Ch6: Analysis of Structures

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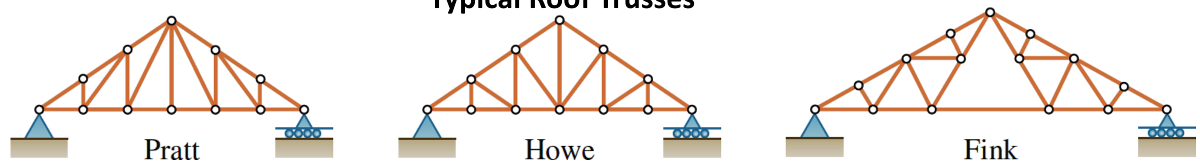
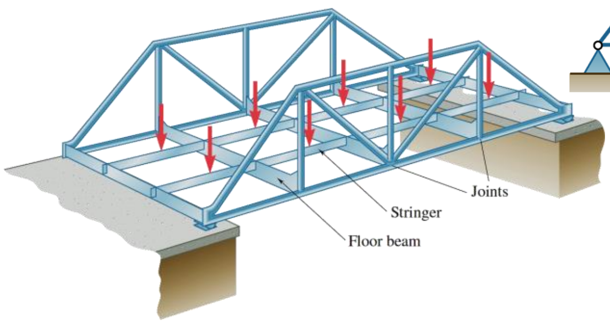
Plane Trusses

Truss

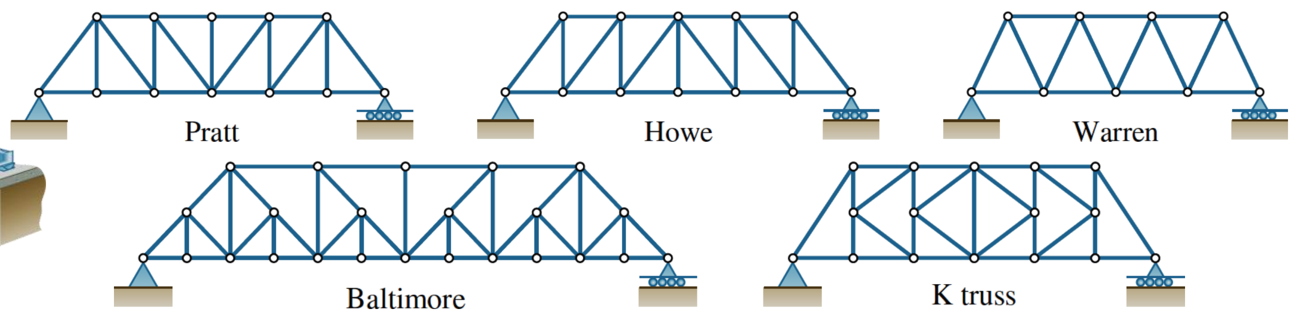
A **Truss** is a stationary structure that consists of straight members connected at joints located at the ends of each member. Planar trusses lie in a single plane and are often used to support roofs and bridges.

Typical Roof Trusses





Typical Bridge Trusses



For large trusses, a **rocker or roller** is commonly used at one of the supports to provide for expansion and contraction due to temperature changes and for deformation from applied loads. Otherwise, the truss is **statically indeterminate**.

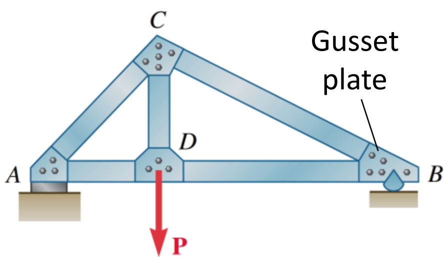
Assumptions for Design & Analysis of Trusses

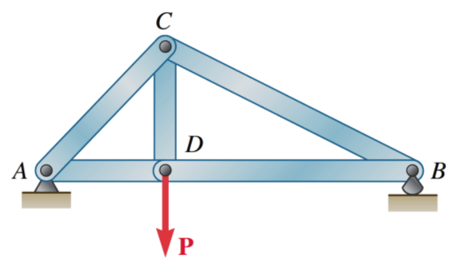
- **All loadings are applied at the joints.**

The weight of the members is neglected (or we can apply it as a vertical force, with half of its magnitude, at each end of the member).

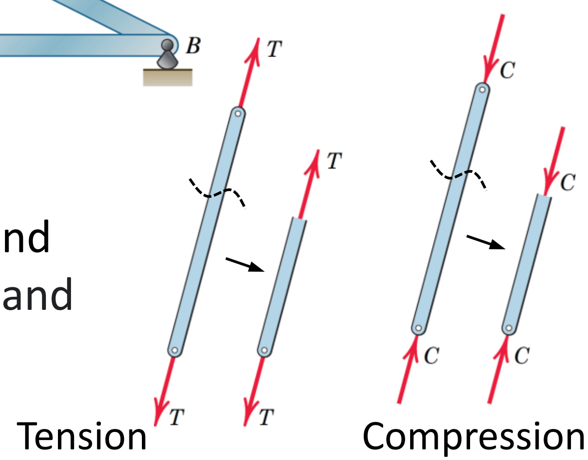
- **The members are joined together by smooth pins.**

The joint connections are usually formed by bolting, welding, or riveting the ends of the members to a common plate (gusset plate) or by simply passing a large bolt or pin through each of the members. We can assume these connections act as pins provided the center lines of the joining members are concurrent.



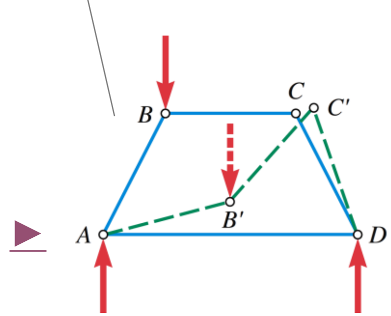


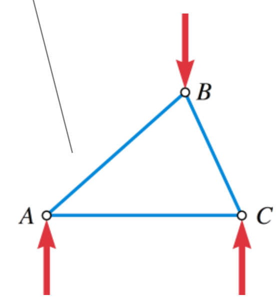
<u>Therefore</u>, we can model the entire truss as a group of **pins** and **two-force members**, i.e., members acted upon by two equal and opposite forces directed along the member.



