Ch7: Internal Forces and Moments

Contents:

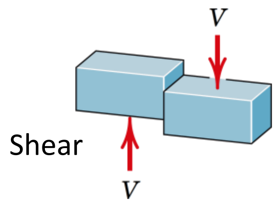
Internal Forces in Members

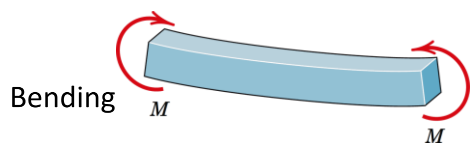
Internal Loadings

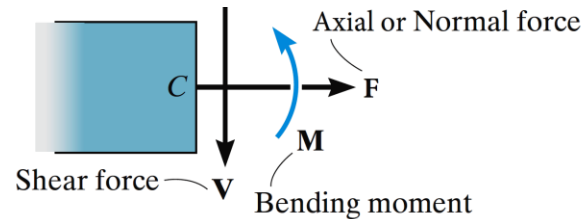
To design a structural member, it is necessary to know the loading acting within the member in order to be sure the material can resist this loading.

★ General internal loadings in two-dimensions:

- Force $\mathbf{F}$ is called **Normal or Axial Force**. It acts perpendicular to the cross section.
- Force $\mathbf{V}$ is called **Shear Force**. It is tangent to the cross section.
- Couple moment $\mathbf{M}$ is called **Bending Moment**.

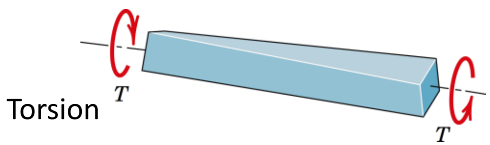


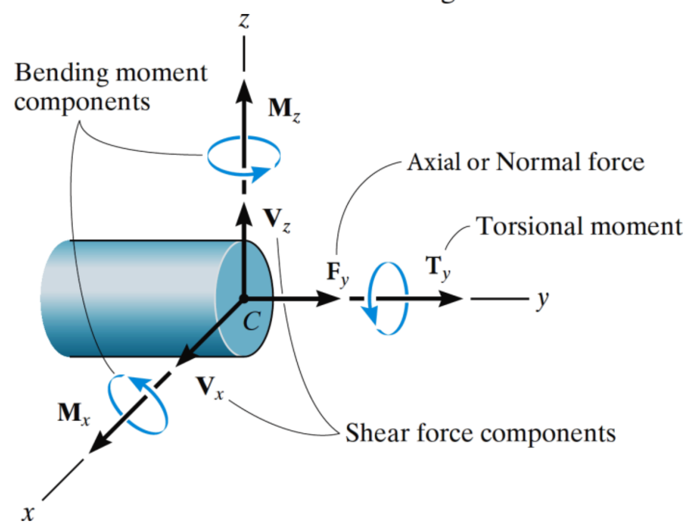




★ General internal loadings in three-dimensions:

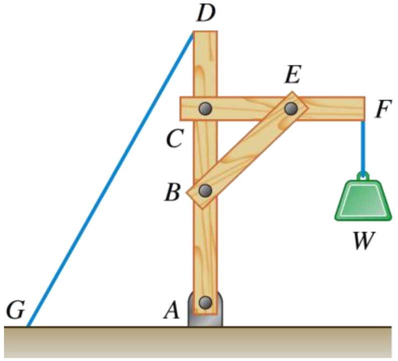
- $\mathbf{F}_y$ is **Normal or Axial Force**.
- $\mathbf{V}_x$ and $\mathbf{V}_z$ are **Shear Force** components.
- $\mathbf{M}_x$ and $\mathbf{M}_z$ are **Bending Moment** components.
- $\mathbf{M}_y$ is a **Torsional or Twisting Moment**.





Procedure for Determining the Internal Forces

Internal loadings can be determined by using the **method of sections**:

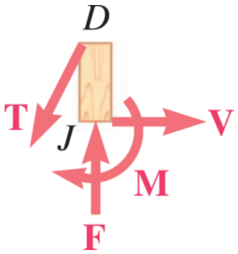


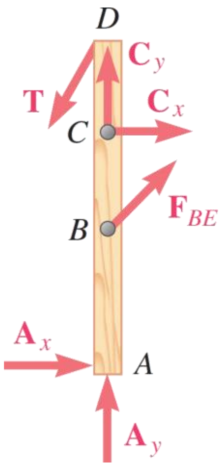
1. **Draw** the FBD and **determine** the reactions at the supports.
2. **Keep** all loadings (including couple moments) acting on the member in their **exact locations**.

   ↓

   • For calculating the <u>internal forces</u>, you should not consider the forces as sliding vectors and couple moments as free vectors. Moreover, you should not replace distributed loads by equivalent concentrated loads.

3. **Pass** an imaginary section through the member, perpendicular to its axis at the point where the internal loadings are to be determined.
4. **Draw** a free-body diagram of the segment that has the least number of loads and unknowns on it, and **apply** the equations of equilibrium.

