Ch8: Friction

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Dry Friction

Friction

Friction is a **tangential force** that resists the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction:

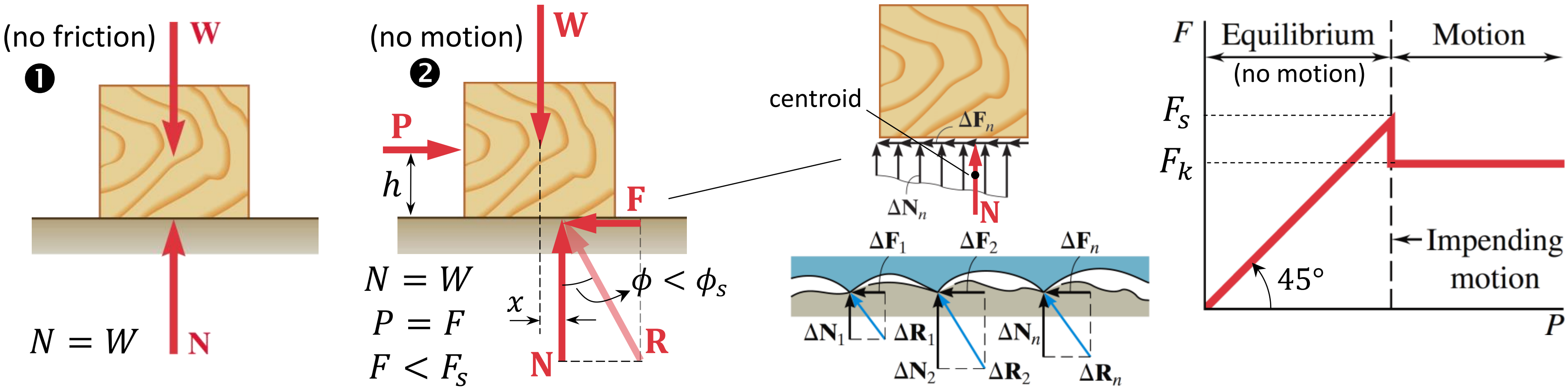
- **Dry friction** (or **Coulomb friction**) is a force that resists the relative lateral motion of two unlubricated solid surfaces in contact. The frictional force always acts tangent to the surface at the points of contact, and is directed so as to oppose the motion (or a tendency to motion) between the surfaces.

- **Fluid friction** describes the friction between either layers of fluid (liquid or gas) or a fluid and the surface of a body that are moving relative to each other [MEC 364].

- **Internal friction** is a force that resists motion between the elements making up a solid material while it undergoes deformation.

Laws of Dry Friction

Suppose that we apply a horizontal force **P** to the block and gradually increase it from zero.



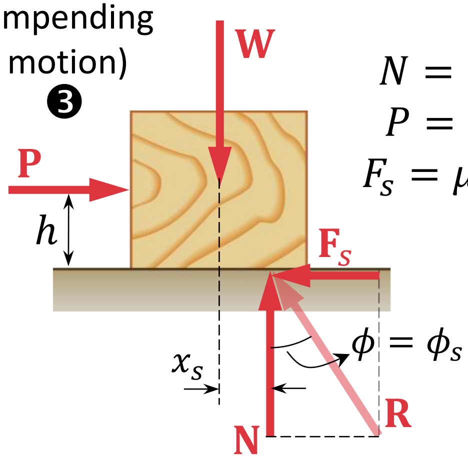
By increasing the force **P**, the **static-friction force F** also increases (until it reaches a maximum value F_s), and the force **N** moves to the right so that the couples formed by **P** and **F** and by **W** and **N** remain balanced ($Wx = Ph$).

The **static-friction force F** is actually the resultant of a great number of forces ΔF_n acting over the entire surface of contact between the block and the plane. These forces are due to the **irregularities** of the surfaces in contact and, to a certain extent, to molecular attraction.

Laws of Dry Friction (cont.)

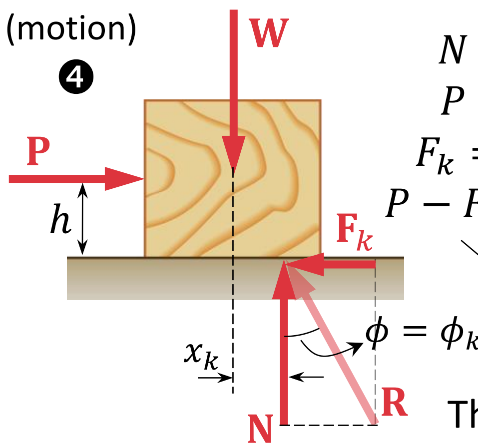
As soon as the block has started in motion, the magnitude of $\mathbf{F}$ drops from F_s to a lower value F_k. $\mathbf{F}_k$ (or $\mathbf{F}_s$) has a sense opposite to the sense of (impending) motion.

