Ch5: Capacitors and Inductors

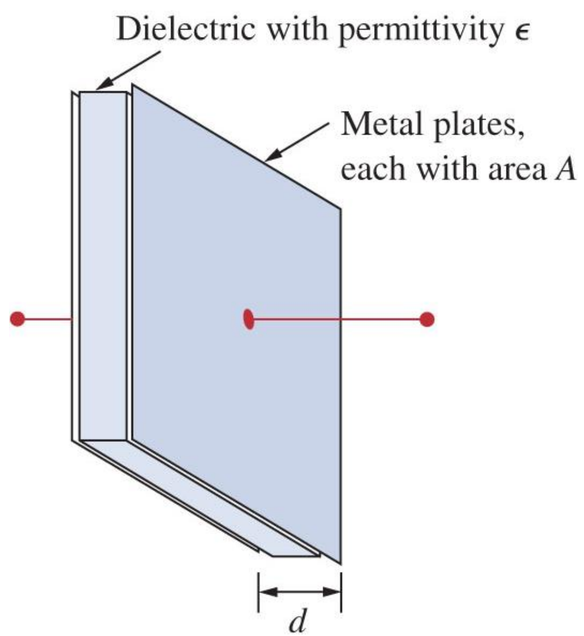
Capacitors

Capacitors

A **Capacitor** is a **passive** element that stores energy in its **electric field**.

It consists of two conducting plates separated by an insulator (or dielectric):

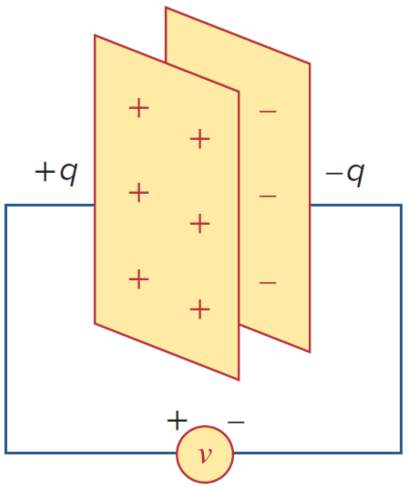
- The plates are typically aluminum foil.
- The dielectric is often air, ceramic, paper, plastic, or mica.



Capacitors

When a voltage source v is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge $-q$ on the other.

- The charges will be equal in magnitude.
- The capacitor is said to store the electric charge.
- The amount of charge stored is proportional to the applied voltage v:



$$q = Cv$$

C is known as the **capacitance** of the capacitor.

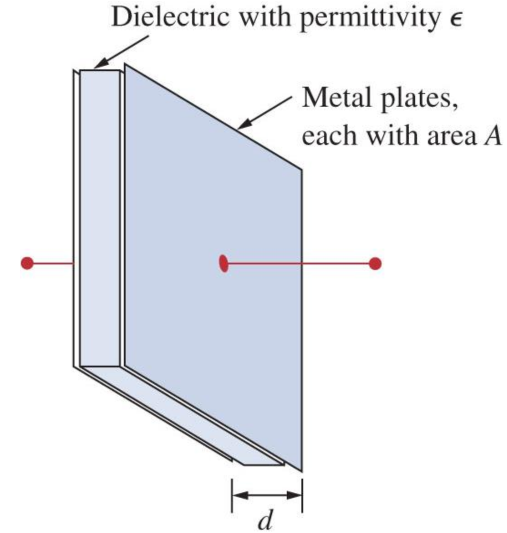
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

- 1 Farad = 1 Coulomb/volt.
- Most capacitors are rated in picofarad (pF) and microfarad (μF).

Capacitance

Capacitance depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor the capacitance is given by:

$$C = \frac{\varepsilon A}{d}$$



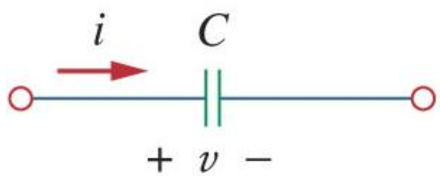
A: the surface area of each plate,
d: the distance between the plates,
ϵ: the **permittivity** of the dielectric material between the plates.

Thus, in general, three factors determine the value of the capacitance:

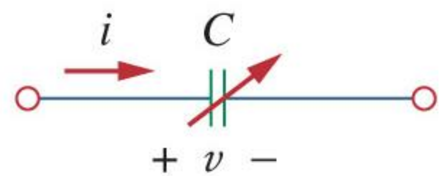
1. The surface **area** of the plates—the larger the area, the greater the capacitance.
2. The **spacing** between the plates—the smaller the spacing, the greater the capacitance.
3. The **permittivity** of the material—the higher the permittivity, the greater the capacitance.

Types of Capacitors

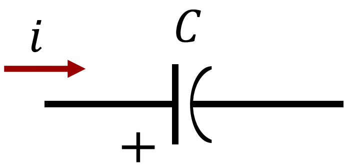
Capacitors are described by the dielectric material they are made of and by whether they are of fixed or variable type.



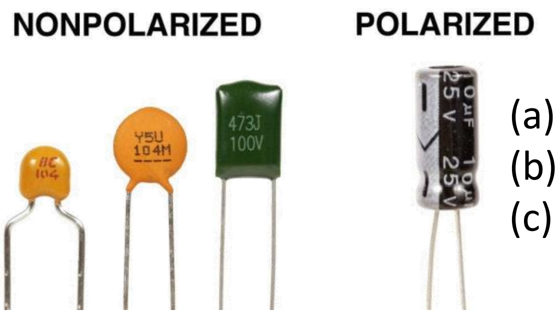
fixed capacitor



variable capacitor



fixed polarized capacitor



(a) (b) (c)

(a) ceramic capacitor,
(b) polyester capacitor,
(c) electrolytic capacitor

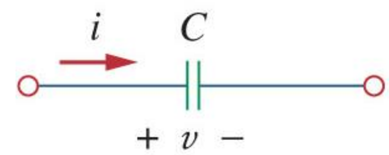
Variable air caps can be adjusted by turning a shaft attached to a set of moveable plates.