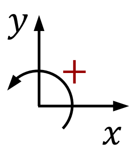


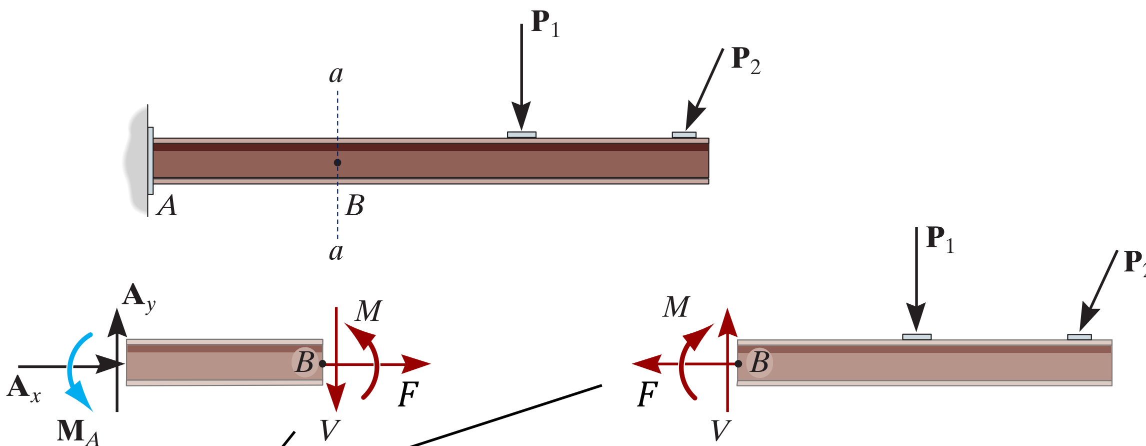


Note: If the solution yields a negative scalar, the sense of the quantity is opposite to that shown on the free-body diagram.

A Simple Example

By passing an **imaginary section a–a** perpendicular to the axis of the member through point B, the member is separated into two segments. The internal loadings (internal **force-couple system**) become external on the free-body diagram of each segment:





According to Newton's third law, these loadings must act in opposite directions on each segment.

The internal **force-couple system** can be determined by applying the equations of equilibrium to the FBD of **either segment**:

$\Sigma M_B = 0 \rightarrow M$ ✓

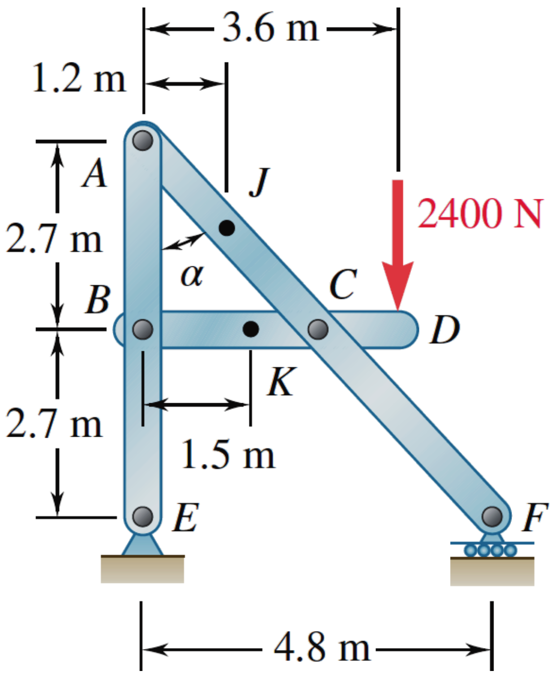
$\Sigma F_x = 0 \rightarrow F$ ✓

$\Sigma F_y = 0 \rightarrow V$ ✓

Note: These loadings generally vary from point to point in a member.

Sample Problem 7.1

In the frame shown, determine the internal forces (a) in member ACF at point J, (b) in member BCD at point K.

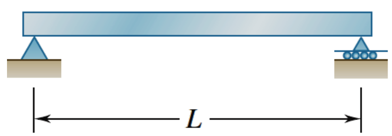


Beams

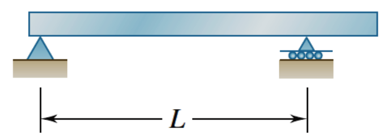
Beams

Beams are usually long, straight prismatic structural members designed to **support loads** applied at various points along them. In most cases, the loads are **perpendicular to the axis** of the beam and cause only **shear and bending** in the beam. Beams are often classified as to how they are supported.

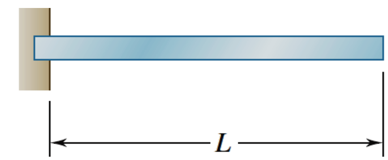
Statically **Determinate** Beams (Unknowns = 3):



(*a*) Simply supported beam

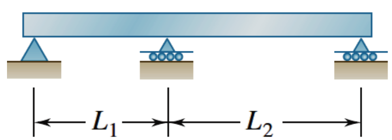


(*b*) Overhanging beam

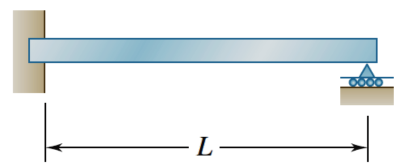


(*c*) Cantilever beam

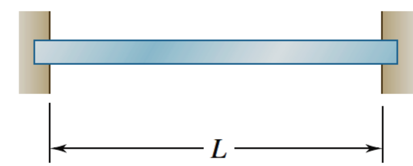
Statically **Indeterminate** Beams (Unknowns > 3):



