



University of Tehran
School of Electrical and Computer Engineering



Pattern Recognition

Assignment 2-2

Due Date: 31st of Farvardin

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PROBLEM 1

Consider five Gaussian pdf's $N_1(0.5, 0.02)$, $N_2(1.2, 0.09)$, $N_3(2.1, 0.06)$, $N_4(3.4, 0.04)$, and $N_5(4.0, 0.01)$ where two numbers indicate mean (μ_i) and variance (σ_i^2), respectively. Generate 5000 samples according to the following rule. The first sample comes from the first Gaussian pdf (N_1), the second one from N_2 , the third one from N_3 , the forth one from N_4 , and the last one from N_5 . This rule repeats until all 5000 samples have been generated. The pdf underlying the random sample is modeled as the following mixture density:

$$\sum_{i=1}^l p_i N(\mu_i, \sigma_i^2)$$

- Consider $I = 5$. Use the EM algorithm and the generated samples to estimate the unknown parameters, (μ_i , σ_i^2 , and p_i , $i = 1, 2, \dots, I$)
- Repeat (a) for $I = 2$. Compare the result.
- Repeat (a) for $I = 4$. Compare the result.
- Repeat (a), using only 1000 generated samples. Compare the result.

PROBLEM 2 (25% Bonus)

Assume that you want to model the future probability that your dog is in one of three states given its current state considering its health and sickness. Compute the probability of the observation sequences PESESEPEPSP and SESPSEPSPSE using **Forward Algorithm** in the following HMM. Which one is more likely?

S	Sleeping
E	Eating
P	Pooping

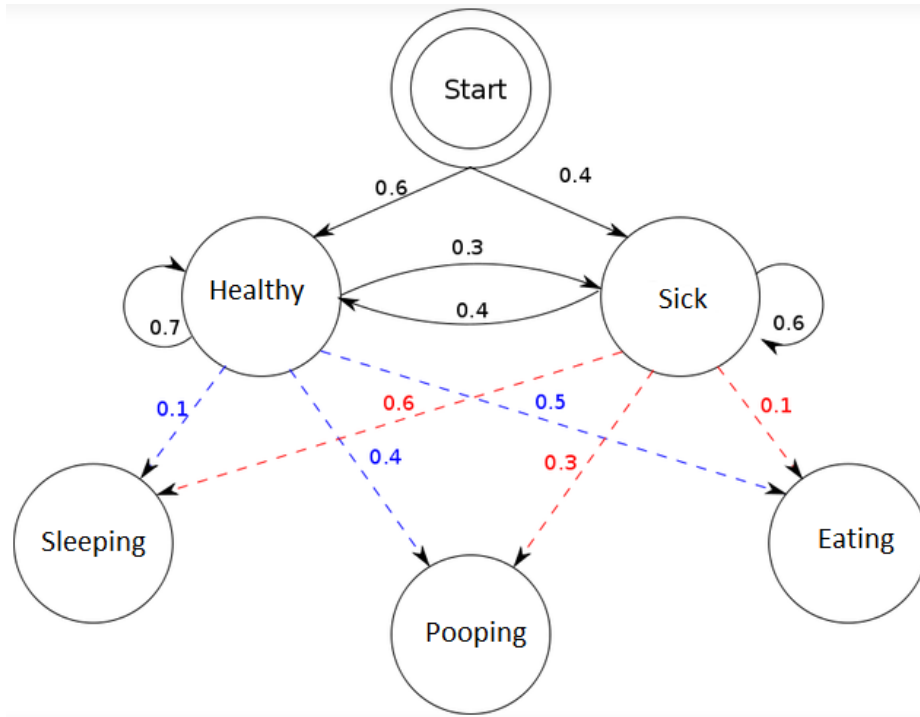


Figure 2- A hidden Markov model for relating numbers of dog behavior to the health level of it (Sick(S) or Healthy(H) are the hidden variables).

** Figure 2 shows a sample HMM for the dog typical tasks when it's healthy or sick. The two hidden states (H and S) correspond to Health and Sickness of dog, and the observations are sleeping, pooping and eating.

PROBLEM 3 (25% Bonus)

- I. Show that for a Markov Chain, $P(q_t | q_{t+1}, \dots, q_T) = P(q_t | q_{t+1})$.
- II. Consider the following 2-state Hidden Markov Models where both states have two possible output symbols A and B.

Model 1:

Transition probabilities: $a_{11} = 0.6$, $a_{12} = 0.4$, $a_{21} = 0.0$, $a_{22} = 1.0$

(a_{ij} is the probability of going from state i to state j)

Output probabilities: $b_1(A) = 0.45$, $b_1(B) = 0.55$, $b_2(A) = 0.5$, $b_2(B) = 0.5$

Initial probabilities: $\pi_1 = 0.4$, $\pi_2 = 0.6$

Model 2:

Transition probabilities: $a_{11} = 0.2$, $a_{12} = 0.8$, $a_{21} = 0.0$, $a_{22} = 1.0$

Output probabilities: $b_1(A) = 0.2$, $b_1(B) = 0.8$, $b_2(A) = 0.6$, $b_2(B) = 0.4$

Initial probabilities: $\pi_1 = 0.7$, $\pi_2 = 0.3$

- a. Sketch the state diagram for two models.
- b. Which model is more likely to produce the observation sequence {A, B, A}?

Hint: For more information on Hidden Markov Models refer to the **third chapter** of [this](#) reference.

Notes

1. Please make sure you reach the deadline because there would be no extra time available.
2. Late policy would be as bellow:
 - Every student has a budget for late submission during the semester. This budget is two weeks for all the assignments.
 - Late submission more than two weeks may cause lost in your scores.
3. Analytical problems can be solved on papers and there is no need to type the answers. The only thing matters is the quality of your pictures. Scanning your answer sheets is recommended. If you are using your smartphones you may use scanner apps such as CamScanner or google drive application.
4. Simulation problems need report as well as source codes and results. This report must be prepared as a standard scientific report.
5. You have to prepare your final report including the analytical problems answer sheets and your simulation report in a single pdf file.
6. Finalized report and your source codes must be uploaded to the course page as a “.zip” file (not “.rar”) with the file name format as bellow:
PR_Assignment #[Assignment Number]_Surname_Name_StudentID.zip
7. Plagiarisms would be strictly penalized.
8. You may ask your questions from corresponding TAs.