Ch3: Nodal and Mesh Analyses

Nodal Analysis

Overview

Two powerful techniques for analyzing any linear circuit:

- **Nodal Analysis**, which is based on Kirchhoff's current law (KCL).
- **Mesh Analysis**, which is based on Kirchhoff's voltage law (KVL).

❖ With the two techniques, we can obtain a set of simultaneous equations that are then solved to obtain the required values of current or voltage.

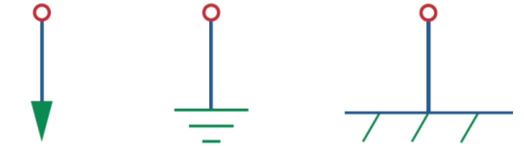
❖ A set of simultaneous equations may be solved by Cramer's rule or computationally (e.g., using MATLAB).

Nodal Analysis (or Node-Voltage Method)

- If instead of using the <u>element voltages</u> as circuit variables we use <u>node voltages</u> as circuit variables, the number of simultaneous equations can be reduced.
- Given a circuit with n nodes, the nodal analysis is accomplished via four steps:

1. Select a node as the reference or datum or ground node. Assign voltages $v_1, v_2, \dots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Label the currents in each branch arbitrarily, except for the branches with current sources.
3. Apply KCL at nodes with unknown voltage. Use Ohm's law to express the branch currents in terms of node voltages.
   Special cases: circuits with voltage sources.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Note: The reference node is commonly called the **ground** since it is assumed to have **zero potential**.



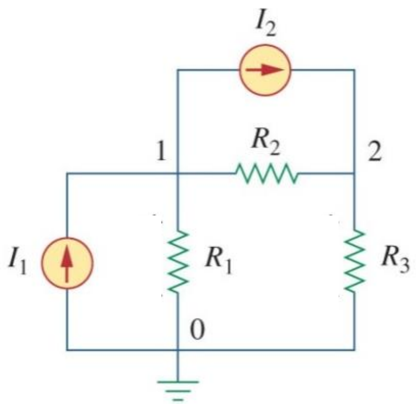
Applying Nodal Analysis

- Let's apply nodal analysis to this circuit to see how it works.
- This circuit has a node that is designed as ground. We will use that as the reference node (node 0). The remaining two nodes are designed 1 and 2 and assigned voltages v_1 and v_2.
- Label the currents in each branch arbitrarily, except for the branches with current sources.
- Now apply KCL to each node:

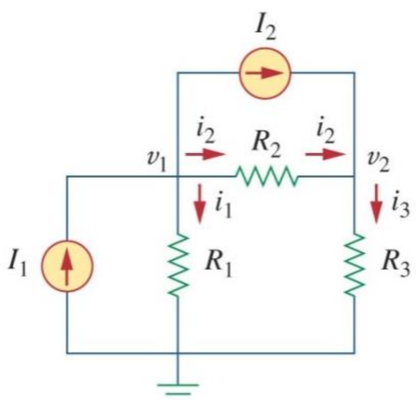
At node 1: $I_1 = I_2 + i_1 + i_2$

At node 2: $I_2 + i_2 = i_3$

Three variables i_1, i_2, and i_3, but only two equations!



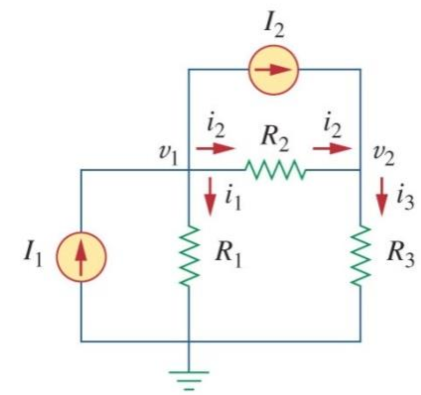
(a)



(b)

Applying Nodal Analysis (cont.)