(*d*) Continuous beam

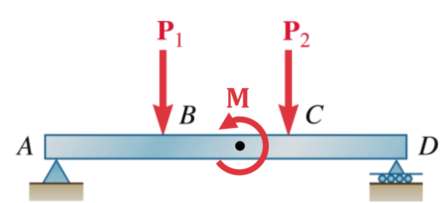


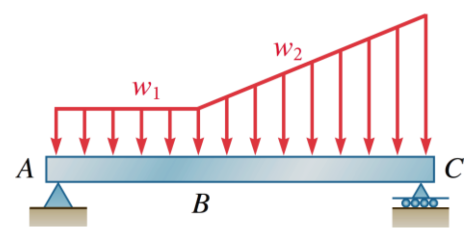
(*e*) Beam fixed at one end and simply supported at the other end



(*f*) Fixed beam

- A beam can be subjected to **concentrated loads** ($\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{M}$), **distributed loads** (w_1, w_2) or a combination of both types of loads:



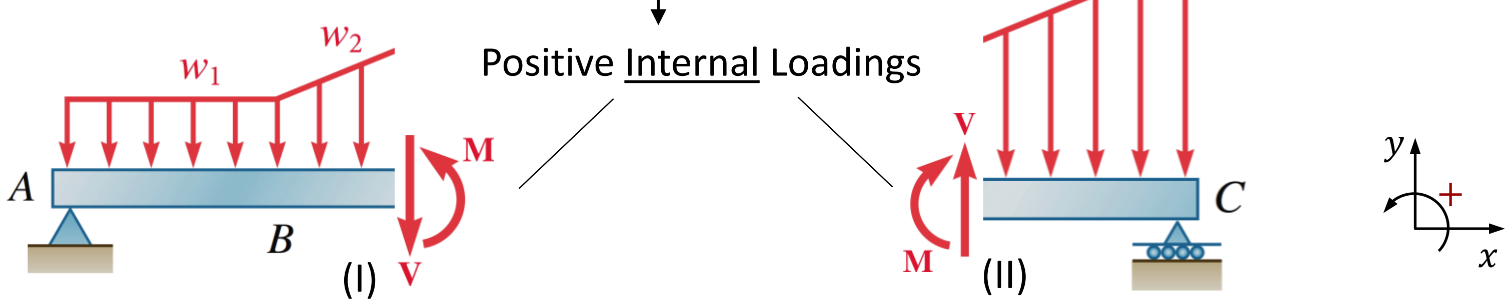


Sign Convention for Beams

For problems in two dimensions engineers generally use a sign convention to report the **internal loadings** V and M at a given point of a beam:

▶ The internal shear **V** is said to be positive if it causes the beam segment on which it acts to rotate clockwise.

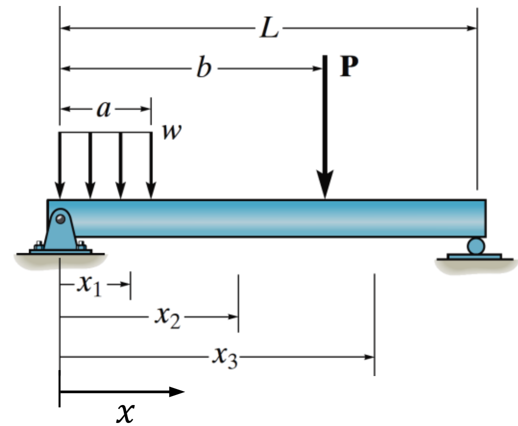
▶ The internal bending moment **M** is said to be positive if it tends to bend the segment on which it acts in a concave upward manner.



Important Note: The sign convention is only for reporting the internal loadings values, and it has nothing to do with the sign of V or M in the equations of equilibrium.

Shear and Moment Diagrams

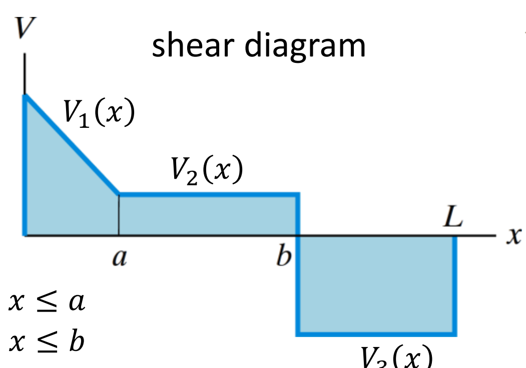
Shear and bending-moment diagrams represent the variations of V and M along the beam's axis, x. They are obtained by using the method of sections at different distances x_1, x_2, x_3,... from one end.

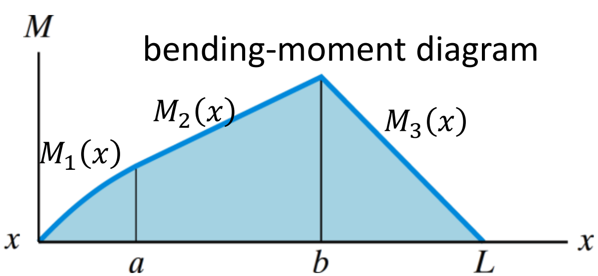


In general, the functions $V(x)$ and $M(x)$ (or their slopes) will be discontinuous, at points where **a distributed load suddenly changes** or where **concentrated forces or couple moments are applied**.

$$\Downarrow$$

Therefore, the regions between these points must be selected to obtain $V(x)$ and $M(x)$.





$$V(x) = \begin{cases} V_1(x) & 0 \le x \le a \\ V_2(x) & a \le x \le b \\ V_3(x) & b \le x \le L \end{cases}, \quad M(x) = \begin{cases} M_1(x) & 0 \le x \le a \\ M_2(x) & a \le x \le b \\ M_3(x) & b \le x \le L \end{cases}$$