(impending motion) ❸

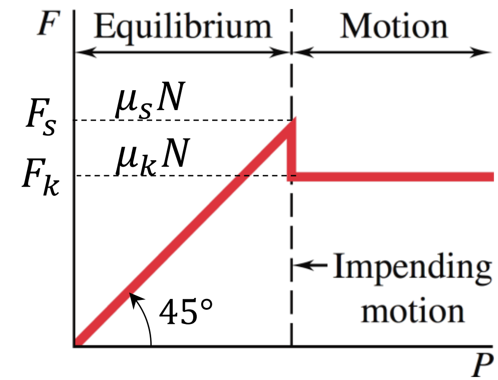


$$N = W$$
$$P = F_s$$
$$F_s = \mu_s N$$

(motion) ❹



$$N = W$$
$$P > F_k$$
$$F_k = \mu_k N$$
$$P - F_k = ma$$



The block keeps sliding with increasing velocity while the **kinetic-friction force** F_k remains approximately constant.

Experimentally:

Static friction: $F_s = \mu_s N$ — μ_s: coefficient of static friction

Kinetic friction: $F_k = \mu_k N$ — μ_k: coefficient of kinetic friction

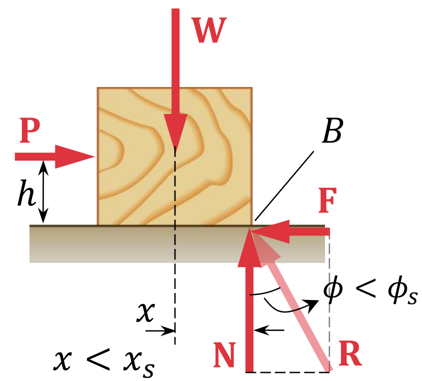
Angle of static friction: $\tan \phi_s = \dfrac{F_s}{N} = \mu_s$

Angle of kinetic friction: $\tan \phi_k = \dfrac{F_k}{N} = \mu_k$

$(F_k < F_s)$
$(\mu_k < \mu_s)$
$(\phi_k < \phi_s)$

Laws of Dry Friction (cont.)

Note: If $\mathbf{N}$ (or $\mathbf{R}$) reaches edge B before F reaches its maximum value F_s, the block starts to **tip** about edge B before it can start sliding.



Coefficients of Friction

- The coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. However, they depend strongly on the **nature** of the surfaces in contact.
- μ_k is approximately 25% smaller than μ_s.
- The coefficients of friction are **dimensionless** quantities.

Contact	μ_s
Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

Problems Involving Dry Friction

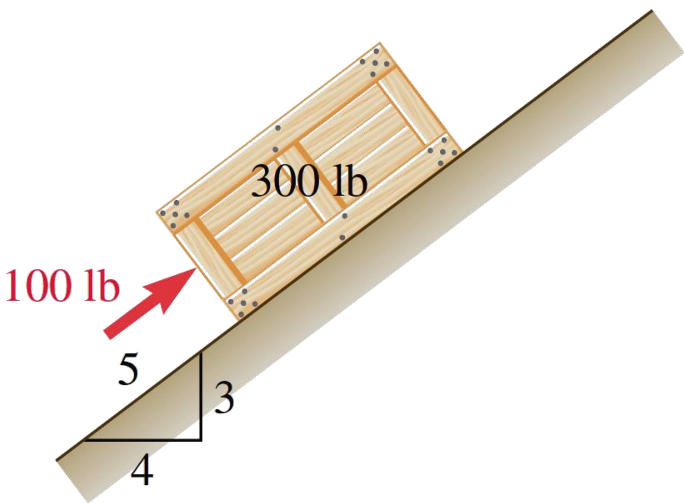
(1) If we **know** that the impending motion (or motion) occurring in a given direction in a contact, the friction force should be shown in the free-body diagram with a sense opposite to that of the impending motion (or motion) and with a magnitude $F_s = \mu_s N$ (or $F_k = \mu_k N$).

(2) If we **do not know** the condition of impending motion (or motion) in a contact, we first assume static equilibrium and solve for the friction force F necessary for equilibrium. If F is a **negative** scalar, the correct sense of F is the reverse of that which was assumed.

- If $|F| \leq \mu_s N$, the body is in static equilibrium as assumed.
- If $|F| > \mu_s N$, the assumption of static equilibrium is invalid, and motion occurs in the opposite direction of F obtained from the assumption. The friction force is $F = \mu_k N$.

