Ch8: Independent Joint Control

Robot Controllers

Robot Controllers

A robot controller generates the required joint inputs (e.g., torques/forces $\boldsymbol{\tau}$, voltage, or current) to perform a given desired task ($\boldsymbol{\theta}_d(t) \in \mathbb{R}^n$ in joint space or $\boldsymbol{x}_d(t) \in \mathbb{R}^m$ or $\boldsymbol{T}_d(t) \in SE(3)$ in task space) while satisfying given transient and steady-state requirements and reducing the effects of the disturbances on the robot.

- **Joint Space Control**
  - Decentralized Control (Independent Joint Control)
  - Centralized Control
- **Task Space Control** (or **Operational Space Control**)

Controller Types for Multi-Joint Robots

Decentralized Control (**Independent Joint Control**): Each joint is controlled separately with no sharing of information between joints. It is based on a single-input/single-output (SISO) approach, since interaction and coupling effects between the joints have been considered as disturbances acting on each single joint drive system. This method is appropriate when the dynamics are decoupled (e.g., in Cartesian robots where mass matrix is diagonal), or approximately decoupled (e.g., in highly geared robots in the absence of gravity where mass matrix is nearly diagonal, as it is dominated by the apparent inertias of the motors).

Centralized Control: In reality, the dynamic equations of a robot manipulator form a complex, nonlinear, and multivariable system. When large operational speeds are required or direct-drive actuation (no gearbox) is employed, the nonlinear coupling terms strongly influence system performance. Therefore, considering the coupling effects between the joints as disturbances may generate large tracking errors (eliminating the causes rather than to reduce the effects induced by them). In this method full, state information for each of the n joints is available to calculate the controls for each joint. Moreover, This approach allows us to design **robust** and **adaptive** nonlinear control laws that guarantee stability and tracking of planned trajectories.

Controller Types

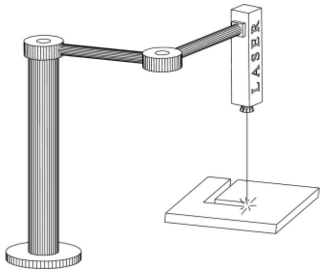
Depending on the task and the robot environment, there are different feedback control strategies including

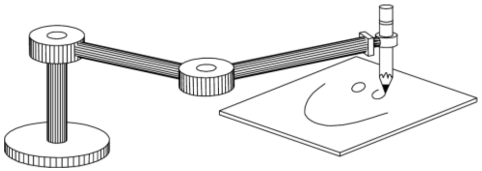
- Position Control (regulation or set-point control)
- Motion Control (tracking control)

$\Big\}$ **when moving in free space**

- Force Control
- Hybrid Motion–Force Control
- Impedance Control

$\Big\}$ **when making contact with environment**





∗ A fundamental constraint imposed by the mechanics:
We cannot independently control the motion and force in the same direction. If the robot imposes a motion, then the environment will determine the force, and if the robot imposes a force, then the environment will determine the motion.

Examples of Desired Tasks

- Pick and place
- Spray painting or laser cutting
- Folding laundry
- Polishing with a polishing wheel
- Back massage
- Erasing a whiteboard
- Shaking hands with a human
- Inserting a peg in a hole

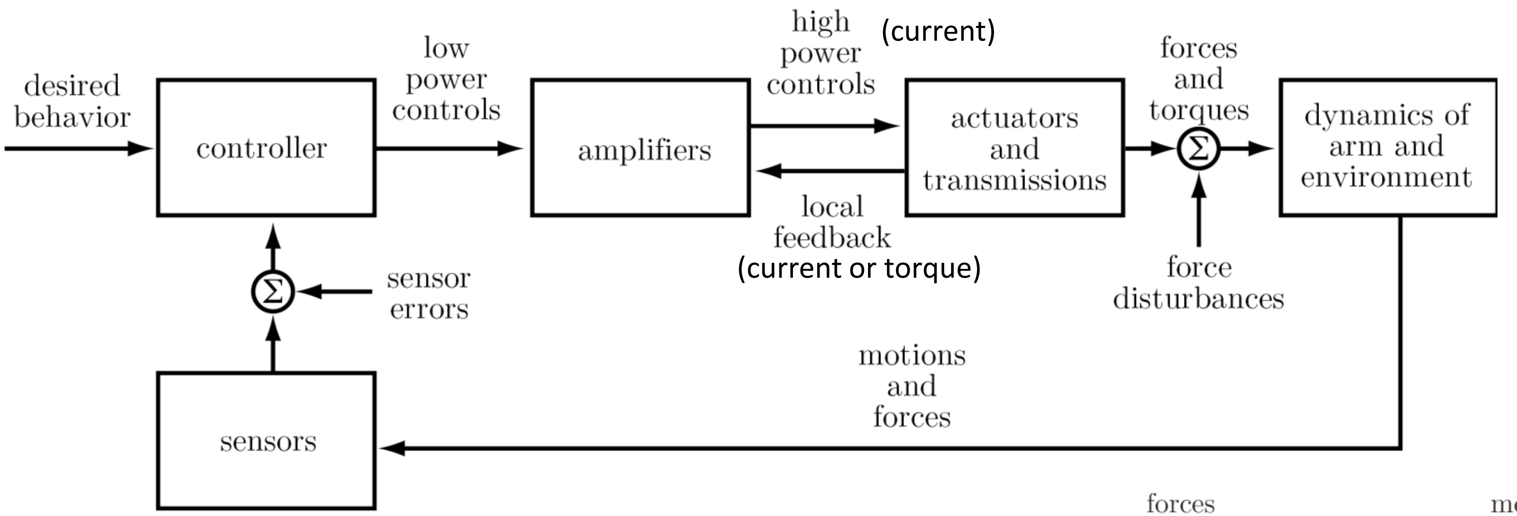
Controller Types

Model-Dependent or **Model-Based Controller**: If a controller (or its design parameters) depends explicitly on the **partial** or **complete** knowledge of the manipulator dynamic model (i.e., $\boldsymbol{M}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, $\boldsymbol{g}(\boldsymbol{q})$).

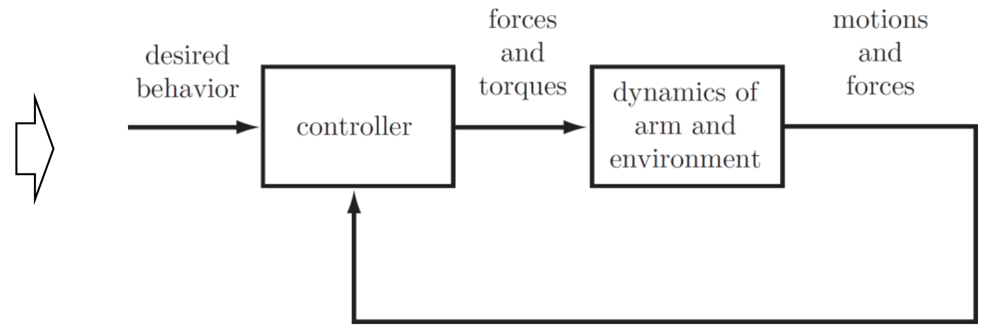
Non-Model-Based Controller: It includes fuzzy controllers, learning controllers, and neural-net-based controllers.

A Typical Robot Control System

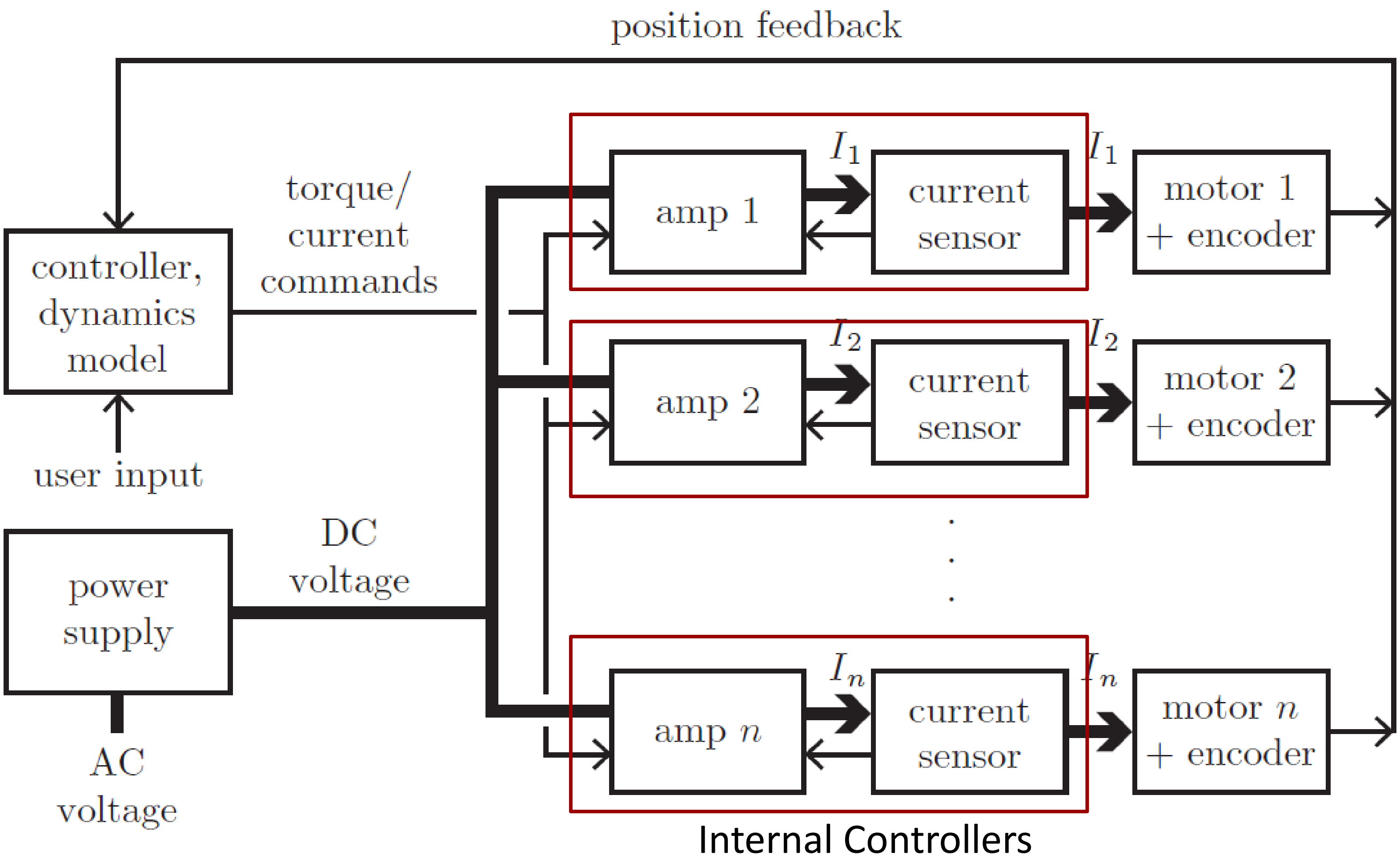
Feedback control (or closed-loop control) uses position, velocity, and/or force sensors to measure the actual behavior of the robot, compares it with the desired behavior, and modulates the control signals sent to the actuators.



A simplified model with ideal sensors and ideal behavior of the amplifier and actuator blocks.



