University of Isfahan
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Fuzzy Systems Homework #2

Identification, Modeling, and the Reason behind the uncertainties available for fuzzy systems

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Abstract

In this exercise, we're going to analyze the different types of uncertainties available and have a comparison between them. Also at the end of this exercise, we're going to discuss fuzzy systems that which certainty can be used in these systems.

1) Types of Uncertainties

In this section, we're going to talk about the different types of uncertainties available. For a better view, we have categorized the uncertainties into 4 groups. Each group can be defined as a subset of the categorizations in groups before.

1-1) Group 1

- **Aleatory variability:** This type of uncertainty is associated with the randomness of a problem. If there are different results for a parameter from time to time, then the type of uncertainty is Aleatory variability [1], [2]. The randomness in this type is caused by the probability of values.
- **Epistemic uncertainty:** epistemic uncertainty is because of the model uncertainty. There are several reasons behind this, it can be the limited knowledge or the limited data available for a model [3]. In this type of uncertainty for a parameter, there may be more than one value at a specific time and there is no assurance which is the most suitable [1]. The uncertainty in this type is caused by the probabilities too.

1-2) Group 2

- **Vagueness:** This term is a bit different from uncertainty but for the sake of completeness we'd added this term to our categorization. In this term, there is no boundary most of the time. Think of the term tall in the sentence "A basketball player needs to be tall", there is no measurement that a person with 1-meter height is tall or a person with 2 meters [4]. In this situation, the context is basketball so it is possible to measure the height of the basketball players in the world and set an average for the height for the term tall. After finding a value for tall, the main question remains unanswered. How to assign value for people having different heights for the term tall?
- **Ambiguity:** For this type, we're applying the values based on context. For example, while humans are speaking there is no distinction between similar sound words like "their" or "they're". Here the answer is pretty straightforward, the most suitable word is chosen for the sentence. The difference between Ambiguity and Vagueness is, Using vagueness there is no straightforward answer for each item but in ambiguity, the answer can be chosen via context [4].

1-3) Group 3

- **Imprecision:** The data can be uncertain, meaning the collected data may be noisy or corrupted which can represent a false distribution for real-world data. With this type of uncertainty, the results of the learned parameter cannot be trusted for real-world problems.
- **Inconsistency:** having many different knowledge bases, or having conflicting information, data or recommendation would represent unuseful information when merging them. This type of uncertainty cannot represent trustful information for real-world problems too.

1-4) Group 4

- **Generalization:** With this generalization term for each topic, the more accepted concept is generalized and the other concepts that can be true for that topic are omitted. Ex: “people say that all birds can fly”, Obviously it is possible to make this sentence false, A dead bird or penguins cannot fly but the sentence is a generalized form of most birds can fly because the population of the flying birds is much higher than the others people say that all birds can fly.

1-5) Comparison

To compare each term with another, first of all, we can define a relation between groups 1 and 2 as:

- Vagueness is a subset of epistemic uncertainty.
- Ambiguity is a subset of Aleatory uncertainty.

Then for groups 3 and 4 from the perspective of computer science is that there may be a relation with vagueness, but it’s not a very strong relationship so there is no need to define it. And for the differences

- Aleatory uncertainty shows a value of random probability for a fact, But Epistemic uncertainty represents a value of probability for more than one fact at a time.
- The difference between Vagueness and Ambiguity is that vagueness cannot specify an exact value for a situation but Ambiguity would specify an exact value for a situation (concerning the context).

- For the terms Imprecision and Inconsistency, the first term has the results but it can be not precise concerning a situation (may not be appropriate for a situation) But the second term Inconsistency, represents that merging data knowledge from different sources is always false (the collected rule is always false).

2) Uncertainties in Fuzzy Systems

In this section, we're going to review the uncertainties used in Fuzzy systems. The random uncertainty is called aleatory uncertainty which is most suited for probability theory [3] and the Epistemic uncertainty is better suited for other mathematical frameworks such as the fuzzy set theory that is the discussion of this exercise.

2-1) Aleatory uncertainty

Aleatory uncertainty or random and irreducible uncertainty [5] can be represented easily with probability theory, that each event can have a probability value. In this context, there is no need to represent the uncertainty with fuzzy theory because the probability theory can cover the uncertainty well [2]. An example of this type of uncertainty is the distribution of a coin-flipping problem. In this problem, the distribution of the events having a head or tail is a type of binomial distribution.

2-2) Epistemic uncertainty

Epistemic uncertainty or reducible uncertainty is a type that cannot be represented with probability theory. Because of the nature of this uncertainty that an exact value cannot be assigned to an event and the events can be correlated, fuzzy logic is used in this type of uncertainty. An example of this type of uncertainty is discussed in [2]. In [2], a transformation method is explained to transform the crisp values into fuzzy intervals and assign a membership value for them. Using this method a crash system for cars is evaluated, which assesses how a front bumper would react to a crash. The epistemic uncertainties in the frontal car crash are introduced as (1) The strength and the elasticness of how the front

bumper would react (2) the dynamic friction behavior of the front bumper and the wall (3) the shell thickness factor that controls the penetration of bumper into the other object

2-3) Aleatory and Epistemic

In fuzzy set theory, both Aleatory and Epistemic uncertainties are also used as mixed-up uncertainty. An example of this category is, for a model both uncertainties are used as inputs that the aleatory uncertainty can be the result of a probability density function and the epistemic can be the result of possibility distribution [6]. The name of the fuzzy system made with these two types is “fuzzy probabilities” which is introduced by Zade in 1984 [7]. To make the idea behind this type of fuzzy system more transparent, it is said that to have the real distribution of a data type, an infinite number of data is needed but in real-world experiments finite data is available. The Aleatory uncertainty is represented by the data distribution and the epistemic uncertainty is to measure how the finite data distribution is the actual real-world distribution [3].

3) References

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