Ch3: Rigid Bodies: Equivalent Systems of Forces

Contents:

Vector Operations

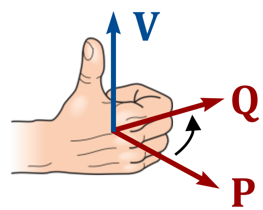
Vector Product (or Cross Product)

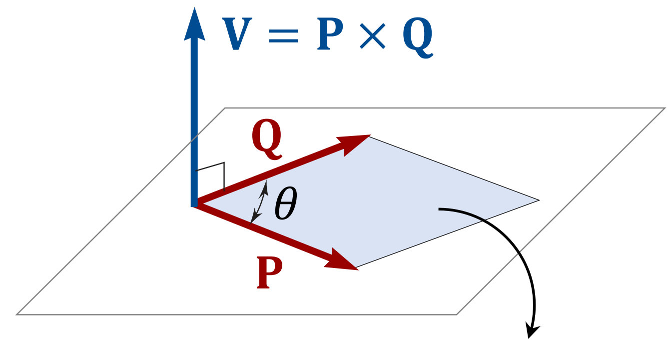
The **Vector Product** of two vectors $\mathbf{P}$ and $\mathbf{Q}$ is defined as the vector $\mathbf{V} = \mathbf{P} \times \mathbf{Q}$ that satisfies:

(1) **Line of action** of $\mathbf{V}$ is perpendicular to the plane containing $\mathbf{P}$ and $\mathbf{Q}$ ($\mathbf{V} \perp \mathbf{P}$, $\mathbf{V} \perp \mathbf{Q}$).

(2) **Magnitude** of $\mathbf{V}$ is $V = PQ\sin\theta$ $\quad (0 \le \theta \le 180°)$

(3) **Direction** of $\mathbf{V}$ is obtained using the **right-hand rule**.





$V = \|\mathbf{P} \times \mathbf{Q}\| =$ Area of the parallelogram

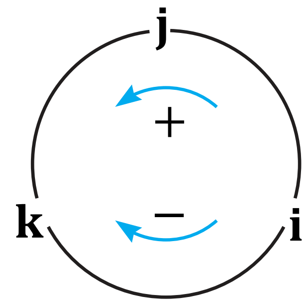
- Vector products are
  - **not commutative**: $\mathbf{Q} \times \mathbf{P} \neq \mathbf{P} \times \mathbf{Q}$; $\quad \mathbf{Q} \times \mathbf{P} = -(\mathbf{P} \times \mathbf{Q})$
  - **distributive**: $\mathbf{P} \times (\mathbf{Q}_1 + \mathbf{Q}_2) = \mathbf{P} \times \mathbf{Q}_1 + \mathbf{P} \times \mathbf{Q}_2$
  - **not associative**: $(\mathbf{P} \times \mathbf{Q}) \times \mathbf{S} \neq \mathbf{P} \times (\mathbf{Q} \times \mathbf{S})$

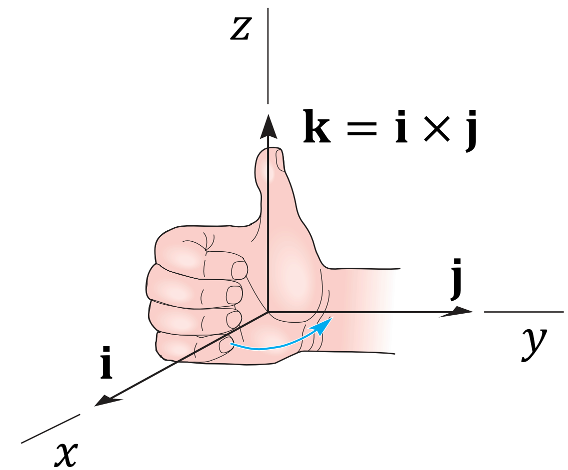
- Multiplication by a scalar a: $\quad a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B})$

Rectangular Components of Vector Products

$$\mathbf{i}\times\mathbf{i}=\mathbf{0} \qquad \mathbf{j}\times\mathbf{i}=-\mathbf{k} \qquad \mathbf{k}\times\mathbf{i}=\mathbf{j}$$
$$\mathbf{i}\times\mathbf{j}=\mathbf{k} \qquad \mathbf{j}\times\mathbf{j}=\mathbf{0} \qquad \mathbf{k}\times\mathbf{j}=-\mathbf{i}$$
$$\mathbf{i}\times\mathbf{k}=-\mathbf{j} \qquad \mathbf{j}\times\mathbf{k}=\mathbf{i} \qquad \mathbf{k}\times\mathbf{k}=\mathbf{0}$$

A quick method to determine the sign:





Given vectors $\mathbf{P}$ and $\mathbf{Q}$ in terms of the rectangular components, find $\mathbf{V}=\mathbf{P}\times\mathbf{Q}$:

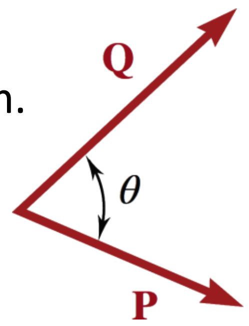
Method 1: Using the distributive property:

$$\mathbf{V}=\mathbf{P}\times\mathbf{Q}=\left(P_x\mathbf{i}+P_y\mathbf{j}+P_z\mathbf{k}\right)\times\left(Q_x\mathbf{i}+Q_y\mathbf{j}+Q_z\mathbf{k}\right)$$
$$=\left(P_yQ_z-P_zQ_y\right)\mathbf{i}+\left(P_zQ_x-P_xQ_z\right)\mathbf{j}+\left(P_xQ_y-P_yQ_x\right)\mathbf{k}$$

Method 2: Using determinant form: $\mathbf{V}=\mathbf{P}\times\mathbf{Q}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{vmatrix}$

Scalar Product (or Dot Product)

The **Scalar Product** of two vectors $\mathbf{P}$ and $\mathbf{Q}$ is defined as the product of the magnitudes of $\mathbf{P}$ and $\mathbf{Q}$ and of the cosine of the angle θ formed between them.



$$\mathbf{P}\cdot\mathbf{Q} = PQ\cos\theta \qquad (0 \le \theta \le 180°)$$

- Scalar products are
  - **commutative**: $\mathbf{P}\cdot\mathbf{Q} = \mathbf{Q}\cdot\mathbf{P}$
  - **distributive**: $\mathbf{P}\cdot(\mathbf{Q}_1 + \mathbf{Q}_2) = \mathbf{P}\cdot\mathbf{Q}_1 + \mathbf{P}\cdot\mathbf{Q}_2$

- Multiplication by a scalar a: $a(\mathbf{A}\cdot\mathbf{B}) = (a\mathbf{A})\cdot\mathbf{B} = \mathbf{A}\cdot(a\mathbf{B})$

Scalar product of unit vectors:

$$\mathbf{i}\cdot\mathbf{i} = 1 \quad \mathbf{j}\cdot\mathbf{j} = 1 \quad \mathbf{k}\cdot\mathbf{k} = 1$$
$$\mathbf{i}\cdot\mathbf{j} = 0 \quad \mathbf{j}\cdot\mathbf{k} = 0 \quad \mathbf{k}\cdot\mathbf{i} = 0$$

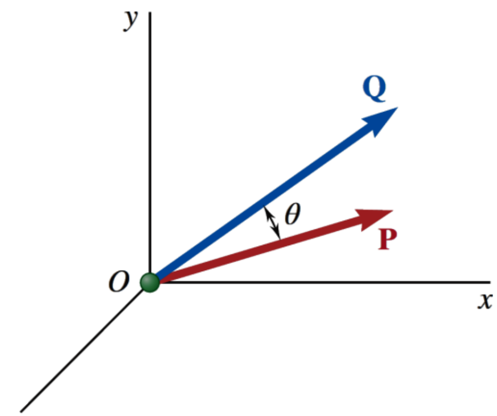
Given vectors $\mathbf{P}$ and $\mathbf{Q}$ in terms of the rectangular components:

$$\mathbf{P}\cdot\mathbf{Q} = \left(P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}\right)\cdot\left(Q_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k}\right) = P_xQ_x + P_yQ_y + P_zQ_z$$

Applications of Scalar Product

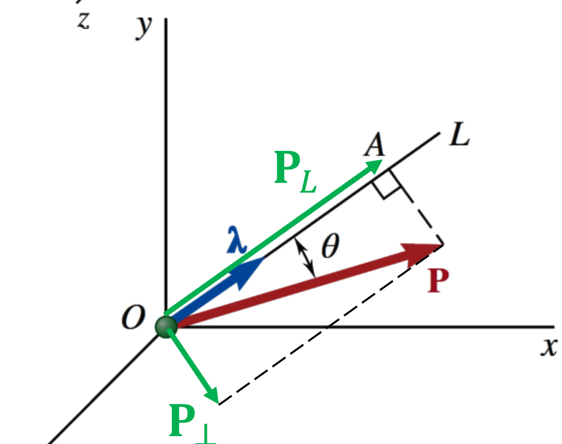
(1) Finding angle formed between two given vectors $\mathbf{P}$ and $\mathbf{Q}$ (or intersecting lines):

$$\theta = \cos^{-1}\left(\frac{\mathbf{P}\cdot\mathbf{Q}}{PQ}\right) \qquad (0 \le \theta \le 180°)$$



(2) Finding projection of a vector $\mathbf{P}$ on a given axis $\boldsymbol{\lambda}$:

