

Let R be an equivalence relation on the set E . The family $(C_x)_{x \in E}$ of the associated equivalence classes create a partition of the set E .

Proof:

1- Prove that $\forall (i, j) \in I^2, C_i \cap C_j \neq \emptyset \Rightarrow C_i = C_j$

Let $x \in C_i, C_i \cap C_j \neq \emptyset$ and R an equivalence relation

$\Rightarrow \exists y \in C_j \mid xRy$

$\Rightarrow \forall z \in C_j, yRz \Rightarrow zRx$

$\Rightarrow C_i = C_j$

2- Prove that $\forall i \in I, C_i \neq \emptyset$

Let $x \in E$ and C_i its equivalence class

$\Rightarrow x \in C_i \Rightarrow \forall i \in I, C_i \neq \emptyset$

3- Prove that $\bigcup_{i \in I} C_i = E$

Let $x \in E$

$\Rightarrow \exists i \in I \mid x \in C_i$

$\Rightarrow E \subset \bigcup_{i \in I} C_i \subset E$

Let $x \in \bigcup_{i \in I} C_i$

$\Rightarrow x \in E$

$\Rightarrow E \supset \bigcup_{i \in I} C_i \subset E$