Let
$$(a,b) \in \mathbb{Z}^2, (q,r) \in \mathbb{N}^2, 0 \le r < b \mid a = bq + r \text{ and } k = gcd(a,b) \Rightarrow k = gcd(b,r)$$

$$k = gcd(a,b) \Rightarrow (k|r)$$
 Let $k' \in \mathbb{Z} \mid k' = gcd(b,r)$
$$\Rightarrow k' \ge k \text{ and}(k'|a)$$

$$\Rightarrow k' = k$$