

Consider the divisibility in \mathbb{N} :

$$\forall (n, m) \in \mathbb{N}^2, n \mid m \iff \exists k \in \mathbb{N} \mid m = kn$$

1. Verify that that relation defines a partial order over \mathbb{N} ;
2. Does \mathbb{N} admit an infimum, supremum for that order?
3. What are the maximums and minimums of $\mathbb{N} \setminus \{0, 1\}$?

1. $(\forall x \in \mathbb{N}, x \mid x)$; $(\forall (x, y, z) \in \mathbb{N}^2, \text{ if } x \mid y \text{ and } z \mid x \Rightarrow z \mid y)$; $(\forall (x, y, z) \in \mathbb{N}^2, x \mid y \text{ and } y \mid x \iff x = y)$; Then \mid is an order relation over \mathbb{N} .

Knowing $11 \mid 13$ and $13 \mid 11$ is false $\Rightarrow \mid$ is partial over \mathbb{N} .

2. $\forall x \in \mathbb{N}, 1 \mid x \Rightarrow 1$ is an infimum.
 $\forall x \in \mathbb{N}, x \mid 0 \Rightarrow 0$ is a supremum.

3. There are no maximums and minimums in $\mathbb{N} \setminus \{0, 1\}$ by \mid .