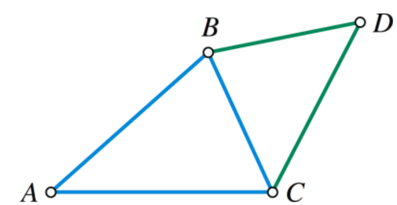
Simple Trusses

If the truss is made of four members and a load is applied at pin B or C, the truss will greatly deform. However, if the truss is made of **three members**, the truss will not deform greatly under a load applied at pin B. Thus, this truss is said to be a **rigid truss**.

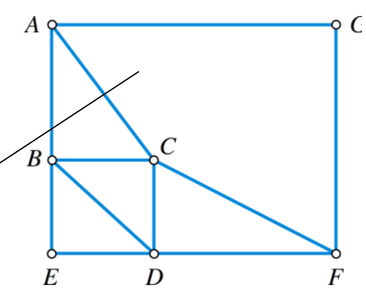




- By attaching <u>two new members to two existing joints</u> of the **basic triangular truss**, and connecting them at a new joint, the larger truss is also **rigid**.



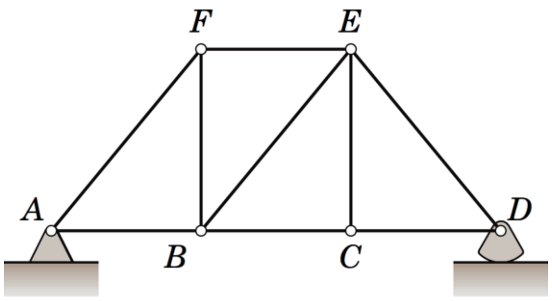
- This procedure can be repeated as many times as desired to form an even larger truss. A truss that can be constructed in this manner is called a **Simple Truss**.



(Note: a simple truss is not necessarily made only of triangles)

Statical Determinacy of Simple Trusses

In a simple truss, the total number of members is $m = 2n - 3$, where n is the total number of joints.



$n = 6$

$m = 9$

On the other hand,

Number of equations: $2n$ ($\Sigma F_x = 0$, $\Sigma F_y = 0$ at each joint)

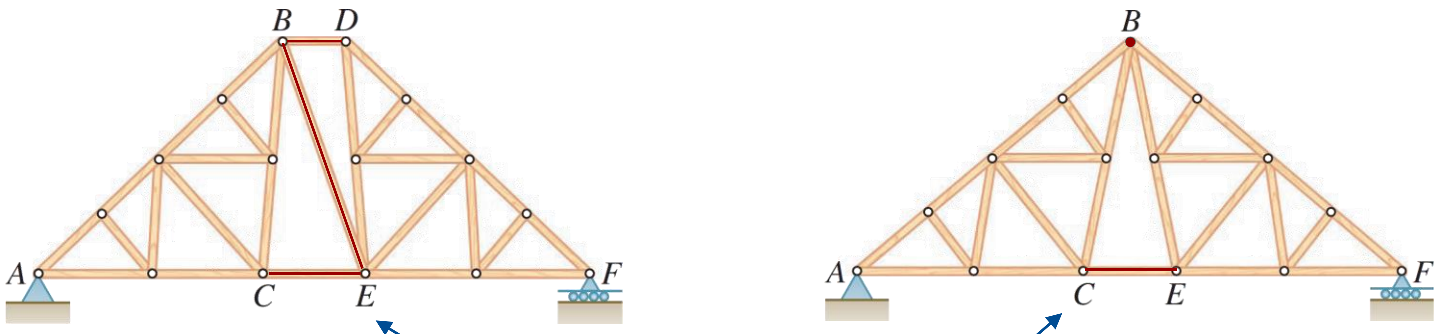
Number of unknowns: m (number of members) + 3 (reactions at supports)

Since in a simple truss $m = 2n - 3$,
number of equations = number of unknowns

Thus, a simple truss is always Statically Determinate.

Compound Trusses

A truss made of **several simple trusses** rigidly connected is known as a **compound truss**.



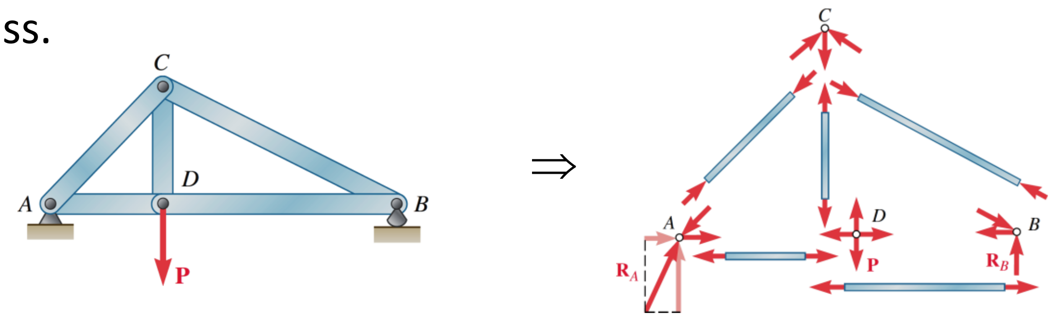
* If the simple trusses are connected by **three bars** or **a single bar and a common joint**, the number of members m and joints n are still related by the formula $m = 2n - 3$ (although this compound truss is not a simple truss anymore).

If these trusses are supported by a **pin** and a **roller/rocker**, they are statically determinate (i.e., we can determine all the unknown reactions and the forces in all the members, and the truss will neither collapse nor move).

