Consider the function $f : \mathbb{R} \mapsto \mathbb{R}$ and those tow propositions:
- A: f is an even and odd function;
- B: f is the null function.
Proove that $A \iff B$.

Proof

'$\Rightarrow$'

Let $x \in \mathbb{R}$ and f is odd and even

$\Rightarrow f(-x) = x and f(-x) = -x$

$\Rightarrow x = -x$

$\Rightarrow \forall x \in \mathbb{R}, f(x) = 0$

Therefore: $A \Rightarrow B$ is true.

'$\Leftarrow$'

Let $\forall x \in \mathbb{R}, f(x) = 0$

$\Rightarrow \forall x \in \mathbb{R}, f(x) = f(-x) = 0 = -f(x)$

Therefore: $A \Leftarrow B$ is true.