Let $f \in F^E$ and $g \in G^F$ tow bijective functions. Then

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

Proof:

Let
$$y \in G$$

 $\Rightarrow \exists! \alpha \in I$

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Let
$$y \in G$$

 $\Rightarrow \exists ! \alpha \in F \mid g^{-1}(y) = \alpha$
 $\Rightarrow \exists ! x \in E \mid f^{-1}(\alpha) = x$
 $\Rightarrow g \circ f(x) = g(\alpha) = y$
 $\Rightarrow (g \circ f)^{-1}(y) = x$

$$\Rightarrow \exists ! x \in E \mid f^{-1}(\alpha) = x$$

$$\Rightarrow g \circ f(x) = g(\alpha) = y$$

$$\Rightarrow (q \circ f)^{-1}(y) = x$$