

ADD-BITS(A, B)

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n = MAX(A.length, B.length) + 1
C = ARRAY(INTEGER, n)
carry = 0
i = C.length
j = A.length
k = B.length
while i <= n
    if j > 0 and k > 0
        C[i] = (A[j] + B[k] + carry) mod 2 + carry
        carry = (A[j] + B[k] + carry) / 2
    else if j == 0 and k == 0
        C[i] = carry
    else if j = 0
        C[i] = (B[k] + carry) mod 2
        carry = (B[k] + carry) / 2
    else
        C[i] = (A[j] + carry) mod 2
        carry = (A[j] + carry) / 2
    if j == 0
        j = j - 1
    if k == 0
        k = k - 1
    i = i - 1

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Loop invariant: at the start of every iteration, $C[i-1 \dots n]$ will always have the sum of bits of $A[j-1 \dots A.length]$ and $B[k-1 \dots B.length]$

Initialization: $C[n-1 \dots n]$ is the null array, therefore it contains the sum of $A[A.length-1, A.length]=\text{null}$ and $B[B.length-1 \dots B.length]=\text{null}$

Maintenance: Suppose $C[i \dots n]$ contains the sum of $B[k \dots B.length]$ and $A[j-1 \dots A.length]$ and the carry, if j and k are still greater than the lower bounds of both A and B then $C[i] = (B[k] + A[j] + \text{carry}) \bmod 2$, which means that $C[i]$ will hold the sum of the tow plus whatever carry out there from the previous operation. If we already summed either of all elements of A , or B or both, $C[i]$ will take the carry, as if $A[j]$ or $B[k]$ or both of them are zero.

Therefore $C[i]$ will hold the result of the sum of $A[j]$ and $B[k]$ plus the carry.

Therefore $C[i-1 \dots n]$ will contain the sum of $B[k-1 \dots B.length]$ and $A[j-1 \dots A.length]$ and the carry

Termination: On termination we have $i = 0, k = 0, j = 0$, therefore $C[1 \dots n]$ contains the sum of $B[1 \dots B.length]$ and $A[1 \dots A.length]$.