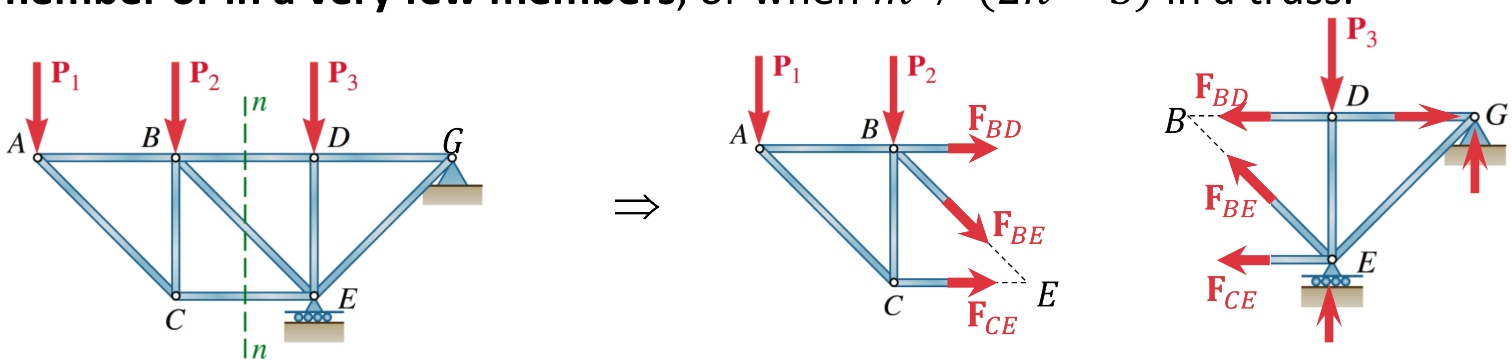
Analysis of Trusses

There are two methods to analyze trusses:

1) **Method of Joints**: It is most effective when we want to determine the forces in **all the members** of a truss.



2) **Method of Sections**: It is most effective when we want to determine the force in **only one member or in a very few members**, or when $m \neq (2n-3)$ in a truss.

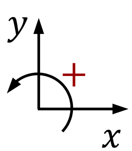


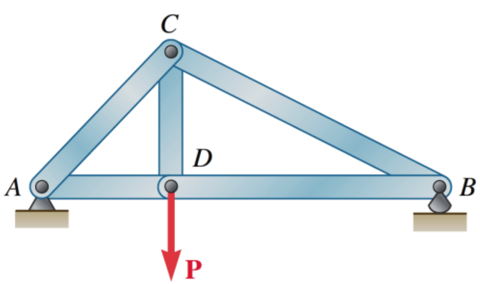
Method of Joints

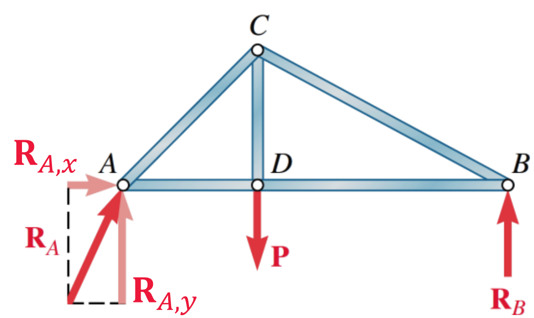
Method of Joints

This method is based on the fact that if the truss is in equilibrium, then **each of its joints** is also in **equilibrium**.

Step 1: Draw a free-body diagram of the entire truss to determine the **reactions** at the supports.
Note: A negative answer means the sense shown on the free-body diagram **must be reversed**, and the **correct sense** must be used for the rest of the analysis.





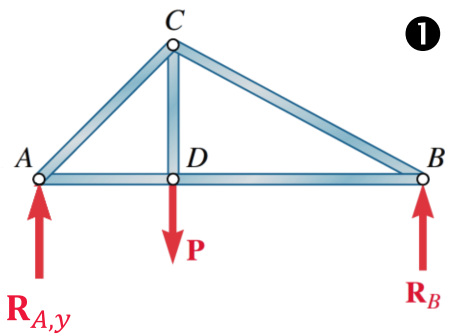


$$\begin{aligned} \Sigma M_A &= 0 \\ \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned} \quad \Rightarrow \quad R_{A,x} = 0,\ R_{A,y},\ R_B\ \surd$$

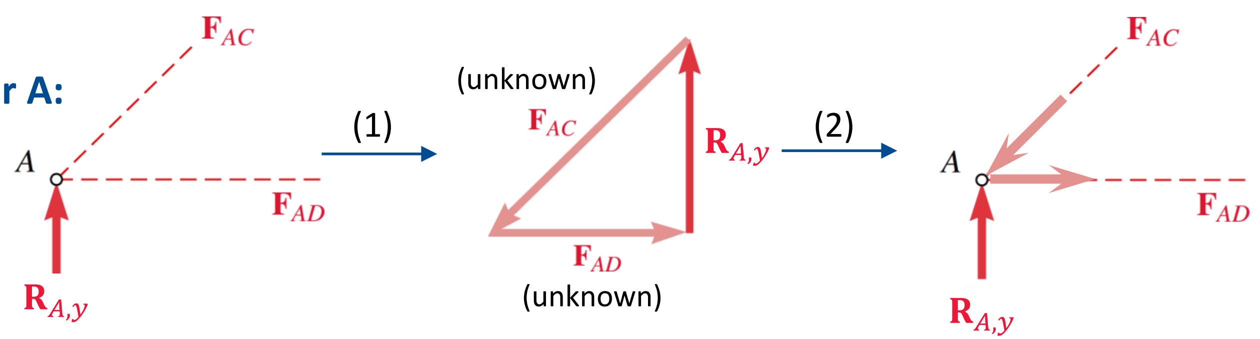
Method of Joints

Step 2: Draw the free-body diagram of an arbitrary joint having **at least one known force and at most two unknown forces** (usually one of the supports) to determine the unknown forces by solving the equilibrium equations (i.e., $\Sigma F_x = 0, \Sigma F_y = 0$) for the pin.

Note: If only **three forces** are involved at a pin, the sense of unknown forces can be found by drawing the corresponding force triangle, otherwise, **assume** that unknown forces are in **tension**. A negative answer means that the member is in compression, and the sense shown on the free-body diagram **must be reversed**, and the correct sense must be used for analysis of the rest of the joints.



