Ch4: Automatic Controllers

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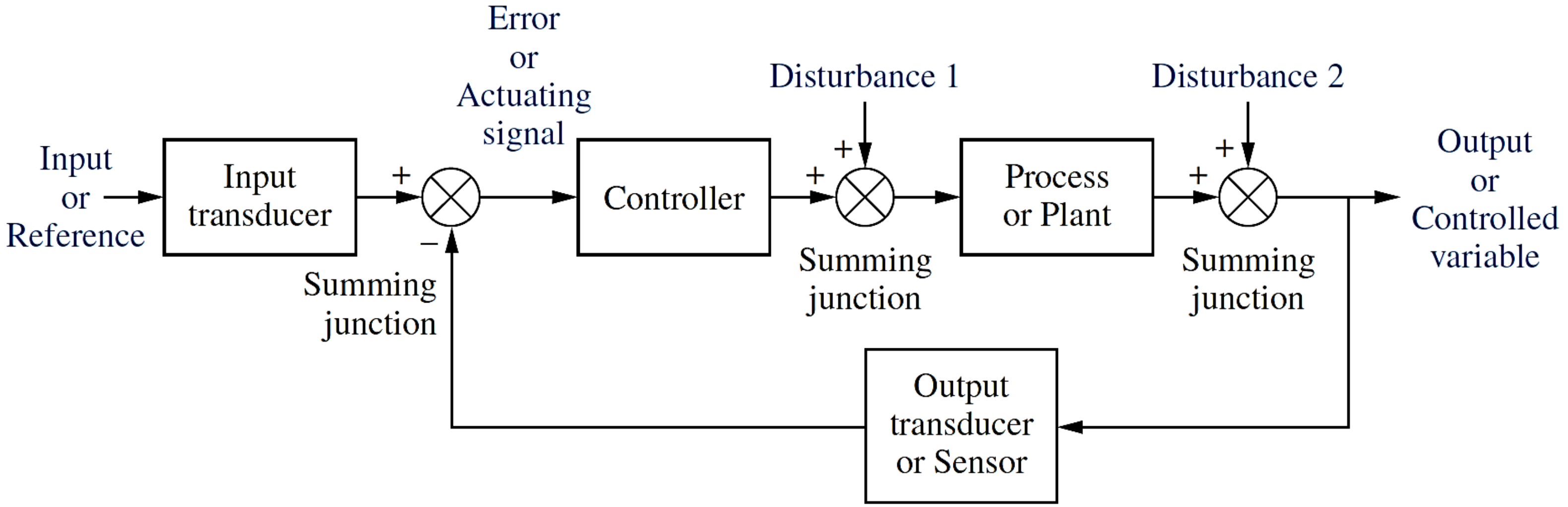
Introduction

Automatic Controllers

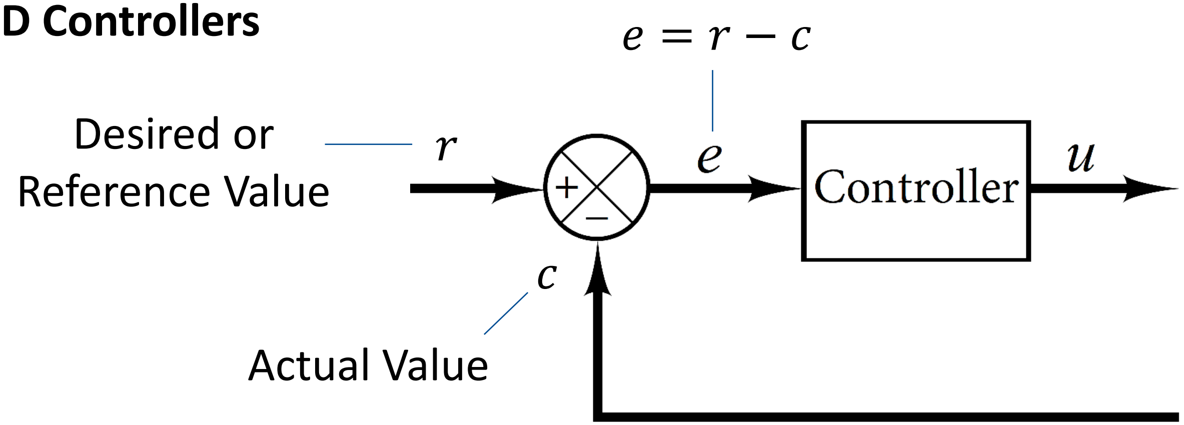
An **Automatic Controller** compares the actual value of the plant output with the reference input, determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value with a desired transient response.