Construction of Shear & Moment Diagrams

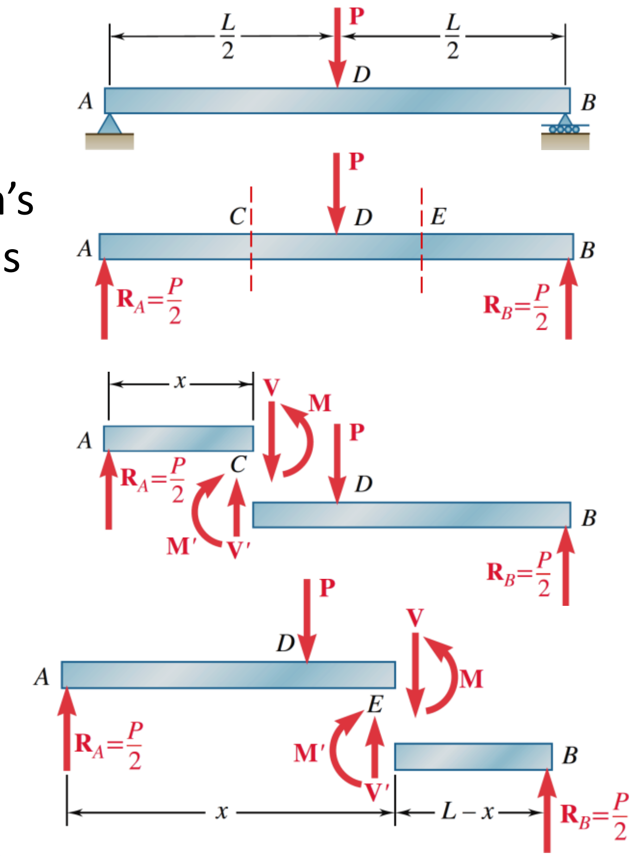
(Method of Sections)

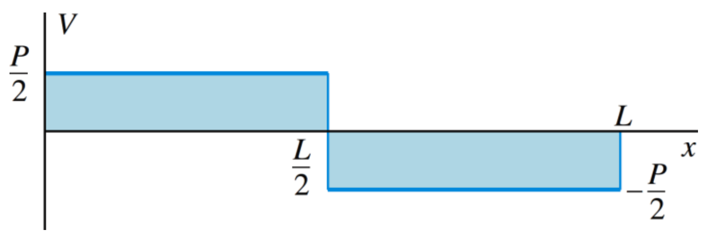
1. Draw the FBD and determine the reactions at the supports.

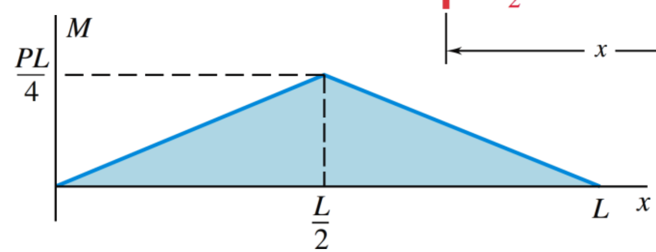
2. Specify separate coordinates x having an origin at the beam's left end and extending to regions between concentrated forces and/or couple moments, or where the distributed loading is continuous.

3. Section the beam at each distance x, draw the FBD of one of the segments (vectors $\mathbf{V}$ and $\mathbf{M}$ act in their positive sense, in accordance with the sign convention), and find V and M with respect to x at each segment.

4. Plot $V(x)$ and $M(x)$.

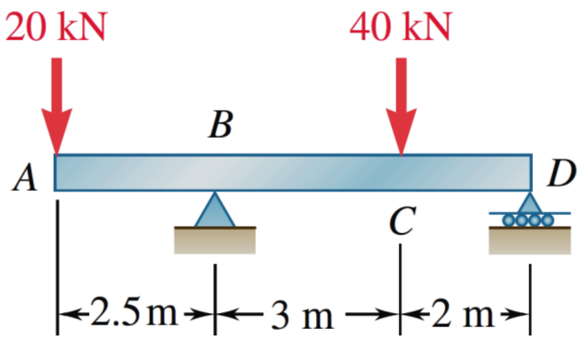






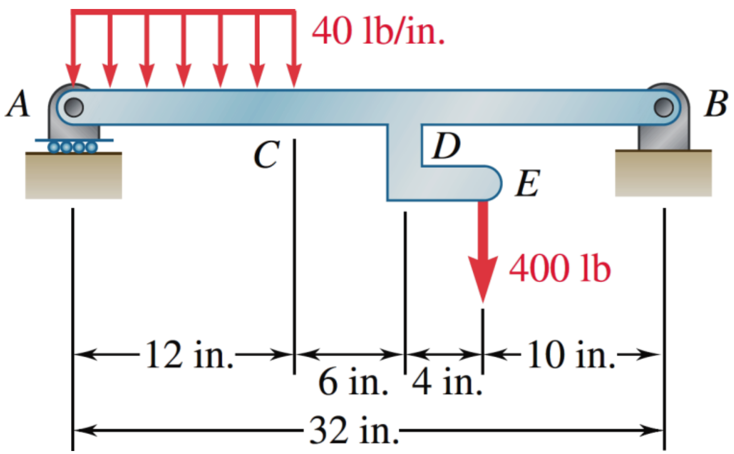
Sample Problem 7.2

Draw the shear and bending-moment diagrams for the beam and loading shown.