❶ **For A:**



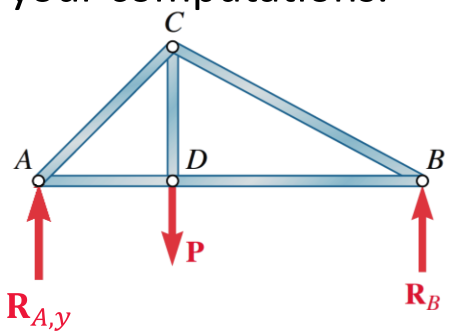
(1): Make a closed triangle using the direction of members to determine the sense of unknown forces.

(2): Reflect the correct sense in the FBD of the pin.

$\Rightarrow F_{AC}, F_{AD}$ √

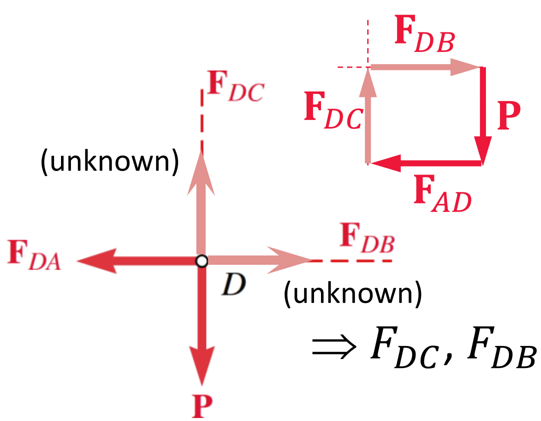
Method of Joints

Step 3: Repeat this procedure until you have found the forces in all the members of the truss. You will end up with three extra equations. These equations can be used to check your computations.



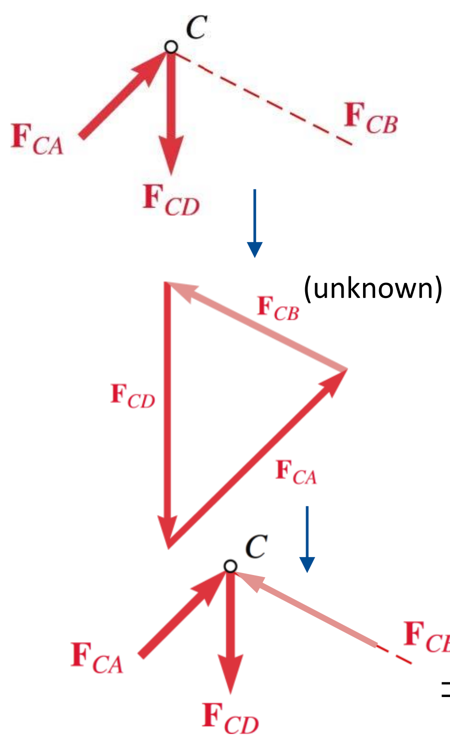
(Because in Step 1, you have used three equations of equilibrium of the entire truss to determine the reactions at the supports.)

❷ **For D:**



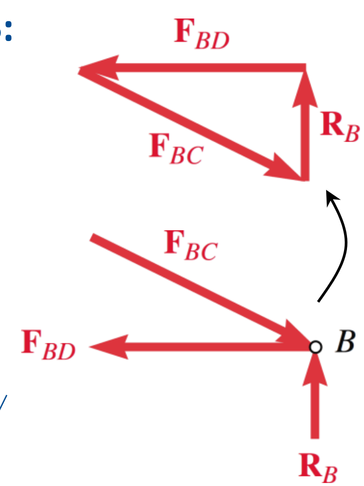
$\Rightarrow F_{DC}, F_{DB}$ √

❸ **For C:**



$\Rightarrow F_{CB}$ √

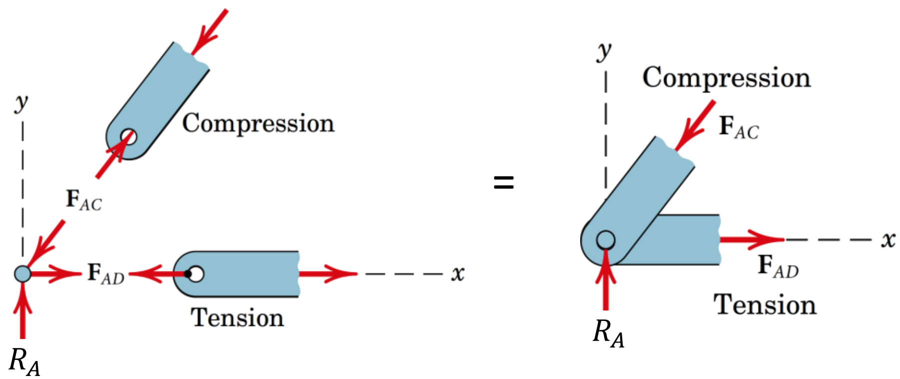
❹ **For B:**



(three extra equations)

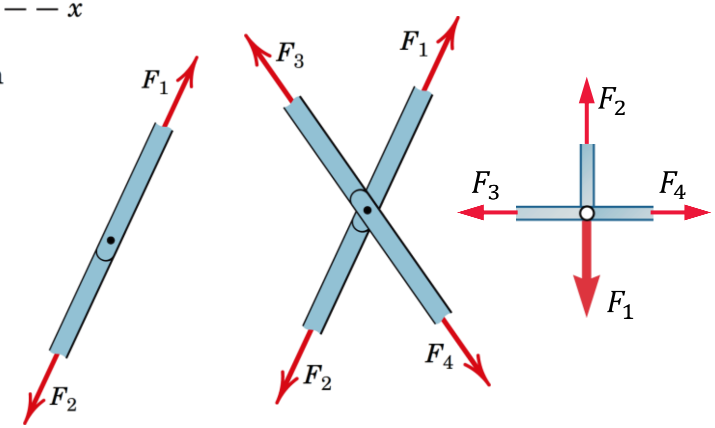
Method of Joints

Note: A member in tension is indicated by an arrow away from the pin, and a member in compression is indicated by an arrow toward the pin. This can be demonstrated by isolating the joint with small segments of the member connected to the pin.



