Let $(a, b) \in \mathbb{N}^2, b \neq 0$. Then $\exists!(q, r) \in \mathbb{N}^2$ such as:
1. $a = bq + r$
2. $0 \leq r < b$

Proof(informal):
$(a, b) \in \mathbb{N}^2$
$\Rightarrow \exists k^{(1)} \in \mathbb{N} \mid (a = b + k^{(1)}$
$\Rightarrow (\exists k^{(1)} \in \mathbb{N} \mid (k^{(1)} < b \text{ and } a = b + k^{(1)}))$ or $(\exists k^{(2)} \in \mathbb{N} \mid a = b + b + k^{(2)})$
Continuing until $0 \leq k^{(q)} < b$, yields
$\Rightarrow \exists (q, r) \in \mathbb{N}, r = k^{(q)} \mid a = qb + r$ and $0 \leq r < b$

Let $((q, r), (q', r')) \in \mathbb{N}^4 \mid a = bq + r$ and $a = bq' + r'$
$\Rightarrow r - r' = b(q - q')$
Having $-b < r - r' < b$
$\Rightarrow -1 < q - q' < 1$
$\Rightarrow q = q'$ and $r = r'$

Proof 2:
Let $A = \{k \in \mathbb{N} : \exists x \in \mathbb{N} : k = xb \leq a\}$
Having $0 \in A, (\forall k \in A, k \leq a)$ and $A \subset \mathbb{N}$
$\Rightarrow \exists! q \in \mathbb{N} \mid max_A = qb$
$\Rightarrow qb \leq a$
$\Rightarrow \exists! r \in \mathbb{N}, r \geq 0 \mid hb + r = a$
Having $qb + b > a$
$\Rightarrow hb + b > hb + r$
$\Rightarrow b > r$