

Let $f \in F^E$ and $g \in E^F$, and $g \circ f \circ g \circ f$ is surjective and $f \circ g \circ f \circ g$ is injective.
Proove that g and f are bijective.

$g \circ f \circ g \circ f$ is surjective and $f \circ g \circ f \circ g$ is injective
 $\Rightarrow g$ is bijective
 $\Rightarrow g^{-1} \circ g \circ f \circ g \circ f$ is surjective and $f \circ g \circ f \circ g \circ g^{-1}$ is injective
 $\Rightarrow f$ is bijective.