Note: According to the passive sign convention, if the direction of i and polarity of v are as shown, the capacitor is being **charged**, otherwise, the capacitor is **discharging**.

Current-Voltage Relationship

By taking the first derivative with respect to time of the formula for the charge stored in a capacitor (i.e., $q = Cv$), we can find the current voltage relationship of the capacitor.

$$q = Cv \quad \Rightarrow \quad \frac{dq}{dt} = C\frac{dv}{dt} \quad \Rightarrow \quad \boxed{i = C\frac{dv}{dt}}$$



The voltage-current relation of the capacitor can be also obtained by integration:

$$i = C\frac{dv}{dt} \quad \Rightarrow \quad \int_{t_0}^{t} dv = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \quad \Rightarrow \quad \boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + \underbrace{v(t_0)}_{\text{The voltage across the capacitor at time } t_0}}$$

- This shows the capacitor's voltage depends on the history of the capacitor's current. Hence, the capacitor has **memory**, which is often exploited in circuits.

Power and Energy

The instantaneous **power** delivered to the capacitor:

$$p = vi = Cv\frac{dv}{dt}$$

The **energy** stored in the electric field that exists between the plates of the capacitor:

$$w = \int p(t)dt = C\int vdv = \frac{1}{2}Cv^2$$

Since $q = Cv$:

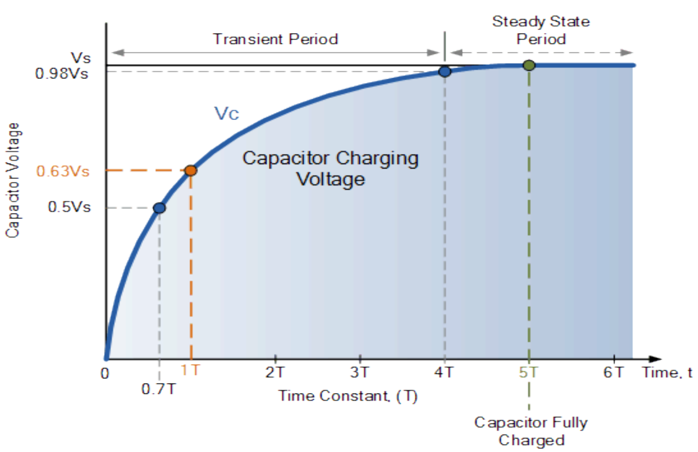
$$w = \frac{1}{2}Cv^2 = \frac{1}{2}\frac{q^2}{C}$$

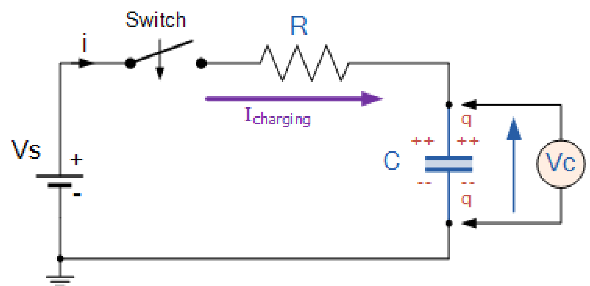
- The word capacitor is derived from this element's capacity to store energy in an electric field.

Properties of Ideal Capacitors

- If a battery (constant DC voltage) is connected across a capacitor (with no charge), the capacitor charges.

- The **voltage across a capacitor cannot change instantaneously** because it is the integral of the current. It takes time to increase or decrease the voltage across a capacitor. Thus, capacitors can be used for timing purposes in electrical circuits using a simple RC circuit, which is a resistor and capacitor in series.



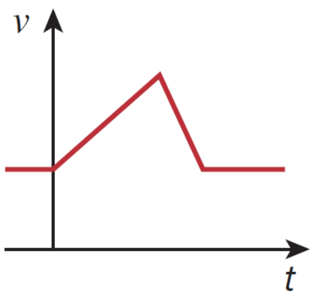


$$i = C\frac{dv}{dt}, \quad v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

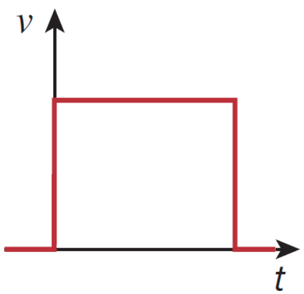
- When the voltage across a capacitor is not changing with time (i.e., constant DC voltage), the current through the capacitor is zero ($i = 0$ or open circuit). For a capacitor to carry current, its voltage must vary with time.

Properties of Ideal Capacitors

- The voltage on a capacitor must be continuous and cannot change abruptly. An abrupt/discontinuous change in voltage requires an infinite current, which is physically impossible!
- If the voltage on the capacitor does not equal the applied voltage, charge will flow, and the voltage will finally reach the applied voltage.
- Conversely, the current through a capacitor can change instantaneously/abruptly.



This voltage across a capacitor is allowed.



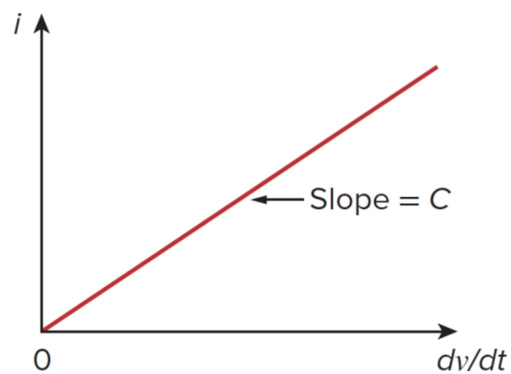
This voltage across a capacitor is not allowable.

- The ideal capacitor does not dissipate energy.