Note: When two pairs of collinear members are joined, the forces in each pair must be equal and opposite.

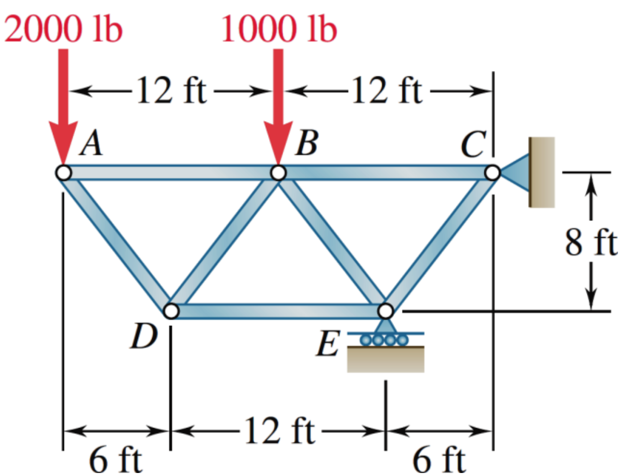
$$F_1 = F_2$$
$$F_3 = F_4$$



Note: In some cases, you may reach a joint (with more than two unknowns) beyond which you cannot proceed. You must then start again from another joint to complete the solution.

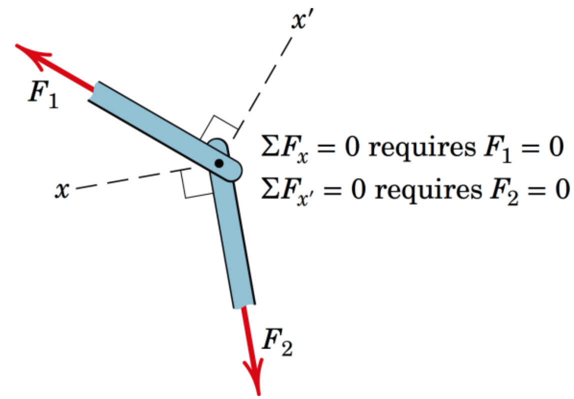
Sample Problem 6.1

Using the method of joints, determine the force in each member of the truss shown.



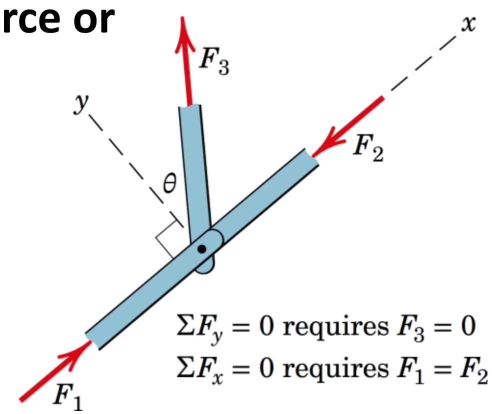
Zero-Force Members

Truss analysis using the method of joints is greatly simplified if we can first identify those members which **support no loading**.

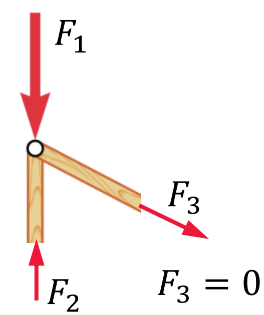


Case 1: If only **two non-collinear members** form a truss joint and **no external load or support reaction** is applied to the joint, the two members must be **zero-force members**.

Case 2: If **three members** form a truss joint for which **two of the members are collinear**, the third member is a **zero-force member** provided **no external force or support reaction** is applied to this joint.

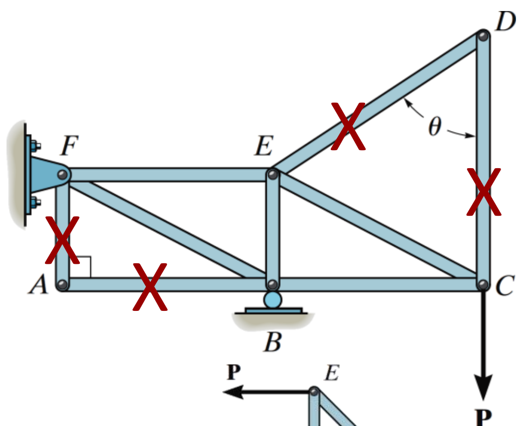


or

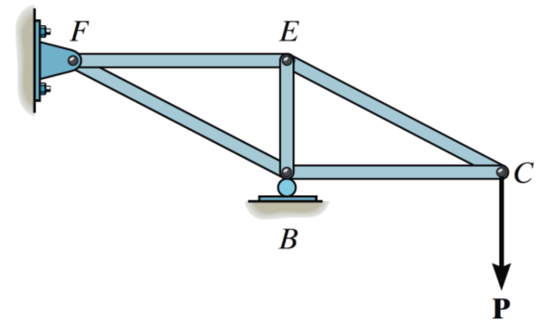


Zero-Force Members

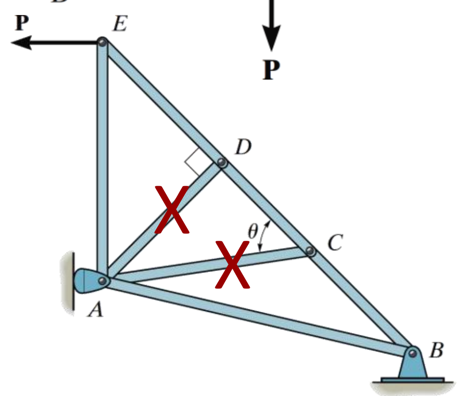
Example 1:



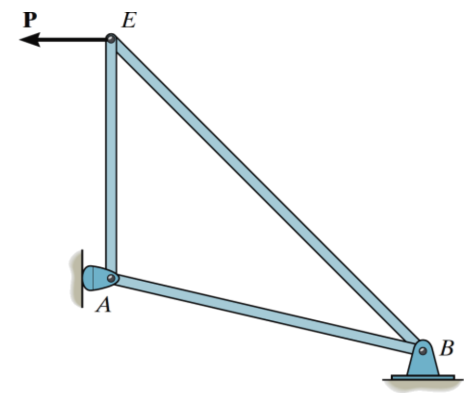
⇒



Example 2:



