

Let  $f : \begin{cases} \mathbb{R}^2 \rightarrow \mathbb{R}^2 \\ (x, y) \mapsto (x + y, x + 2y) \end{cases}$  . Is  $f$  injective? Surjective?

Let  $((x', y'), (x, y)) \in \mathbb{R}^4 \mid f((x', y')) = f((x, y))$   
 $\Rightarrow x + y = x' + y'$  and  $x + 2y = x' + 2y'$   
 $\Rightarrow x + 2y - x - y = y' - y$  and  $x = x'$   
 $\Rightarrow f$  is injective

$\forall (\alpha, \beta) \in \mathbb{R}^2, \exists (x, y) = (2\alpha - \beta, \beta - \alpha) \mid x + y = \alpha$  and  $x + 2y = \beta$   
 $\Rightarrow f$  is surjective