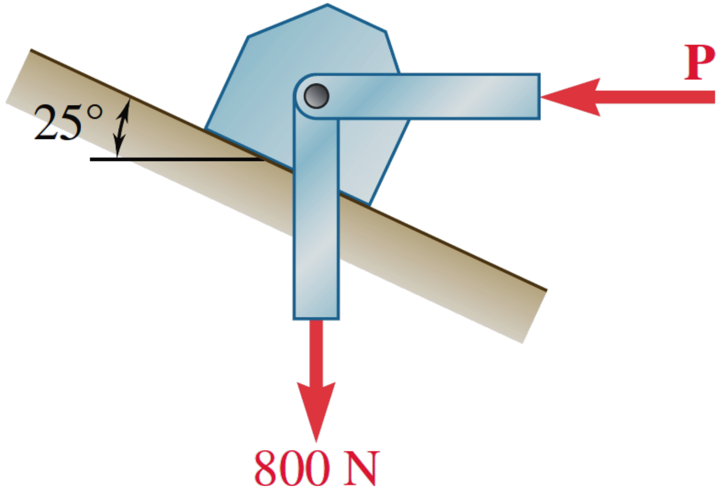
Sample Problem 8.1

A 100-lb force acts as shown on a 300-lb crate placed on an inclined plane. The coefficients of friction between the crate and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the crate is in equilibrium, and find the value of the friction force.



Sample Problem 8.2

A support block is acted upon by two forces as shown. Knowing that the coefficients of friction between the block and the incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the force $\mathbf{P}$ required to (a) start the block moving up the incline, (b) prevent it from sliding down.

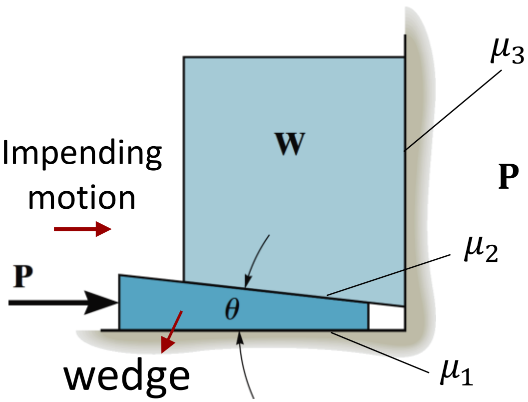


Wedges

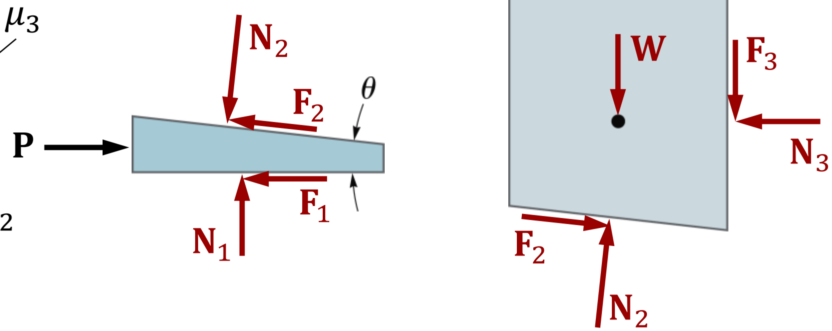
Wedges

A **Wedge** is used to transform an applied force into a force directed at almost right angles to the applied force, and also produce small adjustments in the position of heavy loads. Wedges largely depend on friction to function, and 4 situations may happen:

- Consider a wedge which is used to lift the block by applying a minimum force $\mathbf{P}$. ❶



FBDs:



(Weight of the wedge is usually negligible compared with the other forces involved.)

- If $\mathbf{P}$ is removed and the wedge remains in place, the wedge is **self-locking** ❷, otherwise, a min. $\mathbf{P}$ must act to hold the block ❸ (in this case, the coefficients of friction are very small or θ is large).

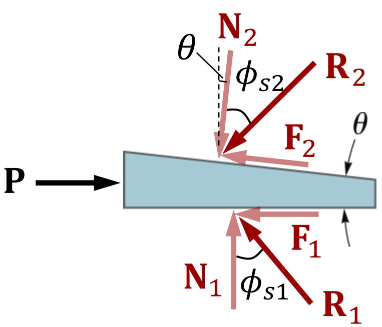
- If the wedge is self-locking and the block is to be lowered ❹, a pull $\mathbf{P}$ on the wedge will be required (and the frictional forces will all act in a sense opposite to that shown).

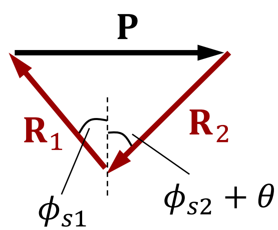
Wedges

Note: It is usually more convenient to replace each pair of normal and friction forces by their resultant $\mathbf{R}$. Now, each free body is subjected to only three forces, and we can solve the problem by drawing the corresponding **force triangles** and using the laws of sine or cosine.

