In The name of God

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Electrical and Computer Department

Pattern Recognition

Homework #1

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Contents

Abstract

In this home work we are going to some calculation and some coding the find the decision boundary of a distribution of different kind of function with one to two dimensions.

And also using the maximum likihood and other tools to make out calculations efficient.

Question 1

Problem 1:

a) $P(x|\omega_i) = \frac{1}{\pi b} \frac{1}{1+(\frac{x-a_i}{b})^2}$ $i=1,2$

↳ for Normalization ↳ $\int_{-\infty}^{\infty} P(x|\omega_i)\,dx = 1 \longrightarrow \frac{1}{\pi b}\int_{-\infty}^{\infty} \frac{dx}{1+(\frac{x-a_i}{b})^2} = \frac{1}{\pi}\, tg^{-1}(\frac{x-a_i}{b})\Big|_{-\infty}^{\infty}$

$\frac{1}{\pi}(\frac{\pi}{2} - (-\frac{\pi}{2})) = 1$ ✓ It is Normalized ✓

b) $P(x \cap \omega_i) = P(x|\omega_i)P(\omega_i) = P(\omega_i|x)P(x) \rightarrow \frac{P(x|\omega_i)P(\omega_i)}{P(x)} = P(\omega_i|x)$

$i=1,2$

↳ $\frac{P(x|\omega_1)P(\omega_1)}{\cancel{P(x)}} = \frac{P(x|\omega_2)P(\omega_2)}{\cancel{P(x)}} \rightarrow P(\omega_1) = P(\omega_2) \longrightarrow P(x|\omega_1) = P(x|\omega_2)$

$\longrightarrow \frac{1}{\pi b}\frac{1}{1+\frac{(x-a_1)^2}{b^2}} = \frac{1}{\pi b}\frac{1}{1+\frac{(x-a_2)^2}{b^2}} \longrightarrow \cancel{1} + (\frac{x-a_1}{b})^2 = \cancel{1} + (\frac{x-a_2}{b})^2 \longrightarrow$

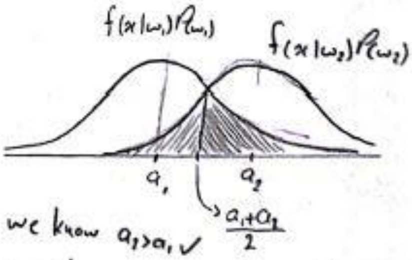
$x^2 - 2a_1x + a_1^2 = x^2 - 2a_2x + a_2^2 \longrightarrow x(2a_2 - 2a_1) = a_2^2 - a_1^2 \longrightarrow$

$x = \frac{(a_2-a_1)(a_2+a_1)}{2(a_2-a_1)} = \frac{(a_2+a_1)}{2}$ ↳ $\boxed{x = \frac{a_2+a_1}{2}}$

The Matlab code are in the file with the figure

c) The error is the black zone in the figure

$P(\omega_1) = P(\omega_2) = \frac{1}{2}$



↳ $P\{error\} = \frac{1}{2}\left(\int_{-\infty}^{\frac{a_1+a_2}{2}} \frac{1}{\pi b}\frac{dx}{1+(\frac{x-a_2}{b})^2} + \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\pi b}\frac{dx}{1+(\frac{x-a_1}{b})^2}\right)$

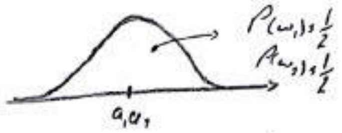
we know $a_2 > a_1$ ✓

$\frac{1}{2\pi}\left(tg^{-1}(\frac{a_1-a_2}{2b}) + \frac{\pi}{2} + \frac{\pi}{2} - tg^{-1}(\frac{a_2-a_1}{2b})\right) = \frac{1}{2\pi}\left(\pi + 2\,tg^{-1}(\frac{a_1-a_2}{2b})\right) = \frac{1}{2} - \frac{1}{\pi}\,tg^{-1}(\frac{|a_2-a_1|}{2b})$

$$\boxed{P\{error\} = \frac{1}{2} - \frac{1}{\pi}\,tg^{-1}\left(\frac{|a_2-a_1|}{2b}\right)}$$

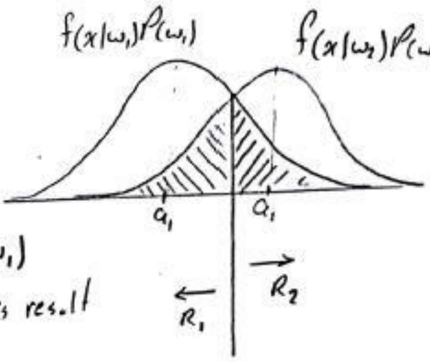
d) The maximum error acourse when $a_1 = a_2$ witch means that the two function are on each other witch means that they both have the same chance $P\{error\} = \frac{1}{2} - \frac{1}{\pi} tg^{-1} \left| \frac{a_2 - a_1}{2b} \right| \rightarrow a_2 = a_1$

$\boxed{P\{error\} = \frac{1}{2}} \rightarrow \boxed{P_{max}\{error\} = \frac{1}{2}} \rightarrow$



for this case the chance for ω_1 or ω_2 is always equal ✓

e) The error is the black part at the Figure



$\rightarrow f(x|\omega_1)P(\omega_1) = g(\omega_1)$

$f(x|\omega_2)P(\omega_2) = g(\omega_2)$

$g(\omega_2) \underset{R_1}{\overset{R_2}{\gtrless}} g(\omega_1)$ The ω_2 is result / The ω_1 is result

$$f(x|\omega_1)P(\omega_1) \underset{R_2}{\overset{R_1}{\gtrless}} f(x|\omega_2)P(\omega_2)$$

$\rightarrow P(\omega_1) = P(\omega_2)$ ✓ $\rightarrow$ $\frac{a_1 + a_2}{2}$

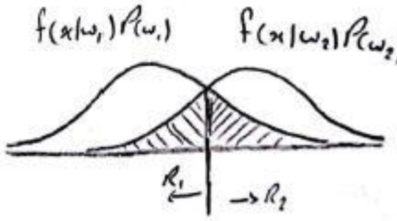
$$\boxed{f(x|\omega_1) \underset{R_2}{\overset{R_1}{\gtrless}} f(x|\omega_2)}$$

decision boundaries Case

$$P\{error\} = \int_{R_2} f(x|\omega_1) P(\omega_1) dx + \int_{R_1} f(x|\omega_2) P(\omega_2) dx$$

$$P\{error\} = 1 - \int_{R_1} f(x|\omega_1)P(\omega_1) dx - \int_{R_2} f(x|\omega_2)P(\omega_2) dx$$

The resaults are ploted as well



$$f(x|\omega_1) \underset{R_2}{\overset{R_1}{\gtrless}} f(x|\omega_2)$$ ✓

f) $Risk = \sum_{i=1}^{2} \int_{R_i} dx \sum_{j=1}^{2} \lambda_{ij} P(\omega_j) f(x|\omega_j)$

$r_1 = \lambda_{11} P(x|\omega_1) P(\omega_1) + \lambda_{21} P(x|\omega_2) P(\omega_2)$

$r_2 = \lambda_2 \lambda_2 P(x|\omega_2) P(\omega_2) + \lambda_{12} P(x|\omega_2) P(\omega_2)$

to chose r_1 / r_2:

The minimum error accours when they are equal

$\lambda_{12} P(x|\omega_2)P(\omega_2) + \lambda_2\lambda_2 P(x|\omega_2)P(\omega_2) \gtrless \lambda_{21} P(x|\omega_1)P(\omega_1) + \lambda_{22} P(x|\omega_1)P(\omega_1)$

$\longrightarrow \begin{matrix}\lambda_{11}=\lambda_{22}=0\\ \lambda_{21}=2 \\ \lambda_{12}=1\end{matrix} \rightarrow P(x|\omega_2)P(\omega_2) \gtrless 2P(x|\omega_1)P(\omega_1) \rightarrow P(\omega_1) = P(\omega_2)$

$\hookrightarrow \frac{1}{\pi b}\frac{1}{1+(\frac{x-a_1}{b})^2} = \frac{1}{\pi b}\frac{2}{1+(\frac{x-a_2}{b})^2} \rightarrow$

$x^2 - 2a_2x + a_2^2 + b^2 = 2x^2 - 4a_1x_1 + 2a_1^2 + 2b_2^2 \rightarrow x^2 + (-4a_1+2a_2)x + (2a_1^2 - a_2^2 - 2b^2) = 0$

$x = (-a_2 + 2a_1) \pm \sqrt{4(a_2-2a_1)^2 - 4(2a_1^2 - a_2^2 - 2b^2)} = (2a_1 - a_2) \pm \sqrt{\underbrace{2a_2^2 + 2a_1^2 - 4a_2a_1 - b^2}_{2(a_2-a_1)^2 - b^2}}$

solve $\Rightarrow$ $\begin{matrix}\gamma_1\\ \gamma_2\end{matrix}$ two ans they be correct of incorrect $\gamma_2 > \gamma_1$

$\hookrightarrow$ in two place they touch each other

$\longrightarrow Risk = \underbrace{\int_{\gamma_1}^{\gamma_2} 2 f(x|\omega_1) P(\omega_1) dx}_{R_2 \text{ is chos.}} + \underbrace{\int_{-\infty}^{\gamma_1} f(x|\omega_2)P(\omega_2) dx}_{R_1 \text{ is the chos}} + \underline{\int_{\gamma_2}^{+\infty} f(x|\omega_2)P(\omega_2) dx} =$

