E10.1/ $y(0) = 1$

prior to $k=0$ we have zero for the inputs so the output before $k=0$ is zero.

we have omitted purelin in computing outputs because it does not change anything for us

$a(0) = w_{11}\, y(0) = 1$

$a(1) = w_{12}\, y(0) + w_{11}\, y(1) = -4 + 1 = -3$

$a(2) = w_{13}\, y(0) + w_{12}\, y(1) + w_{11}\, y(2) = 2 - 4 + 2 = 0$

$a(3) = w_{13}\, y(1) + w_{12}\, y(2) + w_{11}(0) = 2 - 8 = -6$

$a(4) = w_{13}\, y(2) + 0 = 2 \times 2 = 4$

for more than 4 actual output would be zero.

E10.2/

(i)

$$\left.\begin{aligned} p_0^T &= [1 \quad 1 \quad -1 \quad -1] \\ p_1^T &= [-1 \quad -1 \quad 1 \quad 1] \end{aligned}\right\} t_0 = t_1 = +1$$

$$\left.\begin{aligned} p_2^T &= [1 \quad -1 \quad 1 \quad -1] \\ p_3^T &= [-1 \quad 1 \quad -1 \quad 1] \end{aligned}\right\} \rightarrow t_2 = t_3 = -1$$

Consider the initial weights and biases as zer

so

$$w = [0 \quad 0 \quad 0 \quad 0] \quad b = 0$$

we assign α as 0.2

E10.2 Continue(1)

$a(0) = w(0)\, p_0(0) + b(0) = 0$

$e^{(0)} = t - a = 1 - 0 = 1$

$w(1) = w(0) + 2\alpha\, e(0)\, p_0^T(0) \quad , \quad b(1) = b(0) + 2\alpha\, e(0)$

$w(1) = [0\ 0\ 0\ 0] + \frac{2}{5}(1)\,[1 \quad 1 \quad -1 \quad -1] = [0.4 \quad 0.4 \quad -0.4 \quad -0.4]$

$b(1) = 0 + 0.4(1) \Rightarrow b(1) = 0.4$

$$a(1) = w(1)\, p_1(1) + b(1) = \left[\frac{2}{5} \quad \frac{2}{5} \quad -\frac{2}{5} \quad -\frac{2}{5}\right] \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} + 0.4$$

$= -1.6 + 0.4 = -1.2$

$e(1) = t(1) - a(1) = +1 + 1.2 = 2.2$

$$w(2) = w(1) + 2\alpha\, e(1)\, p_1^T(1) = \left[\frac{2}{5} \quad \frac{2}{5} \quad -\frac{2}{5} \quad -\frac{2}{5}\right] + \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{10}}} \cdot \frac{\overset{11}{\cancel{22}}}{\underset{5}{\cancel{10}}}\,[-1 \quad -1 \quad 1$$

$$\Rightarrow w(2) = \left[-\frac{12}{22} \quad -\frac{12}{25} \quad \frac{12}{25} \quad \frac{12}{25}\right]$$

$$b(2) = b(1) + 2\alpha\, e(1) = \frac{2}{5} + \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{11}{\cancel{22}}}{\underset{5}{\cancel{10}}} = \frac{32}{25}$$

$$a(2) = w(2)\, p_2(2) + b(2) = \left[-\frac{12}{25} \quad -\frac{12}{25} \quad \frac{12}{25} \quad \frac{12}{25}\right] \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} + \frac{32}{25}$$

$$\Rightarrow a(2) = \frac{-12}{25} + \frac{12}{25} + \frac{12}{25} - \frac{12}{25} + \frac{32}{25} = \frac{32}{25}$$

E10.2 second continue(

$e(2) = t(2) - a(2) = -1 - \frac{32}{26} = -\frac{57}{26}$

$w(3) = w(2) + 2\alpha e(2) P_2(2) = \dots$

$b(3) = b(2) + 2\alpha e(2) = \dots$

And the iterations goes on and on until convergence.

(ii) for this problem the hebb rule would fit the most. but here using ADALINE network would take alot of time until it converge (it may also not converge).

The reason behind this is ADALINE network uses LMS algorithm and it tries it's best to minimize the problem. we have various inputs so it take much time to learn.

