

$[p, q]$ is $\{k \in \mathbb{N} \mid p \leq k \leq q\}$

Let $(p, q) \in \mathbb{N}^*$, then :

$$(p \leq q) \iff (\exists f : [1, p] \mapsto [1, q] \text{ is injective}) \quad (1)$$

$$(p \geq q) \iff (\exists f : [1, p] \mapsto [1, q] \text{ is surjective}) \quad (2)$$

Proof:

\Rightarrow

Having $(p \leq q)$

$$\Rightarrow \forall k \in [1, p], \exists k \in [1, q] \mid Id(k) = k$$

\Leftarrow

Suppose $\exists f : [1, p] \mapsto [1, q]$ is injective

$$\Rightarrow |[1, p]| \leq |[1, q]|$$

\Rightarrow

Having $(p \geq q)$

$$\Rightarrow \forall k \in [1, q], \exists k \in [1, p] \mid Id(k) = k$$

\Leftarrow

Suppose $\exists f : [1, p] \mapsto [1, q]$ is surjective

$$\Rightarrow |[1, p]| \geq |[1, q]|$$