$\frac{1}{\pi}\left(tg^{-1}\left(\frac{\gamma_2-a_1}{b}\right) - tg^{-1}\left(\frac{\gamma_1-a_1}{b}\right)\right) + \frac{1}{2\pi}\left(tg^{-1}\left(\frac{\gamma_1-a_2}{b}\right) + \frac{\pi}{2}\right) + \frac{1}{2\pi}\left(\frac{\pi}{2} - tg^{-1}\left(\frac{\gamma_2-a_2}{b}\right)\right)$

$Risk = \frac{1}{\pi}\left(tg^{-1}\left(\frac{\gamma_2-a_1}{b}\right) - tg^{-1}\left(\frac{\gamma_1-a_1}{b}\right)\right) + \frac{1}{2\pi}\left(tg^{-1}\left(\frac{\gamma_1-a_2}{b}\right) + \frac{\pi}{2}\right) + \frac{1}{2\pi}\left(\frac{\pi}{2} - tg^{-1}\left(\frac{\gamma_2-a_2}{b}\right)\right)$ ✓

To calculate the error we should get rid of λ_{ij}

$$P\{error\} = \underbrace{\int_{\nu_1}^{\nu_2} f(x|\omega_1)\,dx}_{R_2} + \underbrace{\int_{-\infty}^{\nu_1} f(x|\omega_2)\,dx + \int_{\nu_2}^{+\infty} f(x|\omega_1)\,dx}_{R_1}$$

$$\frac{1}{2\pi}\left(tg^{-1}\left(\frac{\nu_2-a_1}{b}\right) - tg^{-1}\left(\frac{\nu_1-a_1}{b}\right)\right) + \frac{1}{2\pi}\left(tg^{-1}\left(\frac{\nu_1-a_2}{b}\right) + \frac{\pi}{2}\right) + \frac{1}{2\pi}\left(\frac{\pi}{2} - tg^{-1}\left(\frac{\nu_2-a_2}{b}\right)\right)$$

In the case we minimised it with risk the eror Increases $\rightarrow$ if $\nu_1, \nu_2 \notin R$ $\rightarrow$ They don't touch eachother

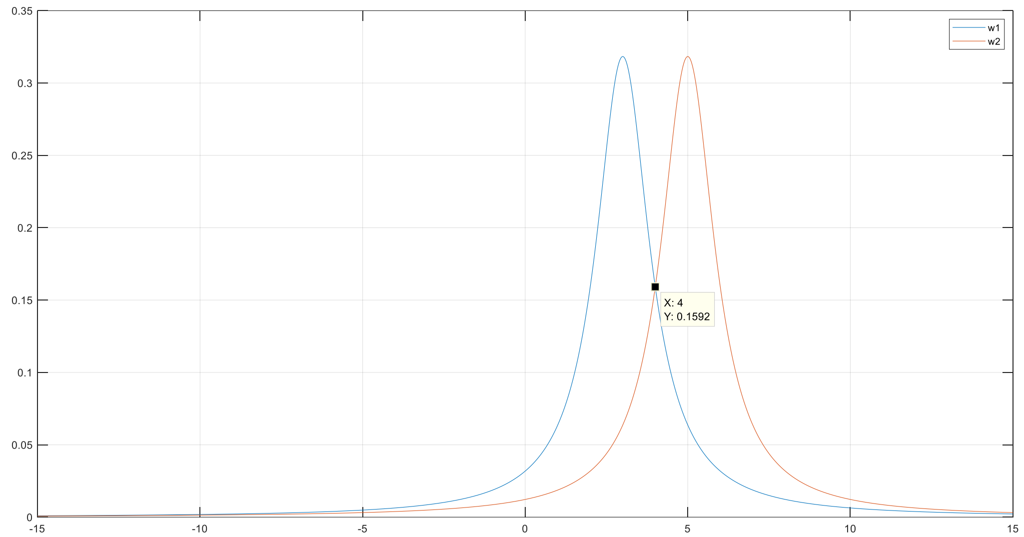


Figure 1

Figure1 shows that where $x = \frac{a_1+a_2}{2}$ in that position $P(\omega_1|x) = P(\omega_2|x)$ and we have showed that $\frac{3+5}{2} = 4$ witch shows that this is correct.

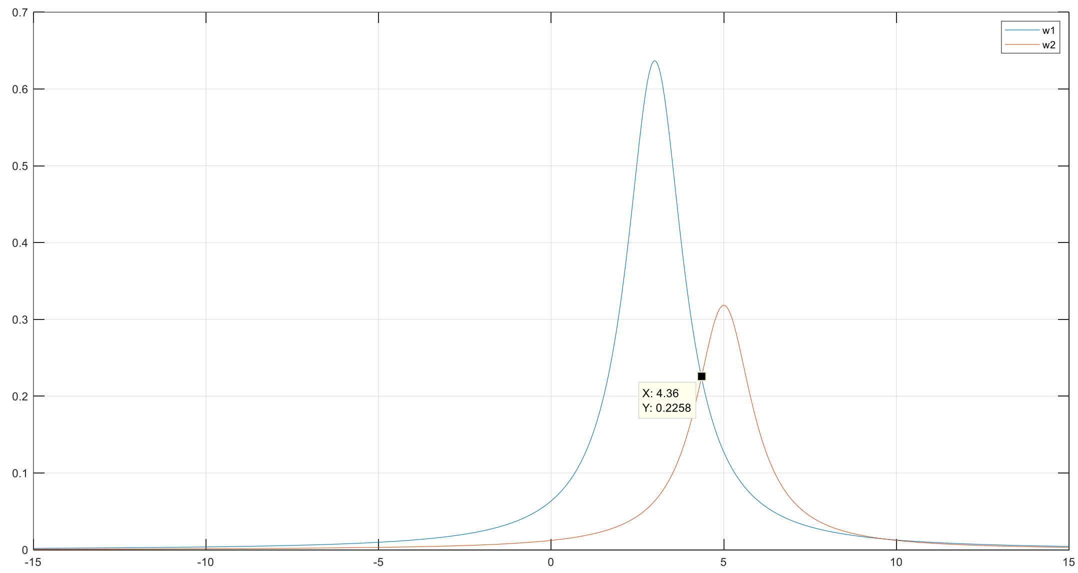


Figure 2

In Figure2 we can see that when we use rick value as well because they are like a constant variable the will change the border of the 2 regions.

Question 2

Problem 2:

a) Risk $= \sum_{i=1}^{c}\sum_{j=1}^{c} \lambda_{ij} P(x|\omega_i)P(\omega_i)$

$$\delta^* = \arg\min_{j=1}^{c} \left(\sum_{i=1}^{c} \lambda_{ij} P(x|\omega_i)P(\omega_i) \right) \rightarrow c=2$$

$$\left.\begin{array}{l} \lambda_{11} P(x|\omega_1)P(\omega_1) + \lambda_{21} P(x|\omega_2)P(\omega_2) \\ \lambda_{12} P(x|\omega_1)P(\omega_1) + \lambda_{22} P(x|\omega_2)P(\omega_2) \end{array}\right\} \rightarrow \text{compaire}$$

$$\lambda_{11} P(x|\omega_1)P(\omega_1) + \lambda_{21} P(x|\omega_2)P(\omega_2) \underset{R_1}{\overset{R_2}{\gtrless}} \lambda_{12} P(x|\omega_1)P(\omega_1) + \lambda_{22} P(x|\omega_2)P(\omega_2)$$

$$(\lambda_{21} - \lambda_{22}) P(x|\omega_2)P(\omega_2) \underset{R_1}{\overset{R_2}{\gtrless}} (\lambda_{12} - \lambda_{11}) P(x|\omega_1)P(\omega_1) \rightarrow$$

The decision $\rightarrow$ $(\lambda_{21} - \lambda_{22}) P(x|\omega_2)P(\omega_2) \underset{R_1}{\overset{R_2}{\gtrless}} (\lambda_{12} - \lambda_{11}) P(x|\omega_1)P(\omega_1)$

$\rightarrow$ Likelihood $\rightarrow$ $\boxed{\dfrac{P(x|\omega_1)}{P(x|\omega_2)} \underset{R_2}{\overset{R_1}{\gtrless}} \dfrac{(\lambda_{21} - \lambda_{22})}{(\lambda_{12} - \lambda_{11})} \cdot \dfrac{P(\omega_2)}{P(\omega_1)}}$

b) Liklihood $\rightarrow$ $\dfrac{P(x|\omega_1)}{P(x|\omega_2)} \underset{R_2}{\overset{R_1}{\gtrless}} \dfrac{(\lambda_{21} - \lambda_{22})}{(\lambda_{12} - \lambda_{11})} \cdot \dfrac{P(\omega_2)}{P(\omega_1)} \rightarrow$

$$\frac{P(x|\omega_2)}{P(x|\omega_1)} \underset{R_1}{\overset{R_2}{\gtrless}} \frac{(\lambda_{12} - \lambda_{11})}{(\lambda_{21} - \lambda_{22})} \cdot \frac{P(\omega_1)}{P(\omega_2)} \rightarrow \frac{P(x|\omega_2)}{P(x|\omega_1)} \overset{R_1}{<} \underbrace{\frac{(\lambda_{12} - \lambda_{11})}{(\lambda_{21} - \lambda_{22})} \cdot \frac{P(\omega_1)}{P(\omega_2)}}_{\text{This upper baund}}$$