For example, impending motion for lifting the block:

1

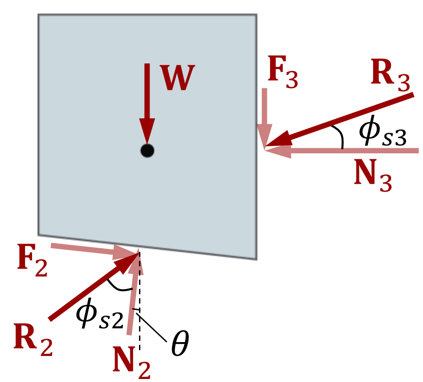


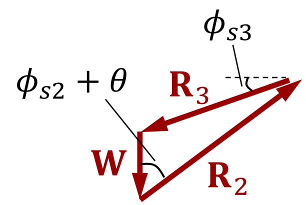


$$\tan\phi_{s1} = \mu_1$$

$$\tan\phi_{s2} = \mu_2$$

$$\tan\phi_{s3} = \mu_3$$



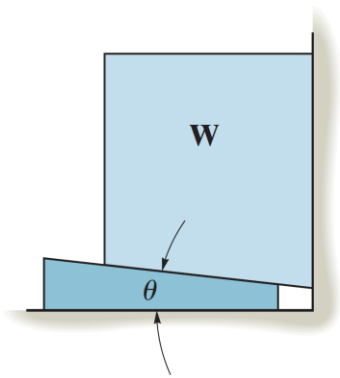


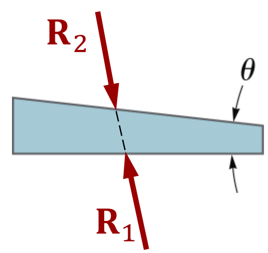
Note: In order for the wedge to lift the block or slide out of its space, slippage must occur at all surfaces simultaneously.

Wedges

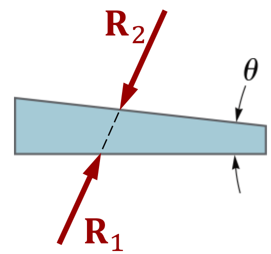
Note: Rection forces $\mathbf{R}_1$ and $\mathbf{R}_2$ when $\mathbf{P}$ is removed and the <u>wedge is **self-locking**</u> are **colinear**:

2





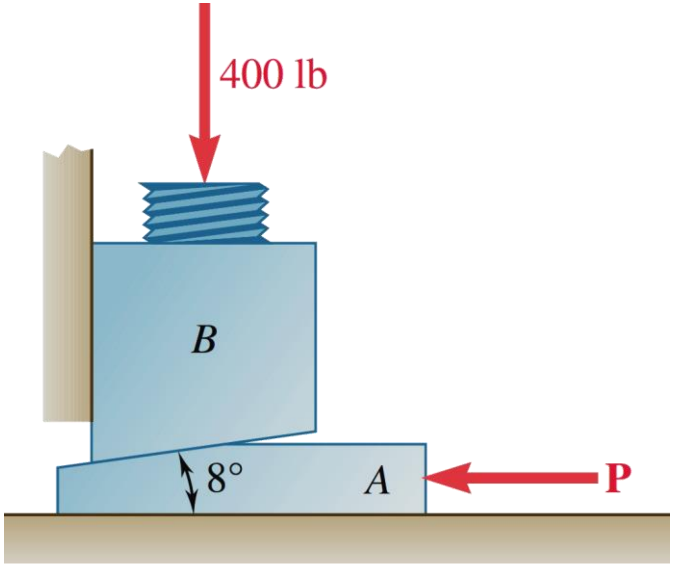
or



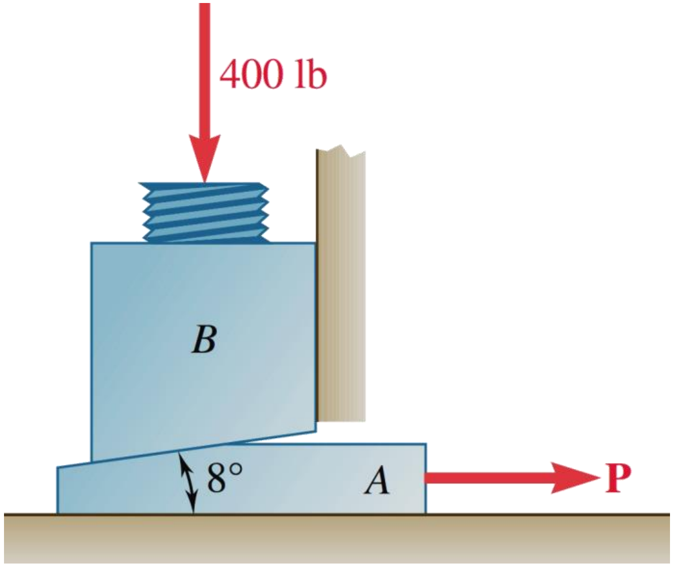
Note: Since the locations of the resultant normal forces are usually not known, the moment equilibrium equations will not be considered in these problems.