Classifications of Common Controllers

Most controllers may be classified according to their **control actions** (u) as:

1. **On-Off Controller (Two-Position or Binary Controller)**
2. **Proportional (P) Controller**
3. **Integral (I) Controller**
4. **Derivative (D) Controller**
5. **PD, PI, and PID Controllers**

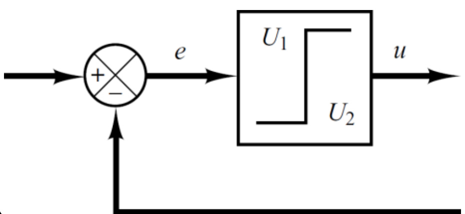


On-Off Controller

On-Off Controller (**Two-Position** or **Binary** Controller): It has only two possible values (usually on and off) at its output $u(t)$, depending on the input $e(t)$ to the controller.

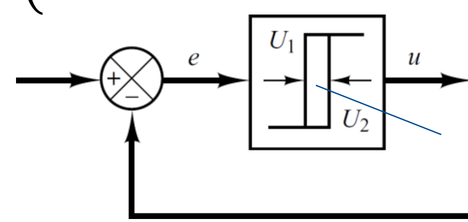
- This control system is relatively simple and inexpensive and is used very widely.
- Output oscillation between two limits is a typical response characteristic of this control system.

$$u(t)=\begin{cases} U_1 & e(t)>0 \\ U_2 & e(t)<0 \end{cases}$$

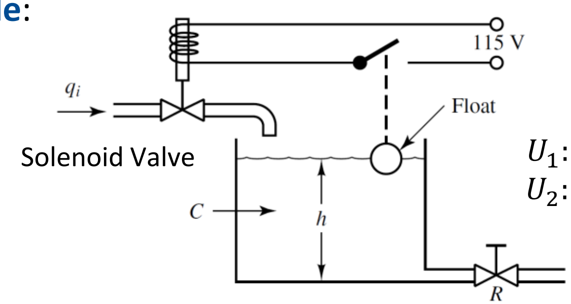


To prevent too-frequent operation of the on–off controller and increase useful life of the system:

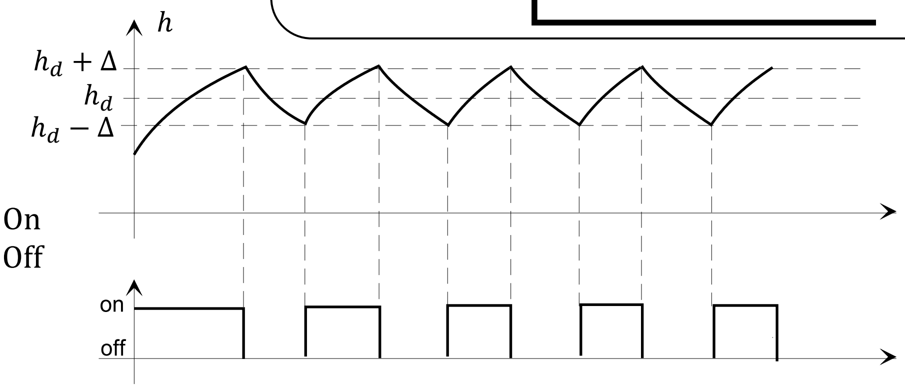
$$u(t)=\begin{cases} U_1 & e(t)>\Delta \\ \text{Keep latest state} & -\Delta\le e(t)\le\Delta \\ U_2 & e(t)<-\Delta \end{cases}$$