- We can now use Ohm's law to express the unknown currents i_1, i_2, and i_3 in terms of node voltages v_1, v_2.
- In doing so, keep in mind that current flows from high potential to low. From this we get:



$$I_1 = I_2 + i_1 + i_2$$
$$I_2 + i_2 = i_3$$

$$i_1 = \frac{v_1 - 0}{R_1}$$
$$i_2 = \frac{v_1 - v_2}{R_2}$$
$$i_3 = \frac{v_2 - 0}{R_3}$$

Substituting back into the node equations

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$
$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \qquad (*)$$

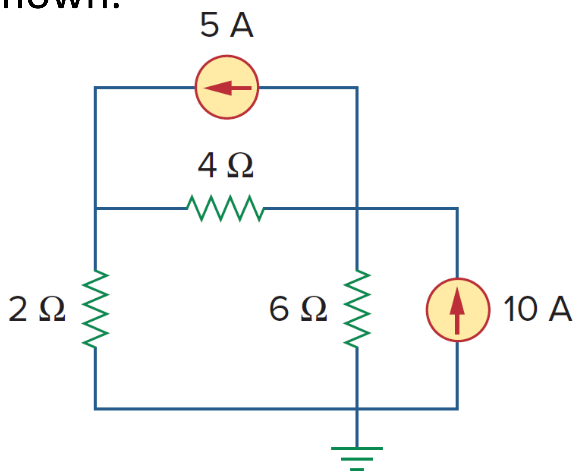
Two variables v_1 and v_2, and two equations!

- The last step is to solve the system of equations $(*)$ using any standard method, such as the substitution method, the elimination method, Cramer's rule, matrix inversion, or MATLAB.

$$\begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_2 + 1/R_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

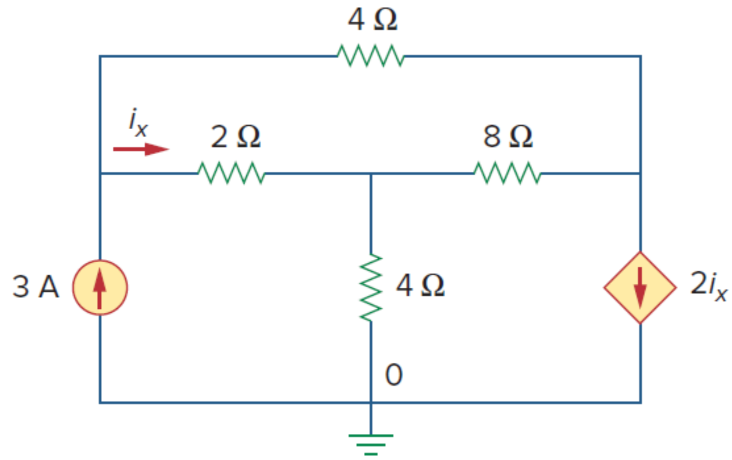
Example

Using nodal analysis, calculate the node voltages in the circuit shown.



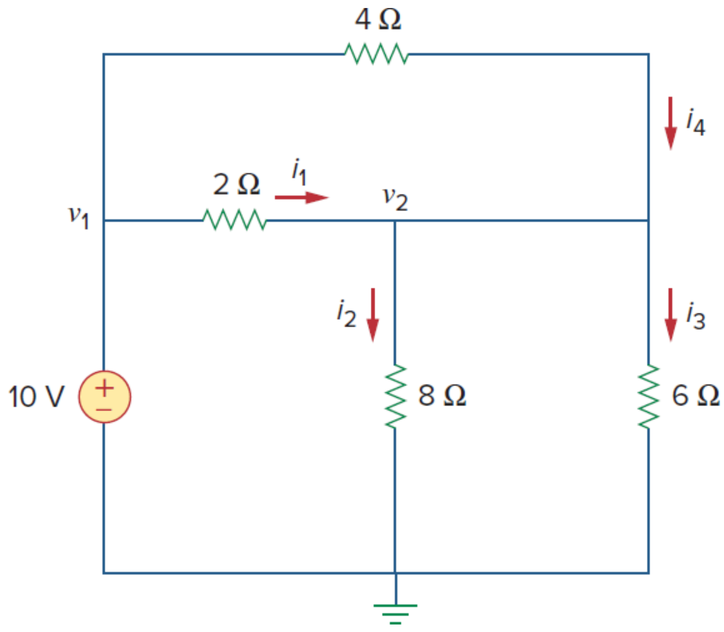
Example

Using nodal analysis, determine the voltages at the nodes.



