

PW2 – Applied Optimization – M2 E3A SAM

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Application of usual optimization methods

The objective of this lab is to use Matlab software (MathWorks) to compare different linear search algorithms for the minimization of a function defined on \mathbb{R}^n .

PART I :

In this first part, one will discover the Particle Swarm Optimization (PSO) applied to the Rastrigin function.

The particle swarm algorithm begins by creating the initial particles, and assigning them initial velocities. It evaluates the objective function at each particle location, and determines the best (lowest) function value and the best location. It chooses new velocities, based on the current velocity, the particles' individual best locations, and the best locations of their neighbors.

It then iteratively updates the particle locations (the new location is the old one plus the velocity, modified to keep particles within bounds), velocities, and neighbors. Iterations proceed until the algorithm reaches a stopping criterion.

In that way, to observe the efficiency and the robustness of PSO method, this part presents an exemple to show how to find the minimum of the Rastrigin function f . This is a 2-variable function x and y defined by :

$$f(x, y) = 20 + x^2 + y^2 - 10(\cos(2\pi x) + \cos(2\pi y))$$

1) With extrema point of view, what can you notice for this particular function ? Plot it for $x \in [-2, 2]$ and $y \in [-2, 2]$ for a first plot, and for $x \in [-0.1, 0.1]$ and $y \in [-0.1, 0.1]$ for a second plot.

2) We propose to solve the following (PO) problem by PSO method :

$$\min_{\substack{-0.1 \leq x \leq 0.1 \\ -0.1 \leq y \leq 0.1}} f(x, y)$$

with the following PSO parametrization :

Number of particles = 40

Maximal iteration = 80

$C_1 = C_2 = 2$

The coefficient w used here will verify :

$$w = w_{max} - \left(\frac{w_{max} - w_{min}}{K_{max}} \right) \cdot K$$

with :

$$w_{min} = 0.4$$

$$w_{max} = 0.9$$

K_{max} is the maximum iterations number, K is the current iterations number

Search demand : $x \in [-0.1, 0.1]$ and $y \in [-0.1, 0.1]$.

- a) Recall the role of each parameter above.
- b) Simulate PSO method to solve (PO) problem. The simulation will be launched twice : for each of these two simulations, plot also Fitness *value* among the current iteration (the Fitness function is the objective function).

Note :

It sometimes happens that, taking into account the current position and the current speed of a particle, the latter tends to leave the search space during its displacement. When this is the case, the algorithm involves a confinement mechanism, called interval confinement, in order to manage the movement of the particle and this, so that it brings it to a new point also belonging to the search space. In general, this mechanism consists in bringing the particle back to the closest admissible point.

- c) Analyze the results obtained with these 2 simulations.
- d) In your opinion, what is the interest in the comparison of these 2 simulations ?
- e) Conclude.

PART II :

It is proposed in this second part to apply the simulated annealing method to the same form as the one of (PO) problem in Part I to compare both methods, but for an objective function g .

- 1) Recall the simulated annealing algorithm
- 2) One suppose that the objective function $g = \alpha f$ (α positive real) represents the energy consumption function of a car vehicle to be minimized on the interval $x \in [0, 0.1]$ and $y \in [0, 0.1]$ where x and y are respectively an image of engine speed and engine torque.
 - a) Simulate the simulated annealing method for this (PO) problem by using Metropolis algorithm.
 - b) Analyze the obtained results and compare with PSO method for this problem.
 - c) Conclude.