

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \\ 2T(\frac{n}{2}) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases} \quad (1)$$

$$T(n) = 2T(\frac{n}{2}) + n$$

and

$$T(\frac{n}{2}) = 2T(\frac{n}{4}) + \frac{n}{2} \Rightarrow T(n) = 4T(\frac{n}{4}) + 2n$$

Knowing

$$T(n/4) = 2T(\frac{n}{8}) + \frac{n}{4} \Rightarrow T(n) = 8T(\frac{n}{8}) + 3n$$

...

$$\Rightarrow T(n) = 2^i T(\frac{n}{2^i}) + in$$

We have :

$$T(n/2^i) = 2 \Leftrightarrow \frac{n}{2^i} = 2$$

$$\frac{n}{2^i} = 2 \Leftrightarrow i + 1 = \frac{\lg_2(n)}{\lg_2(2)}$$

Therefore

$$\begin{aligned} T(n) &= 2^{\lg_2(n)} + (\lg_2(n) - 1)n \Leftrightarrow T(n) = n(1 - 1 + \lg_2(n)) \\ &\Leftrightarrow T(n) = n \lg_2(n) \end{aligned}$$