Having A, B, C. Proove that
$(A \cup B \subset A \cup C$ and $A \cap B \subset A \cup C) \Rightarrow (B \cap C)$
Let $x \in B$
$\Rightarrow x \in A \cup C$
$\Rightarrow x \in B \cap A$ or $x \in B \cap C$
$\Rightarrow x \in A \cap C$ or $x \in B \cap C$
$\Rightarrow x \in C$
$\Rightarrow B \subset C$