Example:



U_1: On
U_2: Off



PID Controllers

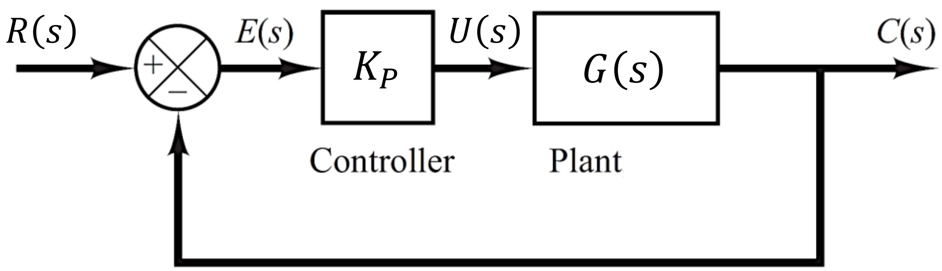
Proportional (P) Controller

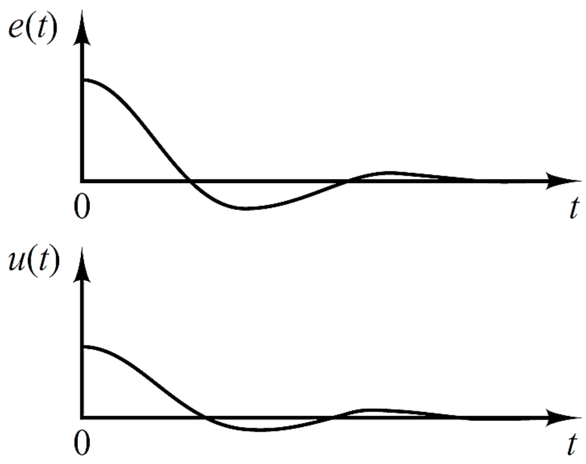
Proportional (P) Controller: Output $u(t)$ is **proportional** to its input $e(t)$.

$$u(t) = K_P e(t) \longrightarrow U(s) = K_P E(s)$$

proportional gain

This controller is essentially an **amplifier** with an adjustable gain.



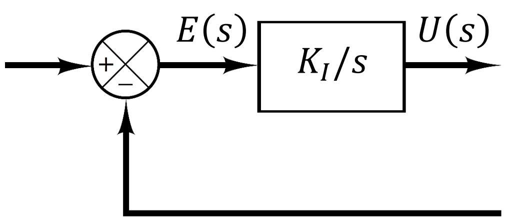


Integral (I) Controller

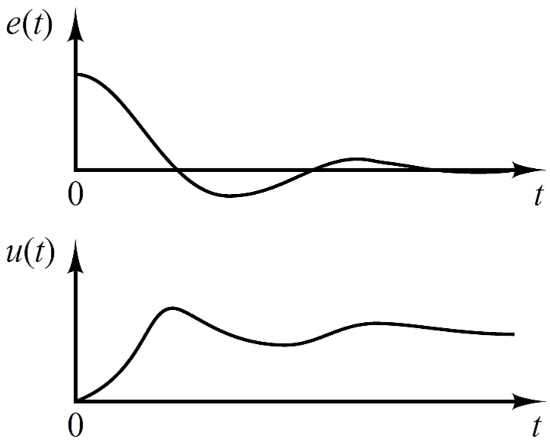
Integral (I) Controller: Output $u(t)$ is **proportional** to the **integral** of its input $e(t)$.

$$u(t) = K_I \int_0^t e(\tau)d\tau \longrightarrow U(s) = \frac{K_I}{s}E(s)$$

integral gain



In this controller, the control action $u(t)$ at any instant is the area under the error signal curve up to that instant. Therefore, the control action is based on the **history** of the system error.