- Projection of $\mathbf{P}$ on the axis OL: $P_L = P\cos\theta = \mathbf{P}\cdot\boldsymbol{\lambda}$
- Component of $\mathbf{P}$ parallel to line L: $\mathbf{P}_L = P_L\boldsymbol{\lambda} = (\mathbf{P}\cdot\boldsymbol{\lambda})\boldsymbol{\lambda}$
- Component of $\mathbf{P}$ perpendicular to line L: $\mathbf{P}_\perp = \mathbf{P} - \mathbf{P}_L$



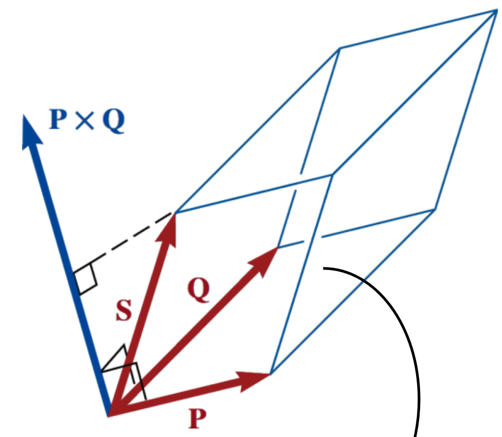
$\boldsymbol{\lambda}$: unit vector of line L

$$\mathbf{P} = \mathbf{P}_\perp + \mathbf{P}_L$$

Scalar Triple Product

Scalar Triple Product (or mixed triple product) of the 3 vectors $\mathbf{S}$, $\mathbf{P}$, and $\mathbf{Q}$ is a scalar as

$$\mathbf{S}\cdot(\mathbf{P}\times\mathbf{Q})$$



$|\mathbf{S}\cdot(\mathbf{P}\times\mathbf{Q})| =$
Volume of the parallelepiped

Method 1: Using the distributive property:

$$\mathbf{S}\cdot(\mathbf{P}\times\mathbf{Q}) = (S_x\mathbf{i}+S_y\mathbf{j}+S_z\mathbf{k})\cdot\left((P_x\mathbf{i}+P_y\mathbf{j}+P_z\mathbf{k})\times(Q_x\mathbf{i}+Q_y\mathbf{j}+Q_z\mathbf{k})\right)$$
$$= S_x(P_yQ_z-P_zQ_y)+S_y(P_zQ_x-P_xQ_z)+S_z(P_xQ_y-P_yQ_x)$$

Method 2: Using determinant form:

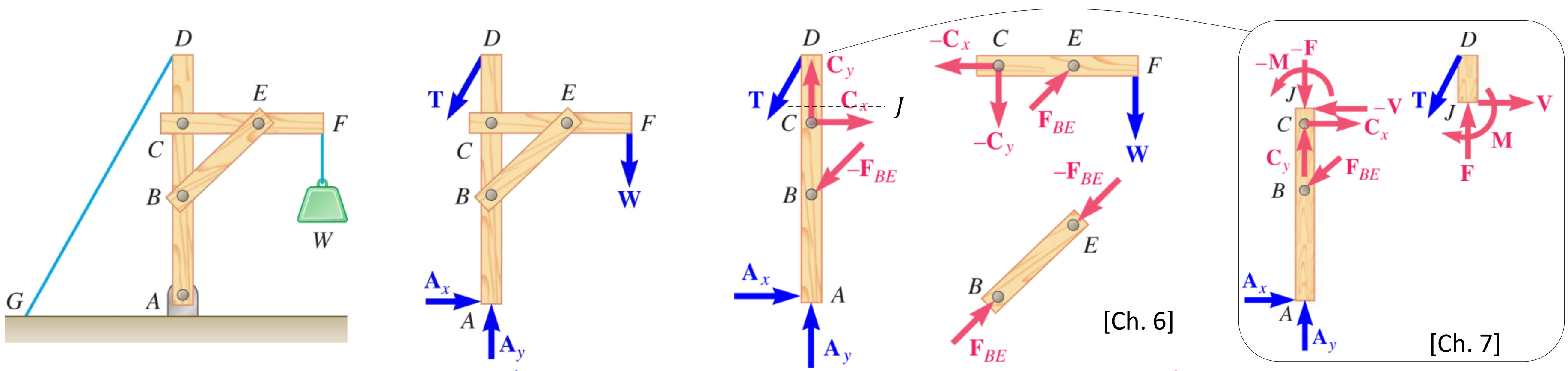
$$\mathbf{S}\cdot(\mathbf{P}\times\mathbf{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment of a Force

External and Internal Forces

Forces acting on rigid bodies can be separated into two groups:

1. **External Forces** are exerted by other bodies on the rigid body. They are entirely responsible for the external behavior of the rigid body, either causing it to move or ensuring that it remains at rest. External forces can be either applied forces or reactive forces at supports [Ch. 3, 4, 5].
2. **Internal Forces** hold together the particles forming the rigid body. If the rigid body is structurally composed of several parts, the forces holding the component parts together are also defined as internal forces [Ch. 6, 7].



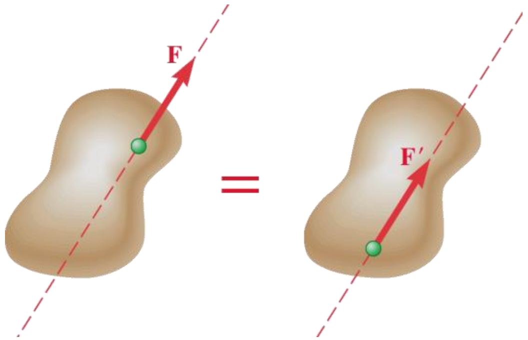
External Forces

Internal Forces

Principle of Transmissibility

Based on experimental evidence, the conditions of equilibrium or motion of a rigid body remain unchanged if a force $\mathbf{F}$ acting at a given point of the rigid body is replaced by any force of the same magnitude and same direction, along the same line of action (at a different point). These forces are called **equivalent forces**.

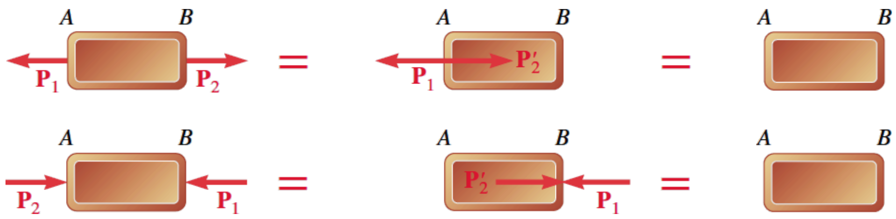
Thus, the external forces acting on a rigid body can be represented by **sliding vectors** (they allowed to slide along their lines of action).



We should be careful in using the principle of transmissibility to determine **internal forces** and deformations. For example, consider two sets of forces acting on a bar:

Set 1:

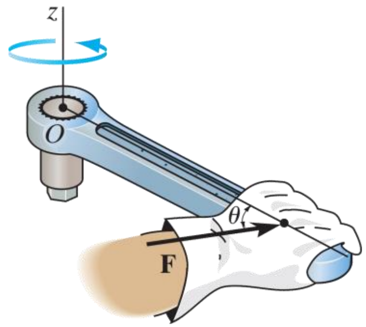
Set 2:

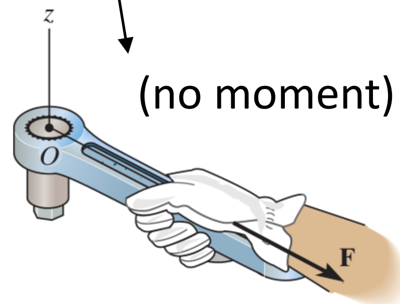


Both sets produce the same external effect, but in Set 1 the bar is in **tension** and in Set 2 the bar is in **compression**. Thus, this is true only in terms of the external behaviors.

Moment of a Force

Moment of a Force is a measure of the tendency of a force to make a body rotate about a specific point or axis (that is not on the line of action of the force). It is sometimes called a **torque**.





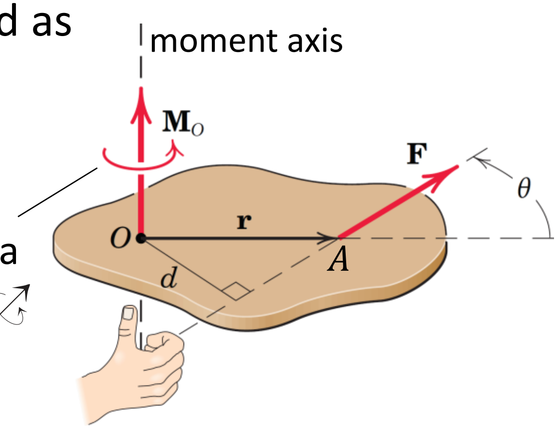
(no moment)

Moment of $\mathbf{F}$ about point O (or actually about the axis passing through O and perpendicular to the plane containing O and $\mathbf{F}$) is a **vector** quantity. It is defined as

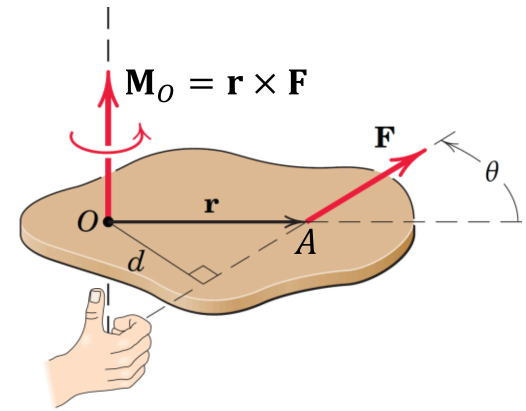
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Position vector of A
(point of application of $\mathbf{F}$)

$\mathbf{M}_O$ is represented by a curl around an arrow to distinguish it from force vectors.