Question 3

Problem 3: $P(x|\omega_i) = \frac{1}{\pi C} \frac{1}{1+\left(\frac{x-a_i}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_i}{b}\right)}$

$$\underbrace{P(x/\omega_1)P(\omega_1)}_{g_1(x)} \underset{R_2}{\overset{R_1}{\gtrless}} \underbrace{P(x/\omega_2)P(\omega_2)}_{g_2(x)} \Rightarrow \frac{P(\omega_1)}{1+\left(\frac{x-a_1}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_1}{b}\right)} \underset{R_2}{\overset{R_1}{\gtrless}} \frac{P(\omega_2)}{1+\left(\frac{x-a_2}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_2}{b}\right)}$$

(#)

$g_1(x) \underset{R_2}{\overset{R_1}{\gtrless}} g_2(x) \Rightarrow$ if $g_1(x) > g_2(x)$ we will choose R_1

" $g_1(x) < g_2(x)$ we will choose R_2

if we believe that $P(\omega_1) = P(\omega_2)$ according to Theodoridis Textbook page 33 $\Rightarrow P(\omega_1) = P(\omega_2)$ and we are going to use that ∴

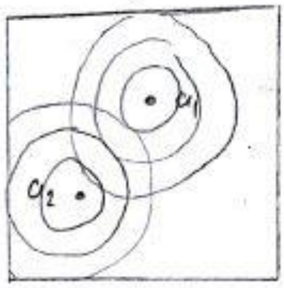
$$\left(\frac{x-a_1}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_1}{b}\right) \underset{R_1}{\overset{R_2}{\gtrless}} \left(\frac{x-a_2}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_2}{b}\right)$$ $\rightarrow$ using the matrix applications

$$\Rightarrow \left\|\left(\frac{x-a_1}{b}\right)^T \Sigma \left(\frac{x-a_1}{b}\right)\right\| \underset{R_1}{\overset{R_2}{\gtrless}} \left\|\left(\frac{x-a_2}{b}\right)^T \Sigma^{-1} \left(\frac{x-a_2}{b}\right)\right\| \longrightarrow$$

$$\left\|\frac{x-a_1}{\cancel{b}}\right\| \underset{R_1}{\overset{R_2}{\gtrless}} \left\|\frac{x-a_2}{\cancel{b}}\right\|$$ $\Rightarrow$ if the distance from $a_1 = \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix}$ is equal to distance from $a_2 = \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix}$

$$\|x-a_1\| \underset{R_1}{\overset{R_2}{\gtrless}} \|x-a_2\|$$

the distance is the comparator



for an amount of Σ → we can say that the base of the decision is the distance from the middle of $\underline{x}$ vector → we said that the b is constant

but lit we do it completly ✓

$$* \to \Sigma^{-1} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{bmatrix} \quad a_1 = \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} \quad a_2 = \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix} \quad x_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix} \quad x_2 = \begin{pmatrix} x_{21} \\ x_{22} \end{pmatrix}$$

$$\frac{P(\omega_1)}{1+\left(\frac{x_1-a_1}{b}\right)^T \Sigma^{-1} \left(\frac{x_1-a_1}{b}\right)} \underset{R_2}{\overset{R_1}{\gtrless}} \frac{P(\omega_2)}{1+\left(\frac{x_2-a_2}{b}\right)^T \Sigma^{-1} \left(\frac{x_2-a_2}{b}\right)}$$

$$\frac{P(\omega_1)}{1-\left(\frac{x_{11}-a_{11}}{b}, \frac{x_{12}-a_{12}}{b}\right)\begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{bmatrix}\begin{pmatrix} \frac{x_{11}-a_{11}}{b} \\ \frac{x_{12}-a_{12}}{b} \end{pmatrix}} \Rightarrow \left(\gamma_1\frac{(x_{11}-a_{11})}{b} + \gamma_3\frac{(x_{12}-a_{12})}{b}, \gamma_2\frac{(x_{11}-a_{11})}{b} + \gamma_4\frac{(x_{12}-a_{12})}{b}\right)$$

$$\times \begin{pmatrix} \frac{x_{11}-a_{11}}{b} \\ \frac{x_{12}-a_{12}}{b} \end{pmatrix} = \frac{\gamma_1}{b^2}(x_{11}-a_{11})^2 + \frac{\gamma_3}{b^2}(x_{12}-a_{12})(x_{11}-a_{11}) + \frac{\gamma_2}{b^2}(x_{12}-a_{12})(x_{11}-a_{11}) + \frac{\gamma_4}{b^2}(x_{12}-a_{12})^2$$

$$\frac{\gamma_1}{b^2}(x_{11}-a_{11})^2 + \frac{\gamma_4}{b^2}(x_{12}-a_{12})^2 + \frac{(\gamma_3+\gamma_2)}{b^2}\left((x_{12}-a_{12})(x_{11}-a_{11})\right) \to$$

$$\frac{b^2 P(\omega_1)}{b^2 + \gamma_1(x_{11}-a_{11})^2 + \gamma_4(x_{12}-a_{12})^2 + (\gamma_3+\gamma_2)\left((x_{12}-a_{12})(x_{11}-a_{11})\right)} \underset{R_2}{\overset{R_1}{\gtrless}} \frac{b^2 P(\omega_2)}{b^2 + \gamma_1(x_{21}-a_{21})^2 + \gamma_4(x_{22}-a_{22})^2 +}$$

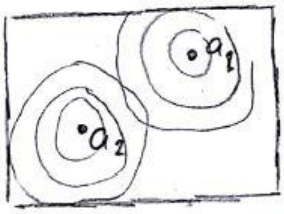
$$\overline{+(\gamma_3+\gamma_2)\left((x_{22}-a_{22})(x_{21}-a_{21})\right)}$$ ↳ In the next page

rest of Prob 3:

$$\frac{P(\omega_1)}{b^2+\gamma_1(x_{11}-a_{11})^2+\gamma_4(x_{12}-a_{12})^2+(\gamma_3+\gamma_2)\big((x_{12}-a_{12})(x_{11}-a_{11})\big)} \underset{R_2}{\overset{R_1}{\gtrless}} \frac{P(\omega_2)}{b^2+\gamma_1(x_{21}-a_{21})^2+\gamma_4(x_{22}-a_{22})}$$

$$\overline{(\gamma_3+\gamma_2)\big((x_{22}-a_{22})(x_{21}-a_{21})\big)}$$

so we showed that in witch case we are going to get the result in any condition we are going to have a result ether R_1 or R_2



→ This could be the view from the top

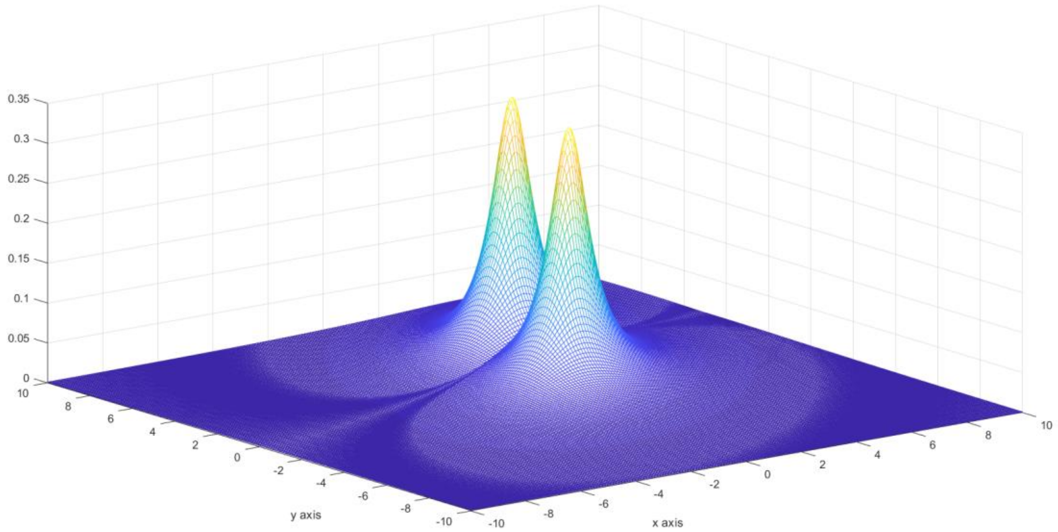


Figure 3

In Figure3 the $a_1 = (2,1)$, $a_2 = (3,5)$, $b = 1, \Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$

