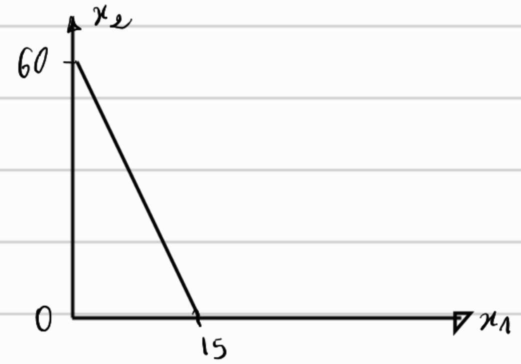
Ex2:

1/ $R = P_1 x_1 + P_2 x_2 \Rightarrow 60 = 4x_1 + x_2 \Rightarrow x_2 = 60 - 4x_1$



2/ $U(x_1, x_2) = 8x_1^2 x_2 \qquad x_2 = 60 - 4x_1$

Méthode de substitution:

$$U(x_1) = 8x_1^2(60 - 4x_1) = 480x_1^2 - 32x_1^3$$

1ère condition:

$$U'(x_1) = 2 \times 480 x_1 - 3 \times 32 x_1^2 = 0$$

$$x_1^2 - \frac{960}{96} x_1 = 0 \Rightarrow x_1^2 - 10x_1 = 0$$

$$\Delta = 10^2 - 4 \times 1 \times 0 = 100$$

$$x_{11} = \frac{-b - \sqrt{\Delta}}{2a} \qquad x_{12} = \frac{-b + \sqrt{\Delta}}{2a}$$

$$= \frac{10 - 10}{2} \qquad = \frac{10 + 10}{2}$$

$$= 0 \qquad = 10$$

$x_1 = 0$ ou $x_1 = 10$

2ème condition:

$$U''(x_1) = 960 - 2 \times 96 x_1 = 960 - 192 x_1 \leq 0$$

$$960 \leq 192 x_1 \quad \text{donc } \boxed{x_1 = 10}$$

or $x_2 = 60 - 4x_1 = 60 - 40 = 20 \quad \boxed{x_2 = 20}$

Par lagrange: $U = 8x_1^2 x_2$

$\exists \lambda$: $L(x_1, x_2, \lambda) = U(x_1, x_2) + \lambda (R - P_1 x_1 - P_2 x_2)$
$= 8x_1^2 x_2 + \lambda(60 - 4x_1 - x_2)$

$\frac{\partial L(x_1, x_2, \lambda)}{\partial x_2} = 8x_1^2 - \lambda = 0 \Rightarrow \lambda = 8x_1^2$

$\frac{\partial L(x_1, x_2, \lambda)}{\partial x_1} = 16 x_2 x_1 - 4\lambda = 0 \Rightarrow \lambda = 4x_1 x_2$

$\Big\} \; 8x_1^2 = 4x_1x_2 \Rightarrow x_2 = 2x_1$

$\frac{\partial L(x_1, x_2, \lambda)}{\partial \lambda} = 60 - 4x_1 - x_2 = 0 \Rightarrow 60 = 4x_1 + x_2$ (D.B)

$\begin{cases} x_2 = 2x_1 \\ x_2 = 60 - 4x_1 \end{cases} \Rightarrow \begin{cases} 6x_1 = 60 \\ x_2 = 2x_1 \end{cases} \Rightarrow \begin{cases} x_1 = 10 \\ x_2 = 20 \end{cases}$

pour savoir combien céder il faut $TMS_{1/2}$.

$Um_1 = \frac{\partial U}{\partial x_1} = 16 x_2 x_1 \quad Um_2 = \frac{\partial U}{\partial x_2} = 8x_1^2$

$TMS = \frac{Um_1}{Um_2} = \frac{16x_1x_2}{8x_1^2} = \frac{2x_2}{x_1} = \frac{2 \cdot 20}{10} = 4$

Le cons est prêt à céder 4 bien 2 pour 1 unité du bien 1.

