# Physics "Motion of a Particle in a Plane" Summary

Friday, January 06, 2023 1:04 PM

1. **Cartesian coordinate system:**

$\rightarrow \vec{r} = x\vec{\imath} + y\vec{\jmath}$: position vector at eat instant in meters (m).

$\rightarrow d_{OM} = \|\vec{r}\| = \sqrt{x^2 + y^2}$: distance between origin and point at given time in meter (m).

$\rightarrow \Delta\vec{r} = \overrightarrow{r_{final}} - \overrightarrow{r_{initial}}$ : Displacement vector in meter (m).
$\Delta\vec{r} = \Delta x\vec{\imath} + \Delta y\vec{\jmath}$ (m)

$\rightarrow d_{M_1M_2} = \|\Delta\vec{r}\| = \sqrt{\Delta x^2 + \Delta y^2}$ : distance between 2 point in meters (m).

$\rightarrow \vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t}$ : average velocity of a particle between 2 instants in (m/sec).

$\vec{V}_{av} = \frac{\Delta x\vec{\imath} + \Delta y\vec{\jmath}}{\Delta t}$ (m/sec).

$\rightarrow \|\vec{V}_{av}\| = \sqrt{Vx^2 + Vy^2}$: magnitude of avegarge velocity (m/sec).

$$\rightarrow \vec{V} = \frac{d\vec{r}}{dt}$$

$$\vec{V} = \frac{dx}{dt}\vec{\imath} + \frac{dy}{dt}\vec{\jmath}$$

$\vec{V} = Vx\vec{\imath} + Vy\vec{\jmath}$ (m/sec)

$\rightarrow$ V= $\sqrt{Vx^2 + Vy^2}$ : speed of the particle at each instant in (m/sec) .

$\rightarrow \vec{a}_{av} = \frac{\Delta\vec{V}}{\Delta t}$ : in (m/$sec^2$)

$\rightarrow \|\vec{a}_{av}\| = \sqrt{ax^2 + ay^2}$ : magnitude of the average acceleration in (m/$sec^2$)

$$\rightarrow \vec{a} = \frac{d\vec{V}}{dt}$$

$$\vec{a} = \frac{dV_x}{dt}\vec{\imath} + \frac{dV_y}{dt}\vec{\jmath}$$

$\vec{a} = ax\vec{i} + ay\vec{j}$ (m/$sec^2$)

$\rightarrow$ a=$\|\vec{a}\|$ = $\sqrt{ax^2 + ay^2}$ (m/$sec^2$)

**REMARK!!!!**

$\rightarrow$ When reaching x-axis

$\Rightarrow$ y=0

$\rightarrow$ When meeting y-axis

$\Rightarrow$ x=0

$\rightarrow$ Reaching max height

$\Rightarrow V_y = 0$

$\rightarrow$ Velocity parallel to x-axis

$\Rightarrow V_y = 0$

$\rightarrow$ Velocity parallel to y-axis

$\Rightarrow V_x = 0$

$\rightarrow$ For a point M($x_m$,$y_m$) to belong to the trajectory, then its coordinates should the equation of trajectory

**2. Types of trajectory:**

$\rightarrow$ **Rectilinear:** y= ax+b

$\rightarrow$ **Curvilinear:**

✓ y= a$x^2$+bx+c

Or

✓ y=a$x^2$+bx (c=0)

Or

✓ y=a$x^2$+c (b=0)

Or

✓ y=a$x^2$(b=c=0)

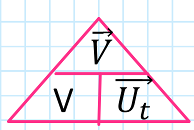
$\rightarrow$ Circular: $(x-x_0)^2 + (y-y_0)^2 = R^2$

Radius

Coordinates of the center

$\rightarrow \vec{V} = V \times \overrightarrow{U_t}$

$$\overrightarrow{U_t} = \frac{\vec{V}}{V}$$




$\rightarrow \vec{a} = a_t.\overrightarrow{U_t} + a_n.\overrightarrow{U_n}$

$$\vec{a} = \frac{dV}{dt}.\overrightarrow{U_t} + \frac{V^2}{R}.\overrightarrow{U_n}$$

**Where: V: speed**

**REMARK!!!**

$\rightarrow$ $(\sqrt{U})' = \frac{U'}{2\sqrt{U}}$

$\rightarrow$ a= $\sqrt{{a_t}^2 + {a_n}^2}$

a= $\sqrt{{a_x}^2 + y^2}$

$\rightarrow$ $\overrightarrow{a_t} = a_t.\overrightarrow{U_t}$

**Where:**

- ✓ $\overrightarrow{a_t}$: tangential acceleration
- ✓ $a_t$: algebric measure of the tangential acceleration.