Sample Problem 7.3

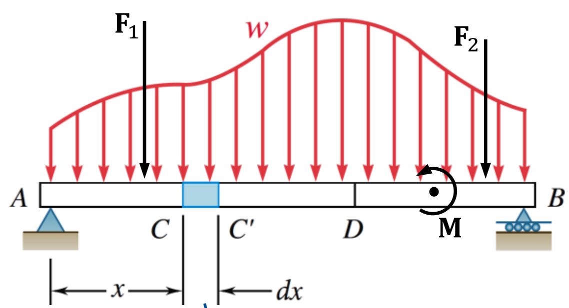
Draw the shear and bending-moment diagrams for the beam AB. The distributed load of 40 lb/in. extends over 12 in. of the beam from A to C, and the 400-lb load is applied at E.



Beams: Relations among Load w, Shear V, and Bending Moment M

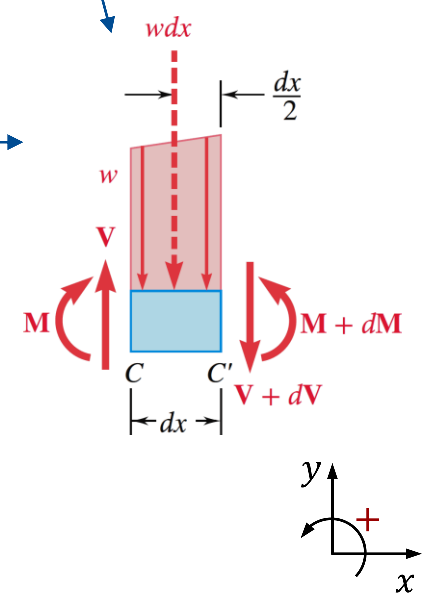
Relations among Load w, Shear V, and Bending Moment M

A quick method for constructing V and M diagrams is based on differential relations that exist between the load w, shear V, and bending moment M.



Consider the beam subjected to an arbitrary load $w = w(x)$ and a series of concentrated forces (e.g., $\mathbf{F}_1$, $\mathbf{F}_2$) and couple moments (e.g., $\mathbf{M}$):

FBD of a small segment of the beam (dx) chosen at x which is **not** subjected to a concentrated load:



Note: Both the shear force and moment acting on the right-hand face must be increased by a small, finite amount (dV and dM) in order to keep the segment in equilibrium.

Now, we write two equations of equilibrium (i.e., $\Sigma F_y = 0$ and $\Sigma M_{C'} = 0$) for the segment.

Relation between Distributed Load w and Shear V

$$\Sigma F_y = 0: \quad V - w(x)dx - (V + dV) = 0 \quad \rightarrow \quad dV = -w(x)dx$$

Result 1: $$\frac{dV}{dx} = -w(x)$$

▶ Slope of shear diagram = − Distributed load intensity

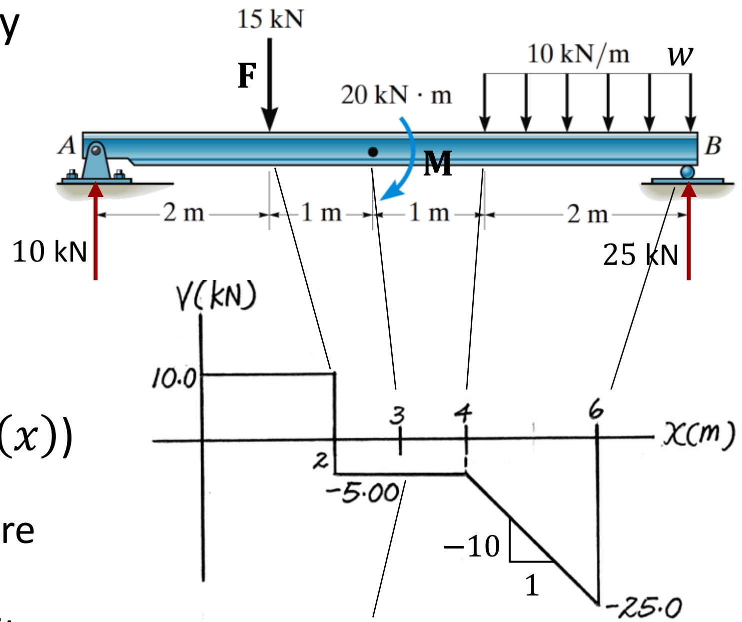
Result 2:

Integration between any two points of each segment:

$$V_2 - V_1 = -\int_{x_1}^{x_2} w(x)dx$$

▶ Change in shear = − (Area under distributed load $w(x)$)

Note: These equations are **valid** only between the points where concentrated forces $\mathbf{F}$ are applied.
Note: The shear diagram is discontinuous at such points, and it **jumps** toward the direction of $\mathbf{F}$ by the magnitude of $\mathbf{F}$.



(**M** has no effect on shear diagram)

Relation between Shear V and Moment M

$$\Sigma M_{C'} = 0: \qquad (M + dM) - M - V dx + \overset{\approx 0}{\cancel{w dx \frac{dx}{2}}} = 0 \quad \rightarrow \quad dM = V(x) dx$$

Result 1: $$\boxed{\frac{dM}{dx} = V(x)}$$

- ▶ Slope of bending-moment diagram = Shear
- ▶ The shear is zero at points of a segment where the bending moment is max or min.