Moment of a Force

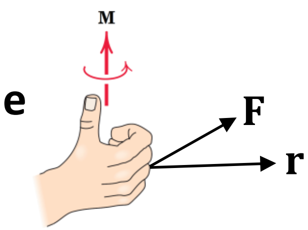


Magnitude: $M_o = rF\sin\theta = Fd$ (Unit: N·m, lb·ft, or lb·in)

moment arm (perpendicular distance from O to the line of action of $\mathbf{F}$)

angle between the tails of $\mathbf{r}$ and $\mathbf{F}$

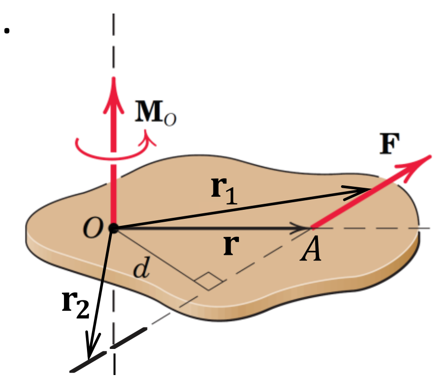
Direction: It is determined using the **right-hand rule** (fingers curl in the direction from $\mathbf{r}$ to $\mathbf{F}$)



Based on the **principle of transmissibility** of force $\mathbf{F}$, the moment $\mathbf{M}_O$ of $\mathbf{F}$ does not depend upon the actual position of the point of application of $\mathbf{F}$. Thus, we can use any position vector $\mathbf{r}$ directed from point O to any point on the line of action of $\mathbf{F}$.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$$

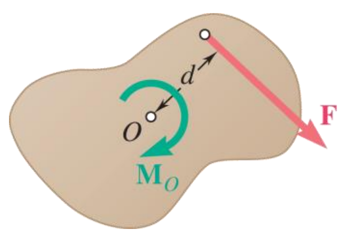
$$M_o = Fd$$



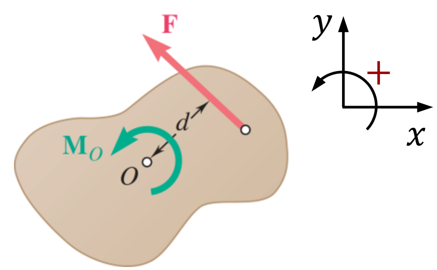
Moment of a Force: Planar Case

In this case, vector $\mathbf{M}_O$ is perpendicular to the plane. To specify the **magnitude** and the **sense** of the moment, by convention, we assign to the magnitude M_O of a counterclockwise ↺ moment a positive sign (since it is directed along the positive z axis, out of the page), and a clockwise ↻ moment a negative sign.

$$M_O = -Fd$$



$$M_O = +Fd$$

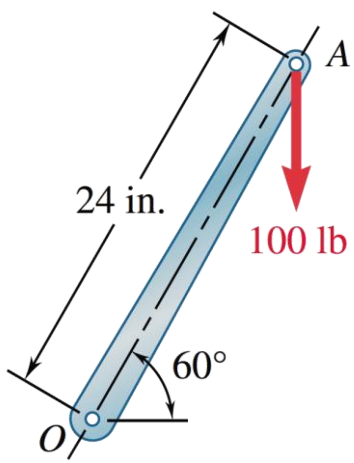


$\mathbf{M}_O$ is represented only by the curl.

Sample Problem 3.1

A 100-lb vertical force is applied to the end of a lever, which is attached to a shaft at O. Determine

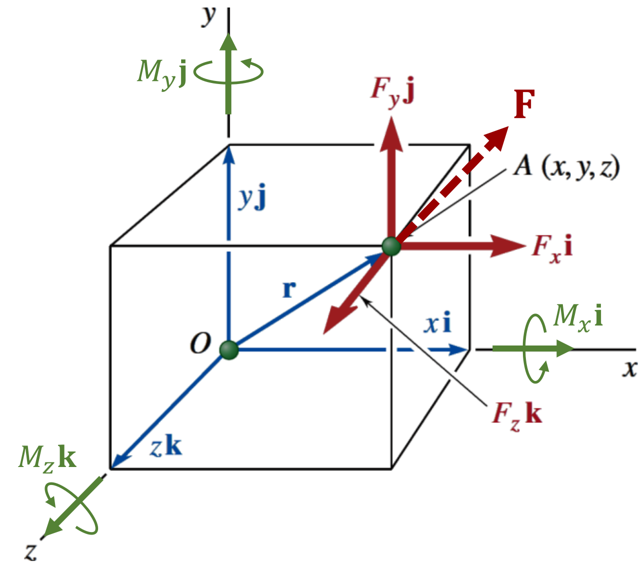
(a) the moment of the 100-lb force about O;
(b) the horizontal force applied at A that creates the same moment about O;
(c) the smallest force applied at A that creates the same moment about O;
(d) how far from the shaft a 240-lb vertical force must act to create the same moment about O;
(e) whether any one of the forces obtained in parts (b), (c), or (d) is equivalent to the original force.



Rectangular Components of Moment

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times \left(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}\right)$$

$$= \left(yF_z - zF_y\right)\mathbf{i} + (zF_x - xF_z)\mathbf{j} + \left(xF_y - yF_x\right)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

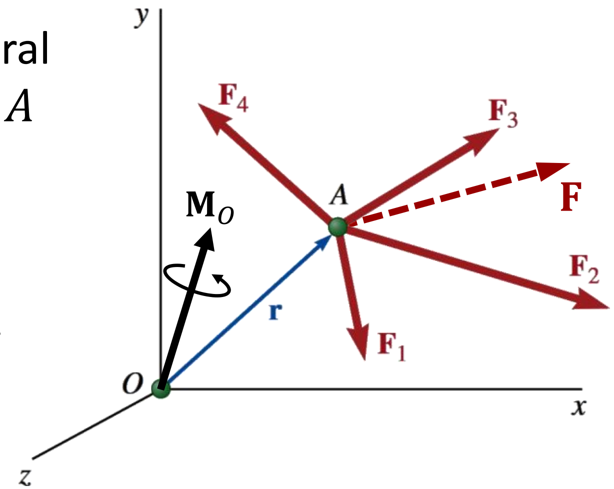
$$= M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

(M_x, M_y, M_z are moments of **F** about the coordinate axes)

Principle of Moments (Varignon's Theorem)

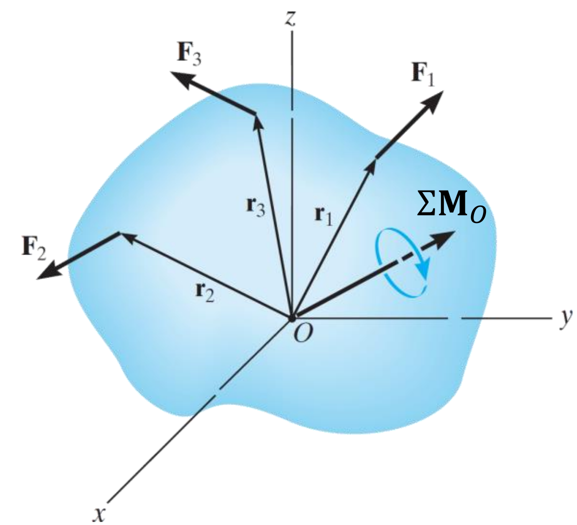
- The moment about a given point O of the resultant of several concurrent forces (like components of a force) at the point A is equal to the sum of the moments of the various forces about the same point O.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \cdots) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \cdots$$



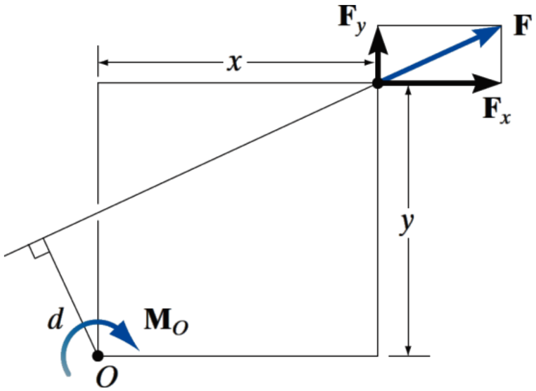
- Resultant Moment of a System of Forces about point O:

$$\Sigma\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$



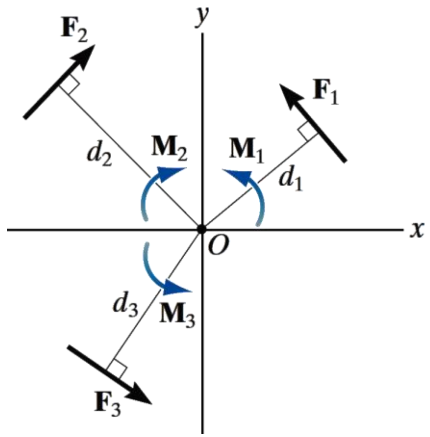
Principle of Moments: Planar Case

- The moment about a given point O of the resultant of several concurrent forces (like components of a force) at the point A is equal to the sum of the moments of the various forces about the same point O.



$$M_O = -Fd = F_y x - F_x y$$

- Resultant Moment of a System of Forces about point O:

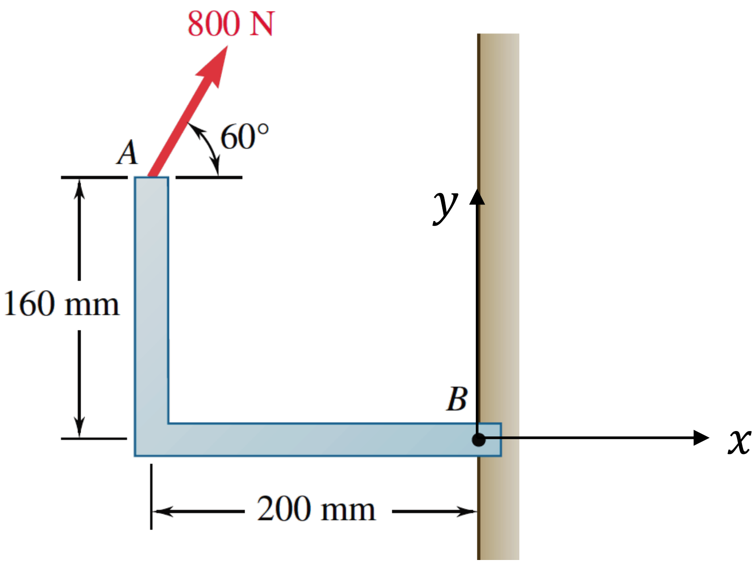


$$\Sigma M_O = F_1 d_1 - F_2 d_2 + F_3 d_3$$

In both cases, if the algebraic sum of the moments is positive, the moment is counterclockwise ↺ (out of the page), and if it is negative, the moment is clockwise ↻ (into the page).

