Consider the function $f : \mathbb{R} \mapsto \mathbb{R}$ and those tow propositions:
- A: f is an even and odd function;
- B: f is the null function.
Proove that $A \iff B$.

Proof
'$\Rightarrow$'
Let $x \in \mathbb{R}$ and f is odd and even
$\Rightarrow f(-x) = x$ and $f(-x) = -x$
$\Rightarrow x = -x$
$\Rightarrow \forall x \in \mathbb{R}, f(x) = 0$
Therefore: $A \Rightarrow B$ is true.
'$\Leftarrow$'
Let $\forall x \in \mathbb{R}, f(x) = 0$
$\Rightarrow \forall x \in \mathbb{R}$, $f(x) = f(-x) = 0 = -f(x)$
Therefore: $A \Leftarrow B$ is true.