

Let $a, b, c \in \mathbb{Z}$

$$\begin{aligned} \text{(a)} \quad & (a|b) \wedge (b|c) \Rightarrow (a|c) \\ & (a|b) \wedge (b|c) \Rightarrow \exists(k, k') \in \mathbb{Z}^2 \mid (b = ka) \wedge (c = k'b) \\ & \Rightarrow c = k'ka \blacksquare \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & (a|b) \wedge (b|a) \Rightarrow a = \pm b \\ & (a|b) \wedge (b|a) \Rightarrow \exists(k, k') \in \mathbb{Z}^2 \mid (a = kb) \wedge (b = k'a) \\ & \Rightarrow k'a = ka \\ & \Rightarrow (k = k' = 1) \vee (k = k' = -1) \blacksquare \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & (a|b) \wedge (a|c) \Rightarrow (a|(b+c)) \wedge (a|(b-c)) \\ & (a|b) \wedge (a|c) \Rightarrow (b+c = a(k+k')) \wedge (b-c = a(k-k')) \blacksquare \end{aligned}$$