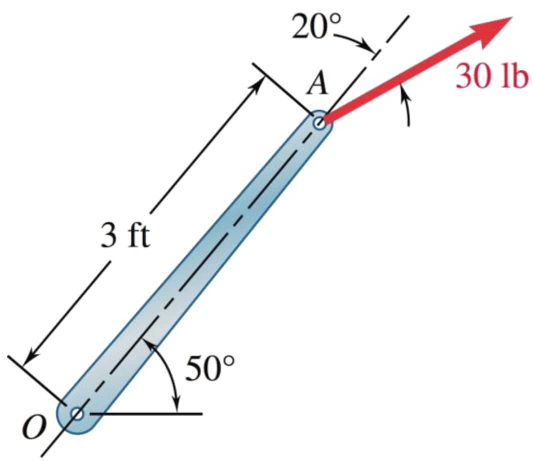
Sample Problem 3.2

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.



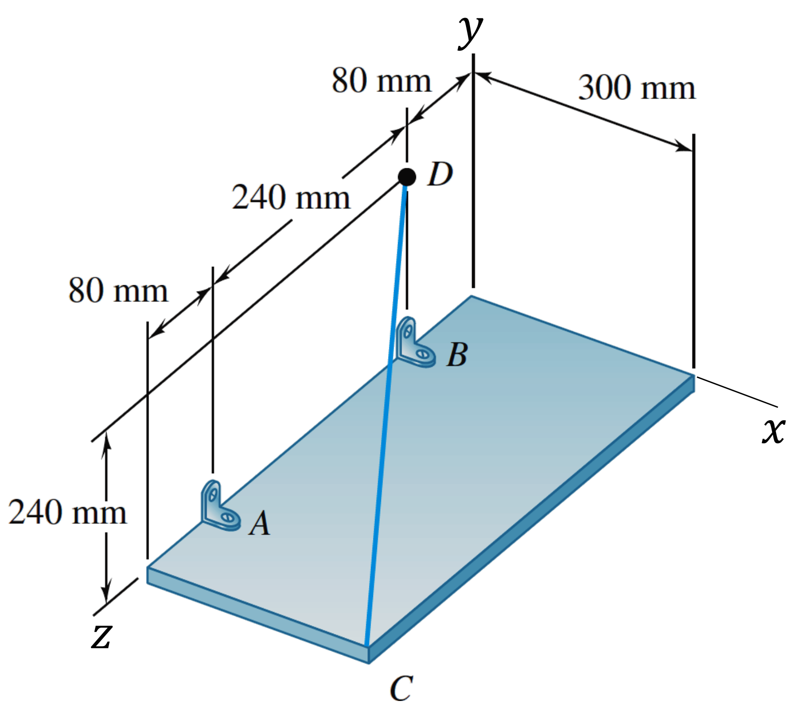
Sample Problem 3.3

A 30-lb force acts on the end of the 3-ft lever as shown. Determine the moment of the force about O.



Sample Problem 3.4

A rectangular plate is supported by brackets at A and B and by a wire CD. If the tension in the wire is 200 N, determine the moment about A of the force exerted by the wire on point C.

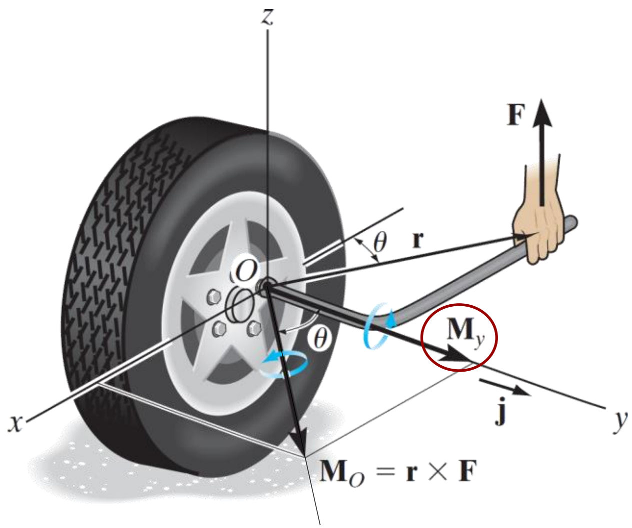


Moment of a Force about an Axis

Moment of a Force about a Specified Axis

Sometimes, the moment produced by a force about a specified axis must be determined.

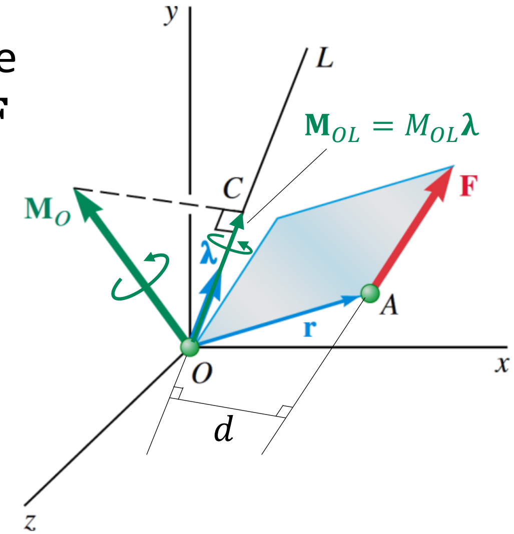
For example, suppose the lug nut at O on the car tire needs to be loosened. The force $\mathbf{F}$ applied to the wrench generates a moment $\mathbf{M}_O$ about the moment axis passing through O, however, the nut can only rotate about the y axis. Therefore, to determine the turning effect of $\mathbf{F}$, only the y component of the moment is needed, and the total moment produced is not important.



Moment axis

Moment of a Force about a Specified Axis

The moment $\mathbf{M}_{OL}$ of $\mathbf{F}$ about OL is defined as the projection of the moment $\mathbf{M}_O$ onto the axis OL. It is a measure of the tendency of $\mathbf{F}$ to make the body rotate about the axis OL.



Magnitude: $M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O = \boldsymbol{\lambda} \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$

Unit vector along OL

$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$
$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
$\boldsymbol{\lambda} = \lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}$

Position vector directed from any point O on the axis OL to any point A on the line of action of $\mathbf{F}$.

d: perpendicular distance from the force line of action to the axis

Vector Form: $\mathbf{M}_{OL} = M_{OL}\boldsymbol{\lambda}$

If M_{OL} is positive, it will have the same sense as $\boldsymbol{\lambda}$, whereas if it is negative, it will act opposite to $\boldsymbol{\lambda}$.

Moment of a Force about a Specified Axis

By resolving $\mathbf{F}$ and $\mathbf{r}$ into components parallel to the axis OL ($\mathbf{F}_1$ and $\mathbf{r}_1$) and in a plane perpendicular to the axis ($\mathbf{F}_2$ and $\mathbf{r}_2$), we can show that the moment $\mathbf{M}_{OL}$ of $\mathbf{F}$ about OL measures the tendency of $\mathbf{F}_2$ to rotate the rigid body about the axis, i.e., only the perpendicular component $\mathbf{F}_2$ of $\mathbf{F}$ tends to make the body rotate about OL. The other component $\mathbf{F}_1$ of $\mathbf{F}$ does not tend to rotate about OL, because $\mathbf{F}_1$ and OL are parallel.