Properties of Ideal Capacitors

- Capacitors whose capacitance is independent of voltage and satisfy this equation are said to be **linear**. We will assume linear capacitors in this course.

$$i = C\frac{dv}{dt}$$



Examples

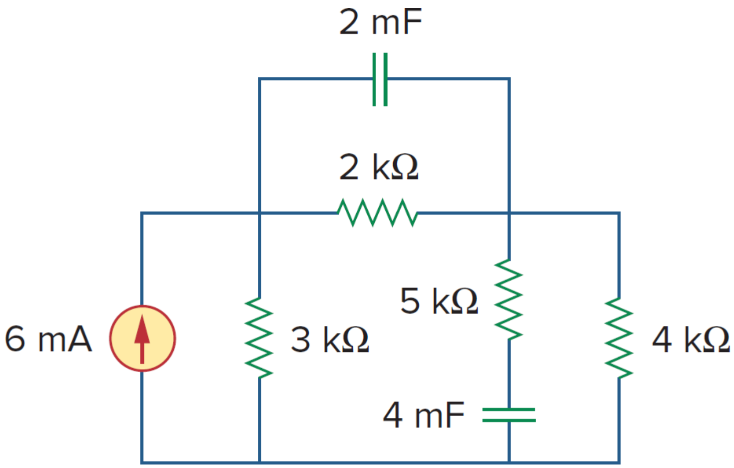
1. Calculate the charge stored on a 3-pF capacitor with 20 V across it. Find the energy stored in the capacitor.

2. The voltage across a 5-μF capacitor is $v(t) = 10\cos 6000t$ V. Calculate the current through it.

3. Determine the voltage across a 2-μF capacitor if the current through it is $i(t) = 6e^{-3000t}$ mA. Assume that the initial capacitor voltage is zero.

Example

Obtain the energy stored in each capacitor under DC conditions.



Series and Parallel Capacitors

Parallel Capacitors

We learned with resistors that applying the equivalent series and parallel combinations can simply many circuits.

Consider obtaining the equivalent capacitor C_{eq} of N capacitors in parallel. Note that the capacitors have the same voltage v across them. Applying KCL

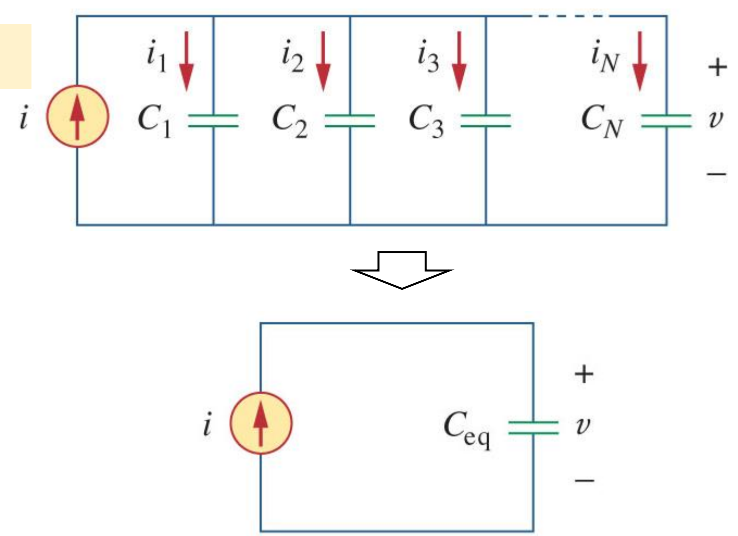
$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$q_{\text{total}} = q_1 + q_2 + \cdots + q_N$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \cdots + C_N \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3 + \cdots + C_N) \frac{dv}{dt} = C_{\text{eq}} \frac{dv}{dt}$$

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots + C_N$$



- The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.
- We observe that capacitors in parallel combine in the same manner as resistors in series.

Series Capacitors

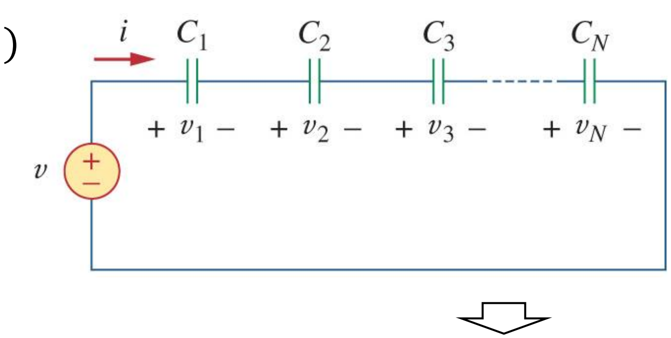
Consider obtaining the equivalent capacitor C_{eq} of N capacitors in series. Note that the same current i flows (and consequently the same charge) through the capacitors. Applying KVL to the loop: $v = v_1 + v_2 + v_3 + \cdots + v_N$

$$q_1 = q_2 = q_3 = \cdots = q_N$$

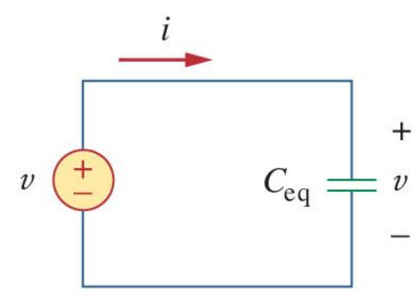
$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0) + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)$$

$$= \frac{1}{C_{\text{eq}}}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$