Sample Problem 8.5

The position of the machine block B is adjusted by moving the wedge A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force $\mathbf{P}$ required to (a) raise block B, (b) lower block B.



(a)



(b)

Screws

Square-Threaded Screws

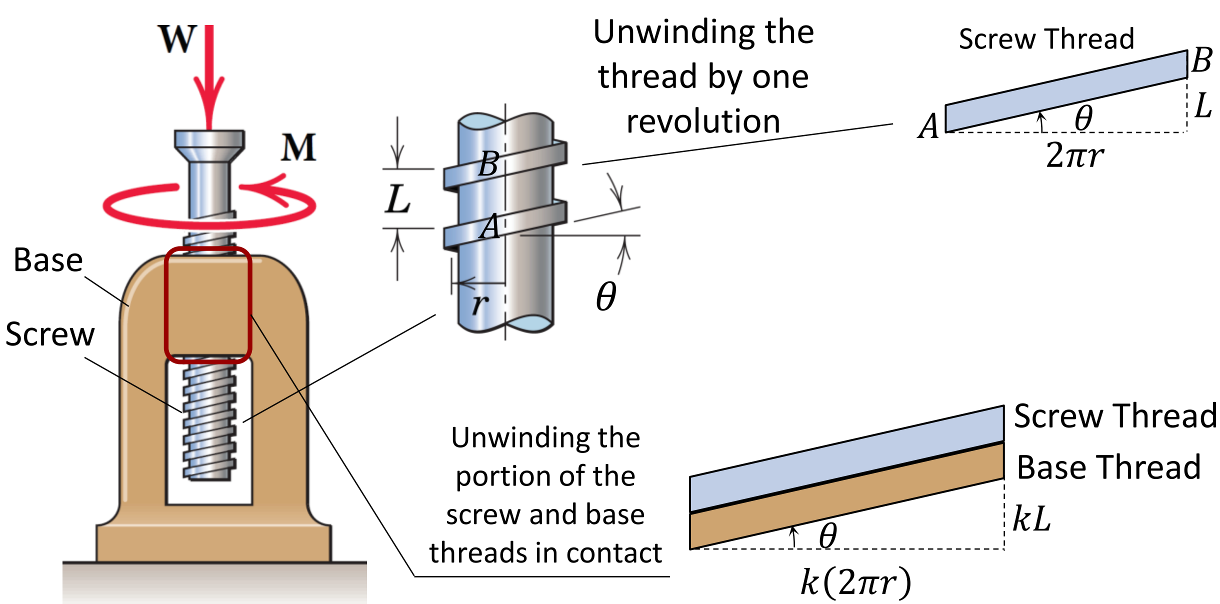
Square-threaded screws are frequently part of jacks, presses, clamps, and other mechanisms to transmit power or motion from one part of the machine to another.

Consider a screw as part of a jack carrying an axial load $\mathbf{W}$. By applying a couple moment $\mathbf{M}$ about the screw axis, the screw can move the load $\mathbf{W}$. Contact between the screw and the base takes place along a portion of their threads.

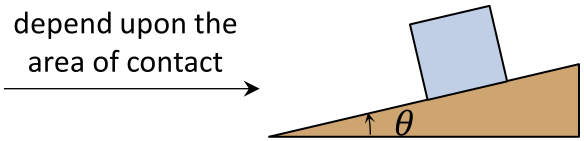
r: mean radius of the thread

$\theta = \tan^{-1}(\frac{L}{2\pi r})$: lead angle

L: lead of the screw (advancement per revolution).