Proof:

$$M_{OL} = \boldsymbol{\lambda}\cdot\mathbf{M}_O = \boldsymbol{\lambda}\cdot(\mathbf{r}\times\mathbf{F}) = \boldsymbol{\lambda}\cdot[\mathbf{r}\times(\mathbf{F}_1+\mathbf{F}_2)]$$

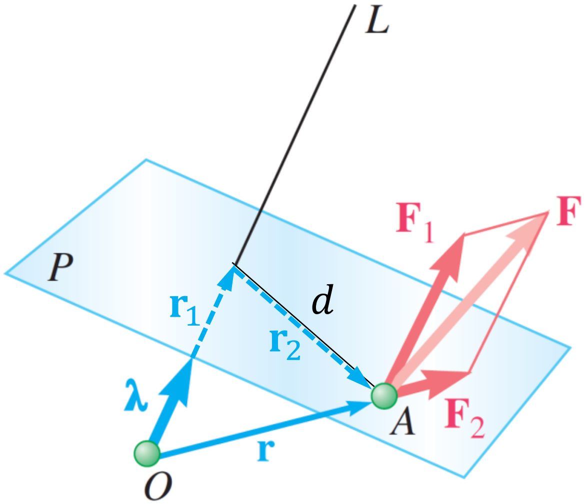
$$= \underbrace{\cancel{\boldsymbol{\lambda}\cdot(\mathbf{r}\times\mathbf{F}_1)}}_{0} + \underline{\boldsymbol{\lambda}\cdot(\mathbf{r}\times\mathbf{F}_2)} = F_2 d$$

$$\boxed{\begin{aligned}&\boldsymbol{\lambda}\cdot[(\mathbf{r}_1+\mathbf{r}_2)\times\mathbf{F}_2]\\ &= \underbrace{\cancel{\boldsymbol{\lambda}\cdot(\mathbf{r}_1\times\mathbf{F}_2)}}_{0} + \boldsymbol{\lambda}\cdot(\mathbf{r}_2\times\mathbf{F}_2)\\ &= (\mathbf{r}_2\times\mathbf{F}_2) = F_2 d\end{aligned}}$$

$$\mathbf{F}_1 = (\boldsymbol{\lambda}\cdot\mathbf{F})\boldsymbol{\lambda} = F_1\boldsymbol{\lambda}$$

$$\mathbf{F}_2 = \mathbf{F} - (\boldsymbol{\lambda}\cdot\mathbf{F})\boldsymbol{\lambda}$$

$$F_2 = \sqrt{F^2 - F_1^2}$$



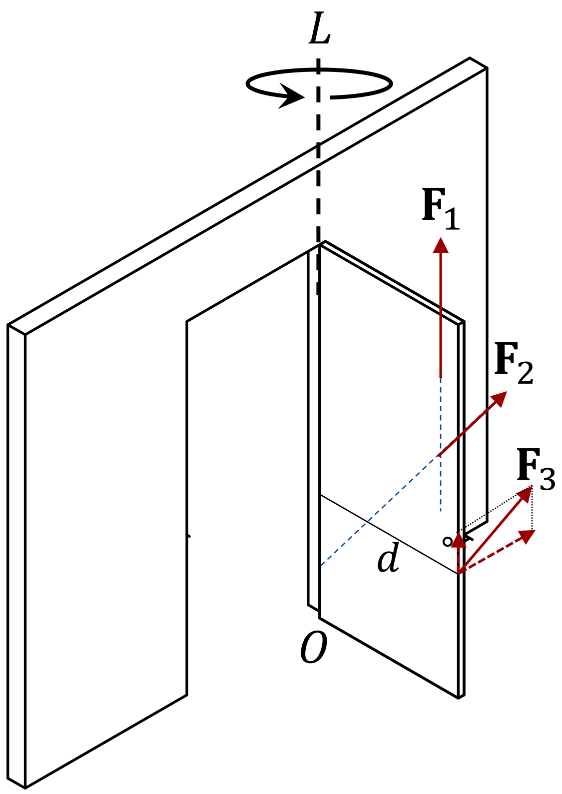
d: perpendicular distance from the force line of action to the axis

Note: The perpendicular distance d between two lines can be determined by $M_{OL} = \boldsymbol{\lambda}\cdot(\mathbf{r}\times\mathbf{F}) = F_2 d$

Moment of a Force about a Specified Axis

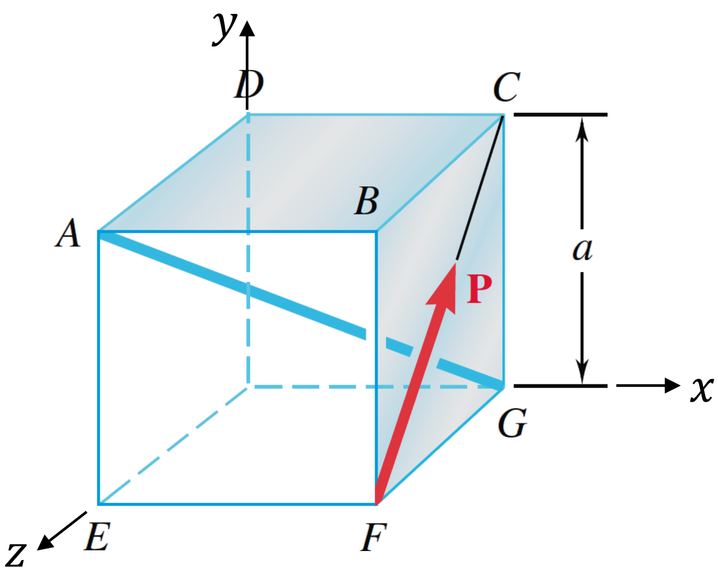
Summary:

- A force whose line of action is parallel to an axis (OL) does not produce moment about that axis (e.g., $\mathbf{F}_1$).
- A force whose line of action intersects an axis (OL) does not produce moment about that axis (e.g., $\mathbf{F}_2$).
- A force that has a component perpendicular to an axis (OL) produces moment about that axis (e.g., $\mathbf{F}_3$); its perpendicular component produces all the moment and its parallel component does not produce any moment about that axis.



Sample Problem 3.5

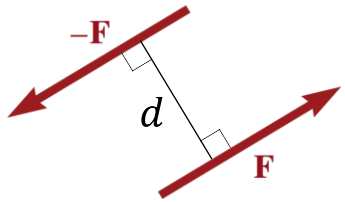
A cube of side a is acted upon by a force $\mathbf{P}$ along the diagonal of a face, as shown. Determine the moment of $\mathbf{P}$ (a) about A, (b) about the edge AB, (c) about the diagonal AG of the cube. (d) Using the result of part c, determine the perpendicular distance between AG and FC.

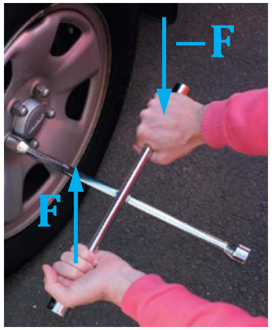


Moment of a Couple

Moment of a Couple

A **Couple** is defined as two parallel, noncollinear forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d.



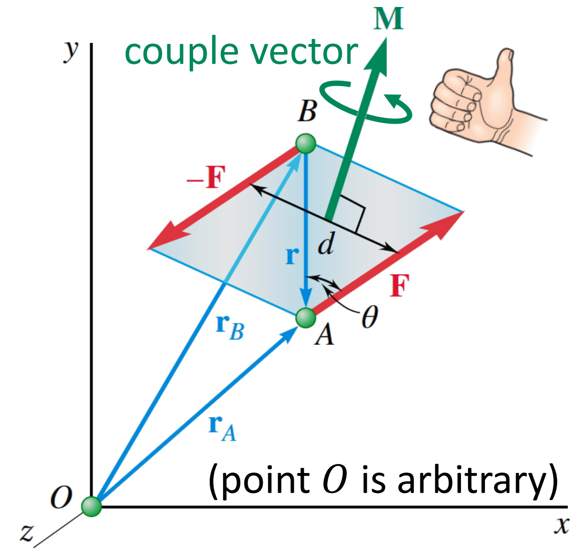


The two forces do not cause the body to move along a line (since $\Sigma\mathbf{F} = \mathbf{0}$), but they do tend to make it rotate. **Moment of the Couple**:

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

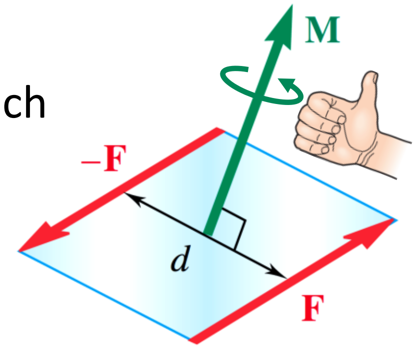
$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

Position vector $\mathbf{r}$ is directed from <u>any point</u> on the line of action of <u>one of the forces</u> to <u>any point</u> on the line of action of <u>the other force</u>.



Moment of a Couple

Since the vector $\mathbf{r}$ in is independent of the choice of the origin O of the coordinate axes, the moment $\mathbf{M}$ of a couple is a **free vector**, which can be applied at any point.

