

Let $(a, b) \in \mathbb{Z}^2, (q, r) \in \mathbb{N}^2, 0 \leq r < b \mid a = bq + r$ and $k = \gcd(a, b) \Rightarrow k = \gcd(b, r)$

$k = \gcd(a, b) \Rightarrow (k \mid r)$
 Let $k' \in \mathbb{Z} \mid k' = \gcd(b, r)$
 $\Rightarrow k' \geq k$ and $(k' \mid a)$
 $\Rightarrow k' = k$ ■