A Typical Robot Driven by DC Electric Motors



Actuator, Load, and Joint Dynamics

Robot Actuators

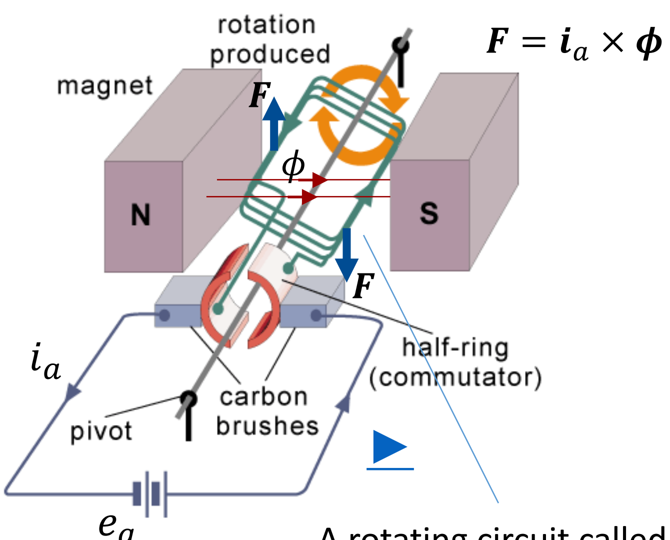
In this chapter, we study the dynamics of **permanent magnet DC motors** with gear reduction, as these are commonly used in robot manipulators. Other types of electric motors used in robot manipulators are **brushless DC motors** and **AC motors**.

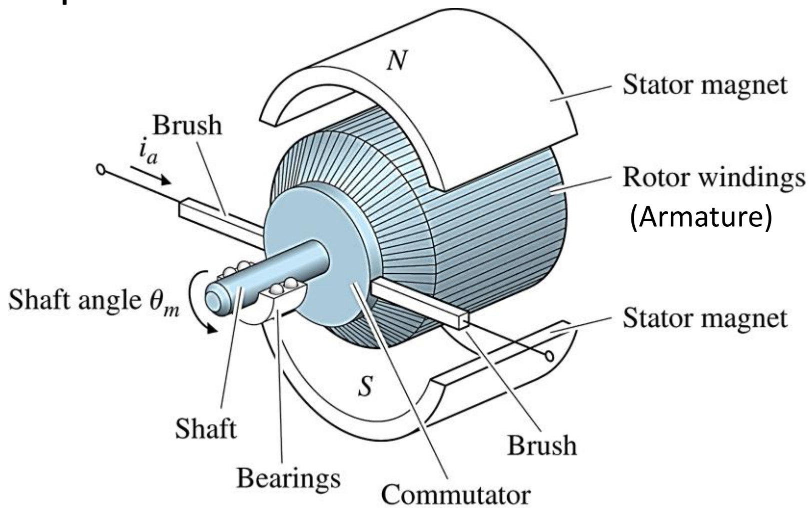
Permanent magnet DC motors with gear reduction: The motor dynamics is linear and well understood and the effect of the gear reduction is largely to decouple the system by reducing the inertia coupling among the joints. However, the presence of the gears introduces **friction**, **drive-train compliance** (or flexibility or elasticity), and **backlash**.

Direct-drive actuation with high-torque motors and no gear reduction: The problems of backlash, friction, and compliance due to the gears are eliminated. However, the inertia coupling among the links is now significant, and the dynamics of the motors themselves may be much more complex. Moreover, the actuators are quite large to create sufficient torque.

DC Motors

A motor is an electromechanical system that provides a rotary motion for a voltage input, i.e., a mechanical output generated by an electrical input.



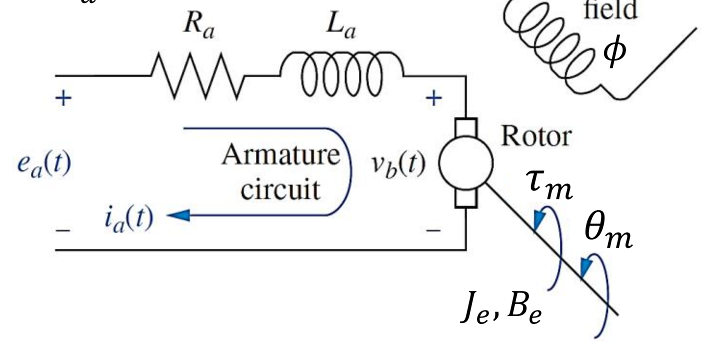


A rotating circuit called the **armature**, through which the current i_a flows, passes through a magnetic field ϕ at right angles and produces a force F which its resulting torque turns the rotor.

There are two ways to control DC Motors:
1. Adjusting the voltage e_a (**Armature Control**) [used for **Permanent-Magnet** DC motors]
2. Adjusting the field flux ϕ (**Field Control**) [used for **Variable-Reluctance** DC motors]

Armature-Controlled DC Motor Dynamics

L_a: Armature Inductance
R_a: Armature Resistance



Electrical Equations:

1- Kirchhoff's Voltage Law (KVL):

Armature circuit equation:

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) \qquad (1)$$

where e_a is the applied armature voltage, i_a is the armature current, and v_b is the back electromotive force (**back EMF**).

Note: For DC motors L_a is small compared to R_a. Thus, it is usually assumed $L_a/R_a \approx 0$.

2- Voltage-Speed Relationship: Since the armature is rotating in a magnetic field, its back EMF voltage v_b is proportional to angular velocity $\dot{\theta}_m$ or ω_m.

$$v_b(t) = K_b \dot{\theta}_m(t) = K_b \omega_m(t) \qquad (2)$$

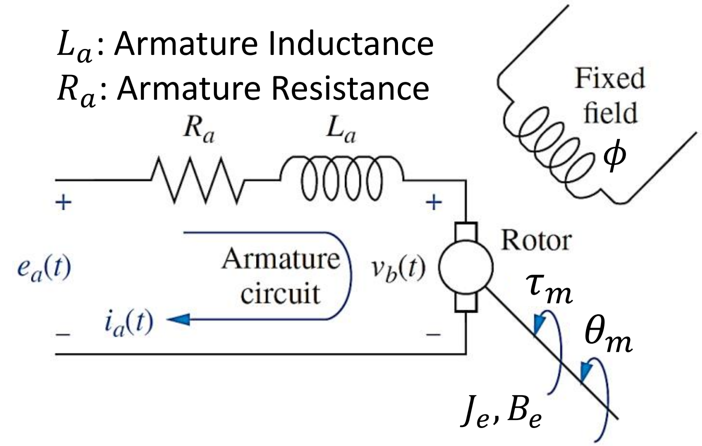
where K_b is the back EMF constant (unit: V·s/rad), and $\dot{\theta}_m(t) = \omega_m(t)$ is the angular velocity of the motor. $K_v = 1/K_b$ is called velocity (or speed) constant.

Armature-Controlled DC Motor Dynamics

Mechanical Equation:

3- Newton's Law of Motion:

The relation between the torque τ_m developed by the motor at the armature, equivalent inertia (J_e) at the armature, and equivalent viscous damping (B_e) at the armature is:



$$\tau_m(t) = J_e\ddot{\theta}_m(t) + D_e\dot{\theta}_m(t) \qquad (3)$$

4- Torque-Current Relationship: The torque τ_m developed by the motor is proportional to the armature current i_a.

$$\tau_m(t) = K_t i_a(t) \qquad (4)$$

where K_t is the motor torque constant, which depends on the motor and magnetic field characteristics (unit: N·m/A). For ideal motor, $K_t = K_b$.

Electrical Constants K_t/R_a and K_b

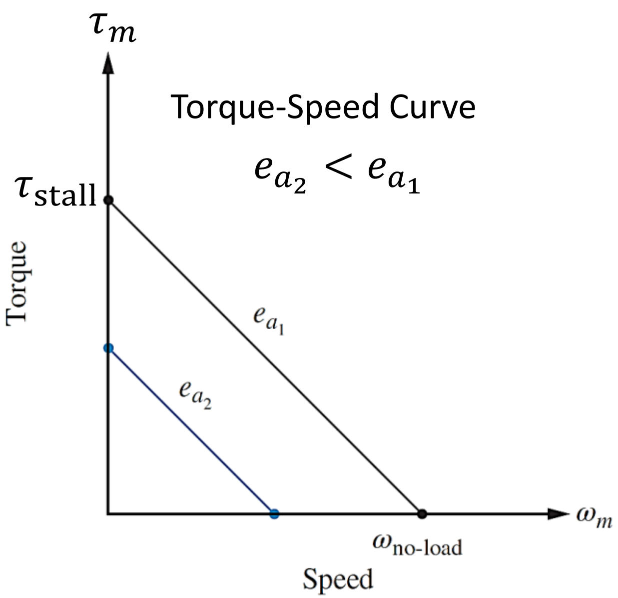
The electrical constants K_t/R_a and K_b can be obtained through a **dynamometer** test of the motor, where a dynamometer measures the **torque** and **speed** of a motor under the condition of a **constant applied voltage** e_a.

Using (1), (2), (4), and assumption $L_a/R_a \approx 0$:

$$e_a = R_a \frac{\tau_m}{K_t} + K_b \omega_m \xrightarrow{\text{or}} \tau_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a} e_a$$

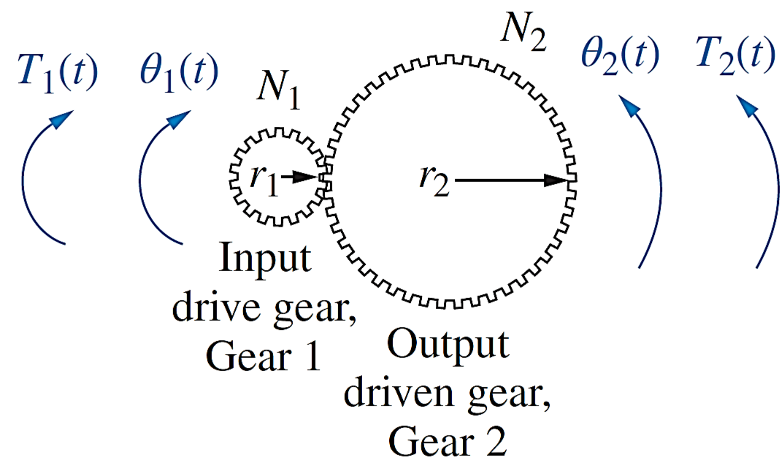
- τ_m when $\omega_m = 0$ is called the Stall Torque (τ_{stall})
- ω_m when $\tau_m = 0$ is called the no-load speed ($\omega_{\text{no-load}}$)

$$\Rightarrow \quad \frac{K_t}{R_a} = \frac{\tau_{\text{stall}}}{e_a} \qquad K_b = \frac{e_a}{\omega_{\text{no-load}}}$$



Mechanical Constants J_e and B_e

The motors are often used in conjunction with gears to reduce the angular velocity and multiply the torque.



Consider a gear set including an input gear with radius r_1 and N_1 teeth and an output gear with radius r_2 and N_2 teeth.

