Let E be a set and $(A_i)_{i\in I}$. Proove that:

$$E \setminus (\bigcap_{i\in I} A_i) = \bigcup_{i\in I}(E \setminus A_i)$$

$$E \setminus (\bigcup_{i\in I} A_i) = \bigcap_{i\in I}(E \setminus A_i)$$

Let $x \in E \setminus (\bigcap_{i\in I} A_i)$
$\iff$ $x \in E$ and $(x \notin A_{i_1}$ or $x \notin A_{i_2}$ or ... or $x \notin A_{i_n})$ for $n = |A_i, \forall i \in I|$
$\iff$ $(x \notin E$ and $x \notin A_{i_1})$ or $(x \notin E$ and $x \notin A_{i_2})$ or ... or $(x \notin E$ and $x \notin A_{i_n})$
$\iff$ $x \in \bigcup_{i\in I}(E \setminus A_i)$
$\iff$ $E \setminus (\bigcap_{i\in I} A_i) = \bigcup_{i\in I}(E \setminus A_i)$

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$\iff$ $x \in \bigcap_{i\in I}(E \setminus A_i)$
$\iff$ $E \setminus (\bigcup_{i\in I} A_i) = \bigcap_{i\in I}(E \setminus A_i)$