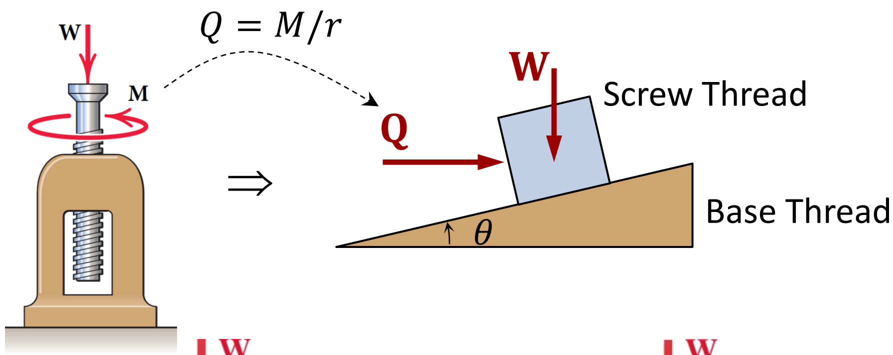


Friction does not depend upon the area of contact



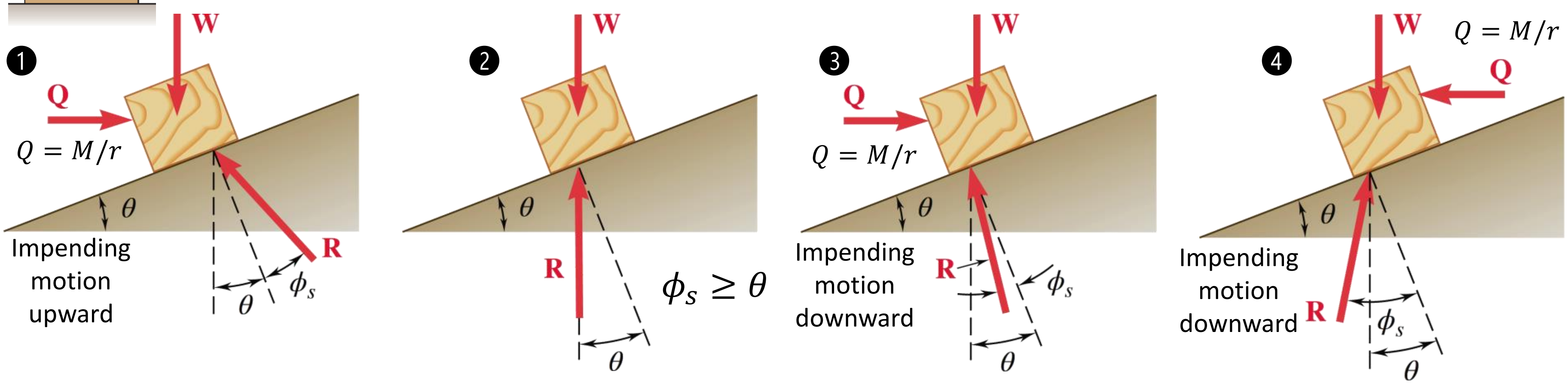
* Similar to the analysis of a block sliding along an inclined plane.

Square-Threaded Screws



The horizontal force **Q** has the same effect as the couple moment **M**; thus, $Q = M/r$.

Four different loading scenarios:



Raising the load:

$$\begin{cases} Q = R\sin(\theta + \phi_s) \\ W = R\cos(\theta + \phi_s) \end{cases}$$

$$\Downarrow$$

$$M = rW\tan(\theta + \phi_s)$$

If $\phi_s \geq \theta$, the screw is **self-locking** (it remains in place under any axial load when the moment is removed).

If $\phi_s < \theta$, the screw unwinds under the load (it is necessary to apply a moment to keep it in place).

$$M = rW\tan(\theta - \phi_s)$$

Lowering the load when the screw is self-locking (i.e., $\phi_s > \theta$)

$$M = rW\tan(\phi_s - \theta)$$

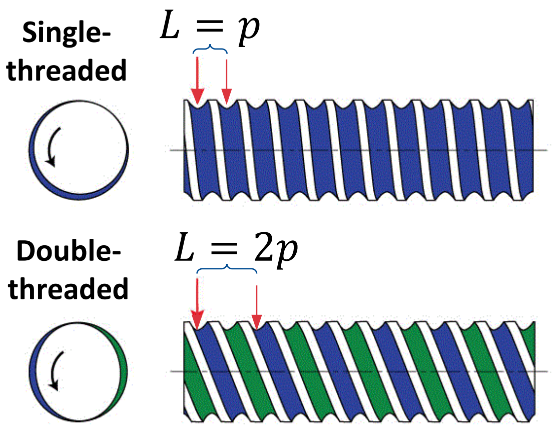
Remark

Do not confuse the **pitch** of a screw p with the **lead** of a screw L!

- The **pitch** p is the distance between two consecutive threads.
- The **lead** L is the distance the screw advances in one full turn.

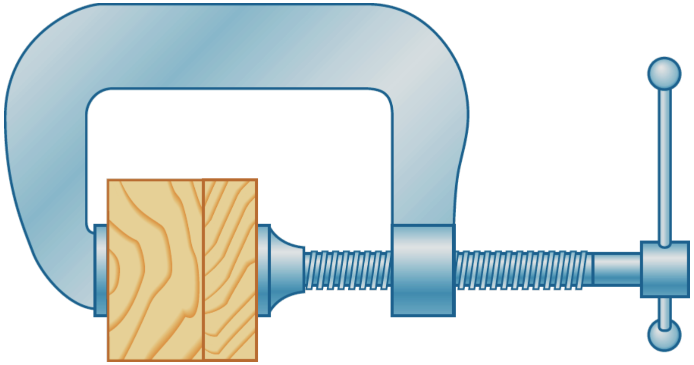
In a single-threaded screw, the lead and the pitch are equal ($L = p$), in a double-threaded screw, the lead is twice the pitch ($L = 2p$), in a triple-threaded screw, it is three times the pitch ($L = 3p$), etc.