According to Figure 3 me can draw a decision graph like the surface below

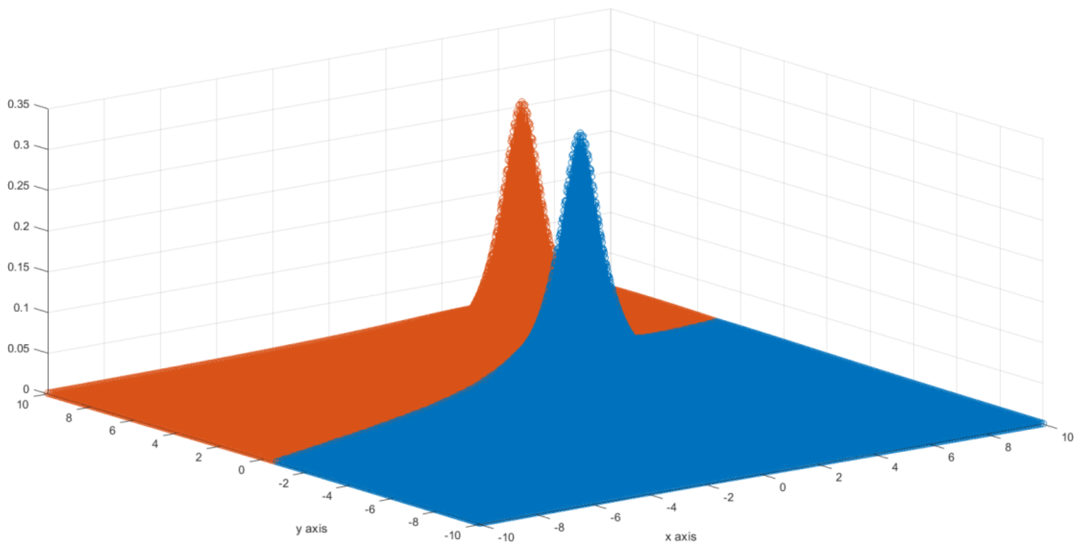


Figure 4

The Figure4 shows that in every location in the map we can make a decision with higher chance of success.

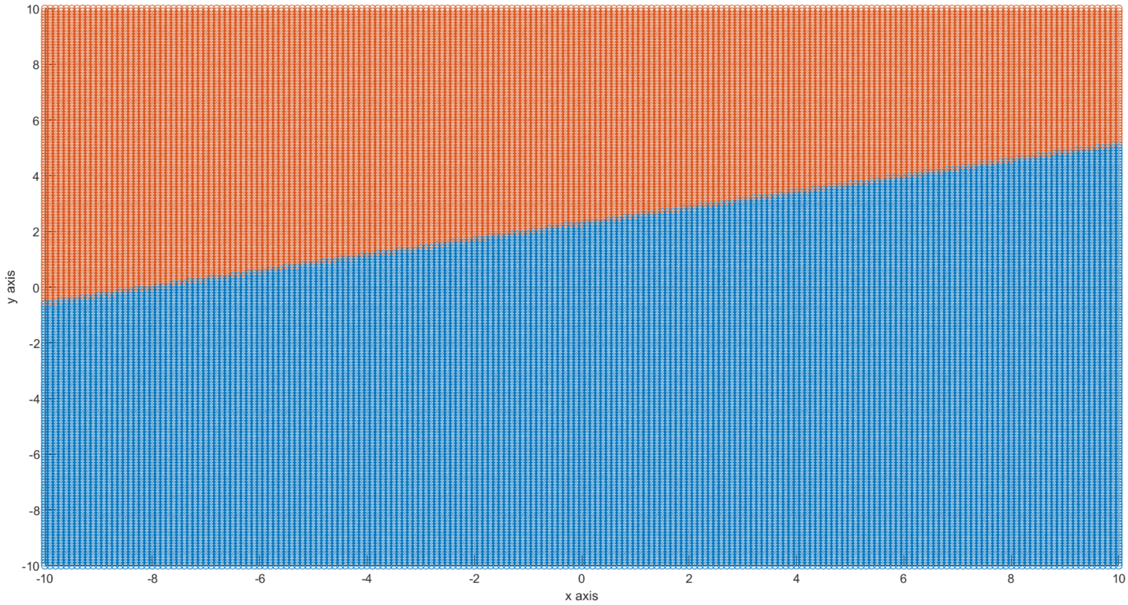


Figure 5

Figure 5 we can see from the top view that with a sample of (X, Y) the location that we choose has superior decision the blue region is for ω_1 and the red region is for region ω_2

Question 4

Problem 4: $P(x|\omega_i) = \begin{cases} \frac{x}{\sigma_i^2}\exp\left(\frac{-x^2}{2\sigma_i^2}\right) & x \geq 0 \\ 0 & x < 0 \end{cases}$ we have two class $i=1,2$

$$P(x|\omega_1) = \begin{cases} \frac{x}{\sigma_1^2}\exp\left(\frac{-x^2}{2\sigma_1^2}\right) & x \geq 0 \\ 0 & ow \end{cases} \quad P(x|$$

$$i^* = \arg\max_{1}^{2} \left(P(x|\omega_i)P(\omega_i)\right)$$

$$\underbrace{P(x|\omega_1)P(\omega_1)}_{g_1(x)} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} \underbrace{P(x|\omega_2)P(\omega_2)}_{g_2(x)} \rightarrow \text{Assume equal priors this means}$$

for $x>0$ ✓

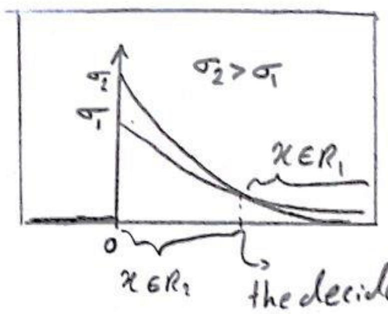
$$P(\omega_1) = P(\omega_2) \longrightarrow P(x|\omega_1) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} P(x|\omega_2) \rightarrow \frac{x}{\sigma_1^2}\exp\left(\frac{-x^2}{2\sigma_1^2}\right) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} \frac{x}{\sigma_2^2}\exp\left(\frac{-x^2}{2\sigma_2^2}\right)$$

$$\frac{\sigma_2^2}{\sigma_1^2} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} \exp\left(x^2\left(\frac{1}{2\sigma_1^2} - \frac{1}{2\sigma_2^2}\right)\right) \longrightarrow \ln \longrightarrow 2\ln\left(\frac{\sigma_2}{\sigma_1}\right) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} x^2\left(\frac{\sigma_2^2-\sigma_1^2}{2\sigma_2^2\sigma_1^2}\right) \rightarrow$$

$$\frac{4\sigma_2^2\sigma_1^2\left(\ln(\sigma_2)-\ln(\sigma_1)\right)}{(\sigma_2^2-\sigma_1^2)} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} x^2 \rightarrow 2\sigma_2\sigma_1\sqrt{\frac{(\ln(\sigma_2)-\ln(\sigma_1))}{(\sigma_2^2-\sigma_1^2)}} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} x$$

$$2\sigma_2\sigma_1\sqrt{\frac{\ln(\sigma_2)-\ln(\sigma_1)}{\sigma_2^2-\sigma_1^2}} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} x$$

so we can seethat x has a range
In this case we can decide
$x\in R_1$ or $x \in R_2$



the decide region $= 2\sigma_2\sigma_1\sqrt{\frac{\ln(\sigma_2)-\ln(\sigma_1)}{(\sigma_2^2-\sigma_1^2)}}$

Question 5

Problem 5:

$$P(x) = \frac{1}{\sqrt{2\pi}\ |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\underline{x}-\underline{\mu})\Sigma^{-1}(\underline{x}-\underline{\mu})\right)$$

a) we used the matlab software to calculate it

stare: $\mu_x = 1.333$ $\mu_y = 1.6111$ $\Sigma_{stor} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} 0.5556 & 0.1852 \\ 0.1852 & 0.9877 \end{bmatrix}$

date: $\mu_x = -0.0909$ $\mu_y = -0.1818$ $\Sigma_{date} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} 1.7677 & -0.0202 \\ -0.0202 & 0.5707 \end{bmatrix}$

$$P_{stor} = \frac{1}{\sqrt{2\pi} \times \sqrt{0.5144}} \exp\left(-\frac{1}{2}\left(\underline{x} - \begin{pmatrix} 1.33 \\ 1.61 \end{pmatrix}\right)^T \times \begin{pmatrix} 1.9200 & -0.36 \\ -0.36 & 1.08 \end{pmatrix} \times \left(\underline{x} - \begin{pmatrix} 1.33 \\ 1.61 \end{pmatrix}\right)\right)$$

$$P_{date} = \frac{1}{\sqrt{2\pi} \times \sqrt{1.0084}} \exp\left(-\frac{1}{2}\left(\underline{x} - \begin{pmatrix} -0.0909 \\ -0.1818 \end{pmatrix}\right)^T \times \begin{pmatrix} 0.5659 & 0.02 \\ 0.02 & 1.752 \end{pmatrix} \times \left(\underline{x} - \begin{pmatrix} -0.0909 \\ -0.1812 \end{pmatrix}\right)\right)$$

b) $P(x|\omega_1)P(\omega_1) \underset{x \in R_2}{\overset{x \in R_1}{\gtrless}} P(x|\omega_2)P(\omega_2) \longrightarrow P(\omega_1) = P(\omega_2) = 0.5 \rightarrow$

$P(x|\omega_1) \underset{x \in R_2}{\overset{x \in R_1}{\gtrless}} P(x|\omega_2)$ $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \checkmark \longrightarrow$ the calculation