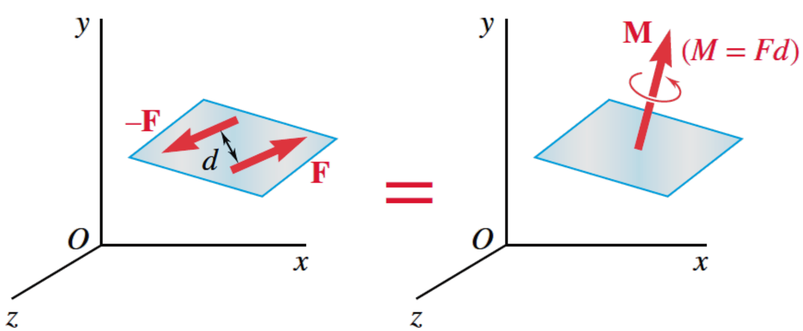


$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Magnitude: $M = rF\sin\theta = Fd$ (d is the perpendicular distance between the lines of action)

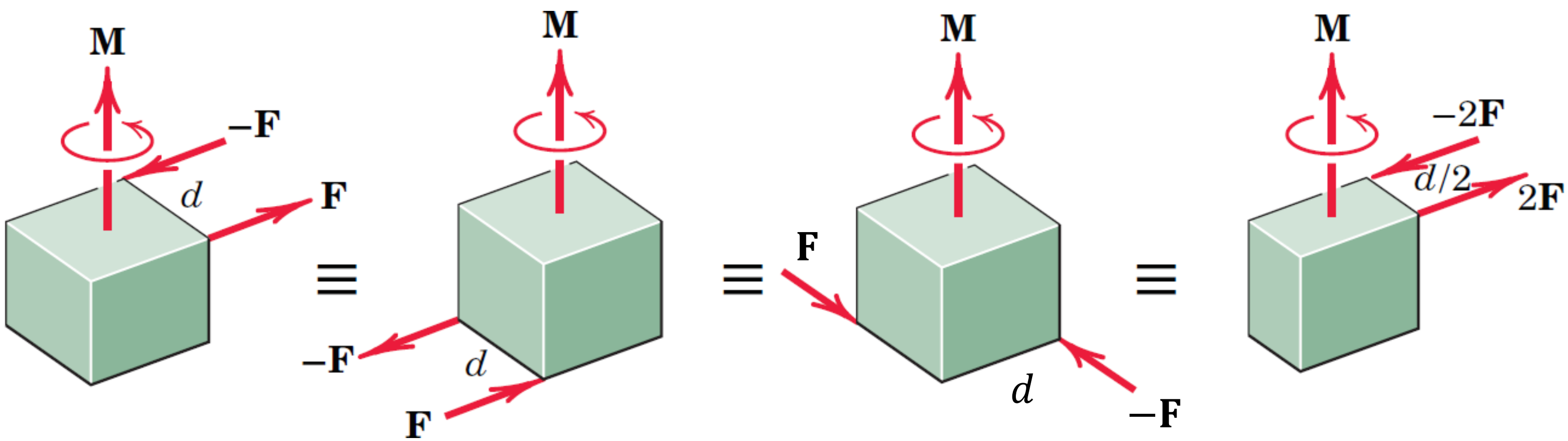
Line of Action: Perpendicular to the plane containing the two forces

Sense: Right-hand rule (fingers are curled with the sense of rotation caused by the couple forces)



Equivalent Couples

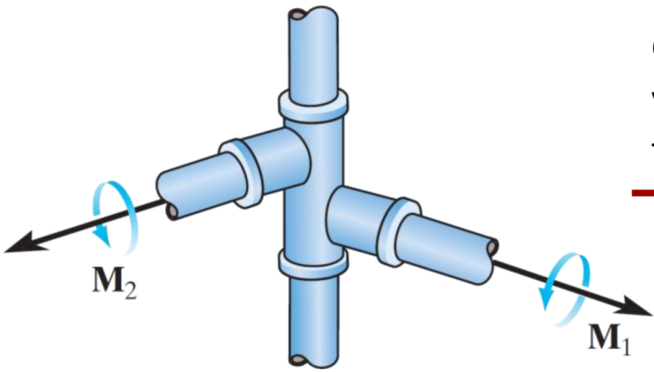
If two couples produce a moment with the same magnitude and direction, then these two couples are **equivalent** (i.e., they have the same external effect on a rigid body).



(Couples lie in parallel planes)

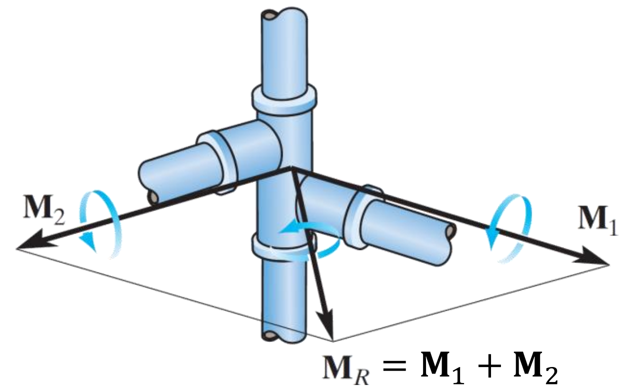
Addition of Couples

Since couple moments are **vectors**, their resultant can be determined by **vector addition**.



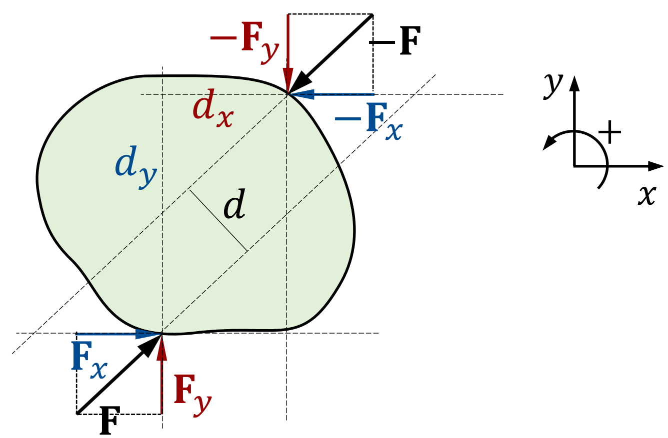
Couple moment is a free vector (we can join their tails at any arbitrary point)

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$



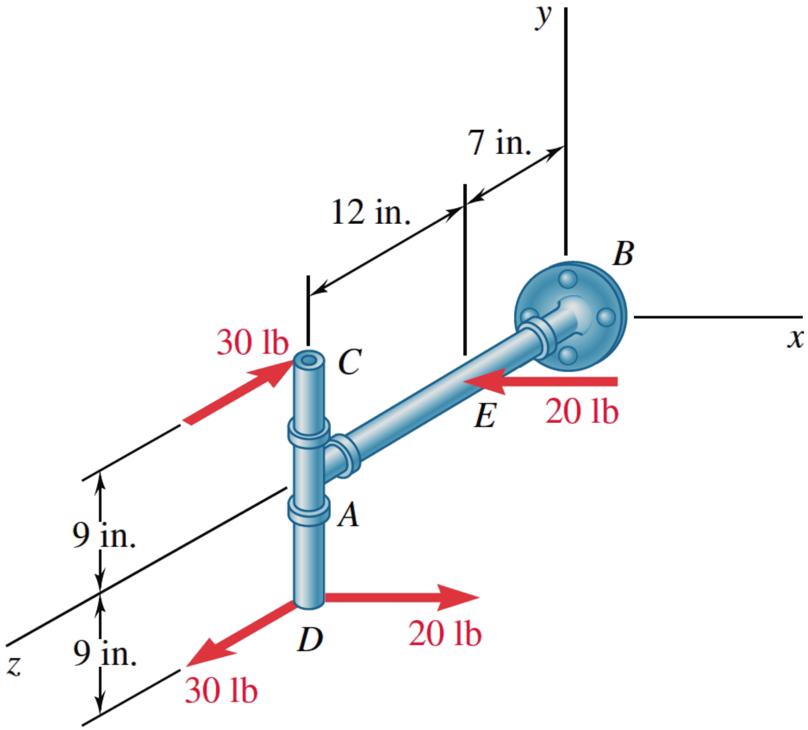
- Using the **principle of moments**, we can determine the moments of the components of the couple, i.e., $(\mathbf{F}_x, -\mathbf{F}_x)$ and $(\mathbf{F}_y, -\mathbf{F}_y)$, rather than the moment of the couple itself, i.e., $(\mathbf{F}, -\mathbf{F})$.

$$M = Fd = F_x d_y - F_y d_x$$



Sample Problem 3.6

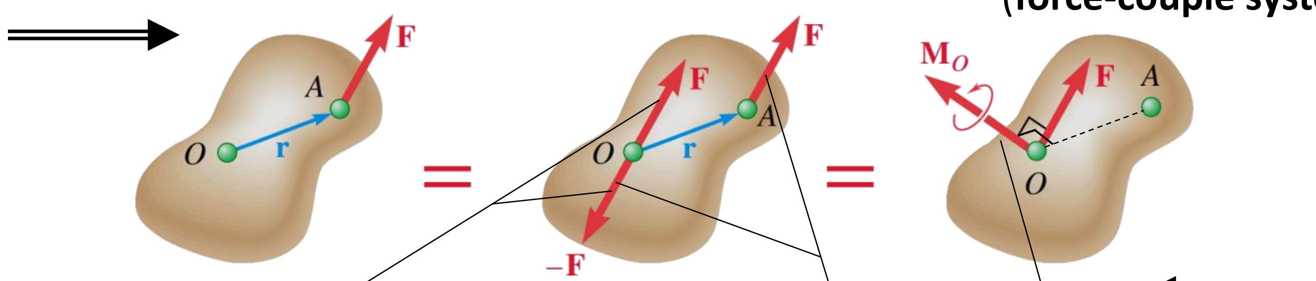
Determine the components of the single-couple equivalent to the two couples shown.



Force-Couple System

Any force $\mathbf{F}$ acting on a rigid body can be moved to an <u>arbitrary point</u> O if we add a couple whose moment is equal to the moment of $\mathbf{F}$ about O.