⇒

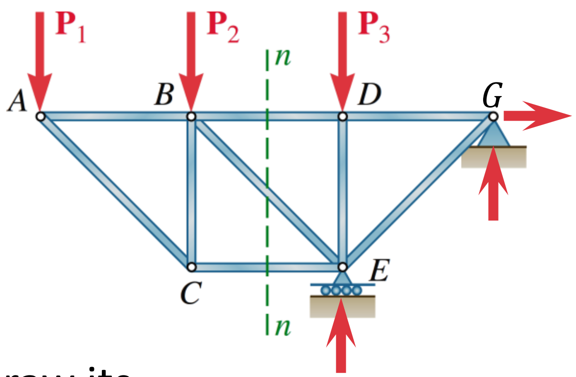


Note: Zero-force members **are not useless**. These members are needed to support the weight of the truss, maintain the truss in the desired shape, and increase the stability the truss during construction. Moreover, there are required <u>if the loading is changed</u>.

Method of Sections

Method of Sections

This method is based on the fact that if the truss is in equilibrium, then **any segment** of the truss is also in **equilibrium**.

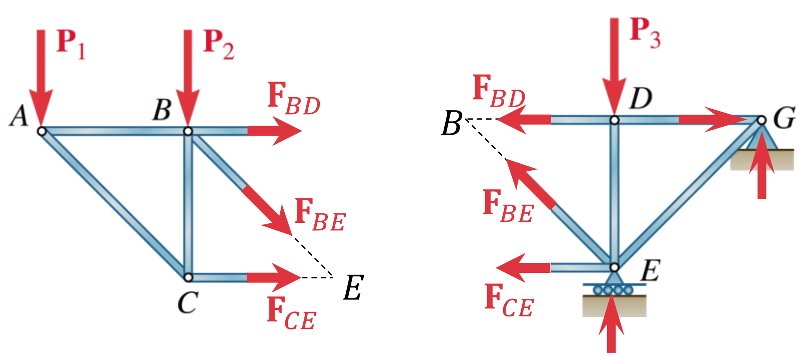


1. Draw a free-body diagram of the entire truss to determine the **reactions** at the supports.

2. **Pass a section** through **<u>three members</u>** of the truss, at least one of which is the member whose force you want to find.

3. Arbitrarily select **one of two separate portions** of truss and draw its FBD (Note: the internal forces in the members now become external forces).

4. Solve the equilibrium equations to find forces in the **three intersected members**.
We assume that these three members are in tension. A negative answer means that the member is in compression, and the sense shown on the free-body diagram **must be reversed**.



$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0 \;\rightarrow\; \mathbf{F}_{BD}, \mathbf{F}_{CE}, \mathbf{F}_{BE} \;\checkmark$$

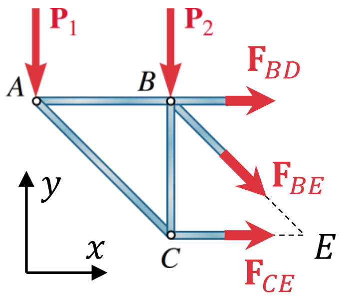
Method of Sections

Note: Choose a segment that has the least number of loads on it.

Note: Sometimes we can write a single equation to find the force in the **desired members**:

a. If the other two unknown forces are **parallel**, write the force equilibrium equation $\Sigma F = 0$ perpendicular to the direction of these unknowns.

$$\Sigma F_y = 0 \rightarrow \mathbf{F}_{BE} \ \checkmark$$



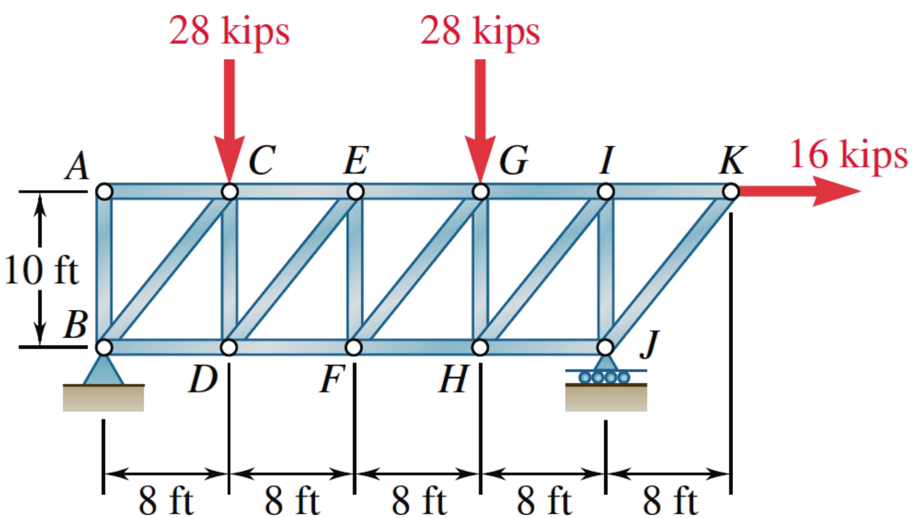
b. If the line of action of the other two unknown forces **intersects at a point**, write the moment equilibrium equation $\Sigma M = 0$ about that point.