$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \checkmark$

$$P_{star} = \frac{1}{\sqrt{2\pi}\times 0.7127} \exp\left(-\frac{1}{2}\begin{pmatrix} x_1 - 1.33 \\ y_1 - 1.61 \end{pmatrix}^T \times \begin{pmatrix} 1.92 & -0.36 \\ -0.36 & 1.08 \end{pmatrix} \times \begin{pmatrix} x_1 - 1.33 \\ y_1 - 1.61 \end{pmatrix}\right) \Rightarrow \text{using Matlab}$$

$$0.5598 \exp\left(-0.5 \times \left(\left(x_1 - \frac{133}{100}\right)\left(\frac{9y_1}{25} - \frac{48x_1}{25} + \frac{987}{500}\right) + \left(y - \frac{161}{100}\right)\left(\frac{9x_1}{25} - \frac{27y_1}{25} + \frac{63}{50}\right)\right)\right) = g(x)$$

$$P_{date} = \frac{1}{\sqrt{2\pi}\times 1.0042} \exp\left(-\frac{1}{2}\begin{pmatrix} x_2 + 0.0909 \\ y_2 + 0.1818 \end{pmatrix}^T \times \begin{pmatrix} 0.5659 & 0.02 \\ 0.02 & 1.752 \end{pmatrix} \times \begin{pmatrix} x_2 + 0.0909 \\ y_2 + 0.1818 \end{pmatrix}\right) \Rightarrow \text{matlab}$$

$$0.3972 \exp\left(-\frac{1}{2}\left(\left(y_2 + \frac{909}{5000}\right)\left(\frac{x_2}{50} + \frac{219y_2}{125} + \frac{800829}{2500000}\right) + \left(x_2 + \frac{909}{10000}\right)\left(\frac{5659x}{10000} + \frac{y}{50} + \frac{5567631}{100000000}\right)\right)\right) = g'(x)$$

So we can use this to decide if $g(x) \geq g'(x)$ $x \in star$, $x \in date$ ✓ this is a classifier that we designed ✓

now we need to check if the classifier works

sample: location → [2,3] → star → reality ✓
↳ star → classify

[1,3] → star → reality ✓
↳ star → classify

[2.5,2] → star → reality ✓
↳ star → classify

[1.5,2] → star → reality ✓
↳ star → classify

[0.5,2] → star → reality ✓
↳ star → classify

[2,1] → star → reality ✓
↳ star → classify

[1,1] → star → reality ✓
↳ star → classify

[0,0.5] → star → reality wrong
↳ date → classify α

[1.5,0] → star → reality wrong
↳ date → classify α

[-1.5,1] → date → reality ✓
↳ date → reality

$[1.5, 0.5] \to$ dote $\to$ reality, star $\to$ classify — wrong α

$[0.5, 0.5] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[-0.5, 0.5] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[-1.5, 0] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[1.5, -0.5] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[0.5, -0.5] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[1, -1] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[-1, -1] \to$ dote $\to$ reality, dote $\to$ classify ✓

$[-2, -1] \to$ dote $\to$ reality, dote $\to$ classify ✓

3 wronge, 16 corret $\to$ all 19 $\to \frac{3}{19} = 15.8\%$ wronge

about $\boxed{15.8\%}$ our classifier may be mis classified ✓

---

c) The drawing has been done in matlab

$$\begin{array}{l} r_1 = \lambda_{21} P(x|\omega_2)P(\omega_2) \\ r_2 = \lambda_{12} P(x|\omega_1)P(\omega_1) \end{array} \to r_1 \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} r_2 \to P(\omega_1) = P(\omega_2) \checkmark$$

$$\begin{array}{l} \lambda_{21} = a \\ \lambda_{12} = 2a \end{array} \to P(x|\omega_2) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} P(x|\omega_1)\times 2 \to$$

$$\frac{1}{\sqrt{2\pi}\,|\Sigma_2|^{\frac{1}{2}}} exp\left(-\tfrac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)\right) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} \frac{2}{\sqrt{2\pi}\,|\Sigma_1|^{\frac{1}{2}}} exp\left(-\tfrac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)$$

Intis case we decide the sample is for R_1 or R_2 ✓

---

d) The equation is like part (c) this part has been it matlab as well

$$\begin{array}{l} g_1 = P(x|\omega_1)P(\omega_1) \\ g_2 = P(x|\omega_2)P(\omega_2) \end{array} \to g_1 \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} g_2 \to P(x|\omega_1)P(\omega_1) \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} P(x|\omega_2)P(\omega_2)$$

$$P(x|\omega_1)\times\frac{1}{3} \underset{x\in R_2}{\overset{x\in R_1}{\gtrless}} P(x|\omega_2)\times\frac{2}{3}$$

Prob 5

$P(x|w_1) \underset{x \in R_2}{\overset{x \in R_1}{\gtrless}} P(x|w_2) \times 2$ ✓ $\Rightarrow$ we repeat part b:

$[2,3]$ → star → reality; ↳ star → classify ✓ $[1,3]$ → star → reality; ↳ star → classify ✓

$[2.5,2]$ → star → reality; ↳ star → classify ✓ $[1.5,2]$ → star → reality; ↳ star → classify ✓

$[0.5,2]$ → star → reality; ↳ star → classify ✓ $[2,1]$ → star → reality; ↳ star → classify ✓

$[1,1]$ → star → reality; ↳ star → classify ✓ $[0,0.5]$ → star → reality; ↳ dote → classify α wrong

$[1.5,0]$ → star → reality; ↳ star → classify ✓ $[-1.5,1]$ → dote → reality; ↳ dote → classify ✓

$[1.5,0.5]$ → dote → reality; ↳ star → classify wrong α $[0.5,0.5]$ → dote → reality; ↳ star → classify wrong α

$[-0.5,0.5]$ → dote → reality; ↳ dote → classify ✓ $[-1.5,0]$ → dote → reality; ↳ dote → classify ✓

$[1.5,-0.5]$ → dote → reality; ↳ dote → classify ✓ $[0.5,-0.5]$ → dote → reality; ↳ dote → classify ✓

$[1,-1]$ → dote → reality; ↳ dote → classify ✓ $[-1,-1]$ → dote → reality; ↳ dote → classify ✓

$[-2,-1]$ → dote → reality; ↳ dote → classify ✓

3 wrong answers we have $\boxed{\frac{3}{19}}$ ✓

$\boxed{15.8\%}$ our classifier misclassifies

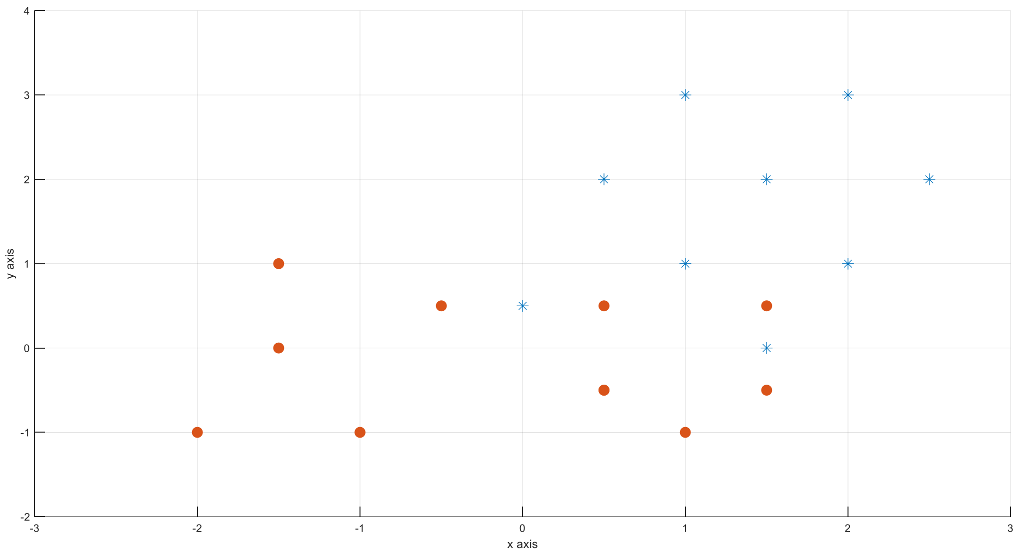


Figure 6

In Figure 6 the samples are showed in the figure and what we are going to find the distribution of the samples.

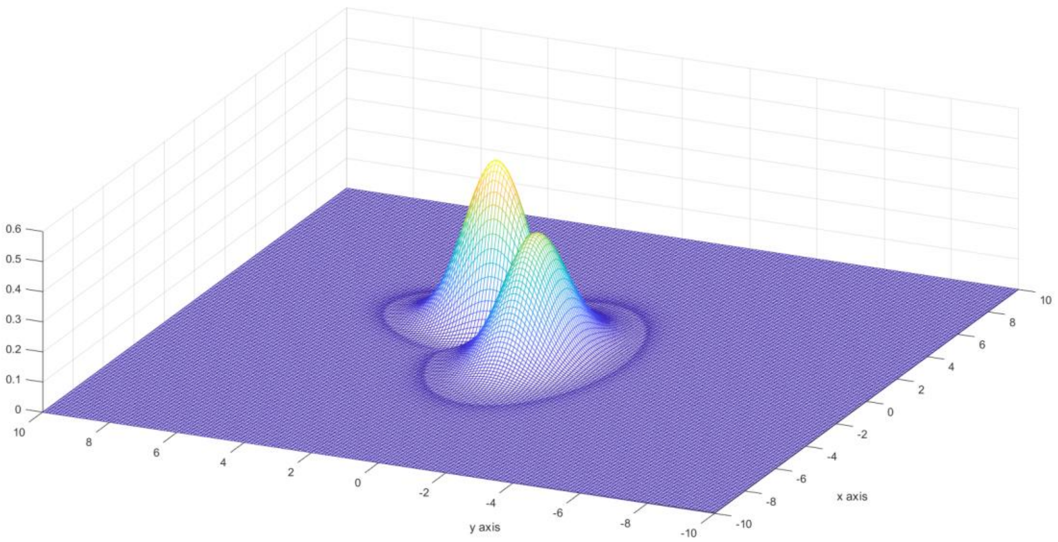


Figure 7

Figure 7 is the 3D view of the Gaussian two dimensional distribution.