$s_1 = s_2$ (traveled distance, no backlash) $\longrightarrow$ $r_1\theta_1 = r_2\theta_2$

$m_1 = m_2$ (modulus: $2r/N$) $\longrightarrow$ $\dfrac{2r_1}{N_1} = \dfrac{2r_2}{N_2}$

$E_1 = E_2$ (lossless gears) $\longrightarrow$ $\tau_1\theta_1 = \tau_2\theta_2$

$$\frac{\theta_1}{\theta_2} = \frac{\tau_2}{\tau_1} = \frac{N_2}{N_1} = \frac{r_2}{r_1}$$

Mechanical Constants J_e and B_e

The equivalent inertia (J_e) and equivalent viscous damping (D_e) (including both the armature inertia/damping and the load inertia/damping) reflected to the armature can be found as follows. Let $r = N_2/N_1$ be the gear ratio (Usually $r \gg 1$, e.g., ~20-200).

$$\tau_m - \tau' = J_m\ddot{\theta}_m + B_m\dot{\theta}_m$$

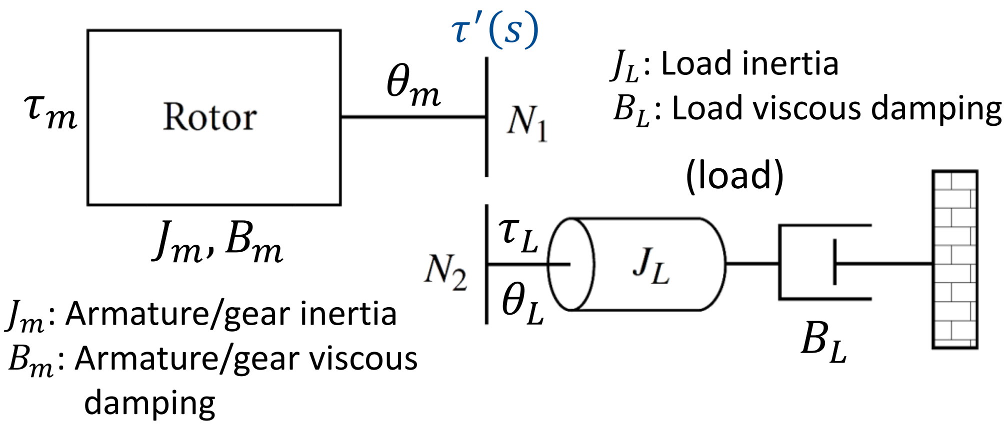
$$\tau_L = J_L\ddot{\theta}_L + B_L\dot{\theta}_L$$

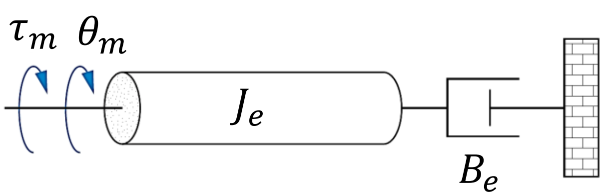
$$\tau' = \tau_L/r$$

$$\theta_m = r\theta_L$$

$$\boxed{\tau_m = J_e\ddot{\theta}_m + B_e\dot{\theta}_m}$$

$$\boxed{J_e = J_m + \frac{J_L}{r^2}; \quad B_e = B_m + \frac{B_L}{r^2}}$$





- These constants can be also determined through laboratory testing using **transient response** or **frequency response** data.

DC Motor Equations in Laplace Domain

Thus, four equations represent the mathematical model of a DC motor:

(1) $e_a = R_a i_a + L_a \frac{di_a}{dt} + v_b$

(2) $v_b = K_b \dot{\theta}_m$

(3) $\tau_m = J_e \ddot{\theta}_m + B_e \dot{\theta}_m$ (or)

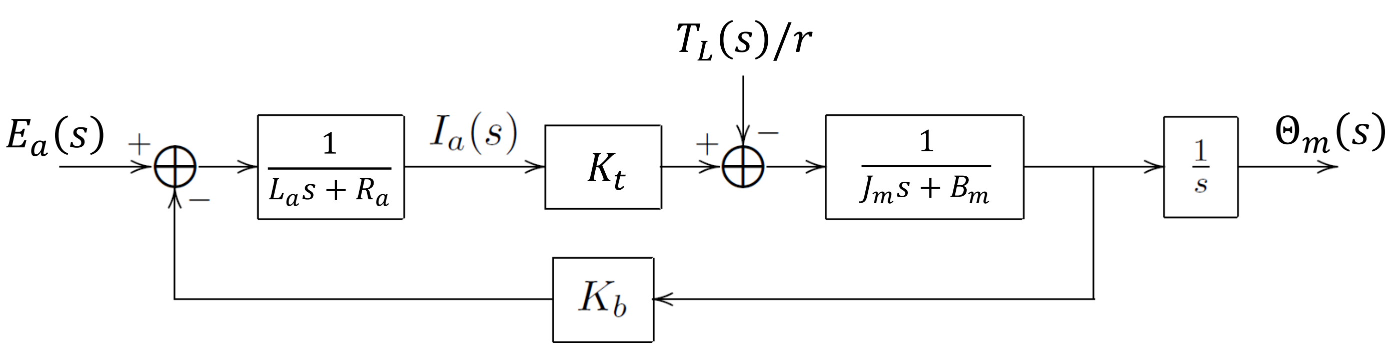
$\tau_m - \tau_L / r = J_m \ddot{\theta}_m + B_m \dot{\theta}_m$

(4) $\tau_m = K_t i_a$

Assuming zero initial conditions:

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_t}{s[(J_m s + B_m)(L_a s + R_a) + K_t K_b]} \quad \text{(when } T_L(s) = 0\text{)}$$

$$\frac{\Theta_m(s)}{T_L(s)} = \frac{-(L_a s + R_a)/r}{s[(J_m s + B_m)(L_a s + R_a) + K_t K_b]} \quad \text{(when } E_a(s) = 0\text{)}$$



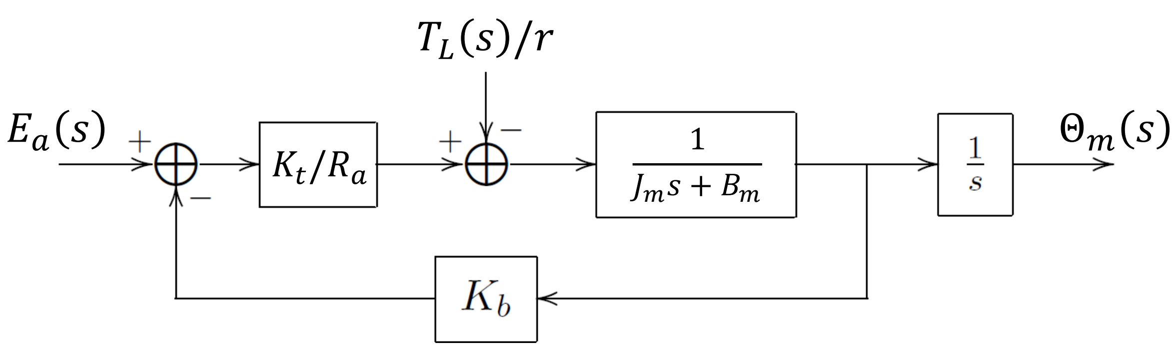
DC Motor Equations in Laplace & Time Domains (Reduced-Order System)

- Since for DC motors L_a is small compared to R_a (i.e., $L_a/R_a \approx 0$), the **reduced-order system** can be written as

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{K_t/R_a}{s(J_m s + B_m + K_t K_b/R_a)}$$

$$\frac{\Theta_m(s)}{T_L(s)} = \frac{-1/r}{s(J_m s + B_m + K_t K_b/R_a)}$$

- In the time domain: $$J_m \ddot{\theta}_m + \left(B_m + \frac{K_t K_b}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} e_a - \frac{1}{r}\tau_L$$



Independent Joint Model

We now assume that the load attached to the DC motor is **a link of a multi-link manipulator** to generate a more accurate description of the manipulator load dynamics.

Consider the dynamic model of an n-DOF open-chain manipulator with no friction at the joints and no external force at the end-effector.

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$$

$$\tau_k = \sum_{j=1}^{n} m_{kj}(\boldsymbol{q})\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}(\boldsymbol{q})\dot{q}_i\dot{q}_j + g_k(\boldsymbol{q}), \qquad k = 1, \dots, n \qquad \text{(componentwise)}$$

If the output side of the gear train is directly coupled to the joint axis:

$$q_k = \frac{\theta_{m_k}}{r_k}, \qquad k = 1, \dots, n$$

$$\tau_k = \tau_{L_k}, \qquad k = 1, \dots, n$$

θ_{m_k}: motor variable
r_k: kth gear ratio
τ_{L_k}: actuator load torques

Independent Joint Model

Thus, the equations of motion of the manipulator including motors can be written componentwise as

$$\begin{cases} \tau_k = \sum_{j=1}^{n} m_{kj}(\boldsymbol{q})\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}(\boldsymbol{q})\dot{q}_i\dot{q}_j + g_k(\boldsymbol{q}), & (1) \\ J_{m_k}\ddot{\theta}_{m_k} + \left(B_{m_k} + \frac{K_{t_k}K_{b_k}}{R_{a_k}}\right)\dot{\theta}_{m_k} = \frac{K_{t_k}}{R_{a_k}} e_a - \frac{1}{r_k}\tau_k & (2) \end{cases}$$

$$q_k = \frac{\theta_{m_k}}{r_k}$$

$$k = 1, \dots, n$$

The simplest approach to the control of this system is to consider the nonlinear term τ_k in (2) as an **input disturbance to the motor** and design an independent controller for each joint/motor. Note that the motor dynamics in (2) is **linear**.

Note: The term τ_k in (2) is divided by the gear ratio r_k. This reduces magnitude of the coupling nonlinearities given by (1), which adds to the validity of the independent joint control approach.

Note: For very high-speed motion or for direct-drive manipulators (without gear reduction), treating the coupling nonlinearities as a disturbance will generally result in larger tracking errors. Thus, more advanced, nonlinear feedback control methods should be used.

Independent Joint Model

Substituting (1) into (2): $$\underbrace{\left(J_{m_k} + \frac{m_{kk}(\boldsymbol{q})}{r_k^2}\right)}_{(*)}\ddot{\theta}_{m_k} + \left(B_{m_k} + \frac{K_{t_k}K_{b_k}}{R_{a_k}}\right)\dot{\theta}_{m_k} = \frac{K_{t_k}}{R_{a_k}}e_a - \frac{d_k}{r_k} \qquad (3)$$

$$k = 1, \dots, n$$

$$d_k = \sum_{j=1, j\neq k}^{n} m_{kj}(\boldsymbol{q})\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}(\boldsymbol{q})\dot{q}_i\dot{q}_j + g_k(\boldsymbol{q})$$

Notes:

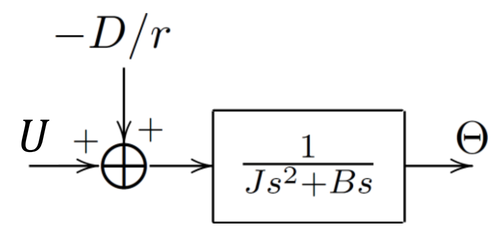
- d_k/r_k represents all the nonlinearities and coupling from the other links.
- The inertia $(*)$ is configuration dependent and may vary over a large range. However, we may approximate it by a constant average or effective inertia J_{eff_k}.