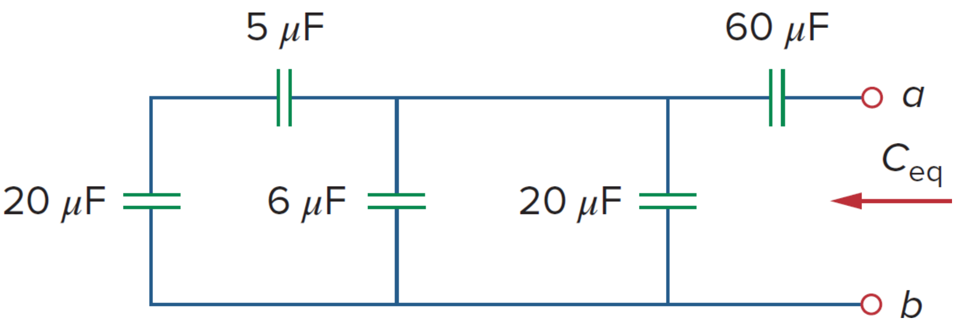
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$



- The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.
- The capacitors in series combine in the same manner as resistors in parallel.
- C_{eq} for two capacitors in series: $$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$$

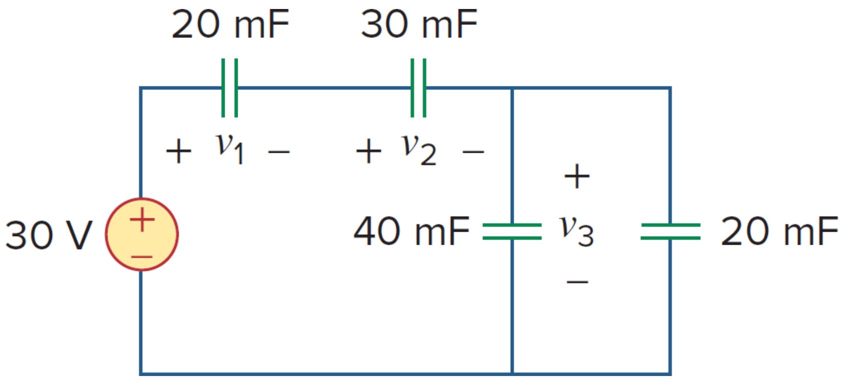
Example

Find the equivalent capacitance seen between terminals a and b of the circuit.



Example

For the circuit, find the voltage across each capacitor.

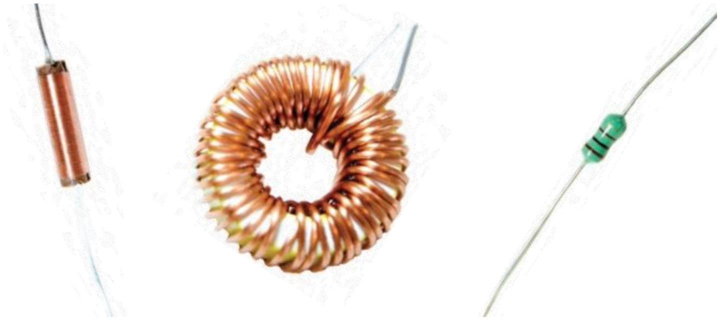


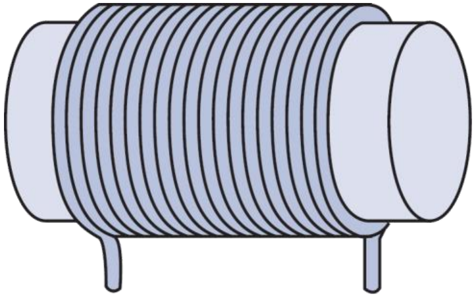
Inductors

Inductors

An **inductor** is a passive element that stores energy in its **magnetic field**.

- They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductive properties, but to enhance the inductive effect, a practical inductor is usually formed into a **cylindrical coil** with many turns of conducting wire.



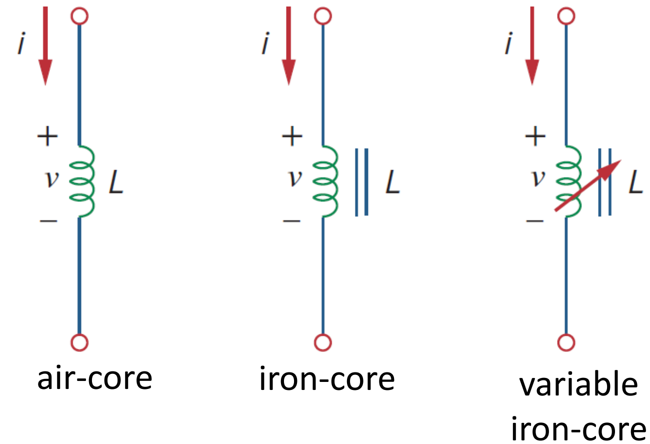


- The core of the cylindrical coil may be made of iron, steel, plastic, or air.
- The terms ***coil*** and ***choke*** are also used for inductors.

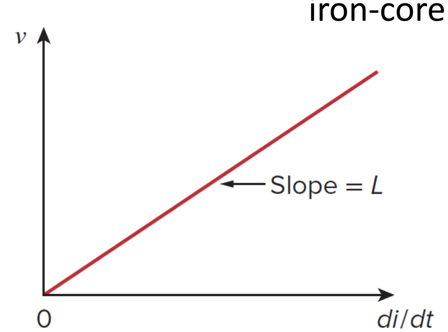
Inductors

If a current i is passed through an inductor, the voltage v across it is directly proportional to the time rate of change in current. Using the passive sign convention,

$$v = L\frac{di}{dt}$$



L is the unit of inductance of the inductor, measured in Henrys H (1 henry equals 1 volt-second per ampere).



- Inductors whose inductance is independent of current and satisfy this equation are said to be **linear**. We will assume linear inductors in this course.