Result 2:

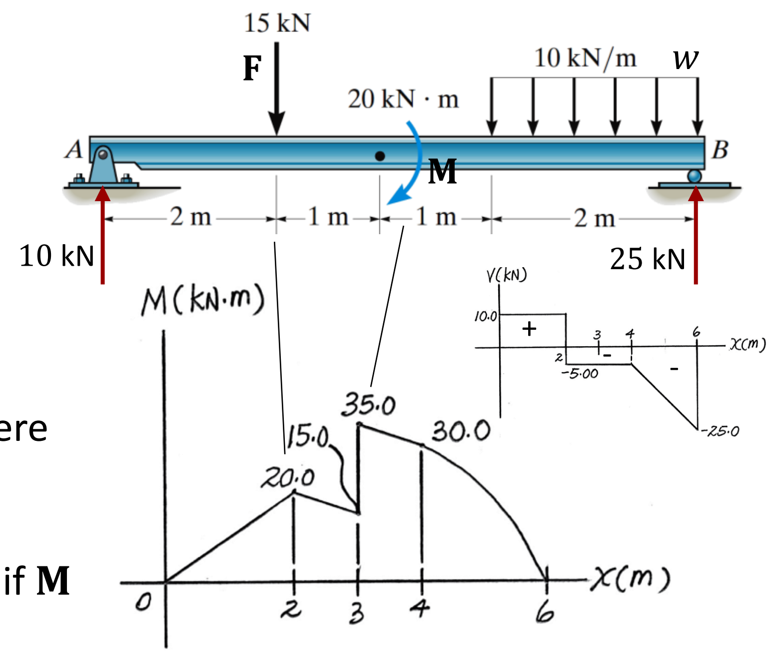
Integration between any two points of each segment:

$$\boxed{M_2 - M_1 = \int_{x_1}^{x_2} V(x) dx}$$

- ▶ Change in moment = Area under shear diagram

Note: These equations are **valid** only between the points where concentrated couple moments $\mathbf{M}$ are applied.

Note: The bending-moment diagram is discontinuous at such points, and it **jumps** upward if $\mathbf{M}$ is clockwise and downward if $\mathbf{M}$ is counterclockwise by the magnitude of $\mathbf{M}$.



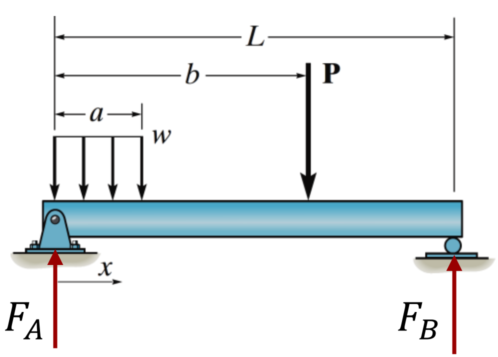
Remarks

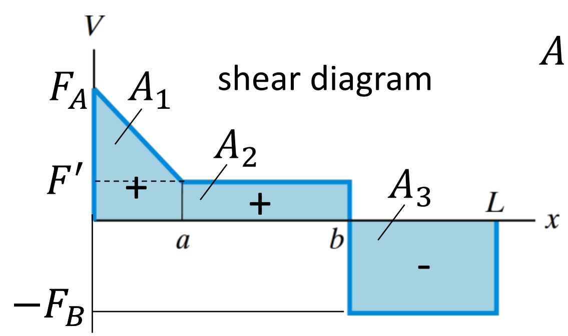
- If the loading curve $w = w(x)$ is a polynomial of degree n, $V = V(x)$ will be a polynomial of degree $n+1$, and $M = M(x)$ will be a polynomial of degree $n+2$.

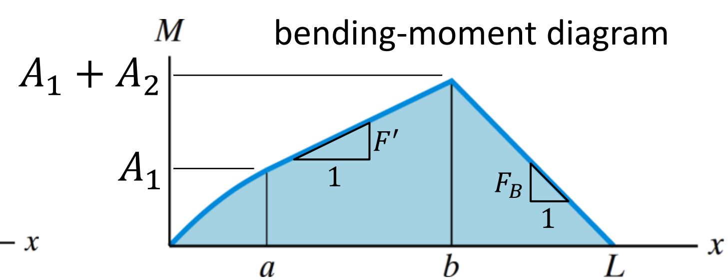
$$\frac{dV}{dx} = -w(x) \qquad \frac{dM}{dx} = V \quad \rightarrow \quad \boxed{\frac{d^2M}{dx^2} = -w}$$

For a segment that w is zero, V is constant, and M is a line (of non-zero slop).
For a segment that w is constant, V is a line (of non-zero slop), and M is a parabola.
…

- The area A under the shear curve should be considered positive where the shear is positive and should be negative where the shear is negative.







Construction of Shear & Moment Diagrams

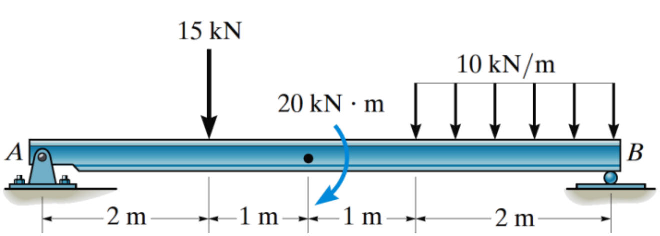
(Quick Method)

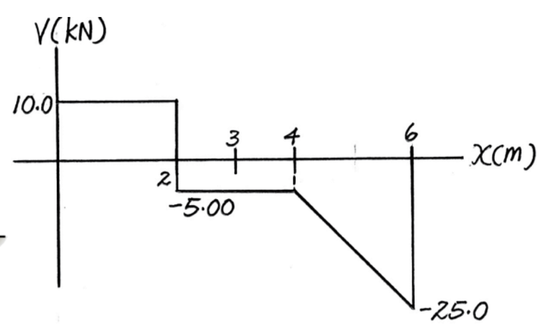
1. Draw the FBD and determine the reactions at the supports.

