

Let E be a set and $(A_i)_{i \in I}$. Prove that:

$$E \setminus \left(\bigcap_{i \in I} A_i \right) = \bigcup_{i \in I} (E \setminus A_i)$$

$$\begin{aligned} & \text{Let } x \in E \setminus \left(\bigcap_{i \in I} A_i \right) \\ \iff & x \in E \text{ and } (x \notin A_{i_1} \text{ or } x \notin A_{i_2} \text{ or } \dots \text{ or } x \notin A_{i_n}) \\ \iff & (x \notin E \text{ and } x \notin A_{i_1}) \text{ or } (x \notin E \text{ and } x \notin A_{i_2}) \text{ or } \dots \text{ or } (x \notin E \text{ and } x \notin A_{i_n}) \\ \iff & x \in \bigcup_{i \in I} (E \setminus A_i) \\ \iff & E \setminus \left(\bigcap_{i \in I} A_i \right) = \bigcup_{i \in I} (E \setminus A_i) \end{aligned}$$