(**force-couple system**)



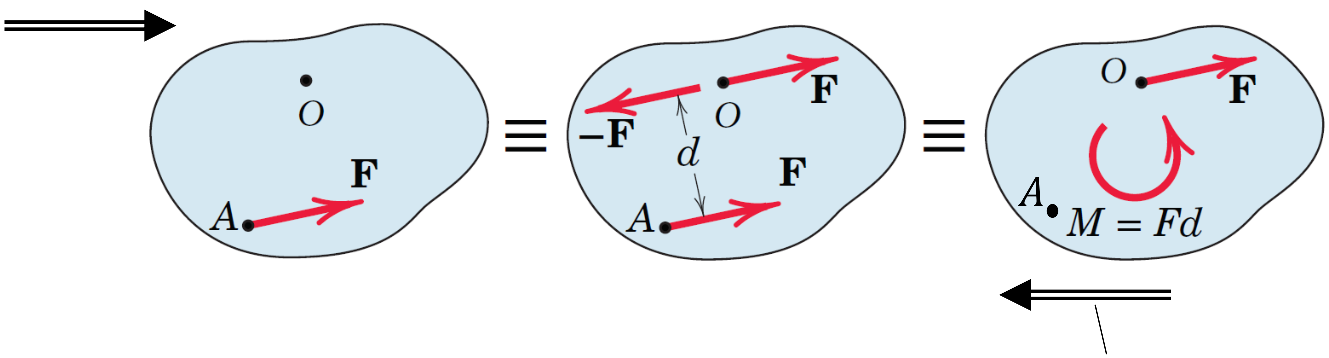
(1) attaching equal and opposite forces at O.

(2) forming a couple of moment $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$.

- Because $\mathbf{M}_O$ is a free vector, it may be applied anywhere; for convenience, however, the couple vector is usually attached at O together with $\mathbf{F}$.

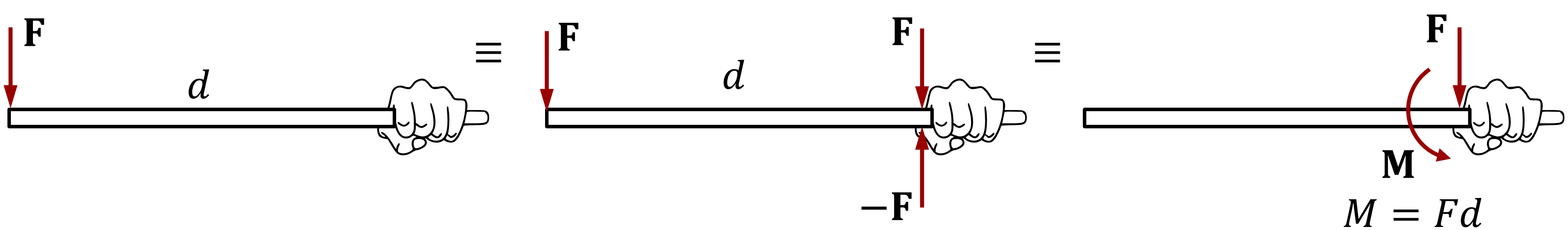
- **Conversely**, any force-couple system consisting of a force $\mathbf{F}$ and a couple vector $\mathbf{M}_O$ that are **mutually perpendicular** can be replaced by <u>a single equivalent force</u>. This is done by moving force $\mathbf{F}$ in the plane perpendicular to $\mathbf{M}_O$ until its moment about O is equal to the moment of the couple being replaced.

Force-Couple System: Planar Case



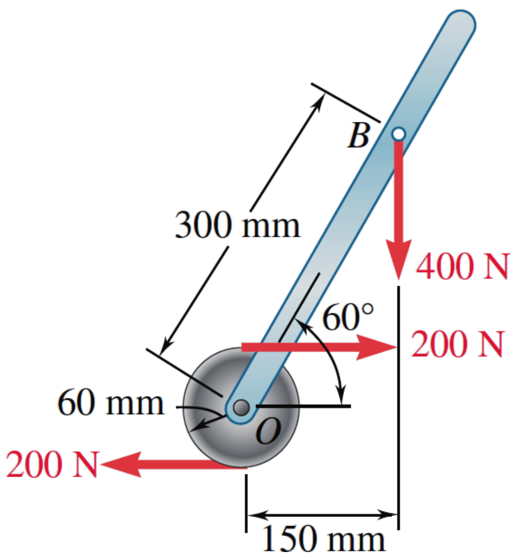
► Since, in planar case, **F** and **M** are always mutually perpendicular, by moving the force of the force-couple system a distance that creates the same moment as the couple, you can always replace the force-couple system with one equivalent force.

Example:



Sample Problem 3.7

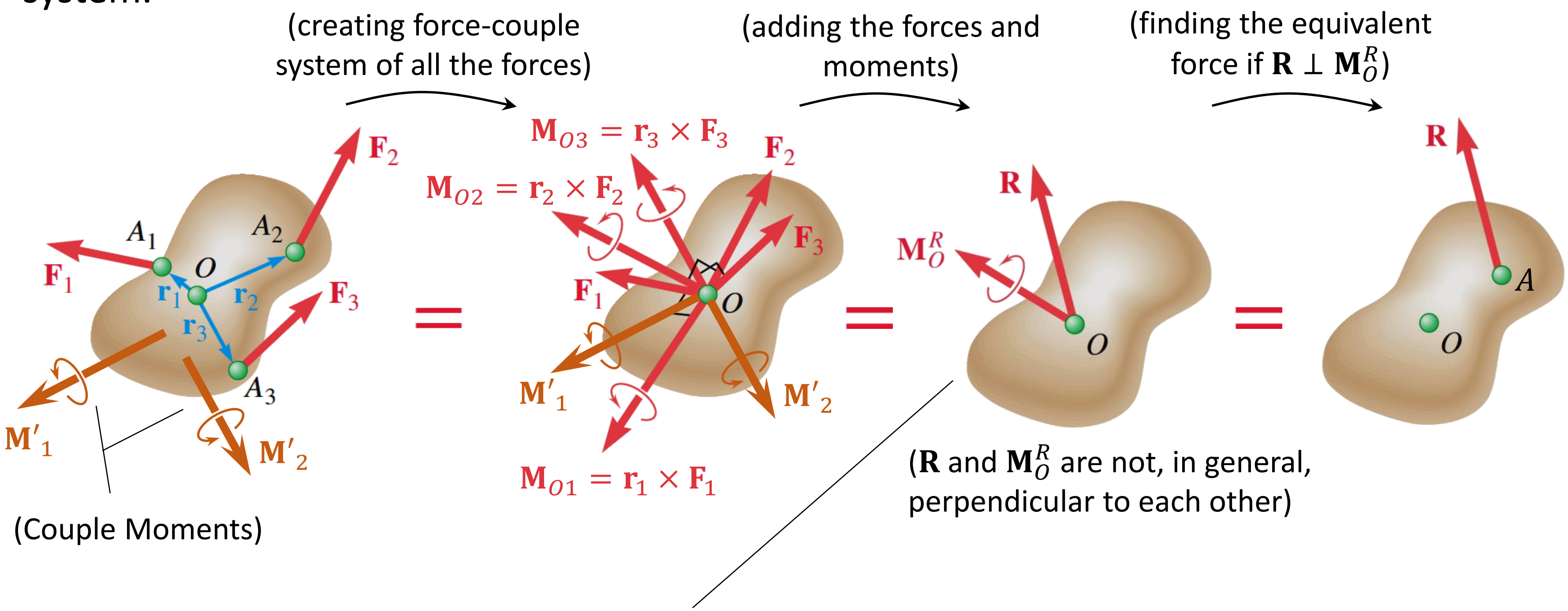
Replace the couple and force shown by an equivalent single force applied to the lever. Determine the distance from the shaft to the point of application of this equivalent force.



Simplifying Systems of Forces & Moments

System of Forces and Couple Moments

We can reduce any system of forces and couple moments to a simpler and equivalent system.

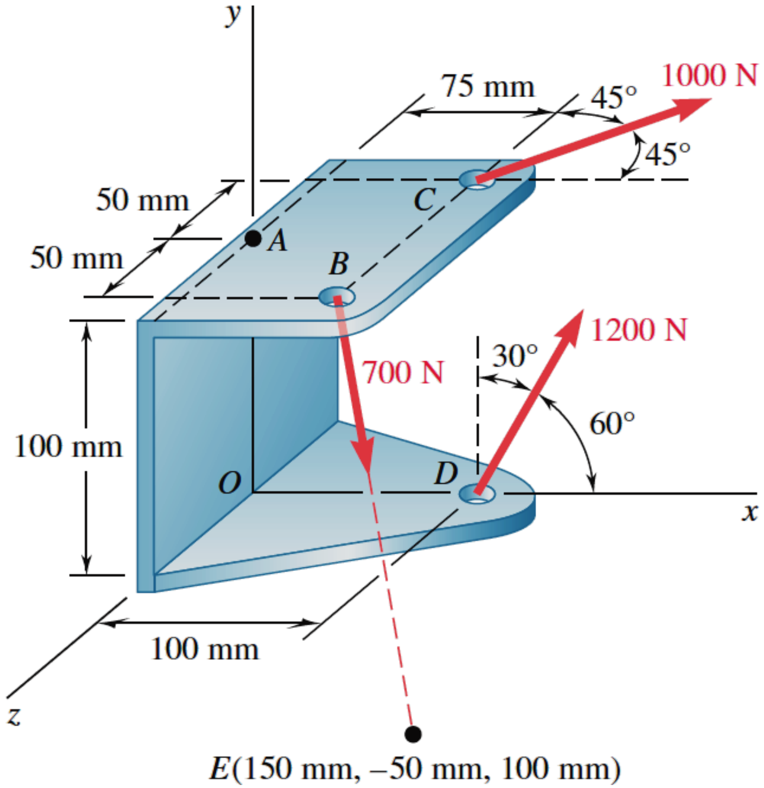


Resultant Force: $\mathbf{R} = \Sigma \mathbf{F}$

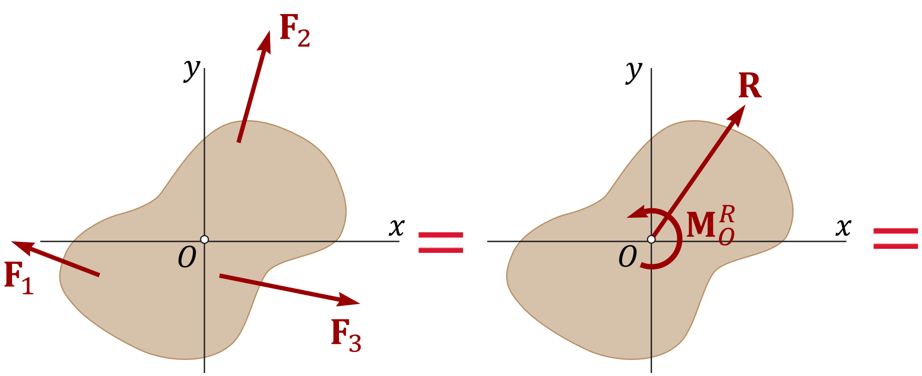
Resultant Couple Moment: $\mathbf{M}_O^R = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}' = \Sigma(\mathbf{r} \times \mathbf{F}) + \Sigma \mathbf{M}'$

Sample Problem 3.10

Three cables are attached to a bracket as shown. Replace the forces exerted by the cables with an equivalent force-couple system at A.