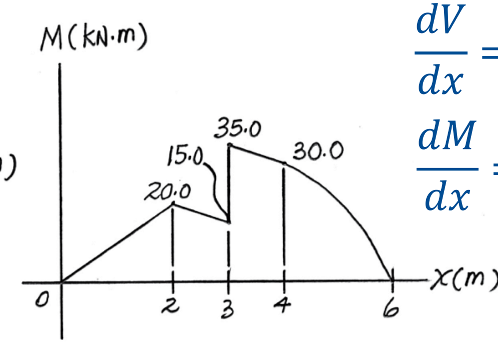
2. Divide the beam into **segments**, between the points where loading changes.

3. Plot $V(x)$ by using the loadings on beam, starting from $x = 0$. For each segment, determine the **function type** (constant, line, parabola,...), **values** of V at two endpoints of each segment (and the **slop** at the endpoints if it is needed). Note that $V(x)$ **jumps** toward the direction of a concentrated force by its magnitude.

4. Plot $M(x)$ by using the $V(x)$, starting from $x = 0$. For each segment, determine the **function type** (constant, line, parabola,...), **values** of M at two endpoints of each segment (and the **slop** at the endpoints if it is needed). Note that $M(x)$ **jumps** upward if a concentrated moment is clockwise and downward if it is counterclockwise, by its magnitude.





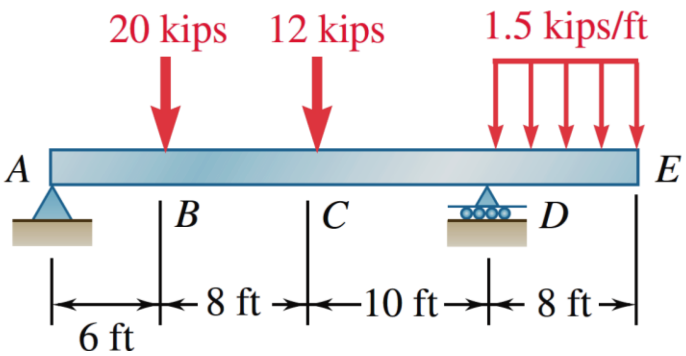


$$\frac{dV}{dx} = -w(x)$$

$$\frac{dM}{dx} = V$$

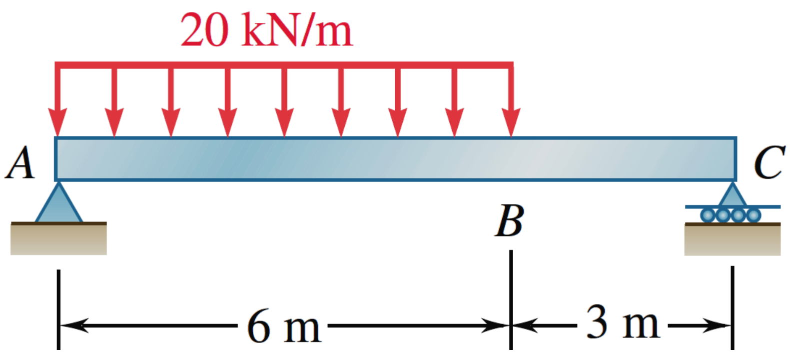
Sample Problem 7.4

Draw the shear and bending-moment diagrams for the beam and loading shown.



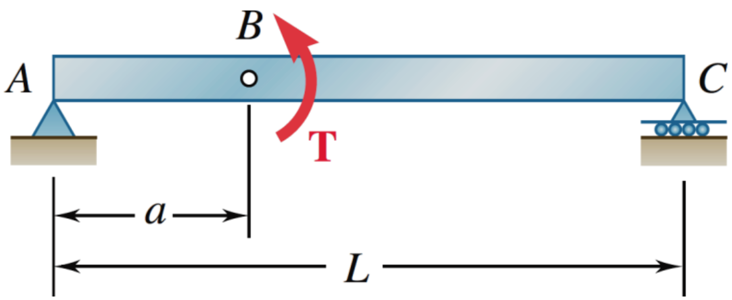
Sample Problem 7.5

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the location and magnitude of the maximum bending moment.



Sample Problem 7.7

The simple beam AC is loaded by a couple of magnitude T applied at point B. Draw the shear and bending-moment diagrams for the beam.



Cables with Concentrated Loads

Cables

Cables (and chains) are flexible members capable of withstanding only tension. They are used in many engineering applications, such as suspension bridges and power transmission lines, aerial tramways, etc.

