Let $(a,b) \in \mathbb{Z}^2, (q,r) \in \mathbb{N}^2, 0 \leq r < b \mid a = bq + r \; and \; k = gcd(a,b) \Rightarrow k = gcd(b,r)$

$k = gcd(a,b) \Rightarrow (k|r)$
Let $k' \in \mathbb{Z} \mid k' = gcd(b,r)$
$\Rightarrow k' \geq k \; and (k'|a)$
$\Rightarrow k' = k \blacksquare$