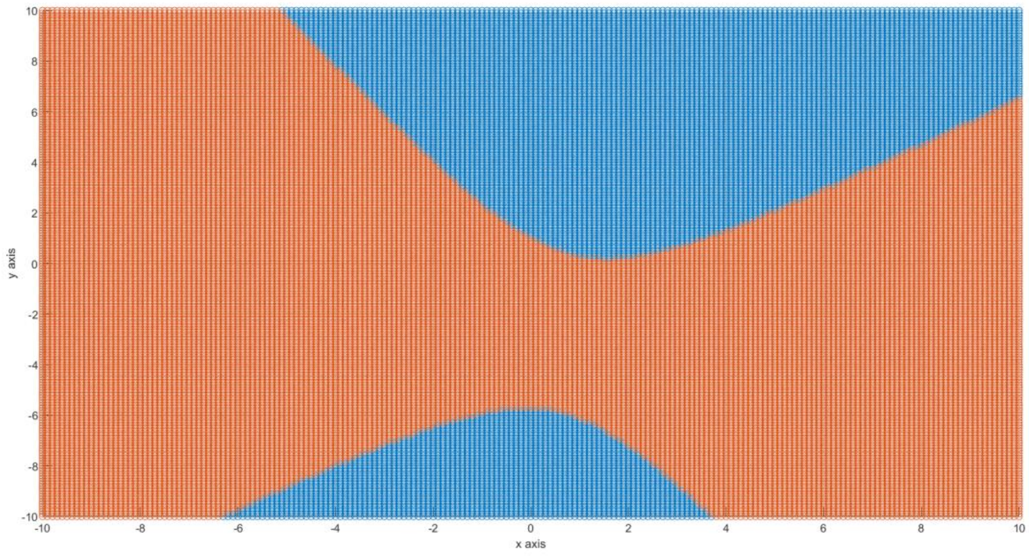


Figure 8

Figure 8 is the top view for the previous distribution we can see the decision boundary of it. (the figure from plot is 3D you should turn it to see this image correctly)

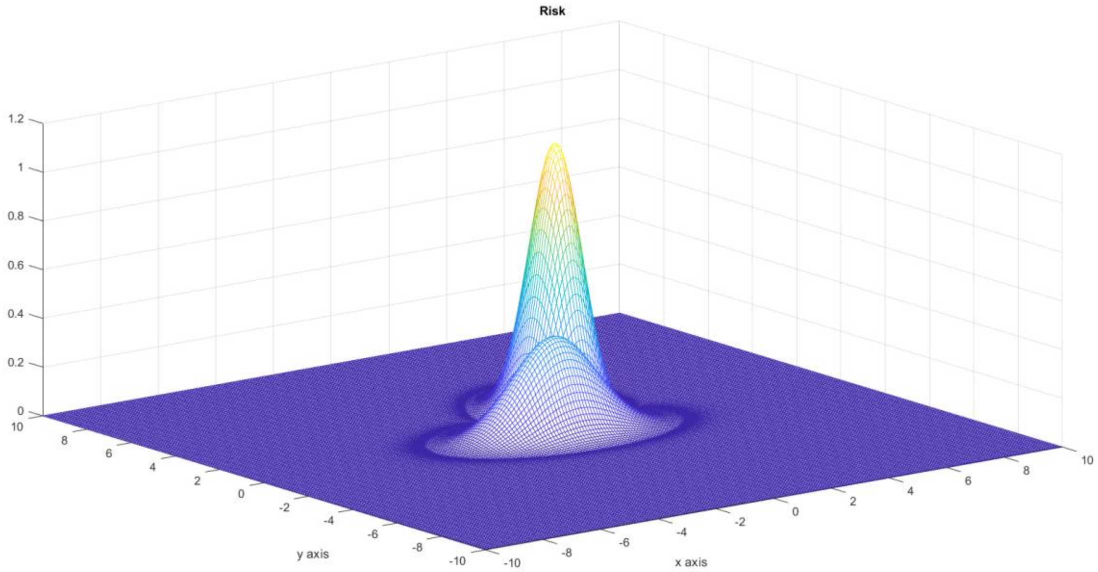


Figure 9

In Figure 9 we can see the rick diagram.

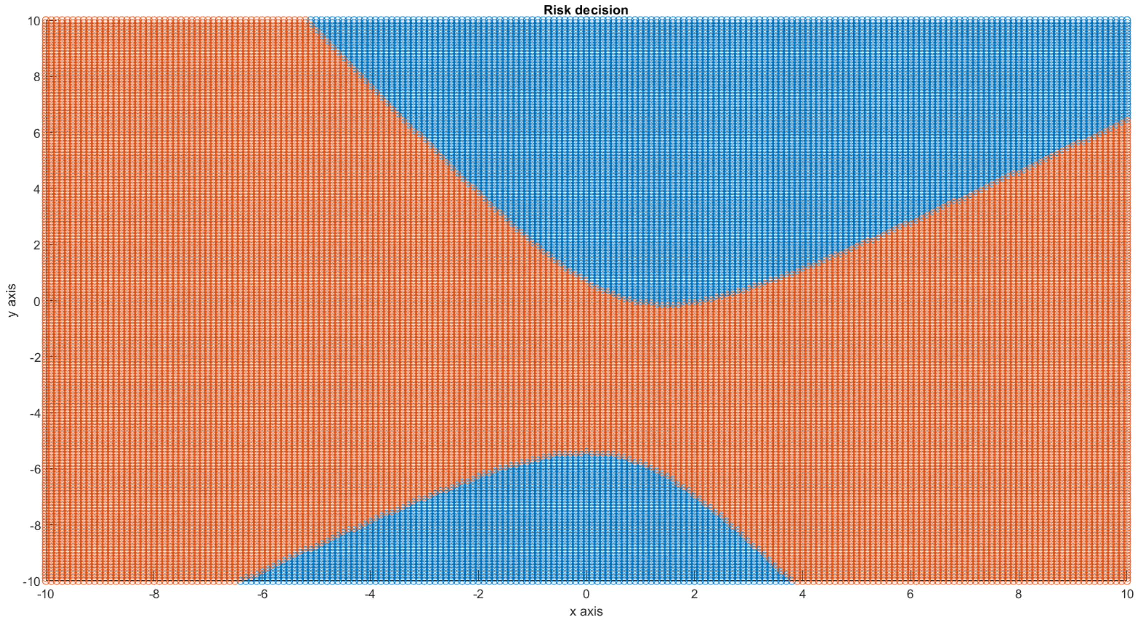


Figure 10

In Figure 10 we can see the decision region that shows in every (X, Y) we can decide either our decision to be ω_1 or ω_2.

If we calculate the decision boundary when $P(\omega_1) = \frac{1}{3}$ and $P(\omega_2) = \frac{2}{3}$ it's like the when we found the region when we had ricks but the difference is the amplitude of the probability

Question 6

Problem 6: $x = (x_1, x_2, \ldots, x_d)^t$ $\quad P_{ij} = Prob(x_i = 1 / \omega_j)$

a) It means the probability of the i'th parameter from x vector to become 1 if we know that we are in the ω_j space and ω_j has accored in that case. and we know the we have c space for ω_j and the x vector has d Parameter

b) $\rightarrow$ we know for deciding $g_j(x) = f(\underline{x} / \omega_j) P(\omega_j)$ $\xrightarrow{\text{another way for showing}}$

$\xrightarrow{\ln}$ $g_i(\underline{x}) = \ln(f(\underline{x}/\omega_i)) + \ln(P(\omega_j)) \rightarrow \underline{x} = (x_1, \ldots, x_d)^T \rightarrow$ Independent

$f(\underline{x}/\omega_j) = \prod_{i=1}^{d} f(x_i / \omega_j) \rightarrow \ln(f(\underline{x}/\omega_j)) = \sum_{i=1}^{d} \ln(f(x_i)/\omega_j)) \rightarrow$

if $x_i = 1 \rightarrow \ln(P_{ij})$
if $x_i = 0 \rightarrow \ln(1 - P_{ij})$ $\Bigg\}\rightarrow$ it depend on the $\underline{x}$ vector to be the number of 1 or 0 that it has

$g_j(\underline{x}) = \ln(f(\underline{x}/\omega_j)) + \ln(P(\omega_j)) \rightarrow \ln(f(\underline{x}/\omega_j)) = \sum^{\text{number of 1}} \ln(P_{ij}) + \sum^{\text{number of 0}} \ln(1 - P_{ij})$