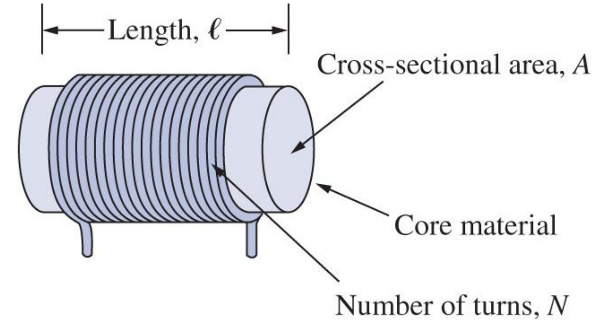
- Inductors may be fixed or variable.
- Typical practical inductors have inductance values ranging from a few microhenrys (μH), as in communication systems, to tens of henrys (H) as in power systems.

Inductance

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension (geometry) and construction. For example, for a solenoid the inductance is

$$L = \frac{N^2 \mu A}{l}$$



N is the number of turns, l is the length, A is the cross-sectional area, and μ is the permeability of the core.

- Inductance can be increased by increasing the number of turns N of coil, using material with higher permeability μ as the core, increasing the cross-sectional area A, or reducing the length l of the coil.

Voltage-Current Relationship, Power, Energy

The current-voltage relation of an inductor can be also obtained by integration:

$$v = L\frac{di}{dt} \quad \Rightarrow \quad \int_{t_0}^{t} di = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau \quad \Rightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + \underbrace{i(t_0)}$$

The current flows the inductor at time t_0

- This shows the inductor's current depends on the history of the inductor's voltage. Hence, the inductor has **memory**.

The **power** delivered to the inductor is

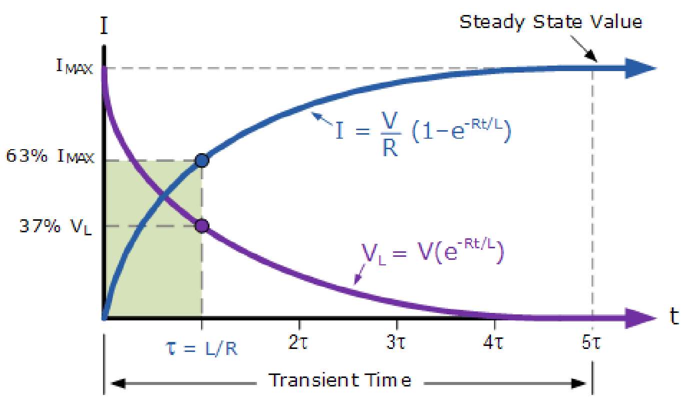
$$p = vi = \left(L\frac{di}{dt}\right)i$$

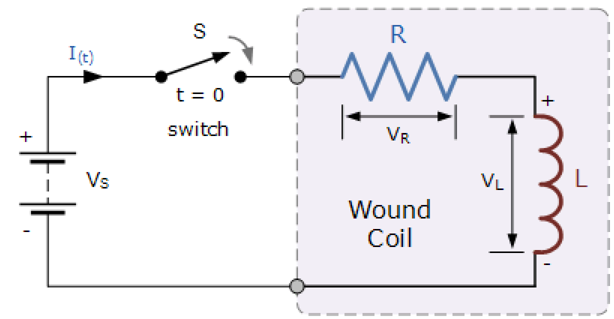
The inductor is designed to store **energy** in its magnetic field. The energy stored is

$$w = \int p(t)dt = L\int i di = \frac{1}{2}Li^2 \quad \Rightarrow \quad w = \frac{1}{2}Li^2$$

Properties of Ideal Inductors

The **current through an inductor cannot change instantaneously** because it is the integral of the voltage. It takes time to increase or decrease the current flowing through an inductor. Thus, it is difficult to start or stop motors, relays, and solenoids very quickly.





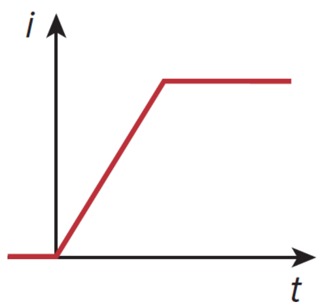
$$v = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

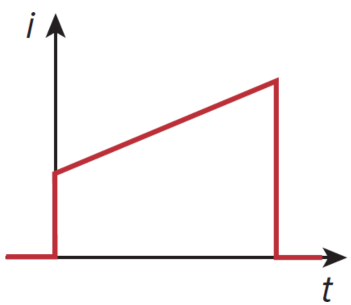
- When the current through an inductor is not changing with time (i.e., constant DC current), the voltage across the inductor is zero ($v = 0$ or short circuit).

Properties of Ideal Inductors

- The current through an inductor must be continuous and cannot change abruptly. An abrupt/discontinuous change in current through an inductor requires an infinite voltage, which is physically impossible! If an inductor is to be turned off abruptly, it will produce a high voltage. Thus, an inductor opposes an abrupt change in the current through it.
- Conversely, the voltage across an inductor can change instantaneously/abruptly.



This current through an inductor is allowed.



This current through an inductor is not allowable.

- The ideal inductor does not dissipate energy.

Examples

1. The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

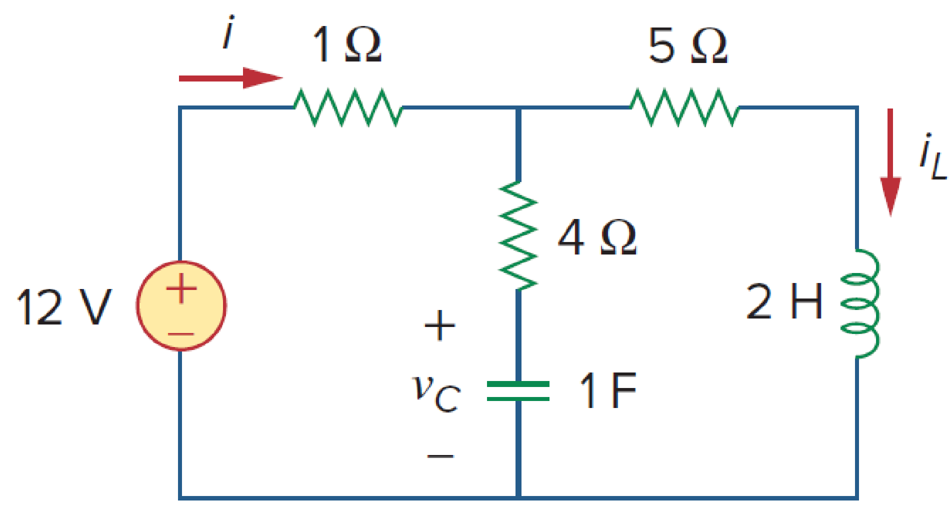
2. Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Example