Thus, (3) can be concisely written as: $$J_{\mathrm{eff}_k}\ddot{\theta}_{m_k} + B_{\mathrm{eff}_k}\dot{\theta}_{m_k} = u_k - \frac{d_k}{r_k}$$

$$B_{\mathrm{eff}_k} = B_{m_k} + \frac{K_{t_k}K_{b_k}}{R_{a_k}}, \qquad u_k = \frac{K_{t_k}}{R_{a_k}}e_a$$

- Now, let's use the following general form for each joint where $u(t)$ is a control input (torque) and $d(t)$ a disturbance:

$$J\ddot{\theta}(t) + B\dot{\theta}(t) = u(t) - \frac{d(t)}{r}$$

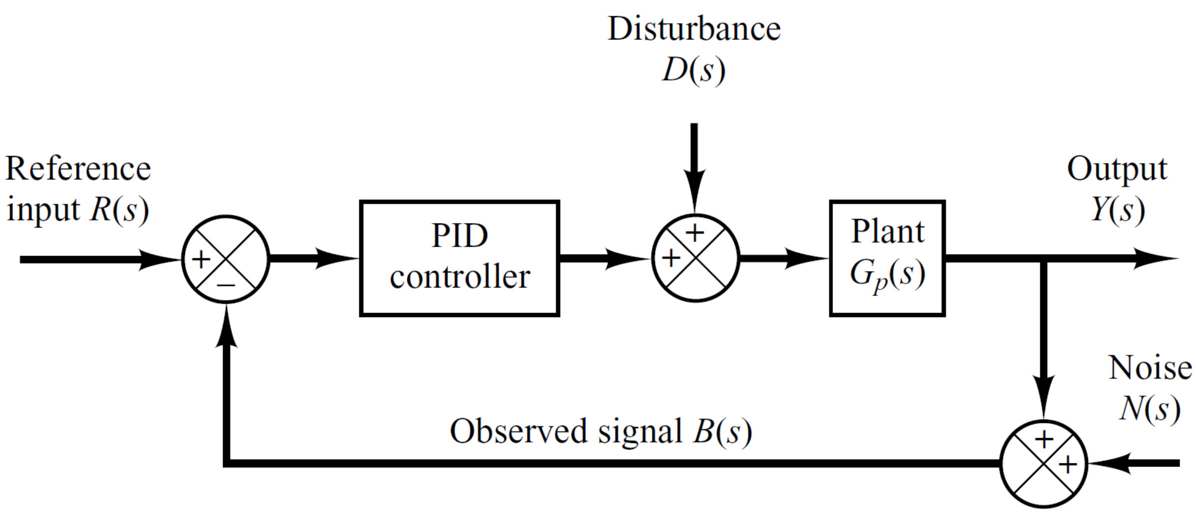


PID Controller

Feedback Control Systems

When you design a controller, you have two main goals, (1) get the system output $Y(s)$ follow the input reference $R(s)$ and/or (2) reject any disturbances $D(s)$ into the system.

In addition to disturbances, there is noise $N(s)$ in the system that makes it difficult to find the actual output of the system. If the noise is high enough frequency, then the feedback controller can filter it out or ignore it completely.



PID Controller

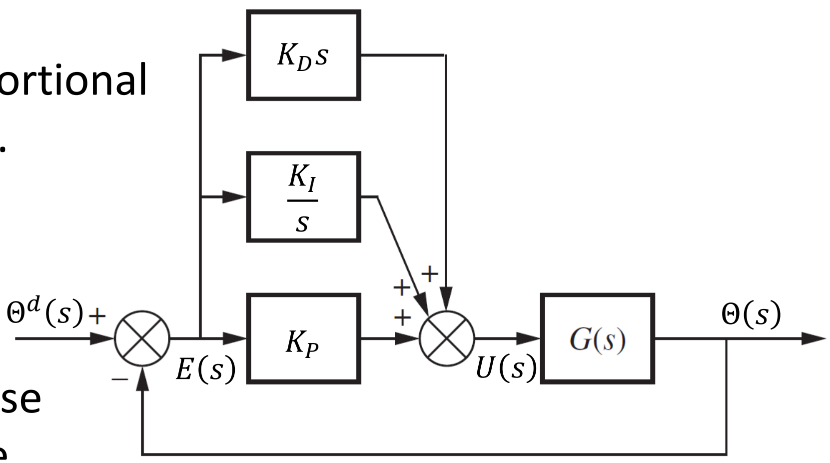
The PID controller is the most common type of controller used in most manipulators.

Time Domain: $$u(t) = K_P e(t) + K_I \int_0^t e(\sigma)d\sigma + K_D \frac{de(t)}{dt}$$

Laplace Domain: $$U(s) = G_{\text{PID}}(s)E(s)\,, \qquad G_{\text{PID}}(s) = K_P + \frac{K_I}{s} + K_D s = K_P\left(1 + \frac{1}{T_I s} + T_D s\right)$$

$$= \frac{K_D s^2 + K_P s + K_I}{s} = K\frac{(s + z_{c_I})(s + z_{c_D})}{s}$$

- $e(t) = \theta^d(t) - \theta(t)$ is tracking error, K_P is proportional gain, K_I is integral gain, and K_D is derivative gain.
- The PID controller adds **one pole** and **two zeros** to the forward transfer function.
- The design problem (known as **tuning**) is to choose the PID gains to achieve the desired performance.



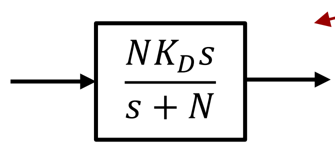
Noise Filtering in PID Control (Derivative Filter)

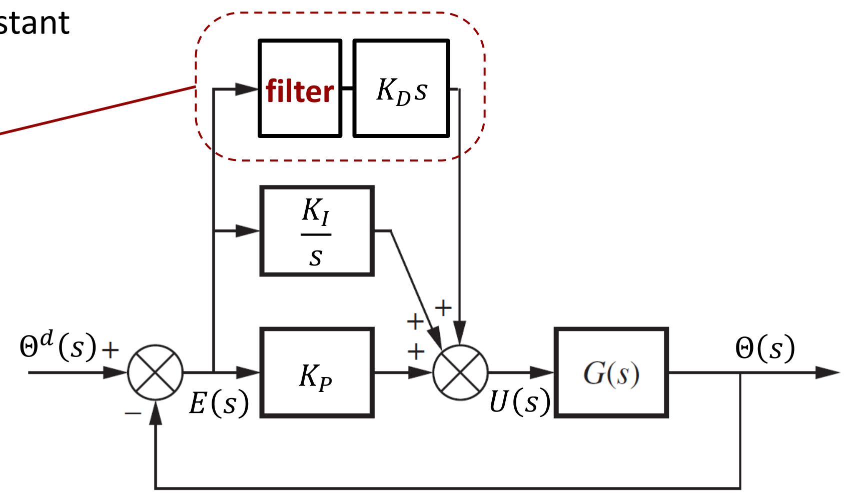
The derivative term $K_D s$ in the PID controller involves differentiation of the signal $\theta(t)$ measured by a position sensor (such as a potentiometer, an encoder, or a resolver). In practice, a differentiator will **amplify high-frequency signals** and thus, will perform poorly in the presence of noise. Two solutions:

- Using velocity sensor, such as a tachometer, to measure $\dot{\theta}(t)$ directly.
- More commonly, using a (first-order) low-pass filter (LPF) to pass signals below a **cutoff frequency** N and attenuates signals above that.

$$G_{\mathrm{LPF}}(s) = \frac{N}{s+N} = \frac{1}{\tau_f s + 1}$$

τ_f: filter time constant

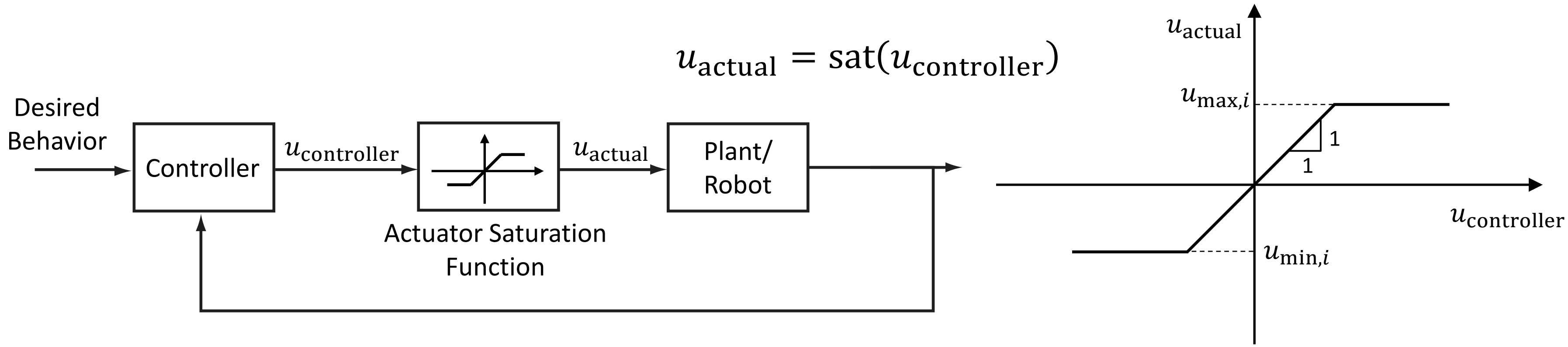




Actuator Saturation and Integrator Windup

In some controllers, choosing large values for the control parameters causes a large (initial) control output which is beyond the actuator's limits (voltage, current, or torque) and **actuator saturation** occurs. In this situation, the control signal to the plant stops changing (which is a nonlinearity) and the feedback path is effectively opened.

$$u_{\text{actual}} = \text{sat}(u_{\text{controller}})$$



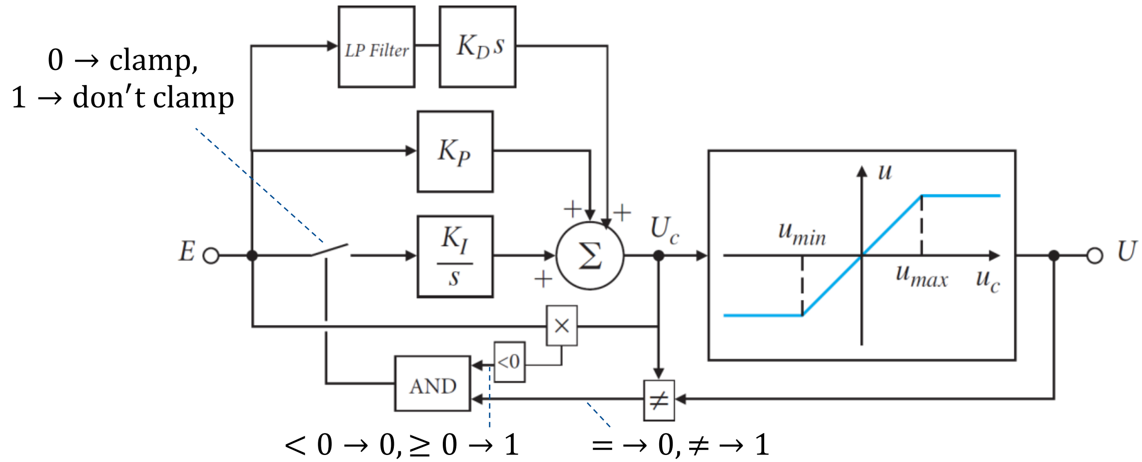
- Under actuator saturation conditions, if the error signal continues to be applied to the integrator input, the **integrator** output will grow (**windup**) until the sign of the error changes and the integration turns around. Thus, the integrator is an unstable element in open-loop and must be stabilized when saturation occurs.
- Actuator Saturation can cause a **large overshoot** and **poor transient response**.

Integrator Antiwindup

The solution to this problem is an **Integrator Antiwindup** circuit, which "turns off" the integral action when the actuator saturates.

