

Let $f \in F^E$ and $g \in G^F$

- If f and g are injective, then $g \circ f$ is injective;

Proof:

Let $(x, x') \in E^2 \mid g \circ f(x) = g \circ f(x')$;

$\Rightarrow f(x) = f(x')$ because g is injective

$\Rightarrow x = x'$ because f is injective

$\Rightarrow (\forall (x, x') \in E^2, g \circ f(x) = g \circ f(x') \Rightarrow x = x')$

- If f and g are surjective, then $g \circ f$ is surjective;

Proof:

Let $y \in G$

$\Rightarrow \exists \alpha \in F \mid g(\alpha) = y$

$\Rightarrow \exists x \in E \mid f(x) = \alpha$ and $g(f(x)) = y$

$\Rightarrow g \circ f$ is surjective;

- If $g \circ f$ is injective, then f is injective;

Proof:

Suppose f is not injective

$\Rightarrow \exists (x, x') \in E^2 \mid f(x) = f(x')$ and $x \neq x'$

$\Rightarrow \exists (x, x') \in E^2 \mid g(f(x)) = g(f(x'))$ and $x \neq x'$

If $g \circ f$ is surjective, then g is surjective

Suppose g is not surjective

$\Rightarrow \exists y \in G, \nexists \alpha \in F \mid g(\alpha) = y$

\Rightarrow (if $\exists x \in E \mid f(x) = \alpha \Rightarrow \exists y \in G, \nexists x \in E \mid g(f(x)) = y$) or (if $\nexists x \in E \mid f(x) = \alpha \Rightarrow \exists y \in G, \nexists x \in E \mid g(f(x)) = y$)