Consider the circuit shown. Under DC conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



Series and Parallel Inductors

Series Inductors

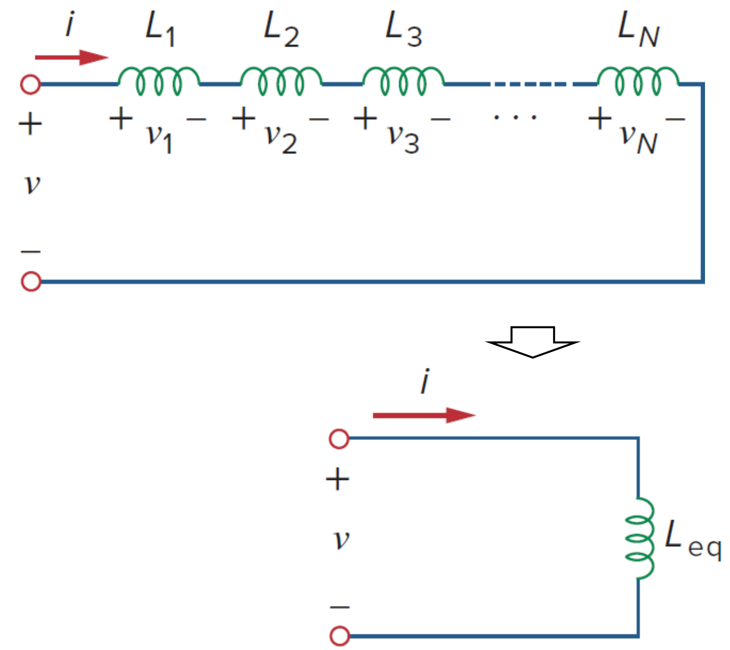
Consider obtaining the equivalent inductor L_{eq} of N inductors in series. Note that the same current i flows through the inductors. Applying KVL to the loop:

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k \right) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$



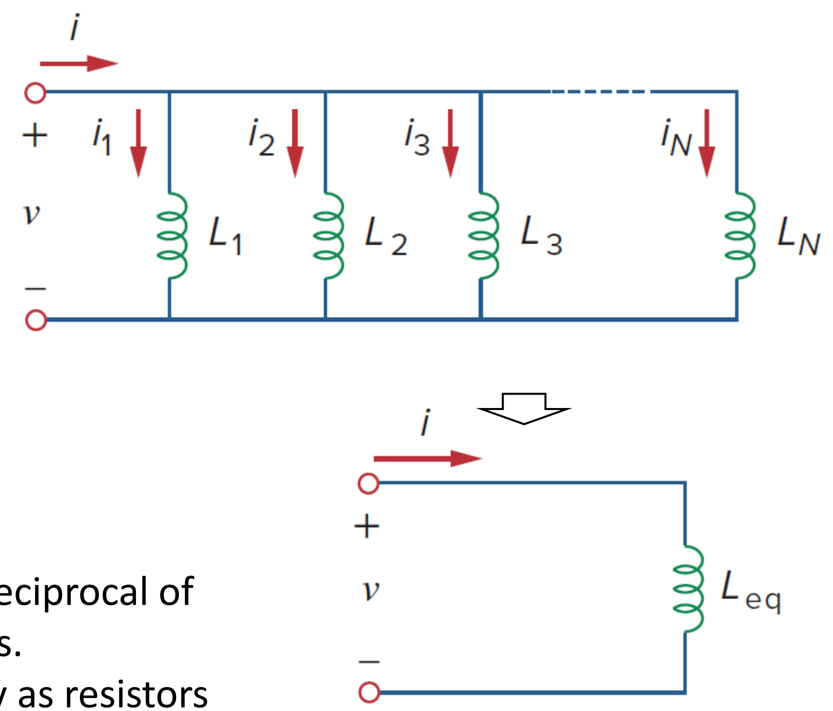
- The equivalent inductance of series-connected inductors is the sum of the individual inductances.
- Inductors in series are combined in exactly the same way as resistors in series.

Parallel Inductors

Consider obtaining the equivalent inductor L_{eq} of N inductors in parallel. Note that the inductors have the same voltage v across them. Applying KCL

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \left(\sum_{k=1}^{N} \frac{1}{L_k}\right) \int_{t_0}^{t} v dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}} \int_{t_0}^{t} v dt + i(t_0)$$



$$\boxed{\frac{1}{L_{\mathrm{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

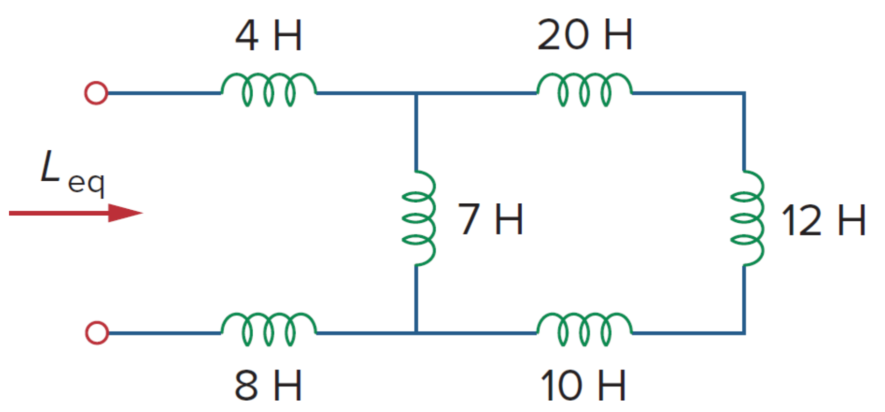
- The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.
- The inductors in parallel are combined in the same way as resistors in parallel.
- L_{eq} for two inductors in parallel: $L_{\mathrm{eq}} = \dfrac{L_1 L_2}{L_1 + L_2}$