$\sum_{i=1}^{d} x_i \ln\left(\frac{P_{ij}}{1 - P_{ij}}\right) + \sum_{i=1}^{d} \ln(1 - P_{ij}) \rightarrow$ if $x_i = 1 \rightarrow$ output $\ln(P_{ij})$

if $x_i = 0 \rightarrow$ output $\ln(1 - P_{ij})$

$$g_j(\underline{x}) = \sum_{i=1}^{d} x_i \ln\left(\frac{P_{ij}}{1 - P_{ij}}\right) + \sum_{i=1}^{d} \ln(1 - P_{ij}) + P(\omega_j)$$

Question 7

Problem 7:

$\mathcal{E}_j$: is the probility to get wrong answere for ω_j spac

$$\mathcal{E}_j = \sum_{i \neq j}^{c} P\{x \in R_i | \omega_j\} = 1 - P\{x \in R_j | \omega_j\}$$

$$\mathcal{E}_1 = 1 - P\{x \in R_1 | \omega_1\} = P\{x \in R_2 | \omega_1\}$$

$$\mathcal{E}_2 = 1 - P\{x \in R_2 | \omega_2\} = P\{x \in R_1 | \omega_2\}$$

$$r = \sum_{j=1}^{c} \sum_{i=1}^{c} \lambda_{ij} P\{x \in R_j \text{ and } x \in \omega_i\} = \lambda_{11} P\{x \in R_1 \text{ and } x \in \omega_1\} +$$

$$\lambda_{21} P\{x \in R_1 \text{ and } x \in \omega_2\} + \lambda_{22} P\{x \in R_2 \text{ and } x \in \omega_2\} + \lambda_{12} P\{x \in R_2 \text{ and } x \in R_1\}$$

$$P\{x \in R_i \text{ and } x \in \omega_j\} = P\{x \in R_i | \omega_j\} P(\omega_j)$$

$P\{x \in R_1 | \omega_1\} = 1 - \mathcal{E}_1$

$P\{x \in R_2 | \omega_1\} = \mathcal{E}_1$

$P\{x \in R_2 | \omega_2\} = 1 - \mathcal{E}_2$

$P\{x \in R_1 | \omega_2\} = \mathcal{E}_2$

$$r = \lambda_{11} P\{x \in R_1 | \omega_1\} P(\omega_1) + \lambda_{21} P\{x \in R_1 | \omega_2\} P(\omega_2)$$

$$+ \lambda_{22} P\{x \in R_2 | \omega_2\} P(\omega_2) + \lambda_{12} P\{x \in R_2 | \omega_1\} P(\omega_1)$$

$$= \lambda_{11} (1 - \mathcal{E}_1) P(\omega_1) + \lambda_{21} \mathcal{E}_2 P(\omega_2) + \lambda_{22} (1 - \mathcal{E}_2) P(\omega_2) + \lambda_{12} \mathcal{E}_1 P(\omega_1)$$

$$r = P(\omega_1) \lambda_{11} + P(\omega_2) \lambda_{22} + P(\omega_1)(\lambda_{12} - \lambda_{11}) \mathcal{E}_1 + P(\omega_2)(\lambda_{21} - \lambda_{22}) \mathcal{E}_2$$

Question 8

Problem 8: a) $Risk = \sum_{i=1}^{C+1}\sum_{j=1}^{C} \lambda_{ij} P\{x|\omega_j \wedge \omega_j\} = \sum_{i=1}^{C+1}\sum_{j=1}^{C} \lambda_{ij} P\{x|\omega_j\} P(\omega_j)$

$\sum_{i=1}^{C}\sum_{j=1}^{C} \lambda_{ij} P\{x|\omega_j\} P\{\omega_j\} + \sum_{j=1}^{C} \lambda_{C+1,j} P\{x|\omega_j\} P(\omega_j) \Rightarrow \underbrace{\sum_{\substack{i=1 \\ i\neq j}}^{C}\sum_{j=1}^{C} \lambda_s P\{x|\omega_j\} P(\omega_j)}_{(1-P(x|\omega_i))}$

$+\sum_{j=1}^{C} \lambda_r P(x|\omega_j\} P\{\omega_j\} = \sum_{i=1}^{C} \lambda_s (1-P(\omega_i|\underline{x})) + \lambda_r P\{\omega_{C+1}|\underline{x}\}$

$$i^* = \arg\min_{i=1}^{C+1} g_i(\underline{x}) \longrightarrow g(x) = \sum_{i=1}^{C} \lambda_s (1-P(\omega_j|\underline{x})\} + \lambda_r P\{\omega_{C+1}|\underline{x}\}$$

$$g_i(x) = \begin{cases} \lambda_s (1-P(\omega_i|\underline{x}\}) & i \neq C+1 \\ \lambda_r \{P(\omega_{C+1}|\underline{x}\}) & i = C+1 \end{cases}$$

the ith sentence

if $\lambda_s P\{\omega_{C+1}|\underline{x}\} > \lambda_r (1-P\{\omega_i|\underline{x}\}) \quad \forall i \quad i < C+1$ ✓

$\arg\min_{i=1}^{C+1} g_i(x) = \arg\max_{i=1}^{C} (P(\omega_i|x)) \longrightarrow$ In the critical condition

$P(\omega_{j+1}|\underline{x}) \geq P\{\omega_i|\underline{x}\}$ & $\lambda_r P\{\omega_{C+1}|x\} \geq \lambda_s (1-P\{\omega_i|\underline{x}\})$

$\frac{\lambda_r}{\lambda_s} P\{\omega_{C+1}|\underline{x}\} \geq (1-P(\omega_i|\underline{x}\}) \longrightarrow P\{\omega_i|\underline{x}\} \geq 1-\frac{\lambda_r}{\lambda_s} P\{\omega_{C+1}|\underline{x}\} \longrightarrow$

$\boxed{P\{\omega_{C+1}|\underline{x}\} \leq 1} \longrightarrow \boxed{P(\omega_i|x\} \geq 1-\frac{\lambda_r}{\lambda_s}}$

Second way:

$R(\omega_i|\underline{x}) = \sum_{j=1}^{C} \lambda_{ij} P(\omega_j|\underline{x}) = \sum_{\substack{j=1 \\ i\neq j}}^{C} \lambda_s P(\omega_j|\underline{x}) = \lambda_s (1-P\{\omega_i|\underline{x}\})$

critical condition $\leadsto \lambda_r \underbrace{P(\omega_{C+1}|\underline{x})}_{as \leq 1} = \lambda_s (1-P(\omega_i|\underline{x})\} \longrightarrow$

$\lambda_r \geq \lambda_s (1-P(\omega_i|\underline{x})) \longrightarrow \frac{\lambda_r}{\lambda_s} \geq 1-P(\omega_i|\underline{x}) \longrightarrow \boxed{P(\omega_i|x) \geq 1-\frac{\lambda_r}{\lambda_s}}$

if $\lambda_r = 0 \leadsto$ it means that there is no threat against rejecting so it's better to reject any way and there is no risk in rejecting.

if $\lambda_r \geq \lambda_s$ It means the risk of rejecting is quit higher then the error so we must never rejeting

In this case we should alway choose and never reject

b) $g(x) = \begin{cases} f(x|\omega_i)P(\omega_i) & i=1,\dots,c \\ \frac{\lambda_s-\lambda_r}{\lambda_s}\sum_{j=1}^{c} f_x(x|\omega_j)P(\omega_j) & i=c+1 \end{cases}$

$$g_i(\underline{x}) \geq g_i(\underline{x}) \to f(\omega_j|\underline{x}) \geq f(\omega_i|x) \to f(\omega|\underline{x})P(\underline{x}) = f(\underline{x}|\omega)P(\omega)$$

$$\frac{f(\underline{x}|\omega_j)P(\omega_j)}{P(\underline{x})} \geq \frac{f(\underline{x}|\omega_i)P(\omega_i)}{P(\underline{x})} \to f(\underline{x}|\omega_j)P(\omega_j) \geq f(\underline{x}|\omega_i)P(\omega_i)$$

from part a $\to$ $P\{\omega_j|\underline{x}\} \geq 1-\frac{\lambda_r}{\lambda_s} \xrightarrow{\times P(\underline{x})} P(\omega_j|\underline{x})P(\underline{x}) \geq (1-\frac{\lambda_r}{\lambda_s})P(\underline{x})$ ($i=c+1$)

$$P(\underline{x}|\omega_j)P(\omega_j) \geq (1-\frac{\lambda_r}{\lambda_s})P(x) \to P(x) = \sum_{j=1}^{c} P(x|\omega_j)P(\omega_j)$$

$$P(\underline{x}|\omega_j)P(\omega_j) \geq (1-\frac{\lambda_r}{\lambda_s})\sum_{j=1}^{c} P(x|\omega_j)P(\omega_j) \to i=c+1$$

$$\Rightarrow g_i(x) = \begin{cases} f(\underline{x}|\omega_i)P(\omega_i) & i=1,2,\dots,c \\ (1-\frac{\lambda_r}{\lambda_s})\sum_{j=1}^{c} f(x|\omega_j)P(\omega_j) & i=c+1 \end{cases}$$

