Math (Powers and Radicals)

1) $a^1 = a$ $a \in \mathbb{R}$

2) $a^0 = 1$ $a \in \mathbb{R}$

3) $0^0 =$ undefined.

4) $a^n \times a^m = a^{n+m}$

5) $(a \times b)^n = a^n \times b^n$

6) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

7) $(a^n)^m = a^{n \times m}$

8) $\frac{a^m}{a^n} = a^{m-n}$

9) $a^{-n} = \frac{1}{a^n}$

10) If $a > 0 \implies a^n > 0$ $n \in \mathbb{Z}$

11) If $a < 0 \implies \begin{cases} a^n > 0 & n \text{ even} \\ a^n < 0 & n \text{ odd} \end{cases}$

Scientific Notation:

$$a \times 10^n \quad\quad n \in \mathbb{Z}$$

↙ decimal number

| ← decimal point comes after the 1st non-zero digit. | ⟹ it's integral part is formed by a non-zero digit. |

Square Root:

1) $\sqrt{0} = 0$

2) $\sqrt{1} = 1$

3) $(\sqrt{a})^2 = \begin{cases} a & \text{if } a > 0 \\ \text{undefined} & \text{if } a < 0 \end{cases}$

4) $\sqrt{a^2} = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$

5) $\sqrt{a^3} = a\sqrt{a}$

6) $\sqrt{a} \times \sqrt{a} = a$

7) $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

8) $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

9) $a\sqrt{c} \times b\sqrt{d} = a \times b \sqrt{c \times d}$

10) $(a\sqrt{b})^2 = a^2 \times b$

11) $(a\sqrt{b})^3 = a^3 \times b\sqrt{b}$

Remarks:

1) IF $a^2 = b^2$

$\Rightarrow a = b \quad$ or $\quad a = -b$

2) If $x^2 = a$

$\Rightarrow \begin{cases} x = \sqrt{a} \quad \text{or} \quad x = \sqrt{a} & \text{if } a > 0 \\ \text{no solution} & \text{if } a < 0 \\ 0 & \text{if } a = 0 \end{cases}$

3) $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

4) $\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

5) $\sqrt{a^2-b^2} \neq a-b$

6) $\sqrt{a^2+b^2} \neq a+b$

7) $\sqrt{(a-b)^2} = \begin{cases} a-b & \text{if } a \geq b \rightarrow a-b \geq 0 \\ -(a-b) & \text{if } a < b \rightarrow a-b < 0 \end{cases}$

Rationalize:

Conjugate: multiplication of the opposite.

Powers of rational exponents:

- $x^2 = a$ Square root of a

- $\sqrt[2]{a} = a^{\frac{1}{2}}$

- $x^3 = a$ Cubic root of a

- $\sqrt[3]{a} = a^{\frac{1}{3}}$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \qquad a \in \mathbb{R}^*$$
$$m, n \in \mathbb{Z}^*$$

n^{th} root of a real number: $x^n = a$

$\begin{cases} x = -\sqrt[n]{a} & \text{or} \quad x = \sqrt[n]{a} \\ \quad \text{2 opposite } n^{th} \text{ root} \\ \text{no real solution} \\ \qquad \text{no } n^{th} \text{ root} \\ x = \sqrt[n]{a} : \text{unique solution} \\ \qquad \qquad \text{one } n^{th} \text{ root only.} \end{cases}$ if n even and $a > 0$

if n even and $a < 0$

if n odd.

Properties:

1) $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ even and } a \geqslant 0 \\ -a & \text{if } n \text{ even and } a \leqslant 0 \\ a & \text{if } n \text{ odd.} \end{cases}$

2) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$

3) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3) $\sqrt[n]{\dfrac{1}{a}} = \dfrac{\sqrt[n]{1}}{\sqrt[n]{a}} = \dfrac{1}{\sqrt[n]{a}}$

4) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n}$

5) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

6) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

7) $\sqrt[n]{a^m} = \sqrt[kn]{a^{km}}$

8) $\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$

9) $\sqrt[n]{a-b} \neq \sqrt[n]{a} - \sqrt[n]{b}$

10) $\sqrt[n]{a^n+b^n} \neq a+b$

11) $\sqrt[n]{a^n-b^n} \neq a-b$

12) $c\sqrt[n]{a} + d\sqrt[n]{a} = (c+d)\sqrt[n]{a}$

13) $\sqrt[n]{(a-b)^n} = \begin{cases} a-b & \text{if } a-b \geqslant 0 \text{ and } n \text{ even.} \\ -(a-b) & \text{if } a-b \leqslant 0 \text{ and } n \text{ even.} \\ a-b & \text{if } n \text{ odd.} \end{cases}$

Remark:

$(a-b)(a^2 + ab + b^2) = a^3 - b^3$

$(a+b)(a^2 - ab + b^2)\; a^3 + b^3$