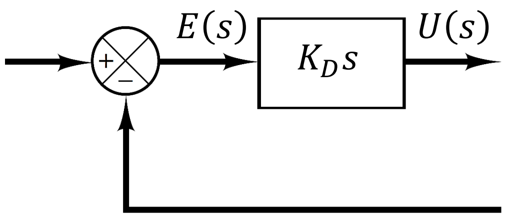
Note: The control action $u(t)$ can have a nonzero value when the error signal $e(t)$ is zero, but this is impossible in a Proportional controller. Thus, constant **disturbances** can be canceled with zero error.

Derivative (D) Controller

Derivative (D) Controller: Output $u(t)$ is **proportional** to the **derivative** of its input $e(t)$.

$$u(t) = K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t} \longrightarrow U(s) = K_D s E(s)$$

derivative gain



- Derivative control is essentially **anticipatory**, measures the instantaneous error, and predicts the large overshoot ahead of time.
- This controller tends to **increase** the **stability** and **sensitivity** of the system. However, it also tends to **amplify noise**.
- This controller does not affect the steady-state error directly, but it adds damping to the system and by increasing the gains the steady-state may be improved.
- It is always used in combination with Proportional (P) or Proportional-plus-Integral (PI) control action, i.e., PD or PID.

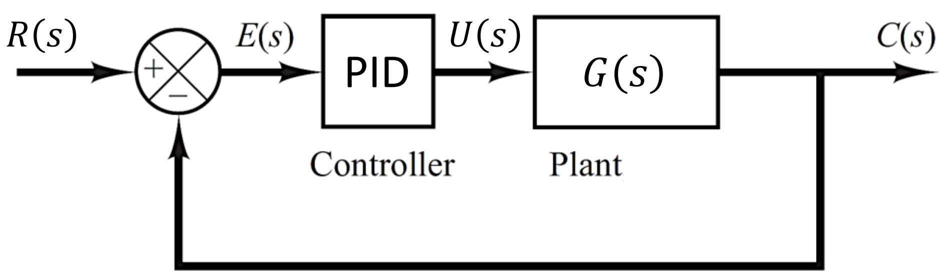
PD, PI, and PID Controllers

PD, PI, and PID Controllers: These controllers are **combinations** of proportional (P), derivative (D), and integral (I) controllers.

For example, PID Controller:

$$u(t) = K_P e(t) + K_I \int e(t)\,dt + K_D \frac{de(t)}{dt} \longrightarrow \begin{cases} U(s) = \left(K_P + \dfrac{K_I}{s} + K_D s\right) E(s) \\ \text{or} \\ U(s) = K_P \left(1 + \dfrac{1}{T_I s} + T_D s\right) E(s) \end{cases}$$

T_I: integral time T_D: derivative time



- This controller has the advantages of each of the three individual control actions.
- If the system is second order or higher the use of PID controller is required if we wish to have arbitrary transient-response behavior and acceptable steady-state behavior.

Parameters of Step Response of Underdamped Second-Order Systems

The parameters defined for the step input response of underdamped second-order systems:

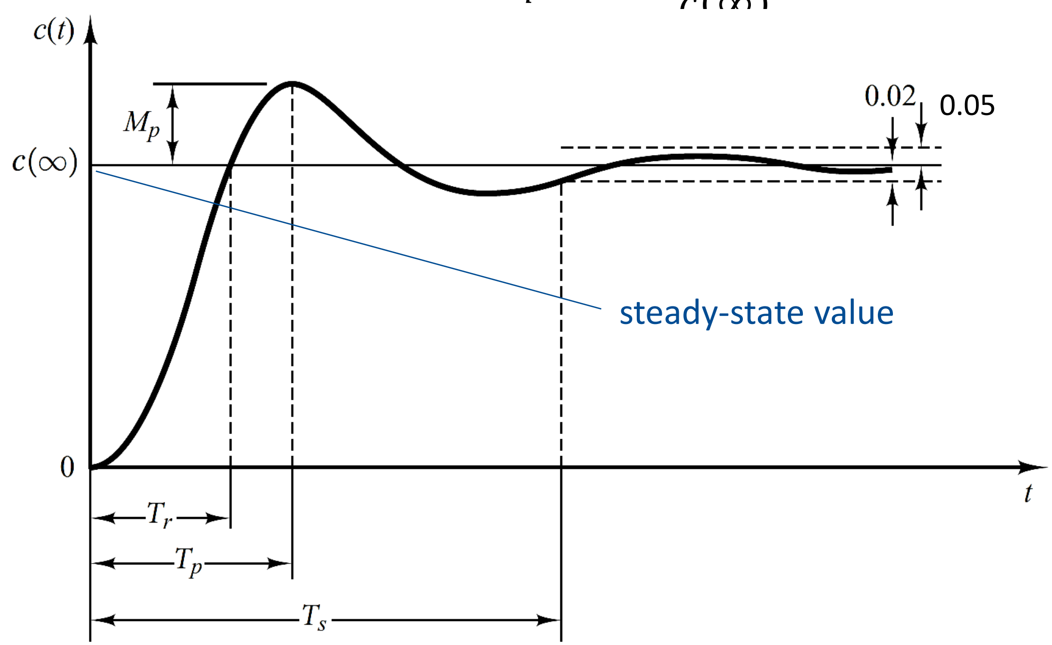
1. Peak Time T_p: The time required for the response to reach the **first** (or maximum) peak.