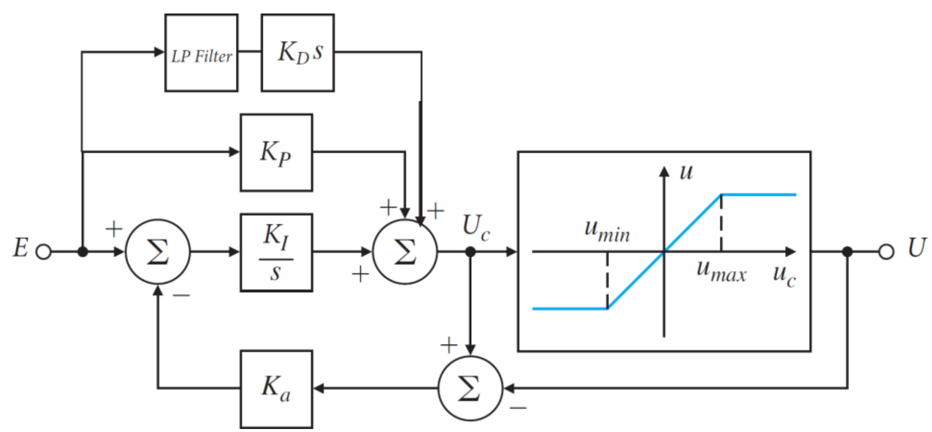
Clamping Method:

Once the actuator saturates AND the error e is the same sign as the controller output u_c, the integral action is turned off.



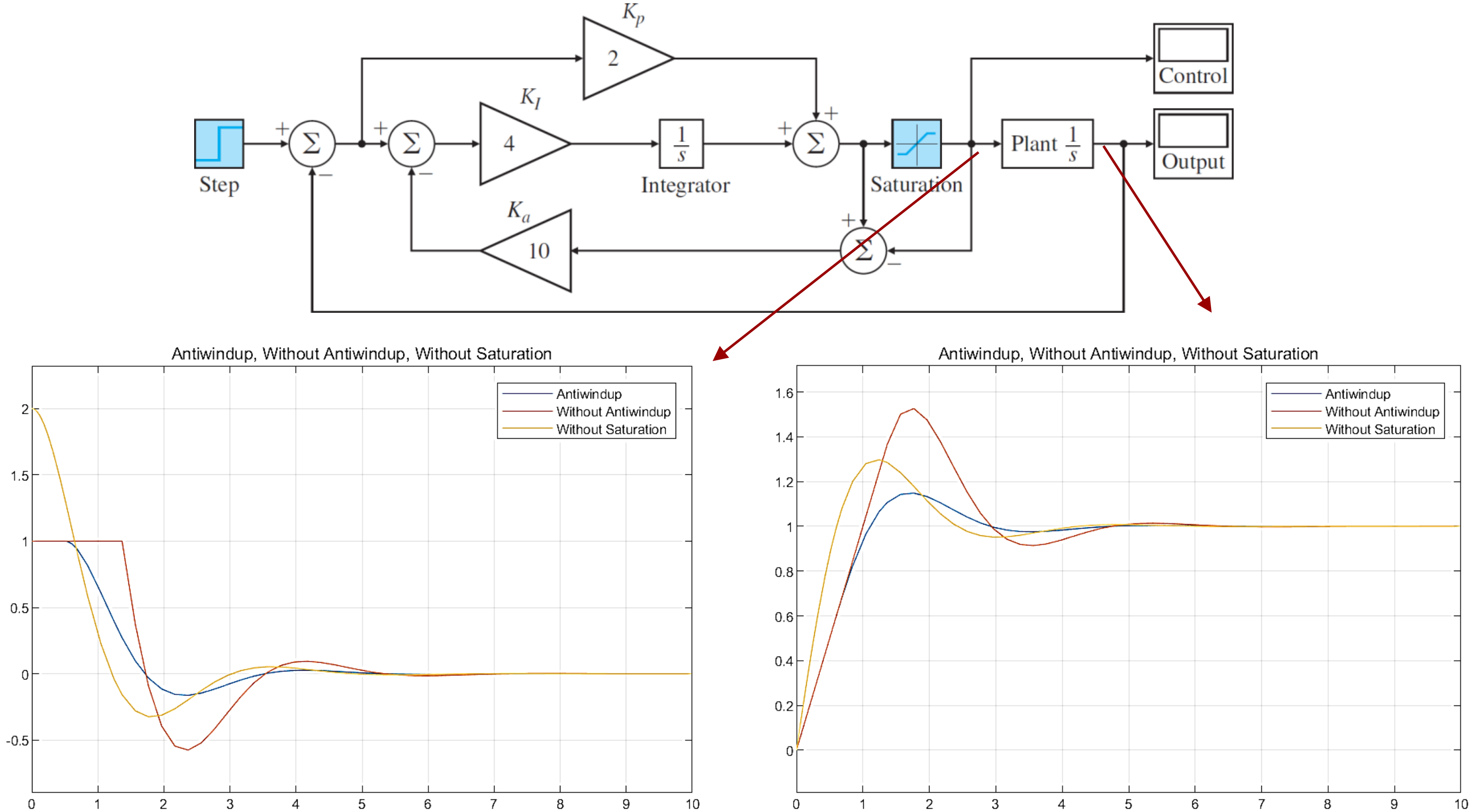
Back Calculation Method:

Once the actuator saturates, the feedback loop around the integrator becomes active and acts to keep the input to the integrator small.
The antiwindup gain, K_a, should be chosen to be large enough



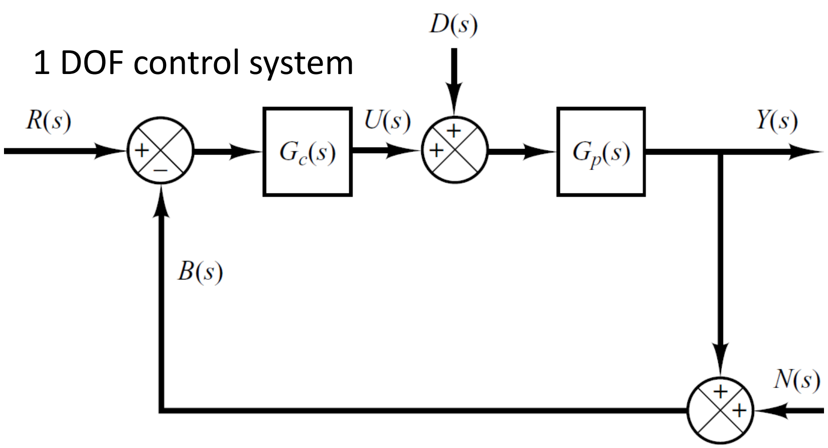
Example

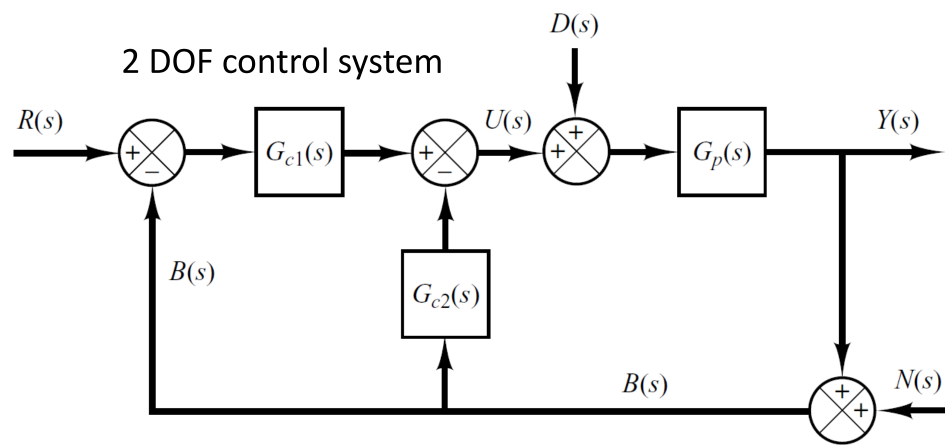
Consider a plant with the transfer function $G(s) = 1/s$ and a PI controller $G_c(s) = K_P + K_I/s = 2 + 4/s$ in the unity feedback configuration. The input to the plant is limited to ± 1.



Two-Degree-of-Freedom Controller

Consider the systems shown, where the system is subjected to the disturbance input $D(s)$ and noise input $N(s)$. The degrees of freedom of a control system refers to how many of closed-loop transfer functions $Y(s)/R(s)$, $Y(s)/D(s)$, and $Y(s)/N(s)$ are independent.

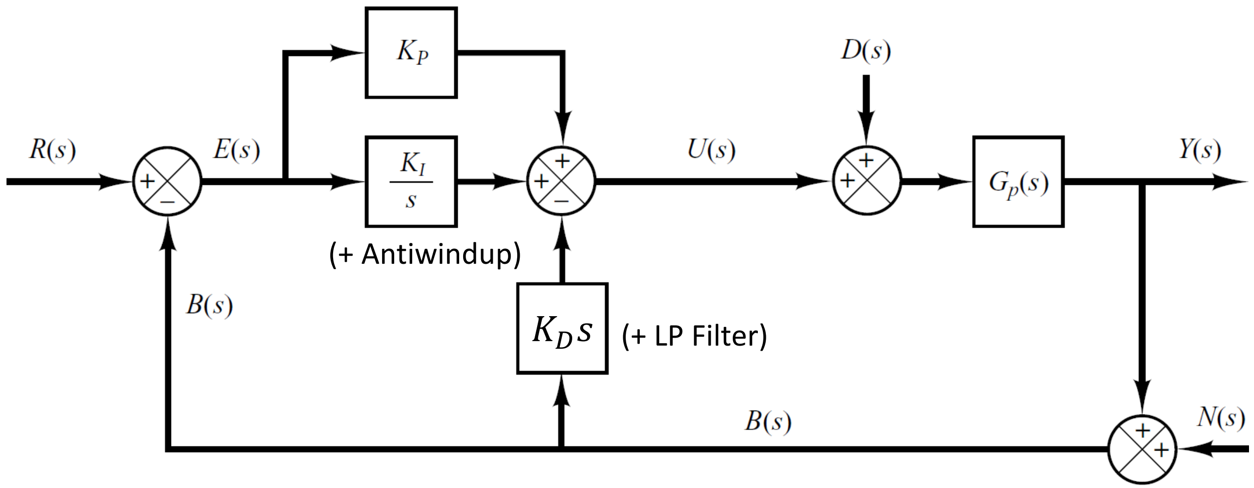




- **Note**: In deriving $Y(s)/R(s)$, we assumed $D(s) = 0$ and $N(s) = 0$. Similar comments apply to the derivations of $Y(s)/D(s)$ and $Y(s)/N(s)$.

2 DOF PI-D Controller

If the reference input is a step function, then, because of the presence of the derivative term in the control action, the control output $u(t)$ will involve an impulse function. To avoid this, we can operate the derivative action only in the feedback path so that differentiation occurs only on the feedback signal and not on the reference signal. This control scheme is called the PI-D control.