**3. Moving on a rectilinear path:**

$\rightarrow$ **U.R.M:**

V=constant
$a_t = 0$
$a_n = 0$(no change in direction)

$\rightarrow$ X = Vt+ $x_0$

$\rightarrow$ U.V.R.M:

V increases or decreases **uniformly**
$a_t \neq 0$
$a_n = 0$

$\rightarrow$
- ✓ V = at + $V_0$
- ✓ X = $\frac{at^2}{2} + V_0 t + x_0$
- ✓ ${V_f}^2 - {V_i}^2 = 2a(x - x_0)$

**4. Moving on a circular path:**

$\rightarrow$ **U.C.M:**

θ' = constant
$a_t = 0$
$a_n \neq 0$

$\rightarrow$ $\theta = \theta' t + \theta_0$

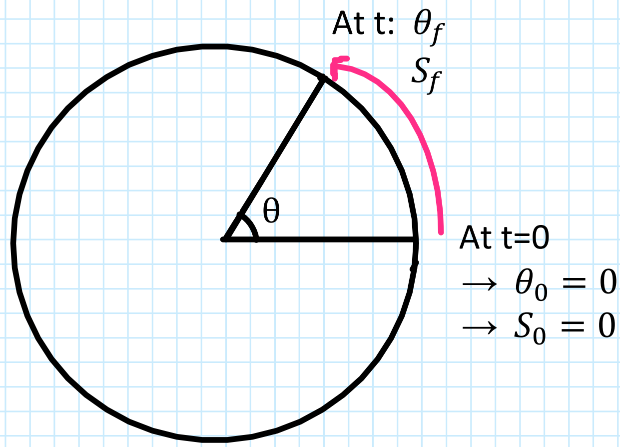
$\rightarrow$ θ: angular abscissa (rad)
$\rightarrow$ θ': instantaneuos angular velocity (rad/sec)
$\rightarrow$ θ": instantaneous angular acceleration (rad/$sec^2$)

$\rightarrow$ **U.V.C.M:**

$\theta' \neq$ constant
$a_t \neq 0$
$a_n \neq 0$

$\rightarrow$
- ✓ $\theta' = \theta'' t + \theta'_0$
- ✓ $\theta = \frac{\theta'' t^2}{2} + \theta'_0 t + \theta_0$
- ✓ ${\theta'_f}^2 - {\theta'_i}^2 = 2\theta''(\theta - \theta_0)$

## 5. Circular trajectory:




→ S: curvilinear abscissa

→ ΔS: curvilinear displacement

→ $V_{av} = \frac{\Delta S}{\Delta t}$: average speed

→ V= $\frac{dS}{dt}$: algebric measure of the velocity

⇒ V>0 → moving along positive sense.

⇒ V<0 → moving along negative sense

→ $a_t = \frac{dV}{dt}$: tangential acceleration

**REMARK!!!**

✓ S = R × $\theta$

✓ V = R × $\theta'$

✓ a = R × $\theta''$

→ θ: angular abscissa (rad)

→ $\Delta\theta = \theta_f - \theta_i$: angular discplacement (variation in θ) in rad

→ $\theta'_{av} = \frac{\Delta\theta}{\Delta t}$: avaerage angular velocity (rad/sec)

→ θ' $= \frac{d\theta}{dt}$: instantaneuos angular velocity (rad/sec)

→ θ" $= \frac{d\theta\prime}{dt}$: instantaneous angular acceleration (rad/$sec^2$)

**REMARK!!!**

θ: angular abscissa (rad) → x: abscissa (rm)

↓ $\Delta\theta$: angular discpalcement (rad) — ↓ $\Delta x$: discpalcement (m)

↓ θ′ $= \frac{\Delta\theta}{\Delta t}$: angular velocity (rad/sec) — ↓ $V = \frac{\Delta x}{\Delta t}$: velocity (m/sec)

↓ θ" $= \frac{\Delta\theta\prime}{\Delta t}$: angular acceleration (rad/$sec^2$) — ↓ $a = \frac{\Delta V}{\Delta t}$: acceleration (m/$sec^2$)