Summary of Resistors, Capacitors and Inductors

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

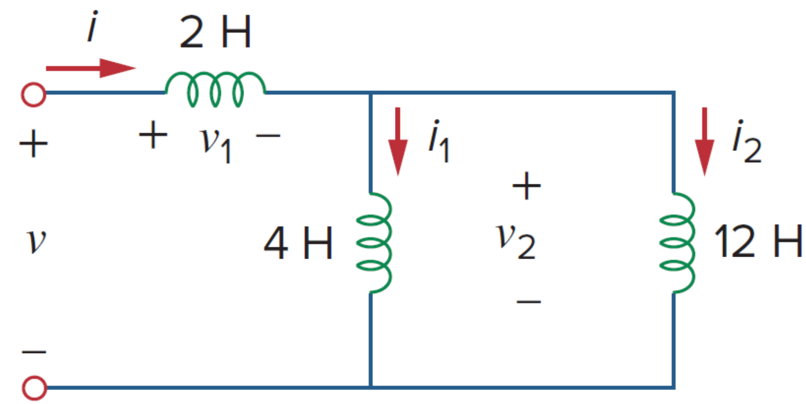
Example

Find the equivalent inductance of the circuit shown.



Example

For the circuit, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.



Linear Voltage Regulators

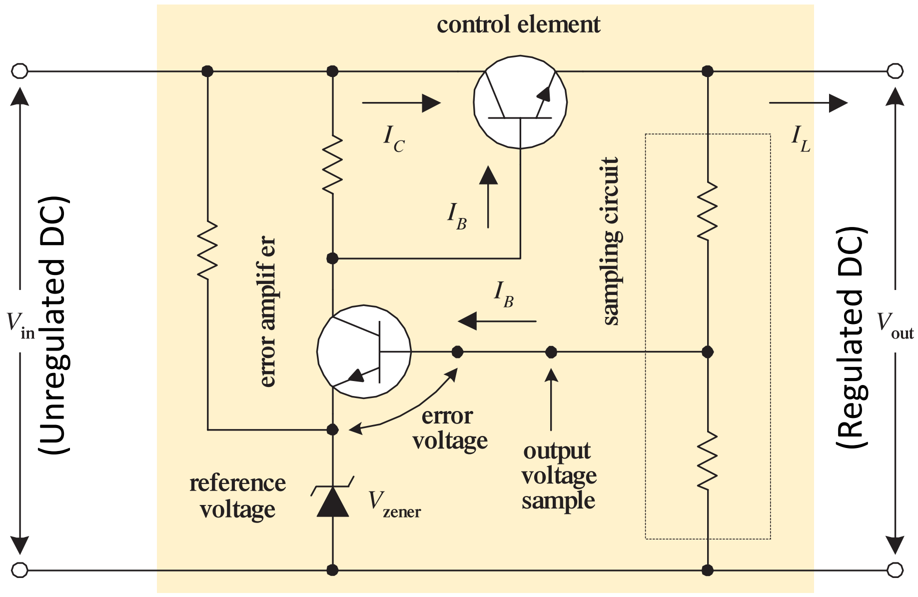
Voltage Regulators

Circuits usually require a DC power supply that can maintain a fixed voltage while supplying enough current to drive a load.

Voltage Regulator is an IC designed to eliminate the **spikes** and maintain a **constant** output voltage with load variations.

- **Fixed-Regulator** ICs
- **Adjustable-Regulator** ICs

Voltage-Regulator IC ($V_{\text{out}} < V_{\text{in}}$)

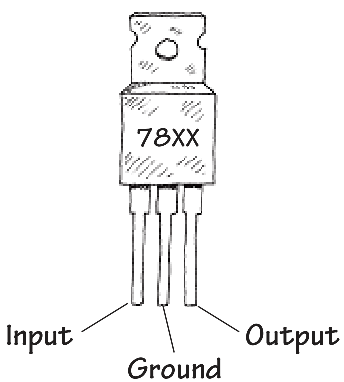


This IC automatically adjust the amount of **current** flowing through a load (so as to maintain a constant output voltage) by comparing the supply's DC output with a fixed or programmed internal reference voltage.

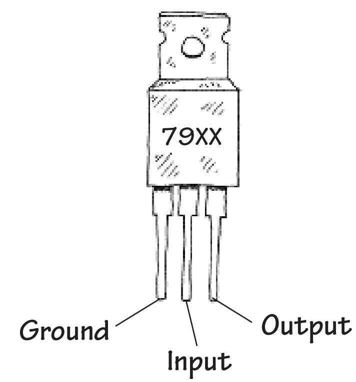
Fixed-Regulator ICs

LM78xx and LM79xx are popular **fixed-regulator** ICs. The "xx" digits represent the output voltage, e.g., 7805 (5 V), 7812 (12 V), etc. These devices can handle a maximum output current of 1.5 A by using a proper heat-sink.