2. Maximum Overshoot M_p: The percentage of the steady-state value that the response overshoots the steady-state value at the peak time T_p.

$$M_p = \frac{c(T_p) - c(\infty)}{c(\infty)} \times 100$$

3. Settling Time T_s: The time required for the response to **reach and stay** within 2% (or 5%) of the steady-state value.

4. Rise Time T_r: The time required for the response to go from 0% to 100% (or 10% to 90% or 5% to 95%) of the steady-state value.

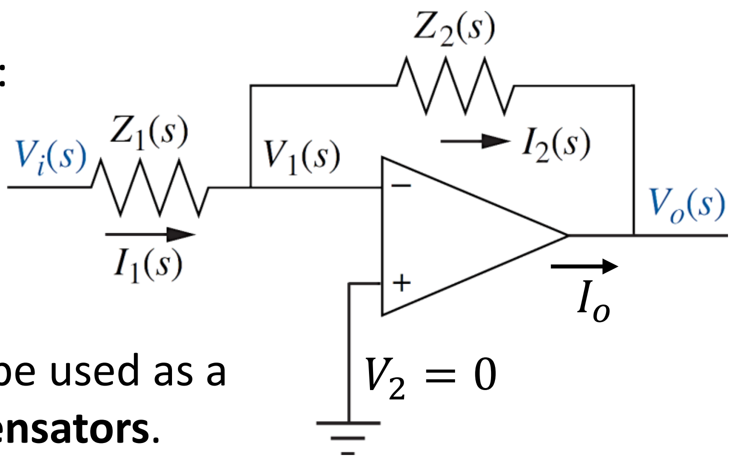


Physical Realization of Controllers/Compensators

Active-Circuit Realization

Transfer Function of an **inverting operational amplifier**:

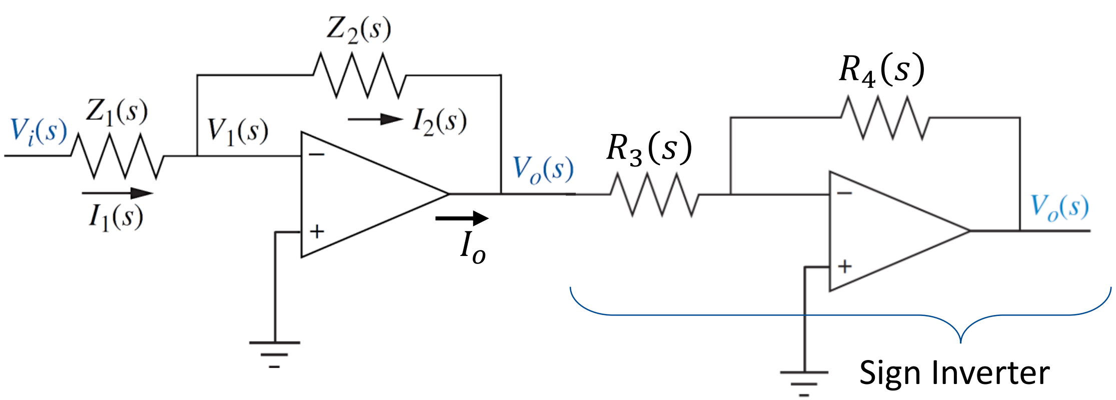
$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



