallest element to the starting index



Try it!
Also try to optimize the algorithm.



Selection Sort

```
public static int[] selectionSort(int[] a){
    /* a[0] to a[n-1] is the array to sort */
    int temp = 0;
    int iMin = 0;
    // advance the position through the entire array
    // (could do j < n-1 because single element is also min element)
    for (int j = 0; j < a.length-1; j++) {
        /* find the min element in the unsorted a[j .. n-1] */
        /* assume the min is the first element */
        iMin = j;
        /* test against elements after j to find the smallest */
        for (int i = j+1; i < a.length; i++) {
            /* if this element is less, then it is the new minimum */
            if (a[i] < a[iMin]) {
                /* found new minimum; remember its index */
                iMin = i;
            }
        }
    }
}
```



Selection Sort

```
/* iMin is the index of the minimum element.  
   Swap it with the current position */  
if ( iMin != j ) {  
    //swap the elements!  
    temp = a[j];  
    a[j] = a[iMin];  
    a[iMin] = temp;  
}  
}  
return a;  
}
```

Counting Sort

- Counting sort is an algorithm for sorting small integers.
- Its main idea is to count the number of different objects.
- Runs in linear running time in the number of items.

Counting Sort Algorithm

Basic implementation

- Find min/max values
- For every element, count in separate array the number of times it was met.
- Then for every *value* from min to max in the array output K times this value, where K is the number calculated in the previous array.

Counting Sort Algorithm

Input Data

0	4	2	2	0	0	1	1	0	1	0	2	4	2
---	---	---	---	---	---	---	---	---	---	---	---	---	---

Count Array

0	1	2	3	4
5	3	4	0	2

Sorted Data

0	0	0	0	0	1	1	1	2	2	2	2	4	4
---	---	---	---	---	---	---	---	---	---	---	---	---	---

Counting Sort Basic Version

```
public static int[] countingSort(int[] numbers, int max)
{
    int[] counts = new int[max + 1];

    for (int i = 0; i < numbers.length; i++)
        counts[numbers[i]]++;

    int[] results = new int[numbers.length];

    int counter = 0;
    for(int i = 0; i <= counts.length-1; i++)
    {
        while(counts[i]>0)
        {
            results[counter++] = i;
            counts[i]--;
        }
    }

    return results;
}
```

Counting Sort Complexity

- Best Case – $O(N+K)$
- Average Case – $O(N+K)$
- Worst Case – $O(N+K)$

Where K is the largest element in sequence.

Sorting Complexity Refresh

	Best	Average	Worst
• Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$
• Selection Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$
• Counting Sort	$O(n+k)$	$O(n+k)$	$O(n+k)$
• Radix Sort	$O(n \lg_b n)$	$O(n \lg_b n)$	$O(n \lg_b n)$

Resources

Counting and Radix Sort Explained :

<http://courses.csail.mit.edu/6.006/spring11/rec/rec11.pdf>

Summary

- Algorithm Complexity
- Most used in practice
- Bubble Sort
- Selection Sort
- Counting Sort