E10.3

let's see how it affect the network

$$R = E(zz^T) = \frac{3}{4}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}$$

$$= \frac{3}{4}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \Rightarrow R = \begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix}$$

$$h = E(tz) = \frac{3}{4}(1)\begin{bmatrix}1\\1\end{bmatrix} + \frac{1}{4}(-1)\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0.5\\1\end{bmatrix}$$

$$x^* = R^{-1}h = \frac{1}{3/4}\underset{2\times 2}{\begin{bmatrix}1 & -0.5\\-0.5 & 1\end{bmatrix}}\underset{2\times 1}{\begin{bmatrix}0.5\\1\end{bmatrix}} = \frac{4}{3}\begin{bmatrix}0\\0.75\end{bmatrix}$$

$$\Rightarrow x^* = \begin{bmatrix}0\\1\end{bmatrix}, \quad c = E(t^2) \Rightarrow c = \frac{3}{4}(1)^2 + \frac{1}{4}(-1)^2 = 1$$

$$f(x) = 1 - 2x^T\begin{bmatrix}0.5\\1\end{bmatrix} + x^T\begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix}x$$

$$x = \begin{bmatrix}w_{11}\\w_{12}\end{bmatrix} \quad 1 - w_{11} - 2w_{12} + \begin{bmatrix}w_{11} & w_{12}\end{bmatrix}\begin{bmatrix}w_{11} + \frac{1}{2}w_{12}\\\frac{1}{2}w_{11} + w_{12}\end{bmatrix}$$

$$f(x) = 1 - w_{11} - 2w_{12} + w_{11}^2 + \frac{1}{2}w_{11}w_{12} + \frac{1}{2}w_{11}w_{12} + w_{12}^2$$

$$\Rightarrow f(x) = w_{11}^2 + w_{12}^2 + w_{11}w_{12} - w_{11} - 2w_{12} + 1$$

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E 10.3 Continue

the hessian matrix of $f(x)$ or $\nabla^2 f(x)$ is 2R so we name it as A and comput it

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

eigen vectors in sumarry can be calculated as

$$(2-\lambda)^2 - 1 = 0 \Rightarrow (2-\lambda-1) = 0 \Rightarrow \lambda_1 = 1$$
$$(2-\lambda+1) = 0 \Rightarrow \lambda_2 = 3$$

eigen values

اسفند

23 February 2018

۶ جمادی الثانیه ۱۴۳۹

جمعه

eigen vectore corresponds to $\lambda_1 = 1$:

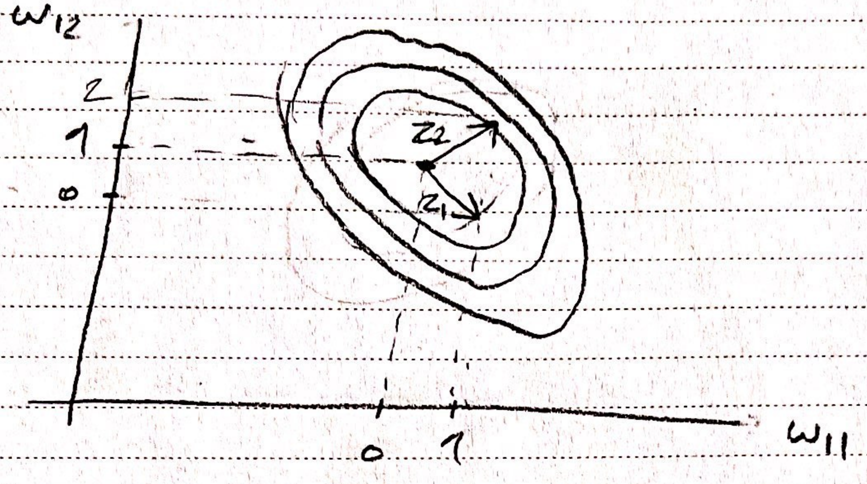
$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = 0 \Rightarrow e_1 = -e_2 \Rightarrow z_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

eigenvector corresponds to $\lambda_2 = 3$:

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \Rightarrow e_1 = e_2 \Rightarrow z_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$z_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \lambda_1 = 1$ $\qquad z_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda_2 = 3$

the contour surface will be like this,



As we can see the circles in contour from P10.3 is converted to ellipses. but the stationary point x^* is the same as before.