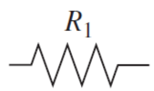
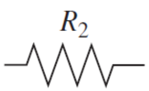
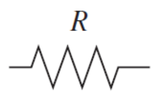
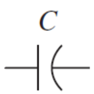
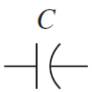
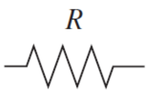
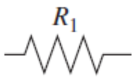
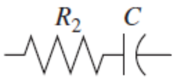
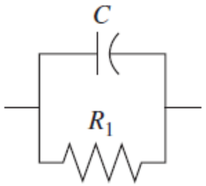
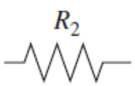
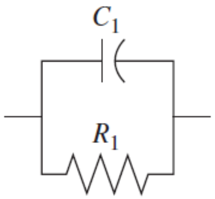
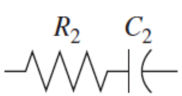
By wisely choosing of $Z_1(s)$ and $Z_2(s)$, this circuit can be used as a building block to implement the **controllers** and **compensators**.

Note: This transfer function contains a minus sign. Another circuit can be connected to either the input or the output of the circuit to act as a **sign inverter** as well as a gain adjuster.

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)}\frac{R_4(s)}{R_3(s)}$$



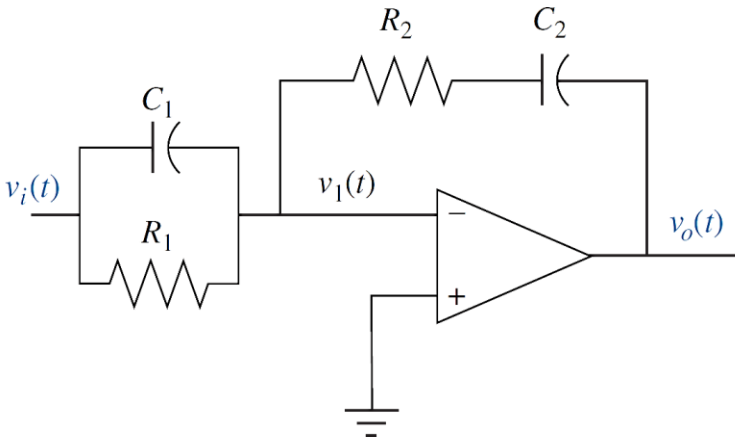
Active-Circuit Realization: PID Controllers

Function	$Z_1(s)$	$Z_2(s)$	$-\frac{Z_2(s)}{Z_1(s)}$
P Controller	 R_1	 R_2	$-\frac{R_2}{R_1}$
I Controller	 R	 C	$-\frac{\frac{1}{RC}}{s}$
D Controller	 C	 R	$-RCs$
PI controller	 R_1	 R_2 C	$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	 C R_1	 R_2	$-R_2C\left(s+\frac{1}{R_1C}\right)$
PID controller	 C_1 R_1	 R_2 C_2	$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$

Example

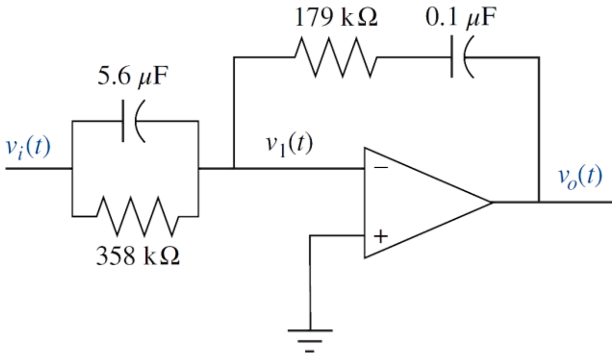
Implement the following PID controller.