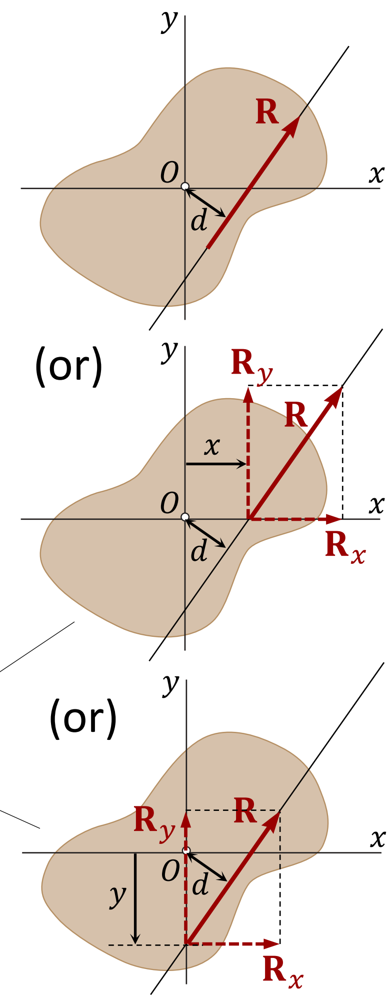


Special Case 1 for $\mathbf{R} \perp \mathbf{M}_O^R$: Coplanar Forces



$$\mathbf{R} = \mathrm{R}_x\mathbf{i} + \mathrm{R}_y\mathbf{j}$$

$$\mathbf{M}_O^R = M_O^R\mathbf{k}$$



$$M_O^R = dR \Rightarrow \quad d \checkmark$$

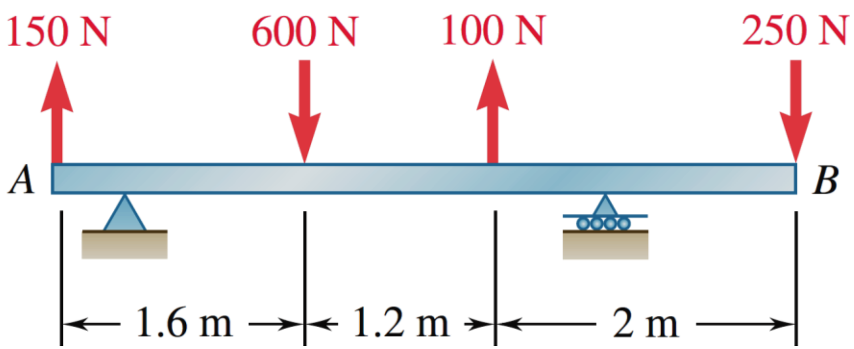
$$M_O^R = xR_y \Rightarrow \quad x \checkmark$$

$$M_O^R = yR_x \Rightarrow \quad y \checkmark$$

Determining the x and y <u>intercepts</u> of the line of action of **R**.

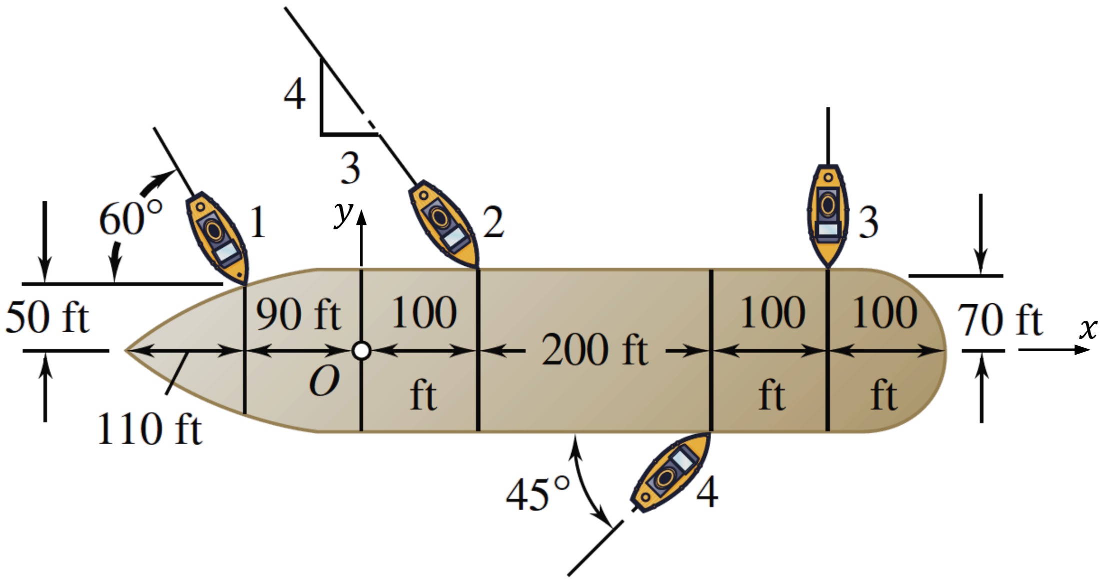
Sample Problem 3.8

A 4.80-m-long beam is subjected to the forces shown. Reduce the given system of forces to (a) an equivalent force-couple system at A, (b) an equivalent force-couple system at B, (c) a single force or resultant.

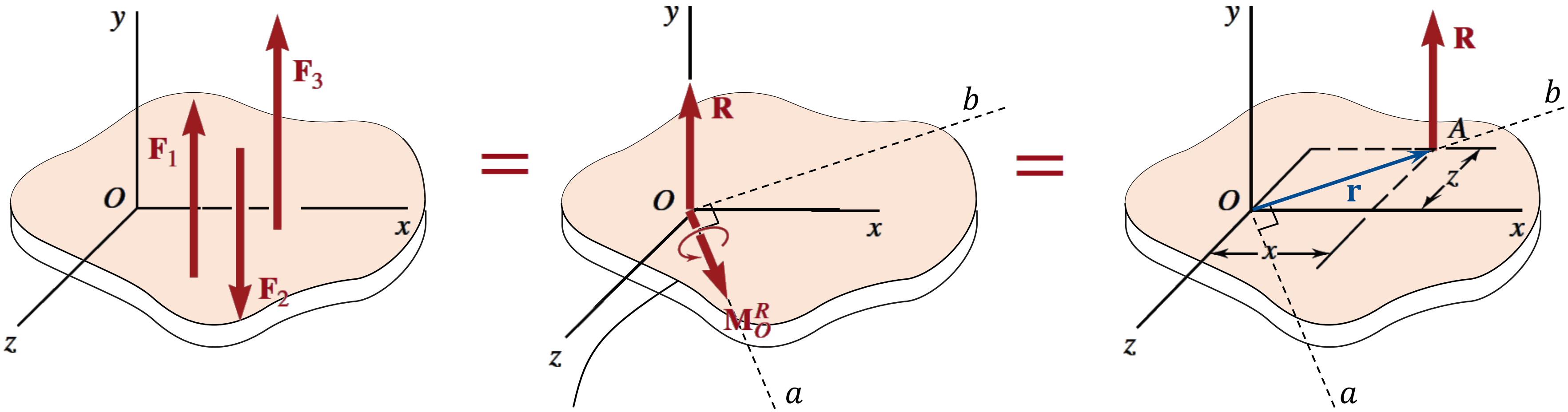


Sample Problem 3.9

Four tugboats are bringing an ocean liner to its pier. Each tugboat exerts a 5000-lb force in the direction shown. Determine (a) the equivalent force-couple system at the foremast *O*, (b) the point on the hull where a single, more powerful tugboat should push to produce the same effect as the original four tugboats.



Special Case 2 for $\mathbf{R} \perp \mathbf{M}_O^R$: Parallel Forces



$\mathbf{M}_O^R$ lies in the $z-x$ plane.

$\mathbf{R} = \mathrm{R}_y\mathbf{j}$ $\Rightarrow$ $\mathbf{M}_O^R = \mathbf{r} \times \mathbf{R}$ $\Rightarrow$ x, z ✓

$\mathbf{M}_O^R = M_x^R\mathbf{i} + M_z^R\mathbf{k}$ $\quad$ $\mathbf{r} = x\mathbf{i} + z\mathbf{k}$

(vector of a point A that
$\mathbf{R}$ creates the same moment $\mathbf{M}_O^R$)

Sample Problem 3.11

A square foundation mat supports the four columns shown. Determine the magnitude and point of application of the resultant of the four loads.