(ii) maximum learning rate is:

$$\alpha < \frac{1}{\lambda_{max}} \Longrightarrow \alpha < \frac{1}{\lambda_2} = \frac{1}{3}$$

So the maximum learning rate is 0.33

E 10,13

(i) $z(k) = \begin{bmatrix} y(k-1) \\ y(k-2) \end{bmatrix}$ $x = \begin{bmatrix} w_{11} \\ w_{12} \end{bmatrix}$

$t(k) = y(k)$

$a(k) = x^T z = w_{11}\, y(k-1) + w_{12}\, y(k-2)$

$\text{error} = t(k) - a(k) = y(k) - w_{11}\, y(k-1) - w_{12}\, y(k-2)$

we now have found the error. to find the mean square error, we need to get the expected of it as

$E(\text{error}) = E(t(k) - a(k)) = E(t(k)) - E(a(k))$

$\Rightarrow E(\text{error}) = t^2(k) - a^2(k) = y^2(k) - w_{11}^2\, y^2(k-1) - w_{12}^2\, y^2(k-2)$
$+ 2 w_{11} w_{12}\, y(k-1)\, y(k-2)$

So to write it as $C_y(n)$ we can right

$E(\text{error}) = C(0) - w_{11}^2\, C(0) - w_{12}^2\, C(0) + 2 w_{11} w_{12}\, C(1)$

MSE → mean square error

E 10.13 Continue(

(ii)

rewrite the MSE as terms of $y(k)$

$$E(error) = y^2(k) - w_{11}^2 y^2(k-1) - w_{12}^2 y^2(k-2) + 2w_{11}w_{12} y(k-1) y(k-2)$$

⇒

$$E(error) = \sin^2\left(\frac{k\pi}{5}\right) - w_{11}^2 \sin^2\left(\frac{(k-1)\pi}{5}\right) - w_{12}^2 \sin^2\left(\frac{(k-2)\pi}{5}\right)$$

$$+ 2w_{11}w_{12} \sin\left(\frac{(k-1)\pi}{5}\right) \sin\left(\frac{(k-2)\pi}{5}\right)$$

(iii)

$$\nabla E(error) = \begin{bmatrix} -2w_{11} y^2(k-1) + 2w_{12} y(k-1) y(k-2) \\ -2w_{12} y^2(k-2) + 2w_{11} y(k-1) y(k-2) \end{bmatrix}$$

hessian matrix,

$$\nabla^2 E(error) = \begin{bmatrix} -2y^2(k-1) & 2y(k-1)y(k-2) \\ 2y(k-1)y(k-2) & -2y^2(k-2) \end{bmatrix}$$

$$\nabla^2 E(error) = \begin{bmatrix} -2\sin^2\left(\frac{(k-1)\pi}{5}\right) & 2\sin\left(\frac{(k-1)\pi}{5}\right)\sin\left(\frac{(k-2)\pi}{5}\right) \\ 2\sin\left(\frac{(k-1)\pi}{5}\right)\sin\left(\frac{(k-2)\pi}{5}\right) & -2\sin^2\left(\frac{(k-1)\pi}{5}\right) \end{bmatrix}$$

E10.13 Contine (2)

To find the eigen values and eigen vectors of hessian matrix:

$$\left(-2\sin^2\left((k-1)\frac{\pi}{5}\right)-\lambda\right)^2-4\sin^2\left((k-1)\frac{\pi}{5}\right)\sin^2\left((k-2)\frac{\pi}{5}\right)=0$$

we again use the term $y(k)$ to simplify our equations:

$$-2y(k-1)-\lambda-2y(k-1)y(k-2)=0 \quad (A)$$

$$-2y(k-1)-\lambda+2y(k-1)y(k-2)=0 \quad (B)$$

$$(A)\rightarrow \lambda_1=-2y(k-1)\left(1+y(k-2)\right)$$

$$(B)\rightarrow \lambda_2=-2(y(k-1))\left(1-y(k-2)\right)$$

to simplify more, we write $y(k-1)$ as u and $y(k-2)$ as v

$$\lambda_1=-2u(1+v)$$

$$\begin{bmatrix}-2u^2+2u+2uv & 2uv\\ 2uv & -2v^2+2u+2uv\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}=0,$$

$$\begin{cases}(-2u^2+2u+2uv)e_1+2uv\,e_2=0\\ 2uv\,e_1+(-2u^2+2u+2uv)e_2=0\end{cases}$$

And this equations goes on ...