Let $f : E \to F$ and a family of F, $(B_i)_{i\in I}$. Proove that

$$f^{-1}(\bigcap_{i\in I} B_i) = \bigcap_{i\in I} f^{-1}(B_i)$$

Let $x \in f^{-1}(\bigcap_{i\in I} B_i)$
$\Rightarrow \exists y \in \bigcap_{i\in I} B_i \mid f(x) = y$
$\Rightarrow \forall i \in I, x \in f^{-1}(B_i)$
$\Rightarrow \forall i \in I, x \in \bigcap_{i\in I} f^{-1}(B_i)$
$\Rightarrow f^{-1}(\bigcap_{i\in I} B_i) \subset \bigcap_{i\in I} f^{-1}(B_i)$

Let $x \in \bigcap_{i\in I} f^{-1}(B_i)$
$\Rightarrow \exists y, \forall i \in I, y \in B_i$ and $f(x) = y$
$\Rightarrow y \in \bigcap_{i\in I} B_i$
$\Rightarrow x \in f^{-1}(\bigcap_{i\in I} B_i)$
$\Rightarrow f^{-1}(\bigcap_{i\in I} B_i) \supset \bigcap_{i\in I} f^{-1}(B_i)$