[1]

[2]

[3]

[4]

Cables may be divided into two categories, according to their loading:

(1) Cables with **Concentrated Loads** [1,2]

(2) Cables with **Distributed Loads**

i. Cable subjected to a distributed load (Parabolic Cables) [3]

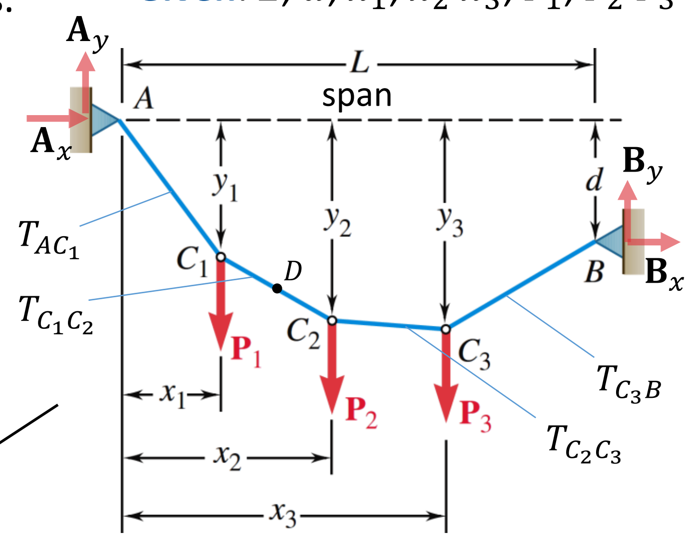
ii. Cable subjected to its own weight (Catenary Cable) [4]

Cables with Concentrated Loads

Consider a cable supporting several **concentrated loads**. We **assume** that:

- The cable is **flexible** (i.e., its resistance to bending is negligible).
- The cable is **inextensible** (i.e., the cable length remains constant).
- The **weight** of the cable is **negligible** compared with the loads.
- Each of the **concentrated loads** lies in a given **vertical** line.

Given: $L, d, x_1, x_2\ x_3, P_1, P_2\ P_3$



$\Downarrow$

Thus, the cable takes the form of several **straight-line segments**, each of which can be approximate as a **two-force member**, and is subjected to a constant tensile force $\mathbf{T}$ directed along the cable.

Unknowns: 11

$$A_x, A_y, B_x, B_y, \underbrace{T_{AC_1}, T_{C_1C_2}, T_{C_2C_3}, T_{C_3B}}_{\text{tension in each segment}}, \underbrace{y_1, y_2, y_3}_{\text{shape of the cable}}$$

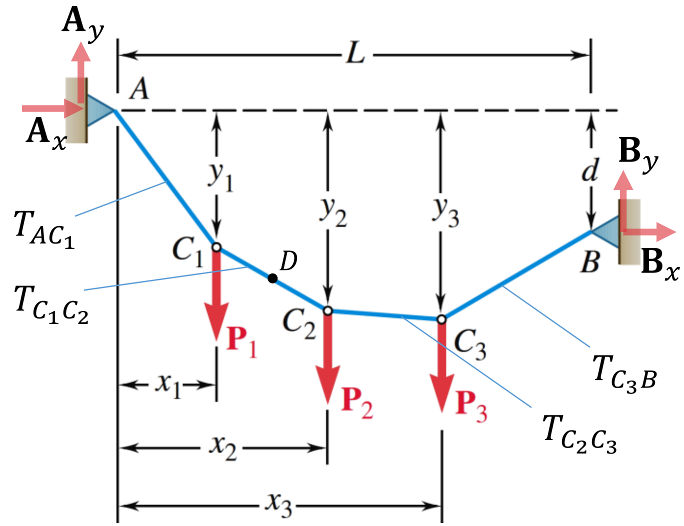
Equilibrium Equations: 10

Two equations at each point A, C_1, C_2, C_3, B.

$\Rightarrow$ We need more information, e.g., cable's total length S, position or slope of a point D, A_x/A_y, B_x/B_y, etc.

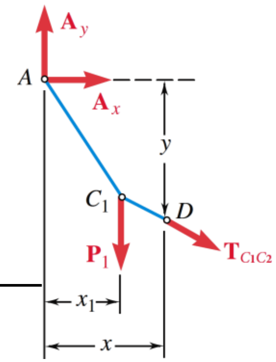
Cables with Concentrated Loads

If the coordinates x and y of a point D of the cable is given, we cut the cable though D:



From entire cable $\begin{cases} \Sigma M_B = 0 \\ \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases}$ $\rightarrow A_x, A_y, B_x, B_y$ ✓

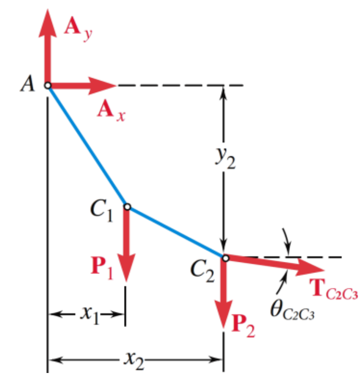
From portion AC_1D $\{ \Sigma M_D = 0$ ⟵



We can now find the vertical distance, slope, and tension of any point of the cable by cutting the cable though that points.

$\Sigma M_{C_2} = 0 \rightarrow y_2$ ✓ $\xrightarrow{\text{similarly}}$ y_1, y_3 ✓

$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned}\right\} \rightarrow T_{C_2C_3}, \theta_{C_2C_3}$ ✓ $\quad T_{C_1C_2}, \theta_{C_1C_2}$ ✓



Note: The horizontal component of the tension force is the same at any point of the cable (i.e., $T\cos\theta = -A_x$). Thus, the tension T is **maximum** in the portion of cable that has the largest angle of inclination θ (i.e., adjacent to one of the two supports of the cable).

Sample Problem 7.8

The cable AE supports three vertical loads from the points indicated. If point C is 5 ft below the left support, determine (a) the elevation of points B and D, (b) the maximum slope and the maximum tension in the cable.