

Proove that $\forall n \in \mathbb{N}, \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$

Proof:

Let $n = 0, P(0)$ is true.

Suppose $\forall n \in \mathbb{N}, \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$

$$\Rightarrow \sum_{k=1}^{n+1} k^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$\Rightarrow \sum_{k=1}^{n+1} k^2 = \frac{(n+1)[n(2n+1)+6(n+1)]}{6}$$

$$\Rightarrow \sum_{k=1}^{n+1} k^2 = \frac{(n+1)[n(2n+7)+6]}{6}$$

$$\Rightarrow \sum_{k=1}^{n+1} k^2 = \frac{(n+1)[n(2n+3)+2(2n+3)]}{6}$$

$$\Rightarrow \sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$$

Proove that $\forall n \in \mathbb{N}, \sum_{k=1}^n k^3 = \frac{[n(n+1)]^2}{4}$

Proof:

Let $n = 0, P(0)$ is true.

Suppose $\forall n \in \mathbb{N}, \sum_{k=1}^n k^3 = \frac{[n(n+1)]^2}{4}$

$$\Rightarrow \sum_{k=1}^{n+1} k^3 = \frac{n(n+1)^2+4(n+1)^3}{4}$$

$$\Rightarrow \sum_{k=1}^{n+1} k^3 = \frac{(n+1)^2(n^2+4(n+1))}{4}$$

$$\Rightarrow \sum_{k=1}^{n+1} k^3 = \frac{[(n+1)(n+2)]^2}{4}$$