$$G_c(s) = \frac{(s + 55.92)(s + 0.5)}{s}$$

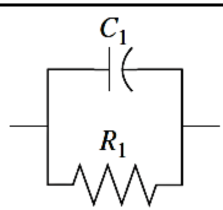
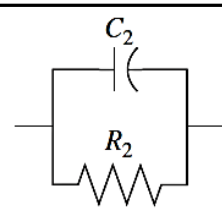
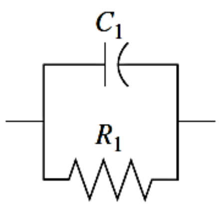
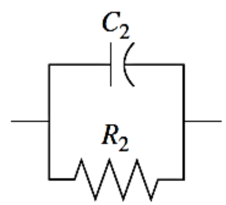


Answer:

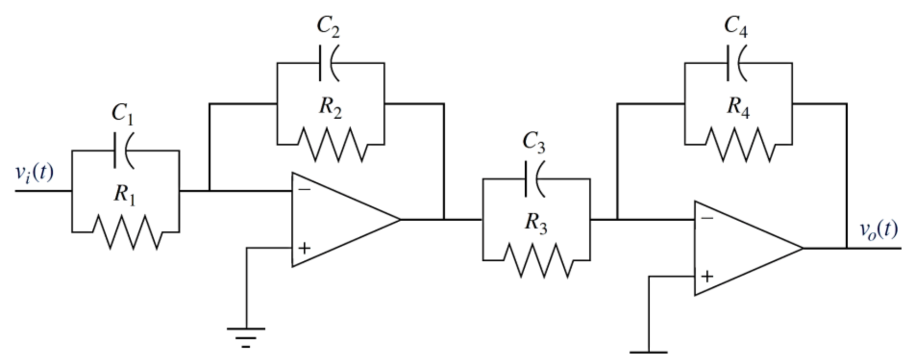
Since there are four unknowns and three equations, we arbitrarily select a practical value for one of the elements, e.g., $C_2 = 0.1$ μF, and find the remaining values.



Active-Circuit Realization: Lag/Lead Compensators

Function	$Z_1(s)$	$Z_2(s)$	$-\dfrac{Z_2(s)}{Z_1(s)}$
Lag compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)}{\left(s+\dfrac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation			$-\dfrac{C_1}{C_2}\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)}{\left(s+\dfrac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

A lag-lead compensator can be formed by cascading the lag compensator with the lead compensator.

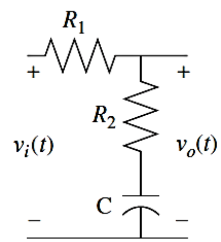
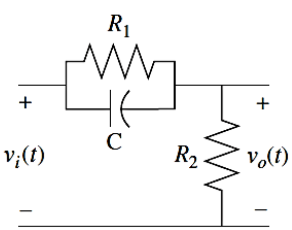
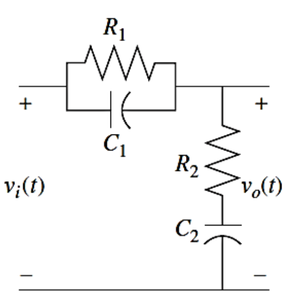


Lag compensator
$R_2C_2 > R_1C_1$

Lead compensator
$R_3C_3 > R_4C_4$

Passive-Circuit Realization: Lag/Lead Compensators

Lag, lead, and lag-lead compensators can also be implemented with passive networks.

Function	Network	Transfer function, $\dfrac{V_o(s)}{V_i(s)}$
Lag compensation		$\dfrac{R_2}{R_1+R_2}\dfrac{s+\dfrac{1}{R_2C}}{s+\dfrac{1}{(R_1+R_2)C}}$
Lead compensation		$\dfrac{s+\dfrac{1}{R_1C}}{s+\dfrac{1}{R_1C}+\dfrac{1}{R_2C}}$
Lag-lead compensation		$\dfrac{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}{s^2+\left(\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_2}+\dfrac{1}{R_2C_1}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$