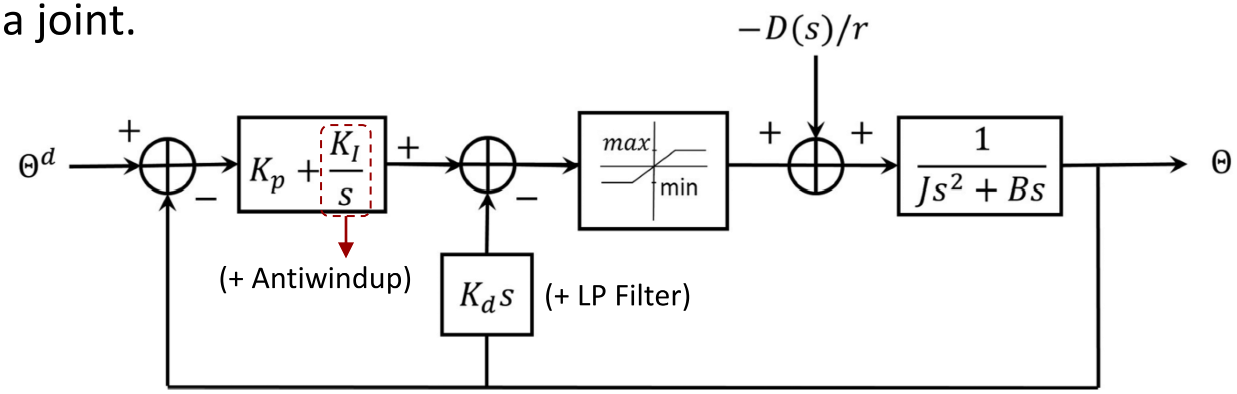
$$U(s) = \left(K_P + \frac{K_I}{s}\right)E(s) - K_D sB(s) = \left(K_P + \frac{K_I}{s}\right)R(s) - \left(K_P + \frac{K_I}{s} + K_D s\right)B(s)$$

With LP Filter: $$U(s) = \left(K_P + \frac{K_I}{s}\right)E(s) - \frac{NK_D s}{s+N}B(s)$$

Position or Set-Point Control

Two DOF PID Controller of a Joint

Let's consider the problem of set-point tracking or tracking a constant or step reference command θ_d of a joint.



$$G_1(s) = \frac{\Theta(s)}{\Theta^d(s)} = \frac{K_P s + K_I}{Js^3 + (B + K_D)s^2 + K_P s + K_I}$$

$$G_2(s) = \frac{\Theta(s)}{D(s)} = \frac{-s/r}{Js^3 + (B + K_D)s^2 + K_P s + K_I}$$

The overall system response by the Principle of Superposition:

$$\Theta(s) = G_1(s)\Theta^d(s) + G_2(s)D(s)$$

PD Controller

Let's $K_I = 0$:
$$\Theta(s) = \frac{K_P}{Js^2 + (B + K_D)s + K_P}\Theta^d(s) - \frac{1/r}{Js^2 + (B + K_D)s + K_P}D(s)$$

The closed-loop system is second-order and will be **stable** for all positive values of K_P and K_D and bounded disturbance $D(s)$.

The **tracking error** $E(s)$ for a step reference input $\Theta^d(s) = \theta_d/s$ and a constant disturbance $D(s) = D/s$:

$$E(s) = \Theta^d(s) - \Theta(s) = \frac{Js^2 + Bs}{Js^2 + (B + K_D)s + K_P}\frac{\theta_d}{s} + \frac{1/r}{Js^2 + (B + K_D)s + K_P}\frac{D}{s}$$

Using the final value theorem:
$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{D/r}{K_P}$$

e_{ss} is smaller for larger gear ratio and larger K_P.

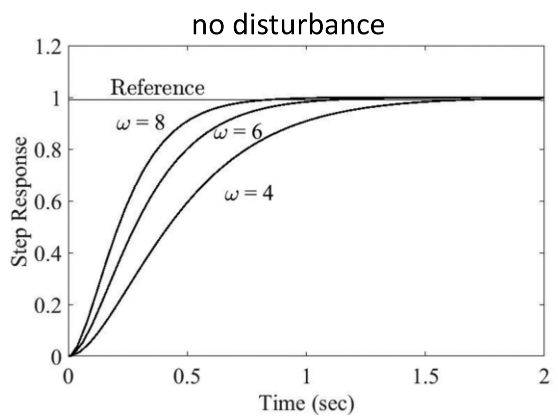
PD Controller

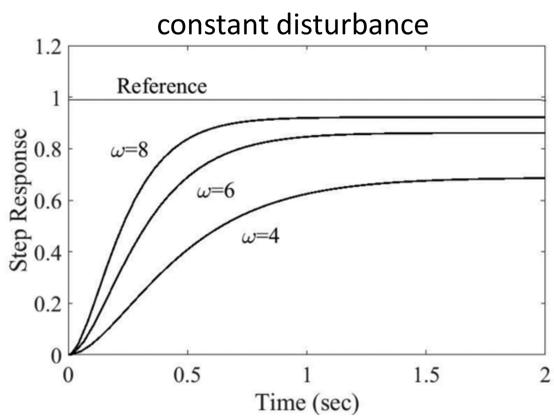
Since the closed-loop system is 2nd-order, the step response is determined by the natural frequency ω and damping ratio ζ.

$$s^2 + \frac{(B + K_D)}{J}s + \frac{K_P}{J} \equiv s^2 + 2\zeta\omega_n s + \omega_n^2 \qquad \Rightarrow \qquad K_P = \omega_n^2 J, \qquad K_D = 2\zeta\omega_n J - B$$

Given a desired value for ω_n and ζ, the gains K_P and K_D can be found.

- It is customary in robotics applications to take the damping ratio $\zeta = 1$ so that the response is critically damped. In this context, ω_n determines the speed of response.
- Let k_r be the effective **joint stiffness**. The joint resonant frequency is then $\omega_r = \sqrt{k_r/J}$. It is common engineering practice to limit ω_n to no more than half of ω_r to avoid excitation of the joint resonance.





PID Controller

To remove the steady-state error e_{ss} due to the disturbance entirely, we can add an integral control:

$$\Theta(s) = \frac{(K_P s + K_I)}{Js^3 + (B + K_D)s^2 + K_P s + K_I}\Theta^d(s) - \frac{s}{Js^3 + (B + K_D)s^2 + K_P s + K_I}D(s)$$

Applying the Routh-Hurwitz criterion to $(Js^3 + (B + K_D)s^2 + K_P s + K_I)$, it follows that the closed-loop system is **stable** if the gains are positive, and in addition,

$$K_I < \frac{(B + K_D)K_P}{J} \qquad (*)$$

- A common design rule-of-thumb for PID control is to first set $K_I = 0$ and design the proportional and derivative gains, K_P and K_D, to achieve the desired transient behavior (maximum overshoot, settling time, etc.) and then to choose K_I within the limits imposed by $(*)$ to remove the steady-state error.

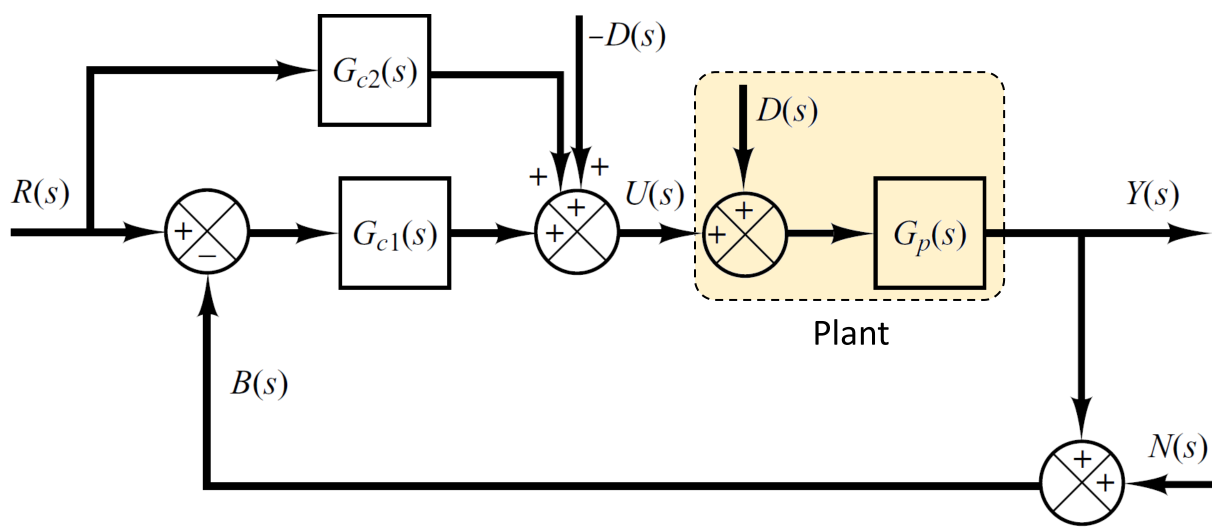
Feedforward Control

Feedforward Control

Feedback control is powerful in practice; however, a feedback-only-controller is not necessarily the best architecture because it generates the necessary control output only after an error (due to change in the reference input or disturbance) is generated. Adding one or more **Feedforward** paths to a feedback controller can **predict** ahead of time what control output is required before the effect of the changes shows up in the system output, and consequently, improve the system performance (e.g., accuracy and speed).

The feedforward control can be used when

1. The system model $G_p(s)$ is known,
2. the reference signal $R(s)$ is a time-varying signal, and/or
3. the disturbance $D(s)$ is measurable.



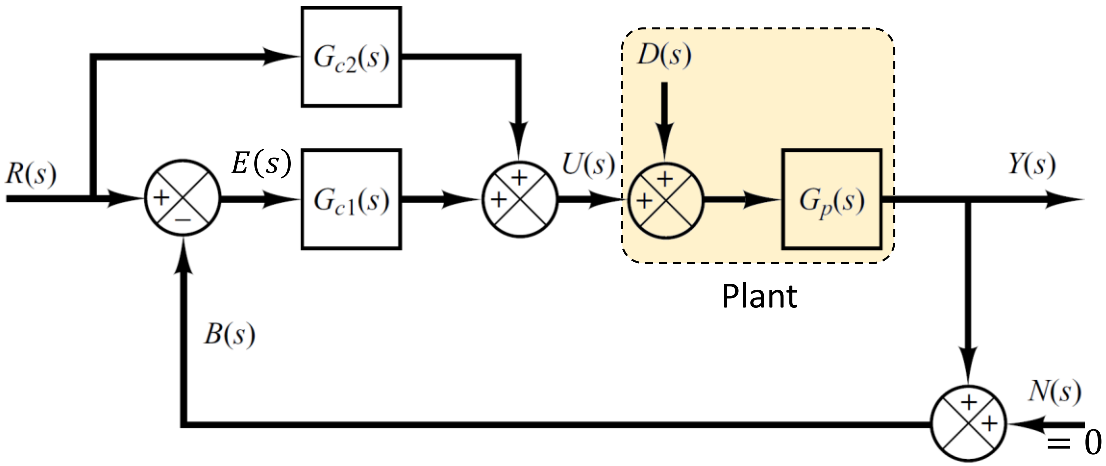
Reference Input Feedforward Control

For $E(s) = 0$ (when $D(s) = 0$):

$$R(s) = Y(s) = U(s)G_p(s)$$

$$\Rightarrow \quad U(s) = R(s)\frac{1}{G_p(s)}$$

$$\Rightarrow \quad G_{c2}(s) = \frac{1}{G_p(s)} = G_p^{-1}(s)$$



- For stability of the closed-loop system, we require that in addition to stability of the feedback system, the feedforward transfer function $G_{c2}(s)$ must itself be stable (or **Hurwitz**). Since $G_{c2}(s) = G_p^{-1}(s)$, $G_p(s)$ must be also **minimum phase**. Moreover, since $G_p(s)$ is strictly **proper**, $G_{c2}(s)$ is not proper.