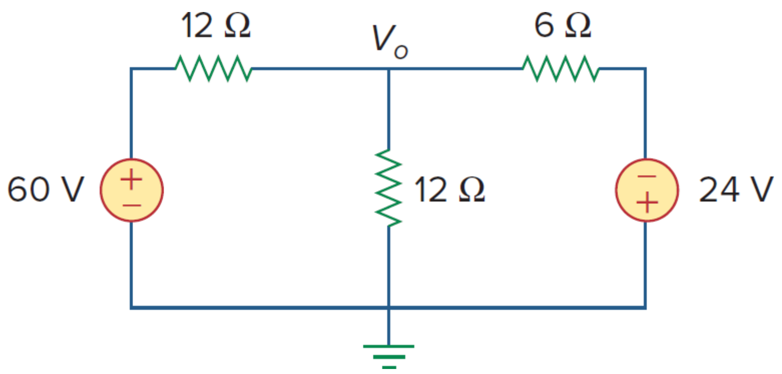
Nodal Analysis with Voltage Sources

If a voltage source is connected between the reference (or ground) node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source (for example, in this circuit, $v_1 = 10V$). Then, we apply KCL at nodes with unknown voltage.



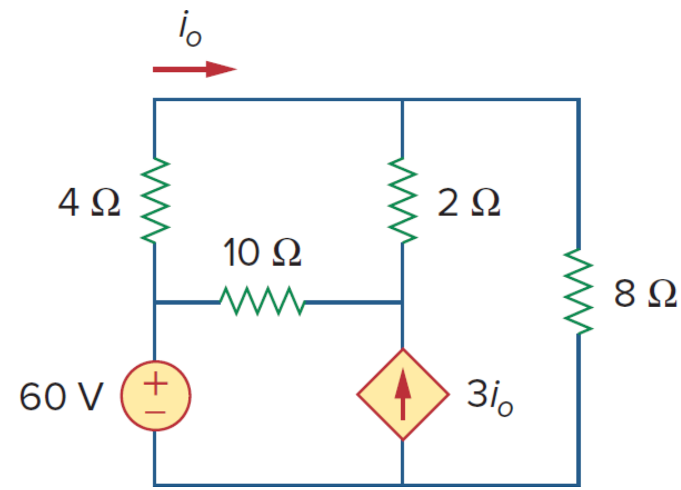
Example

Using nodal analysis, find V_o and the power dissipated in all the resistors in the circuit.



Example

Using nodal analysis, find current i_o in the circuit.

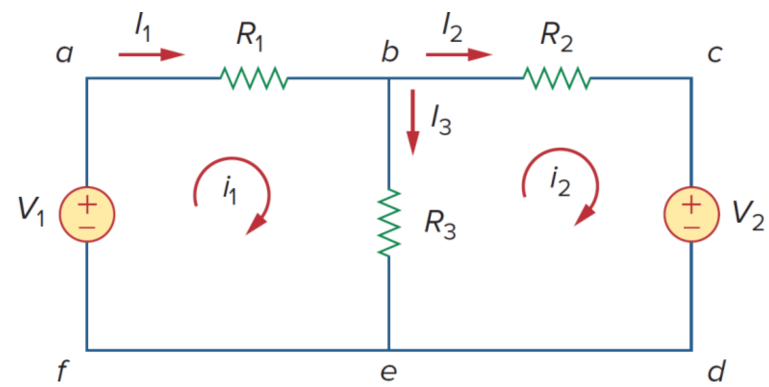


Mesh Analysis

Mesh Analysis

Recall:
- A loop is a closed path with no node passed more than once.
- A mesh is a loop that does not contain any other loop within it.

In this example, paths *abefa* and *bcdeb* are meshes, but path *abcdefa* is not a mesh.



Another general procedure for analyzing circuits is to use the **mesh currents** as the circuit variables (the current through a mesh is known as **mesh current**). Using mesh currents instead of branch/element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.