$$\Sigma M_E = 0 \rightarrow \mathbf{F}_{BD} \ \checkmark$$

$$\Sigma M_B = 0 \rightarrow \mathbf{F}_{CE} \ \checkmark$$

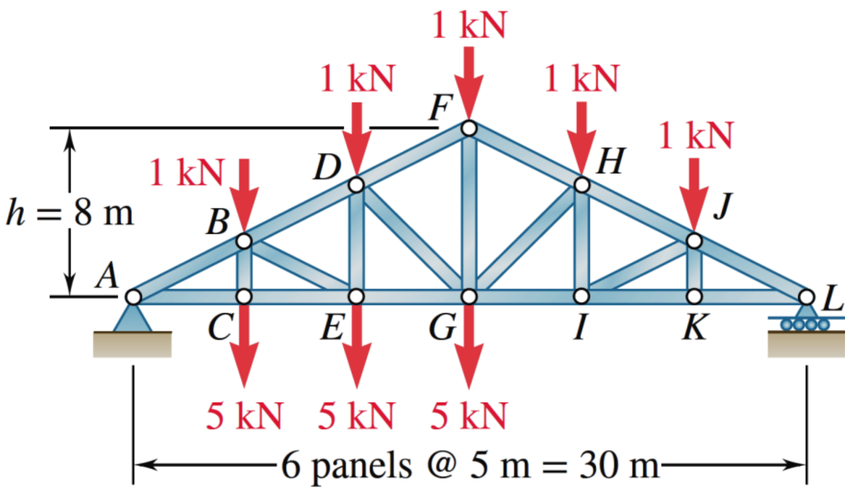
Sample Problem 6.2

Determine the forces in members EF and GI of the truss shown.



Sample Problem 6.3

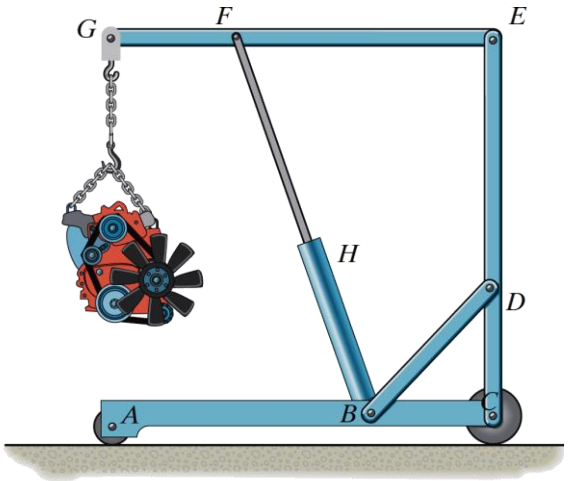
Determine the forces in members FH, GH, and GI of the roof truss shown.

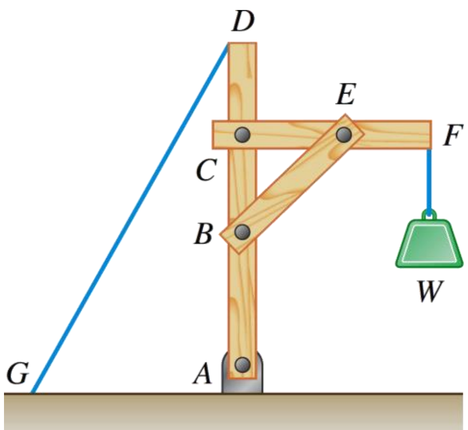


Frames and Machines

Frames

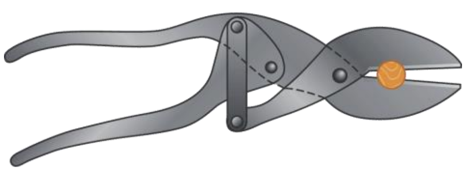
Frames are designed to support loads and are usually stationary, fully constrained structures. Frames always **contain at least one pin-connected multi-force member**, i.e., a member acted upon by three or more forces that, in general, are not directed along the member.

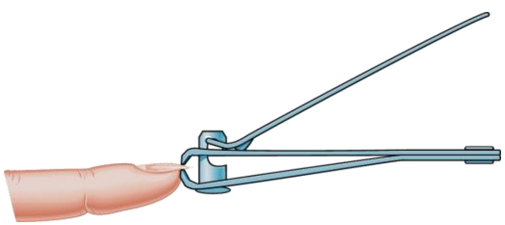


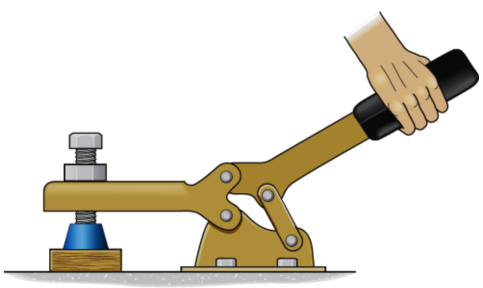


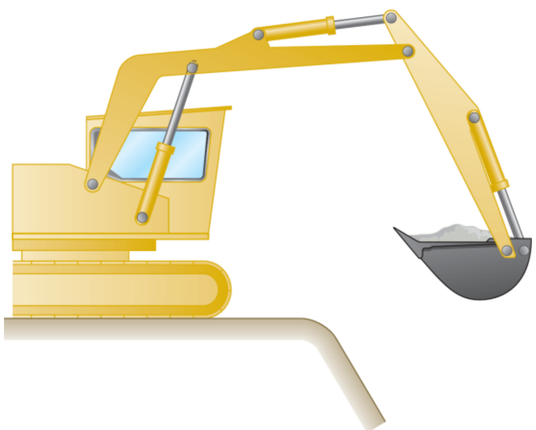
Machines

Machines are structures designed to transmit and modify forces. Their main purpose is to transform **input forces** into **output forces**. They may or may not be stationary and always contain moving parts. However, the machines considered here are always at rest, and you will be working with the set of forces required to maintain the equilibrium of the machine.







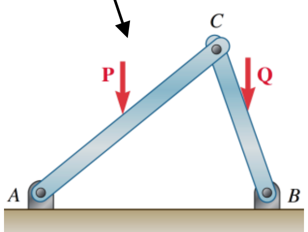


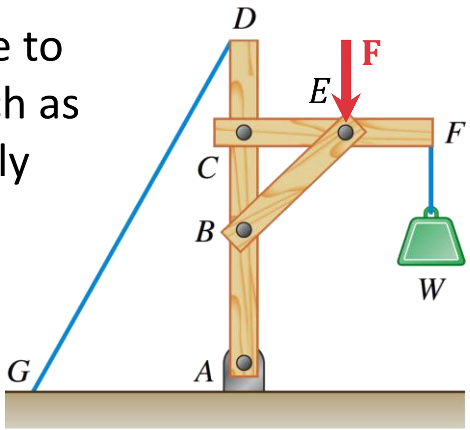
Machines, like frames, always contain **at least one multi-force member**. Thus, the same procedure is used to analyze machines.