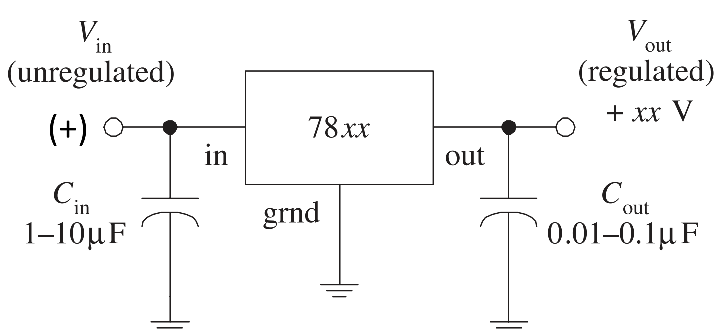
Positive Voltage Regulator

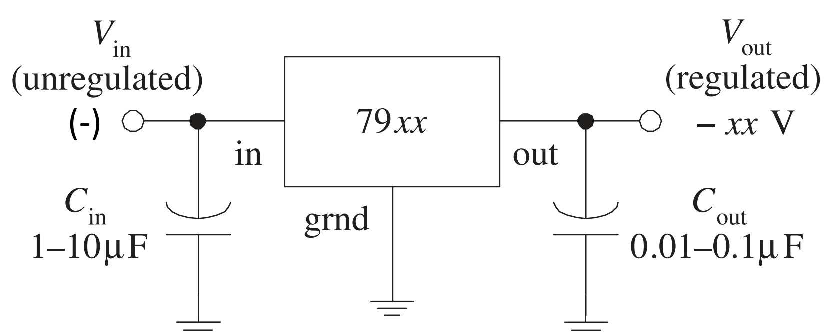


Negative Voltage Regulator



- V_{in} should be at least 2 V higher than V_{out} (smaller difference $\Rightarrow$ less energy dissipation).
- All capacitors are used to remove unwanted input or output spikes/noise.

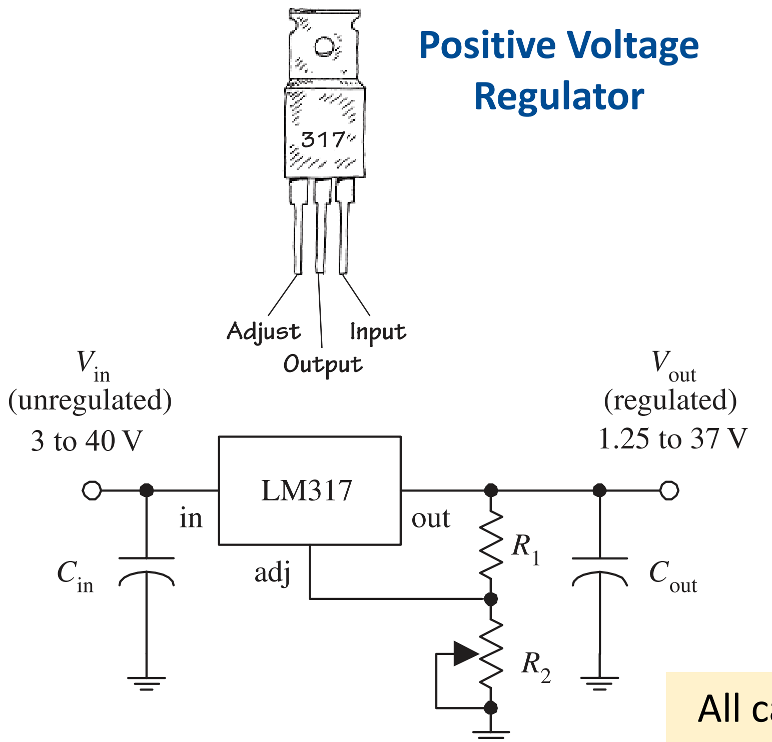




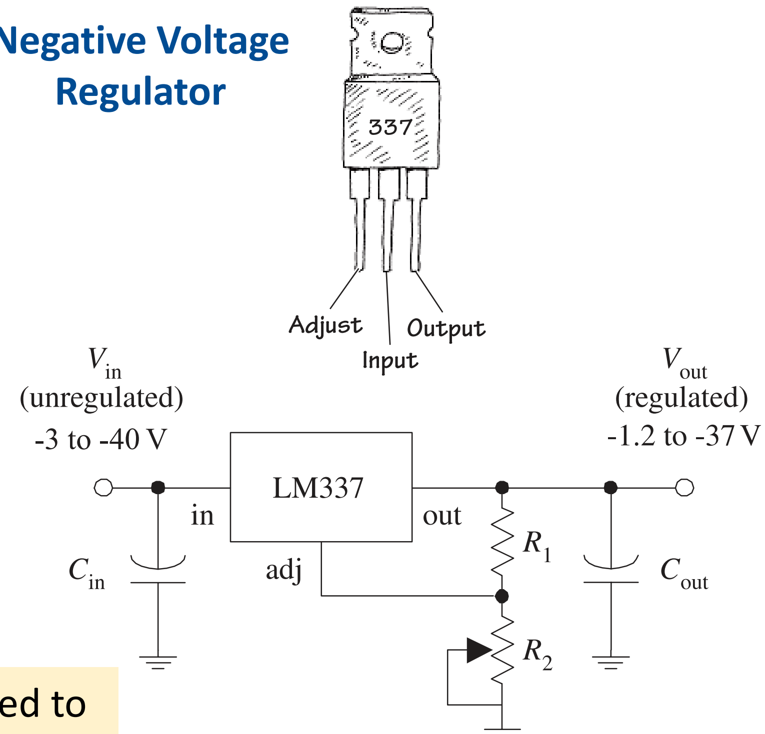
Adjustable-Regulator ICs

LM317 and LM337 are popular **adjustable-regulator** ICs for providing different voltages.

Positive Voltage Regulator



Negative Voltage Regulator



$$V_{out} = 1.25\text{V}\left(1 + \frac{R_2}{R_1}\right)$$

All capacitors are used to remove unwanted input or output spikes/noise (0.1μF or larger)

$$V_{out} = -1.25\text{V}\left(1 + \frac{R_2}{R_1}\right)$$