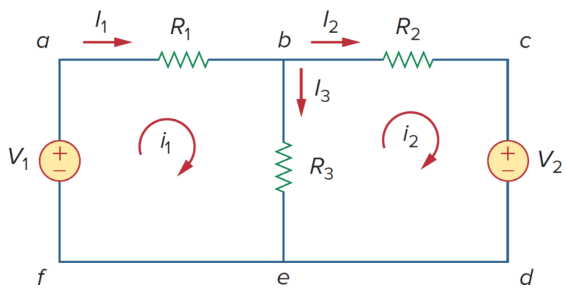
- **Mesh analysis** uses **KVL** to find unknown mesh currents.

Mesh Analysis Steps

Mesh analysis of a circuit with n meshes follows these steps:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n mesh currents. Use Ohm's law to express the voltages in terms of the mesh currents.
   Special cases: circuits with current sources.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Example:



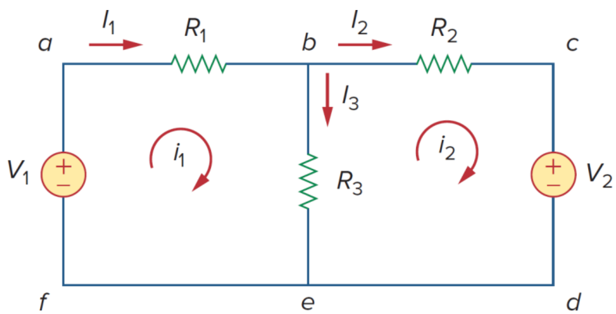
- First, mesh currents i_1 and i_2 are assigned to the two meshes.

  Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use i for a mesh current and I for a branch current. The current elements I_1, I_2, and I_3 are algebraic sums of the mesh currents.

$$I_1 = i_1, \qquad I_2 = i_2, \qquad I_3 = i_1 - i_2$$

Mesh Analysis Steps

Applying KVL to the meshes:



$$V_1 - R_1 i_1 - R_3(i_1 - i_2) = 0 \quad\Downarrow\quad (R_1 + R_3)i_1 - R_3 i_2 = V_1$$

$$-R_2 i_2 - V_2 - R_3(i_2 - i_1) = 0 \quad\Downarrow\quad -R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

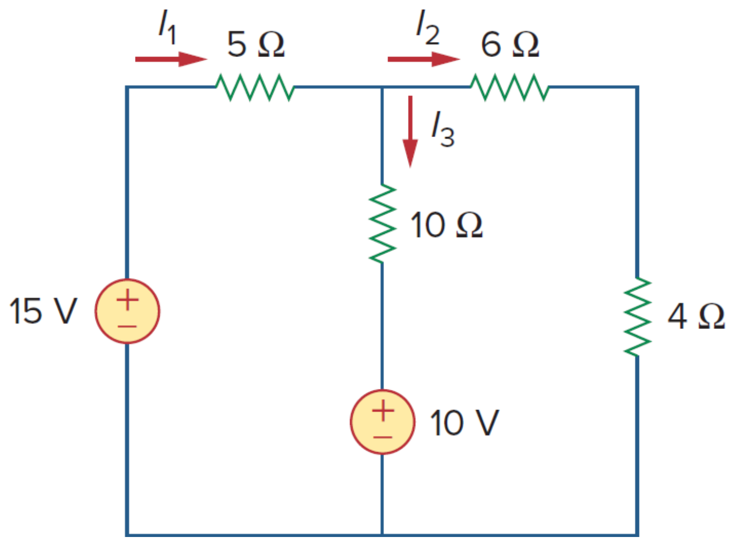
$$(*)$$

- The last step is to solve the system of equations $(*)$ using any standard method, such as the substitution method, the elimination method, Cramer's rule, matrix inversion, or MATLAB.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