One full turns:



Sample Problem 8.6

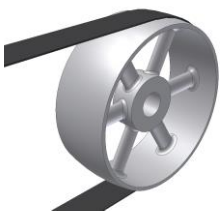
A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread with a mean diameter of 10 mm and a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$. (a) If a maximum couple of 40 N·m is applied in tightening the clamp, determine the force exerted on the pieces of wood, (b) the couple required to loosen the clamp.



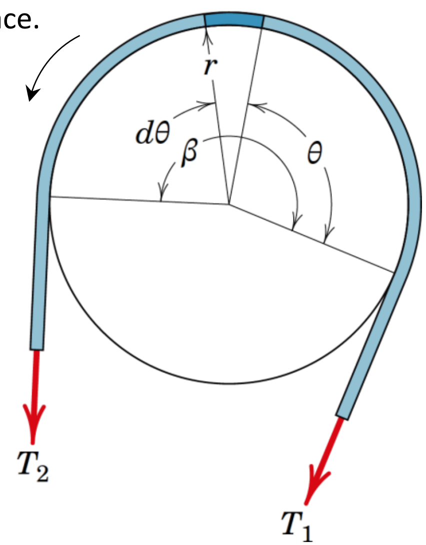
Belts

Flat Belts

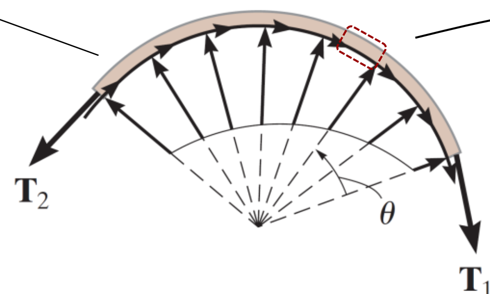
Consider a flat belt passing over a fixed curved surface. The total angle of belt-to-surface contact is β, and the coefficient of friction between the two surfaces is μ. We want to determine the relation between the tensions T_1 and T_2 when the belt is just about to slide ($\mu = \mu_s$) or slides ($\mu = \mu_k$) toward T_2.



Motion or impending motion of belt relative to surface.

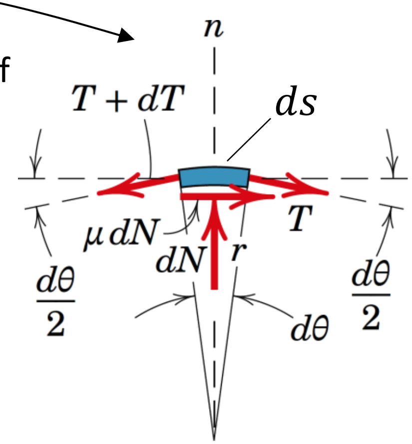


The normal and frictional forces, acting at different points along the belt, will vary both in magnitude and direction.



Because of friction, $T_2 > T_1$

A differential element ds of the belt.

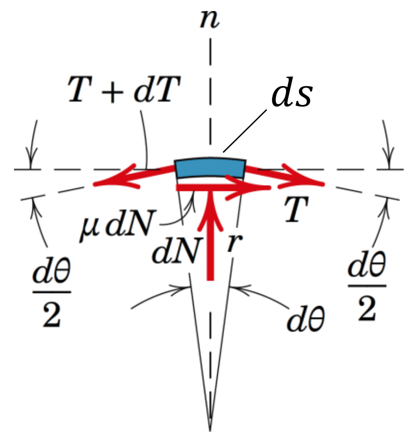


Flat Belts

$$\Sigma F_t = 0: \; T\cos\frac{d\theta}{2} + \mu dN = (T + dT)\cos\frac{d\theta}{2}$$
$$\Sigma F_n = 0: \; dN = (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$$

$$\Rightarrow \quad \begin{aligned} \mu dN &= dT \\ dN &= T d\theta \end{aligned} \quad \Rightarrow \quad \frac{dT}{T} = \mu d\theta$$

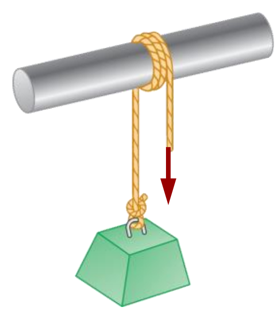
$\sin d\theta/2 \approx d\theta/2$,
$\cos d\theta/2 \approx 1$,
Dropping second-order terms.