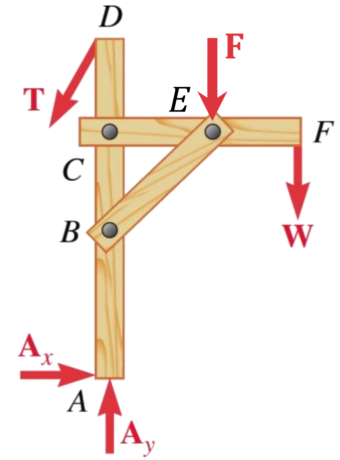
Analysis of Frames (and Machines)

1. Draw a free-body diagram of the entire frame to determine the reactions at the supports as much as possible (all the reactions <u>may not</u> be completely determined in this step).

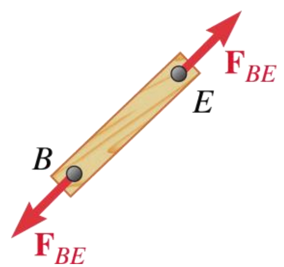
(for example)







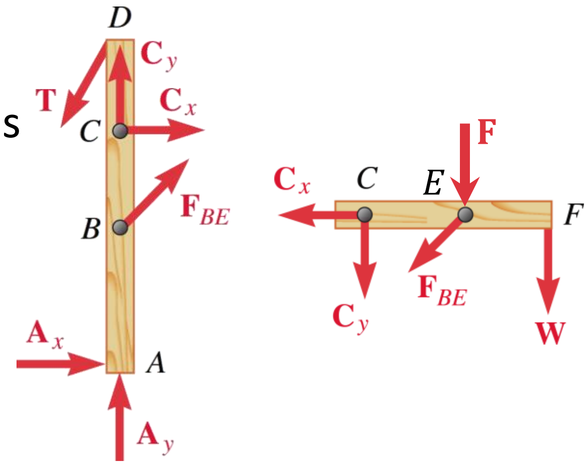
2. Dismember the frame and draw a free-body diagram of each member. **First** consider the **two-force members** and assume that these members are in tension (direct both forces away from the member).



• When a pin connects 2 or 3 members (including **multi-force members**), always assume that the pin belongs to **one** of the **multi-force members** (e.g., in this figure, we assume $\mathbf{F}$ is applied to CEF, not the two-force member, in its FBD). [<u>See Sample Problem 6.6</u>]

Analysis of Frames (and Machines) (cont.)

3. Consider the **multi-force members**. Arbitrarily direct the x any y components of the unknown forces. Note that the forces common to two members which are in contact act with <u>equal magnitude but opposite sense</u> on the respective free-body diagrams of the members.

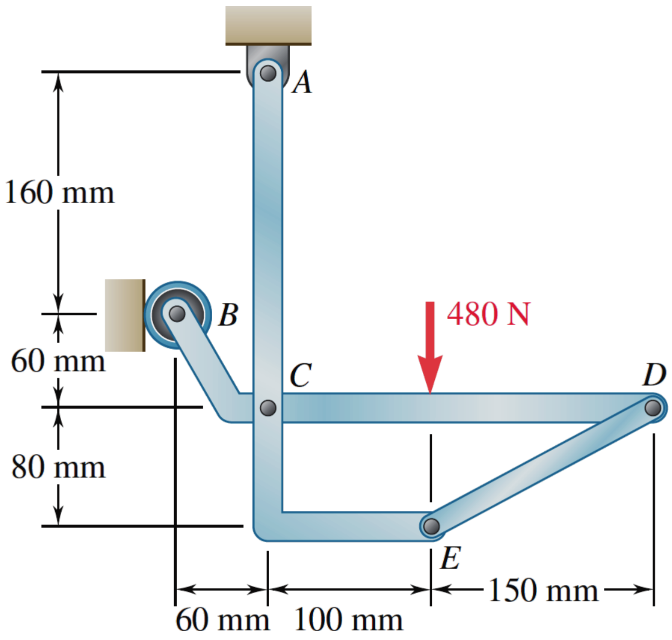


4. Use three equilibrium equations (e.g., $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0$) for each **multi-force member** to determine the internal forces (and remaining reactions).

- A negative sign means the direction is opposite to the direction you assumed.
- Use any remaining equations to check the accuracy of your computations

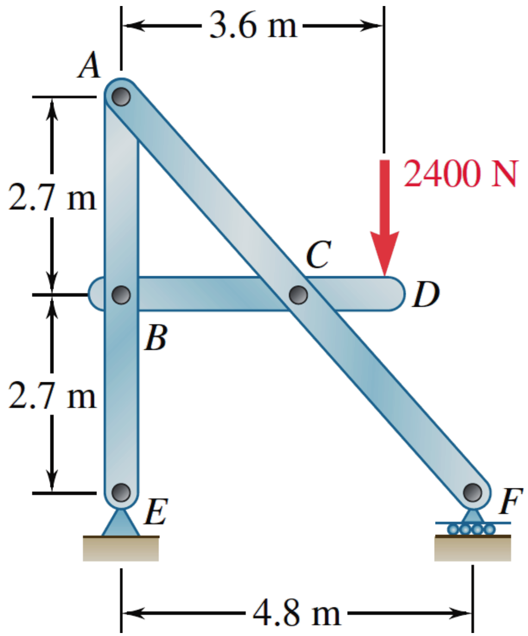
Sample Problem 6.4

In the frame shown, members ACE and BCD are connected by a pin at C and by the link DE. For the loading shown, determine the force in link DE and the components of the force exerted at C on member BCD.



Sample Problem 6.5

Determine the components of the forces acting on each member of the frame shown.



Sample Problem 6.6

A 600-lb horizontal force is applied to pin A of the frame shown. Determine the forces acting on the two vertical members of the frame.