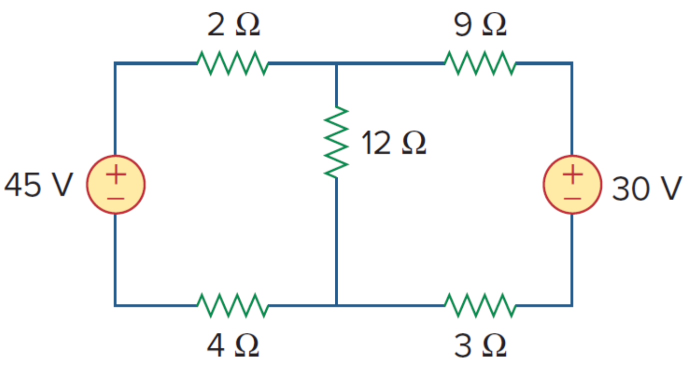
Example

For the circuit shown, find the branch currents I_1, I_2, and I_3 using mesh analysis.



Example

Using mesh analysis, calculate the mesh currents of the circuit.



Mesh Analysis with Current Sources

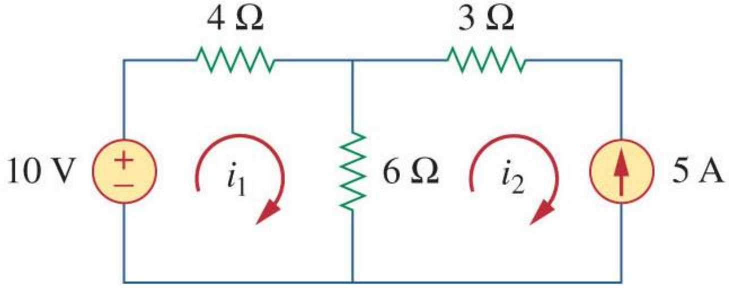
The presence of a current source makes the mesh analysis simpler in that it reduces the number of equations.

- If the current source is located on only one mesh, the current for that mesh is defined by the source. For example:

Here, we set $i_2 = -2$ A

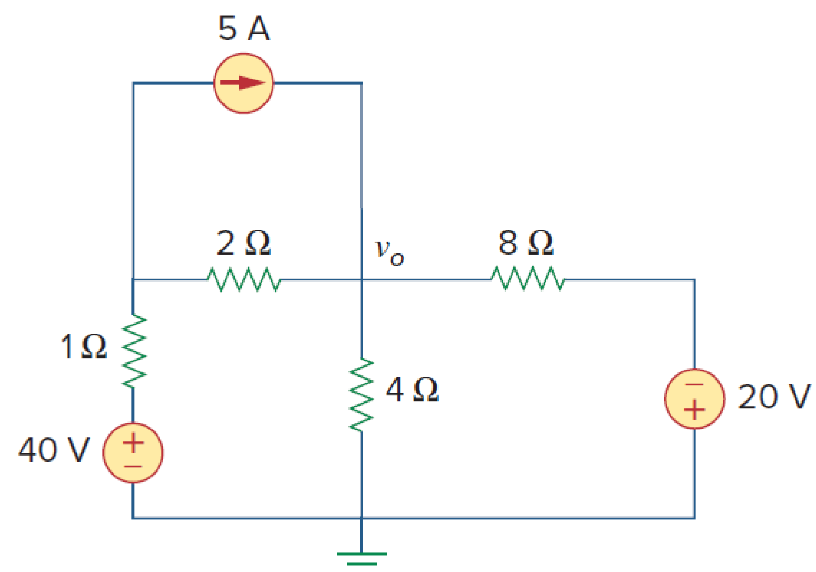
Then,

$$10 - 4i_1 - 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2\text{A}$$



Example

Apply mesh analysis to find v_o in the circuit.



Nodal vs. Mesh Analysis

Nodal vs. Mesh Analysis

Both nodal and mesh analyses provide a systematic way of analyzing a complex network. It is helpful to be familiar with both methods of analysis, for at least two reasons. First, one method can be used to check the results from the other method, if possible. Second, since each method has its limitations, only one method may be suitable for a particular problem.

Nodal analysis is more suitable when the network contains:

- Circuits with fewer nodes than meshes.
- If node voltages are what are being solved for.
- This format is easier to solve by computer.

Mesh analysis is more suitable when the network contains:

- A circuit with fewer meshes than nodes.
- If branch or mesh currents are what is being solved for.