- In the presence of disturbance $D(s)$:

$$E(s) = R(s) - Y(s) = R(s) - \left(R(s)\frac{Y(s)}{R(s)} + D(s)\frac{Y(s)}{D(s)}\right)$$

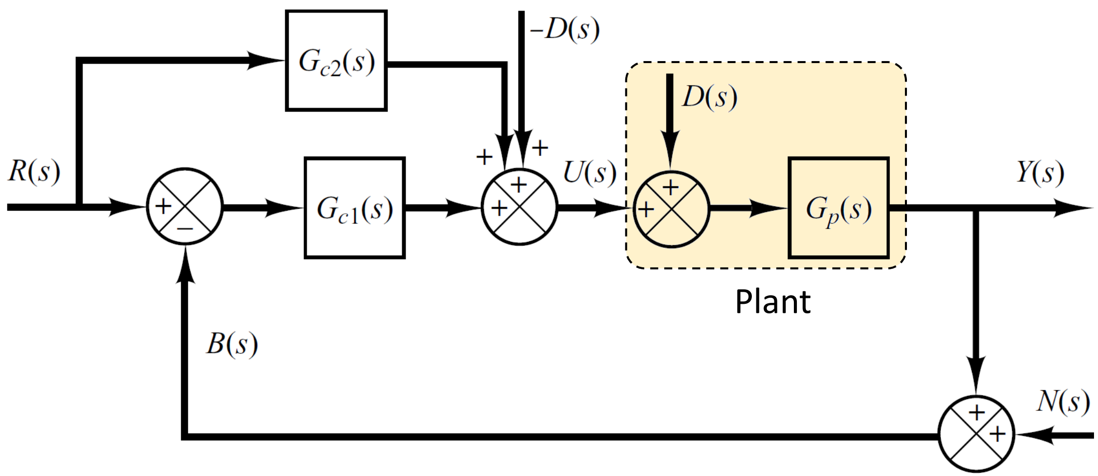
$$\frac{Y(s)}{R(s)} = \frac{\left(G_{c1}(s) + G_{c2}(s)\right)G_p(s)}{1 + G_p(s)G_{c1}(s)}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + G_p(s)G_{c1}(s)}$$

Transfer function $G(s)$ is **Hurwitz** if all its poles are strictly in the left-half complex plane.
Transfer function $G(s)$ is **Minimum Phase** if all its zeros are strictly in the left-half complex plane.
Transfer function $G(s)$ is **Proper** if the degree of the numerator does not exceed the degree of the denominator.

Feedforward Disturbance Rejection Control

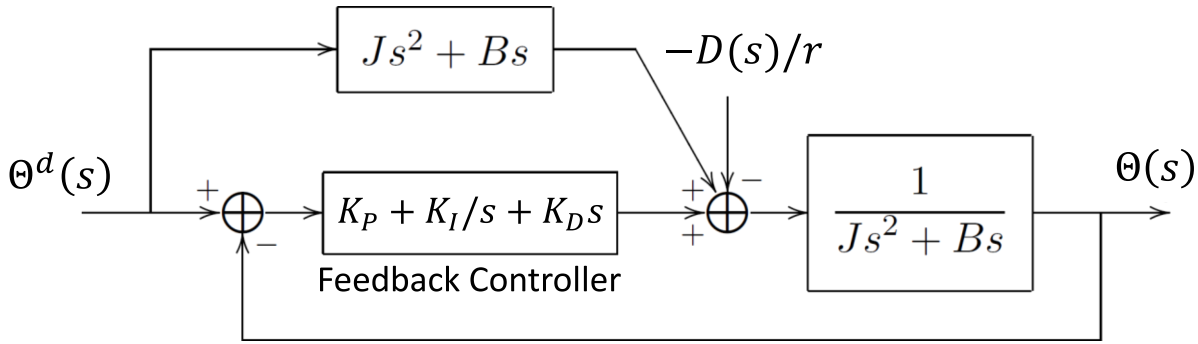
If the disturbance $D(s)$ is measurable in the system-input-level, it can be fed forward as $-D(s)$.



Motion or Tracking Control

Motion or Tracking Feedforward Control

Let's consider the problem of tracking a reference time-varying trajectory $\theta^d(t)$ of a joint using the notion of feedforward control.



- We can show that, in the absence of disturbances, the closed-loop system will track any desired trajectory $\theta^d(t)$ provided that the closed-loop system is stable.
- If the feedback controller is a PID controller, steady-state tracking error $e_{ss} = \lim_{s \to 0} sE(s)$ to a **step disturbance** will approach zero asymptotically (i.e., $\theta(t)$ will track any reference trajectory $\theta^d(t)$).

Motion or Tracking Feedforward Control

- The control law in the time domain:

$$u(t) = \underbrace{J\ddot{\theta}^d(t) + B\dot{\theta}^d(t)}_{\text{feedforward signal}} + K_P e(t) + K_I \int_0^t e(\sigma)d\sigma + K_D \dot{e}(t)$$

$$\underbrace{e(t) = \theta^d(t) - \theta(t)}_{\text{tracking error}}$$

Note: $Js^2 + Bs$ is not a proper transfer function. However, since the derivatives of the reference trajectory $\theta^d(t)$ are known and precomputed, the implementation of the feedforward control does not require differentiation of an actual signal.

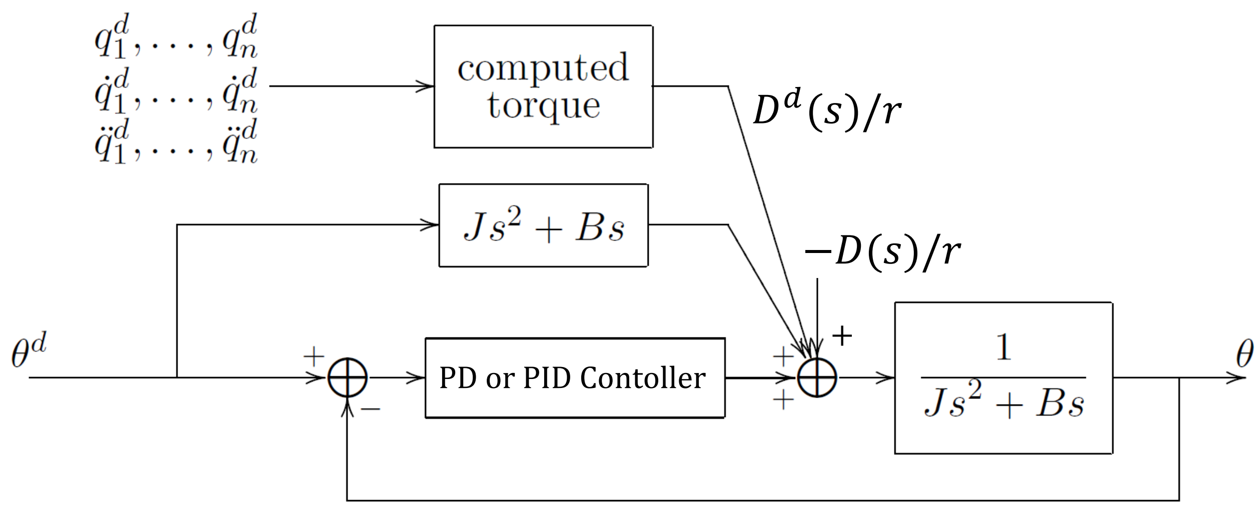
- By considering the joint model $J\ddot{\theta}(t) + B\dot{\theta}(t) = u(t) - d(t)/r$, the close-loop dynamic is

$$J\ddot{e}(t) + (B + K_D)\dot{e}(t) + K_P e(t) + K_I \int_0^t e(\sigma)d\sigma = d(t)/r$$

Computed-Torque Feedforward Control
(Feedforward Disturbance Rejection)

In the presence of a time-varing disturbance $-d(t)/r$, the previous method cannot asymptotically track the reference trajectory. Since, here, this term is not completely unknown, and we can feed forward an approximation of that (i.e., $d_k^d(t)/r$) to anticipate and cancel the effects of this disturbance which is due to the nonlinear coupling inertia, Coriolis, centripetal, and gravitational forces arising from the motion of the manipulator. Note that $d_k^d(t)$ is computed using the desired joint positions, velocities, and accelerations trajectories (i.e., $\boldsymbol{q}^d(t)$, $\dot{\boldsymbol{q}}^d(t)$, and $\ddot{\boldsymbol{q}}^d(t)$).

$$d_k^d(t) = \sum_{j=1, j\neq k}^{n} m_{kj}\left(\boldsymbol{q}^d(t)\right)\ddot{q}_j^d(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ijk}\left(\boldsymbol{q}^d(t)\right)\dot{q}_i^d(t)\dot{q}_j^d(t) + g_k\left(\boldsymbol{q}^d(t)\right)$$



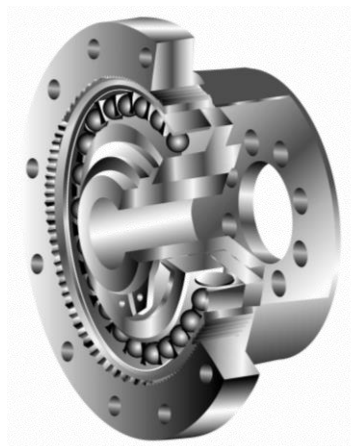
Joint Flexibility

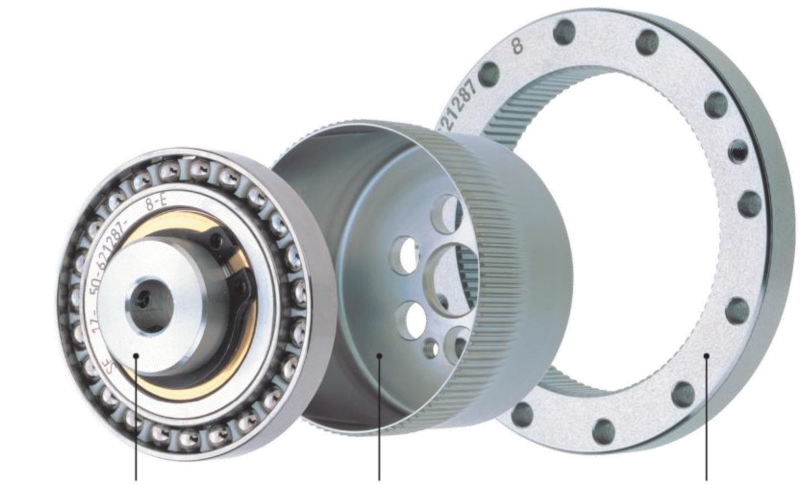
Joint Flexibility or Elasticity

An effect that limits the achievable performance of a manipulator is flexibility in the motor shaft and/or drive train, which is known as **Joint Flexibility** or **Joint Elasticity**.

- For many manipulators, particularly those using **strain wave gears** or **harmonic gears**, for torque transmission, the torsional flexibility in the gears is significant.

Harmonic gears have no/low backlash, high torque transmission, and compact size.





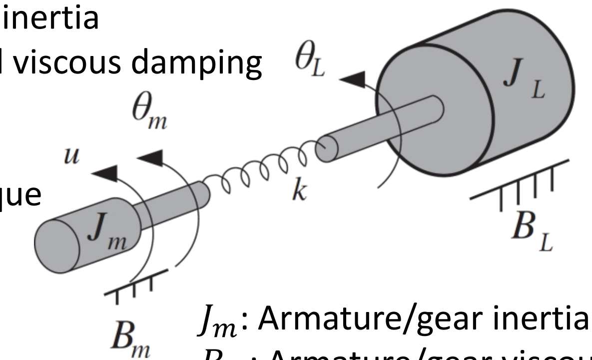
- In addition, joint flexibility can be caused by effects such as shaft windup, bearing deformation, and compressibility of the hydraulic fluid in hydraulic robots.