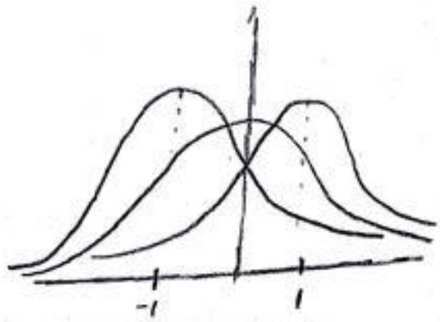
$P(\omega_1)=P(\omega_2)=\frac{1}{2}$ $\quad f_x(x|\omega_1) \sim N(1,1)$ $\quad f_x(x|\omega_2) \sim N(-1,1)$

$$g_1(x) = f(x|\omega_1)P(\omega_1) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{(x-1)^2}{2}\right)} \times \frac{1}{2} = \frac{1}{2\sqrt{2\pi}} e^{-\left(\frac{(x-1)^2}{2}\right)} \quad i=1$$

$$g_2(x) = f(x|\omega_2)P(\omega_2) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{(x-1)^2}{2}\right)} \times \frac{1}{2} = \frac{1}{2\sqrt{2\pi}} e^{-\left(\frac{(x+1)^2}{2}\right)} \quad i=2$$

$$i=3 \to g_3(x) = (1-\frac{1}{4})\sum_{i=1}^{2} f(x|\omega_i)P(\omega_i) = \frac{3}{4} \times \frac{1}{2\sqrt{2\pi}}\left(e^{-\frac{(x-1)^2}{2}} + e^{-\frac{(x+1)^2}{2}}\right) ✓$$

How does $\frac{\lambda_r}{\lambda_s}$ change it starts at $\frac{\lambda_r}{\lambda_s}$ so → It means that rejecting has no risk and cost It means thatthe g_{c+1} case is bigger the the other g_i witch means it's the best case → therfor in this case it's better to reject while the $\frac{\lambda_r}{\lambda_s}$ increases to 1 → in some cases it would be better to not take a risk and in some case $g_i(x)$ might be bigger then $g_{c+1}(x)$ → if $\frac{\lambda_r}{\lambda_s}$ s1 in this case g_{c+1} are all smaller than $g_i(x)$ so we should take any risk at all



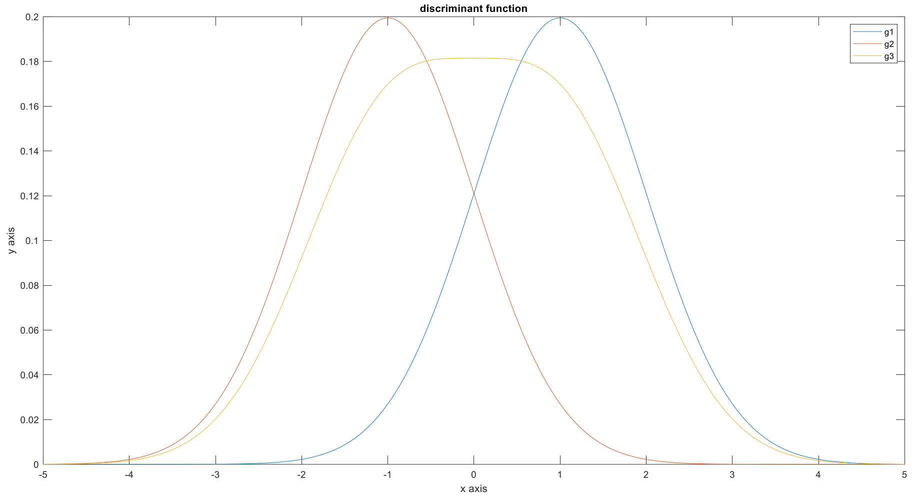


Figure 11

The Figure 11 shows all the $g_i(x)$ from $g_1(x)\ to\ g_3(x)$ and the calculation are mentioned in the previous page

Question 9

Problem 9:

a) $Risk = \sum_{i=1}^{c} \int_{R_i} d\underline{x} \sum_{j=1}^{c} \lambda_{ij} P(\omega_j) f(x|\omega_j) \longrightarrow \lambda = \begin{pmatrix} \lambda_{11} & \lambda_{21} \\ \lambda_{12} & \lambda_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$Risk = \int_{R_1} \lambda_{11} P(\omega_1) f(x|\omega_1) d\underline{x} + \int_{R_1} \lambda_{12} P(\omega_2) f(x|\omega_2) d\underline{x} + \int_{R_2} \lambda_{21} P(\omega_1) f(x|\omega_1) d\underline{x}$

$+ \int_{R_2} \lambda_{22} P(\omega_2) f(x|\omega_2) d\underline{x} \longrightarrow \left.\begin{matrix} \lambda_{11} = 0 \\ \lambda_{22} = 0 \end{matrix}\right\} \Longrightarrow Risk = \int_{R_1} \lambda_{12} P(\omega_2) f(x|\omega_2) d\underline{x} + \int_{R_2} \lambda_{21} P(\omega_1) f(x|\omega_1) d\underline{x}$

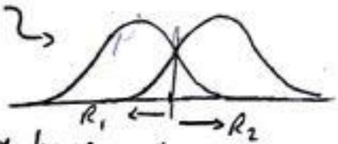
$\lambda_{12} = \lambda_{21} \rightarrow \int_{R_1} P(\omega_2) f(x|\omega_2) d\underline{x} + \int_{R_2} P(\omega_1) f(x|\omega_1) d\underline{x}$

I know $P(\omega_1) + P(\omega_2) = 1 \rightarrow$

$P(\omega_2) \int_{R_1} f(x|\omega_2) d\underline{x} + (1 - P(\omega_2)) \int_{R_2} f(x|\omega_1) d\underline{x}$ to minimize the error we should $\longrightarrow$

$P(\omega_2) \underbrace{\int_{R_1} f(x|\omega_2) d\underline{x}}_{P(x|\omega_2)} \underset{x \in R_1}{\overset{x \in R_2}{\gtrless}} P(\omega_1) \underbrace{\int_{R_2} f(x|\omega_1) d\underline{x}}_{P(x|\omega_1)} \longrightarrow P(\omega_2) P(x|\omega_2) \underset{x \in R_1}{\overset{x \in R_2}{\gtrless}} P(\omega_1) P(x|\omega_1)$

if $P(\omega_1) = P(\omega_2)$ $\rightarrow$ witch mean they have similar possiblty then we are having



$\longrightarrow \int_{R_2} P(x|\omega_1) dx = \int_{R_1} P(x|\omega_2) dx$

if they have a same distribution ✓

b) no if the two distribotion where not the or the chance for $P(\omega_1) \neq P(\omega_2)$ $\rightarrow$ we couldn't get the same result so if we don't have the conditions from the past part statmet would not satisfy the desigion regions.

Question 10

Problem 9:

a) $$\text{Risk} = \sum_{i=1}^{c} \int_{R_i} d\underline{x} \sum_{j=1}^{c} \lambda_{ij} P(w_j) f(x|w_j) \longrightarrow \lambda : \begin{pmatrix} \lambda_{11} & \lambda_{21} \\ \lambda_{12} & \lambda_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Risk} = \int_{R_1} \lambda_{11} P(w_1) f(x|w_1) d\underline{x} + \int_{R_1} \lambda_{12} P(w_2) f(x|w_2) d\underline{x} + \int_{R_2} \lambda_{21} P(w_1) f(x|w_1) d\underline{x}$$

$$+ \int_{R_2} \lambda_{22} P(w_2) f(x|w_2) d\underline{x} \longrightarrow \left.\begin{matrix} \lambda_{11} = 0 \\ \lambda_{22} = 0 \end{matrix}\right\} \Longrightarrow \text{Risk} = \int_{R_1} \lambda_{12} P(w_2) f(x|w_2) d\underline{x} + \int_{R_2} \lambda_{21} P(w_1) f(x|w_1) d\underline{x}$$

$$\lambda_{12} = \lambda_{21} \longrightarrow \int_{R_1} P(w_2) f(x|w_2) d\underline{x} + \int_{R_2} P(w_1) f(x|w_1) d\underline{x}$$

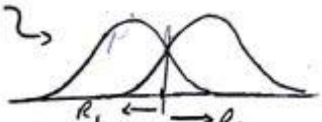
I know $P(w_1) + P(w_2) = 1 \longrightarrow$

$$P(w_2) \int_{R_1} f(x|w_2) d\underline{x} + (1 - P(w_2)) \int_{R_2} f(x|w_1) d\underline{x}$$

to minimize the error we should $\longrightarrow$

$$P(w_2) \underbrace{\int_{R_1} f(x|w_2) d\underline{x}}_{P(x|w_2)} \underset{x \in R_1}{\overset{x \in R_2}{\gtrless}} P(w_1) \underbrace{\int_{R_2} f(x|w_1) d\underline{x}}_{P(x|w_1)} \longrightarrow P(w_2) P(x|w_2) \underset{x \in R_1}{\overset{x \in R_2}{\gtrless}} P(w_1) P(x|w_1)$$

if $P(w_1) = P(w_2)$ $\longrightarrow$ witch mean they have similar possiblity then we are having



$$\longrightarrow \int_{R_2} P(x|w_1) dx = \int_{R_1} P(x|w_2) dx$$

if they have a same distribution ✓

b) no if the two distribution where not the or the chance for $P(w_1) \neq P(w_2)$ $\longrightarrow$ we couldn't get the same result so if we don't have the conditions from the past part statmet would not satisfy the desigion regions.

Reference

Pattern Classification,Duda, 2nd ed

Pattern Recognition, Theodoridis, 4th ed

Pattern Recognition and Machine Learning, Bishop