$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\beta} d\theta \quad \Rightarrow \quad \boxed{\ln\frac{T_2}{T_1} = \mu\beta} \quad \text{(or)} \quad \boxed{\frac{T_2}{T_1} = e^{\mu\beta}}$$

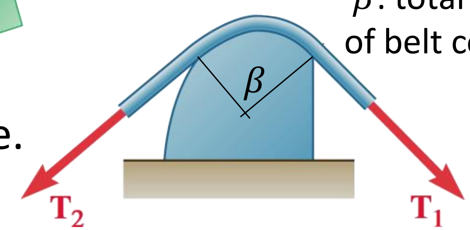
β must be expressed in radians, and
$e \approx 2.718$ is base of the natural logarithm.

Note: If a belt/rope is wrapped around a drum n times, the angle β would be $2\pi n$ radians.



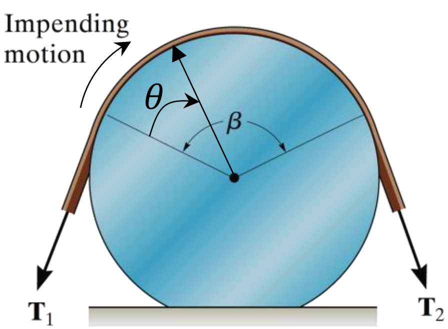
Note: This equation is independent of the shape of the surface (e.g., radius r), thus, it is valid for any noncircular smooth curve.

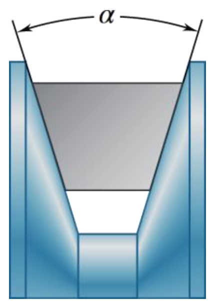
β: total angle of belt contact

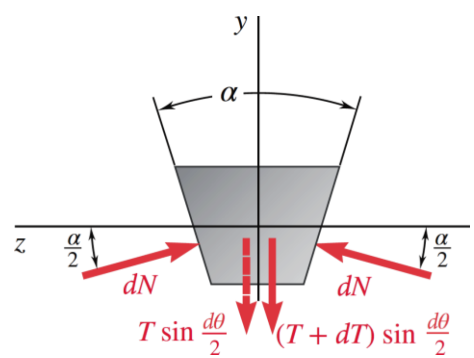


V Belts

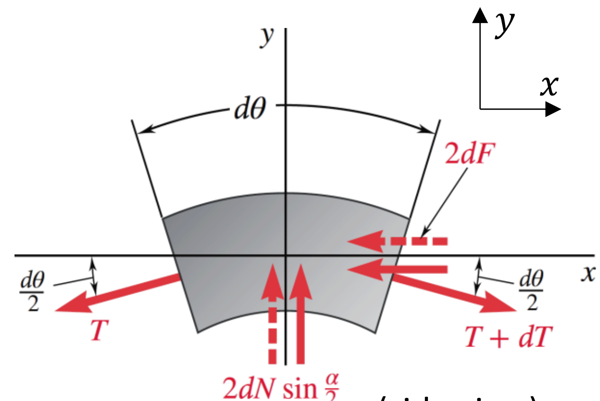
In a **V belt**, contact between belt and pulley takes place **along the sides** of the groove. Similarly, we can determine the relation between the tensions T_1 and T_2 when the belt is just about to slide ($\mu = \mu_s$) or slides ($\mu = \mu_k$) toward T_2.







(cross-section view)



(side view)

$$\Sigma F_x = 0:\ T\cos\frac{d\theta}{2} + 2\mu dN = (T + dT)\cos\frac{d\theta}{2}$$

$$\Sigma F_y = 0:\ -T\sin\frac{d\theta}{2} + 2dN\sin\frac{\alpha}{2} = (T + dT)\sin\frac{d\theta}{2}$$

$\Rightarrow$
$\sin d\theta/2 \approx d\theta/2$,
$\cos d\theta/2 \approx 1$,
Dropping second-order terms.

$$\frac{dT}{T} = \frac{\mu d\theta}{\sin(\frac{\alpha}{2})}$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu}{\sin(\frac{\alpha}{2})} \int_0^{\beta} d\theta \quad \Rightarrow \quad \boxed{\ln\frac{T_2}{T_1} = \frac{\mu\beta}{\sin(\frac{\alpha}{2})}} \quad \text{(or)} \quad \boxed{\frac{T_2}{T_1} = e^{\left(\frac{\mu\beta}{\sin\frac{\alpha}{2}}\right)}}$$

Sample Problem 8.8

A hawser (a thick docking rope) thrown from a ship to a pier is wrapped two full turns around a bollard. The tension in the hawser is 7500 N; by exerting a force of 150 N on its free end, a dockworker can just keep the hawser from slipping. (a) Determine the coefficient of friction between the hawser and the bollard. (b) Determine the tension in the hawser that could be resisted by the 150-N force if the hawser were wrapped three full turns around the bollard.