3/ pnt optimal $(10, 20)$ $\quad U = 8 \cdot 10^2 \cdot 20 = 800 \cdot 20 = 1600$

Ex3 : $U = x_1 x_2 \quad 10 = x_1 + 2x_2$

1/ Méthode de Lagrange : $L(x_1, x_2, \lambda) = U + \lambda(10 - 2x_1 - x_2) =$

$\frac{\partial L}{\partial x_1} = x_2 - 2\lambda = 0 \Rightarrow \lambda = \frac{x_2}{2}$

$\frac{\partial L}{\partial x_2} = x_1 - \lambda = 0 \Rightarrow \lambda = x_1$

$\Big\} \; x_2 = 2x_1$

$\frac{\partial L}{\partial \lambda} = 10 - 2x_1 - x_2 = 0 \Rightarrow x_2 = 10 - 2x_1$

$\Rightarrow \begin{cases} x_2 = 2x_1 \\ x_2 = 10 - 2x_1 \end{cases}$

$\begin{cases} x_2 = 5 \\ 4x_1 = 10 \Rightarrow x_1 = 2{,}5 \end{cases}$

2/ $(2{,}5 ; 5)$

$U = 2{,}5 \times 5 = 12{,}5$

Ex4: $U = x_1^2 + \frac{1}{4}x_2^2 + 2x_1x_2$ $\qquad 40 = 4x_1 + 2x_2$

Méthode de Lagrange: $L(x_1, x_2, \lambda) = x_1^2 + \frac{1}{4}x_2^2 + 2x_1x_2 + \lambda(40 - 4x_1 - 2x_2)$

$\frac{\partial L}{\partial x_1} = 2x_1 + 2x_2 - 4\lambda = 0 \Rightarrow 2\lambda = x_1 + x_2$

$\frac{\partial L}{\partial x_2} = \frac{1}{2}x_2 + 2x_1 - 2\lambda = 0 \quad 2\lambda = \frac{1}{2}x_2 + 2x_1$

$\Rightarrow \frac{1}{2}x_2 - x_1 = 0 \Rightarrow x_2 = 2x_1$

$\frac{\partial L}{\partial \lambda} = 40 - 4x_1 - 2x_2 = 0$

donc $\begin{cases} x_2 = 2x_1 \\ 40 - 4x_1 - 2x_2 = 0 \end{cases}$

$\begin{cases} x_2 = 2x_1 \\ 40 - \underbrace{2x_2 - 2x_2}_{4x_2} = 0 \end{cases}$

$\Rightarrow x_2 = 10$

$\Rightarrow x_1 = 5$

$(5, 10)$

$U = 5^2 + \frac{1}{4}100 + 2.5.10$

$= 25 + 25 + 10.10$

$= 50 + 100 = 150.$

Ex5: $U = (x_1 + 100)^{3/2} x_2^{1/2}$ $\qquad 500 = x_1 + 3x_2$

1/ Par la méthode de Lagrange: $L = (x_1 + 100)^{3/2} . x_2^{1/2} + \lambda(500 - x_1 - 3x_2)$

$\frac{\partial L}{\partial x_1} = \frac{3}{2}(100 + x_1)^{1/2} . x_2^{1/2} - \lambda = 0 \Rightarrow 2\lambda = 3 \times \sqrt{x_2(100 + x_1)}$

$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_2^{-1/2} (x_1 + 100)^{3/2} - 3\lambda = 0 \Rightarrow 2\lambda = \frac{1}{3}\sqrt{\frac{x_1 + 100}{x_2}}$

$\frac{\partial L}{\partial \lambda} = 500 - x_1 - 3x_2 = 0 \Rightarrow 500 = x_1 + 3x_2$

donc $3 \; x_2^{1/2} \; \cancel{(100 + x_1)}^{1/2} = \frac{1}{3} \; x_2^{-1/2} \; \cancel{(x_1 + 100)}^{1/2}$

$x_2 = 1/9$

$x_1 = 500 - 3 \times 1/9 = \frac{1499}{3}$

$\left(\frac{1499}{3}, 1/9\right)$