Idealized Model for Joint Flexibility
(Drive-Train Dynamics in Laplace Domain)

Consider an idealized model, consisting of an actuator connected to a load through a torsional spring representing the joint flexibility with effective stiffness k of the harmonic gear with gear ratio 1.

J_L: Load inertia
B_L: Load viscous damping

u: input motor torque

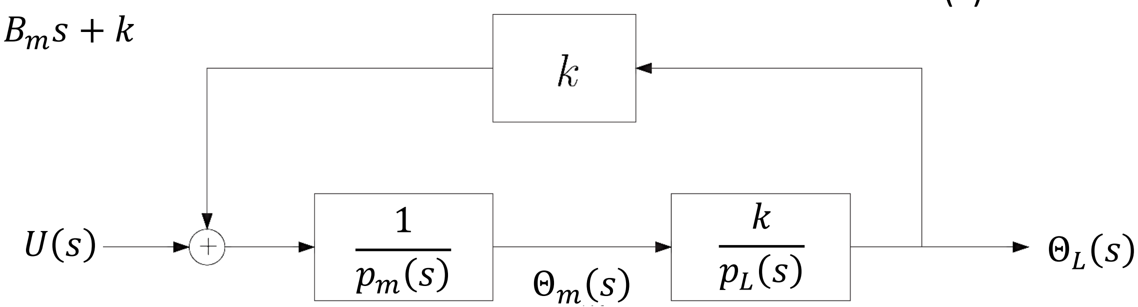


J_m: Armature/gear inertia
B_m: Armature/gear viscous damping

$$J_L\ddot{\theta}_L + B_L\dot{\theta}_L + k(\theta_L - \theta_m) = 0$$
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m + k(\theta_m - \theta_L) = u$$
$$\Downarrow$$
$$p_L(s)\Theta_L(s) = k\Theta_m(s)$$
$$p_m(s)\Theta_m(s) = k\Theta_L(s) + U(s)$$

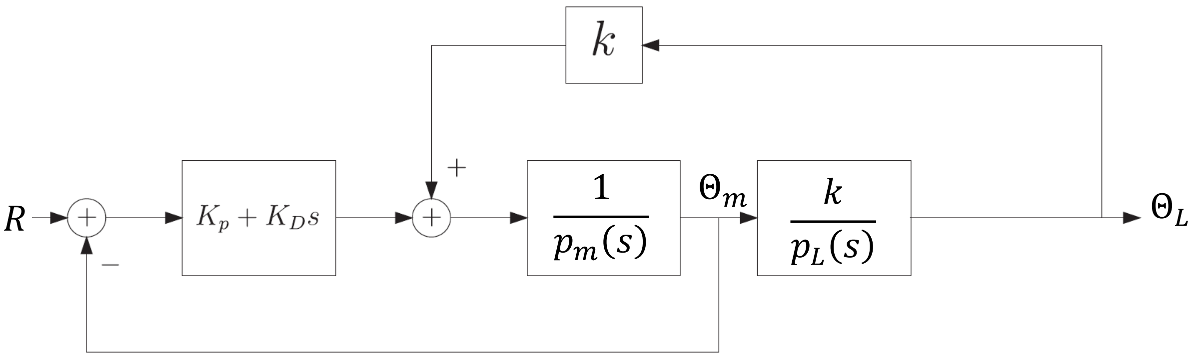
$$\Rightarrow \frac{\Theta_L(s)}{U(s)} = \frac{k}{p_L(s)p_m(s) - k^2} \quad (*)$$

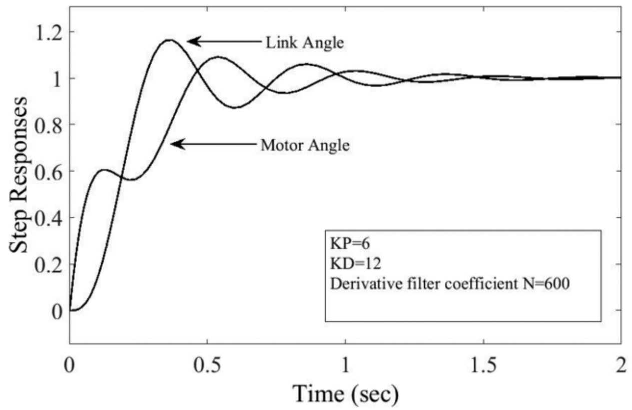
$$p_L(s) = J_L s^2 + B_L s + k$$
$$p_m(s) = J_m s^2 + B_m s + k$$



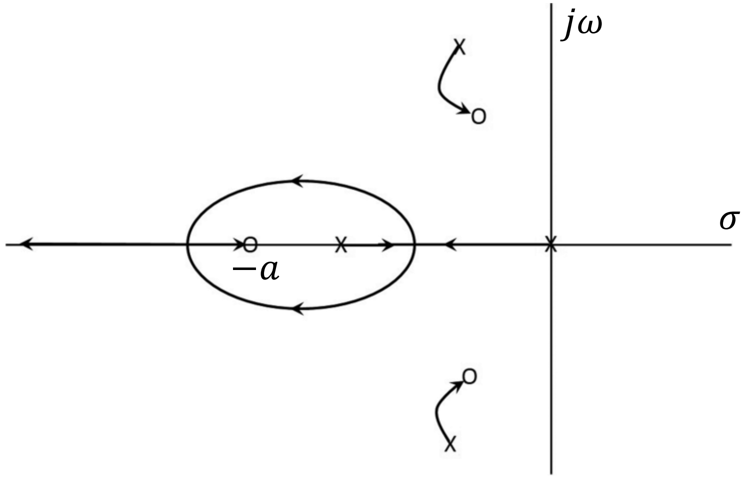
- Depending on whether the position/velocity sensors are placed on the motor shaft or on the load shaft, we can have two types of feedback controller.

PD Control with Motor Angle θ_m Feedback



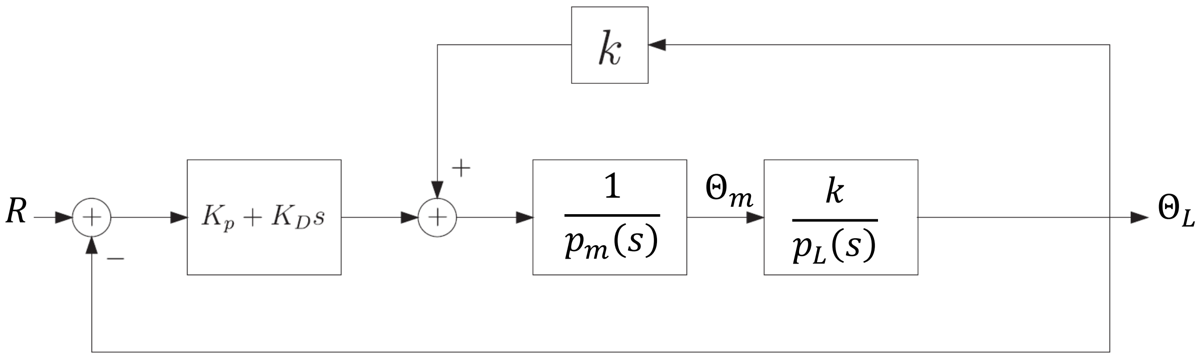


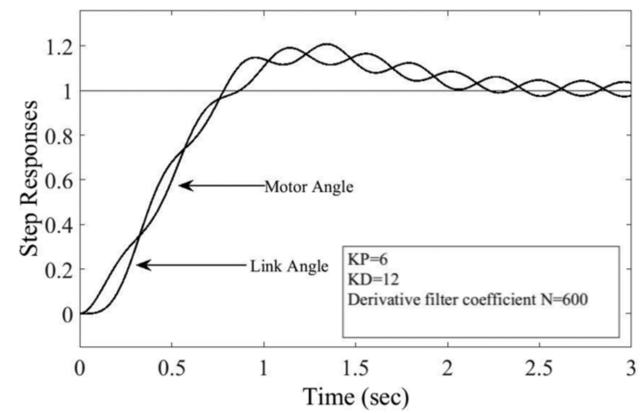
Let's rewrite the PD control $K_P + K_D s$ as $K_D(s + a)$. Then, for any given a we can plot the root locus for the closed-loop systems in terms of K_D.



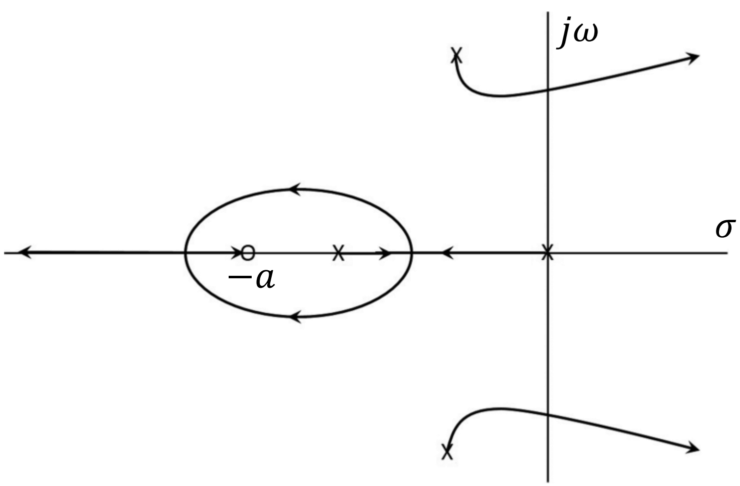
- The closed-loop system is **stable** for all values of the gain K_D, however, the presence of the open-loop zeros near the $j\omega$-axis may result in undesirable oscillations.
- The poor relative stability means that disturbances and other unmodeled dynamics could render the system unstable.
- In practice, k is large and B_L and B_m are small. This places the open-loop poles of the system near the $j\omega$-axis and results in a difficult system to control.

PD Control with Load Angle θ_L Feedback





Let's rewrite the PD control $K_P + K_D s$ as $K_D(s + a)$. Then, for any given a we can plot the root locus for the closed-loop systems in terms of K_D.



- The closed-loop system is **unstable** for large K_D. The critical value of K_D (i.e., the value of K_D for which the system becomes unstable), can be found from the Routh-Hurwitz criterion.
- In this case, we should limit the K_D so that the closed-loop poles remain within the left half-plane with a reasonable stability margin.
- In practice, k is large and B_L and B_m are small. This places the open-loop poles of the system near the $j\omega$-axis and results in a difficult system to control.

Drive-Train Dynamics in State-Space Domain

$$J_L\ddot{\theta}_L + B_L\dot{\theta}_L + k(\theta_L - \theta_m) = 0$$
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m + k(\theta_m - \theta_L) = u$$

$$x_1 = \theta_L \quad x_3 = \theta_m$$
$$x_2 = \dot{\theta}_L \quad x_4 = \dot{\theta}_m$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{k}{J_L}x_1 - \frac{B_L}{J_L}x_2 + \frac{k}{J_L}x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{k}{J_m}x_1 - \frac{B_\ell}{J_m}x_4 - \frac{k}{J_m}x_3 + \frac{1}{J_m}u$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u \quad (**)$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_L} & -\frac{B_L}{J_L} & \frac{k}{J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & -\frac{k}{J_m} & \frac{B_m}{J_m} \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix}$$

If we choose $y = \theta_L = x_1$, then $y = \boldsymbol{C}\boldsymbol{x}$ $(***)$ $\quad \boldsymbol{C} = [1,0,0,0]$