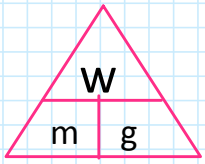
Phy "Newton 2^{nd} Law" Summary

Saturday, January 07, 2023 3:44 PM

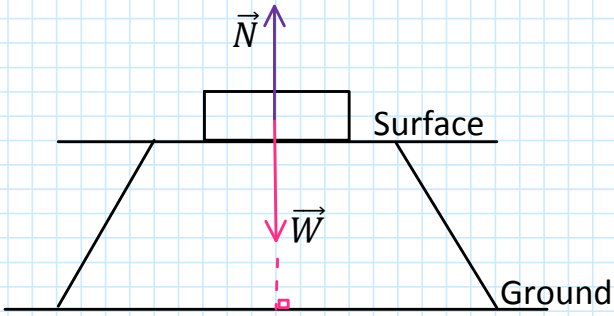
1. Weight: force of gravity , exerted by earth on anybody having mass.

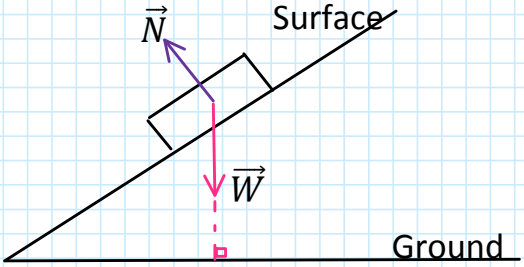
w= m x g

(N) (kg) (g)



★ g $\times 10^{-3}$ → kg





1. **Normal reaction of the support:**$(\vec{N})$
   → Applied by the surface of contact on the body
   → Always perpendicular to the surface of contact

2. **Tension :**$(\vec{T})$
   → Exists whenever an object its suspended from a rope, wire, spring….
   → Tension is always along the rope

4. **Friction:**$(\overrightarrow{Fr})$
   → Against tendency of slipping

5. **Air resistance:**
   → Friction with air particles

6. **Magnetic force:**
   → north / south

7. **Electric force:**
   → +ve / -ve

8. **Newton's 1^{st} Law: law of equilibrium (law of inertia):**

$$\Sigma \vec{F}_{ext} = \vec{0}$$

The body is at equilibrium

1. Static
2. Dynamic

9. **Inertia:**

Resistance to the change in the state of motion

→ **At rest:** inertia related to your mass.

→ In motion: inertia related to mass x density.

$$\sum \vec{F}_{ext} = \vec{0}$$

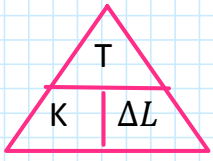
In motion → in motion

At rest → at rest

→ No change in state of motion

10. **Hook's Law:**

$T = K.\Delta L$



$$K(N/m) = \frac{T(N)}{\Delta L(m)}$$

11. **Newton's 2^{nd} Law:**

$$\sum \vec{F}_{ext} = m\vec{a}$$

12. **Integrals**:

$$\int t^n dt = \frac{t^{n+1}}{n+1} + constant$$

13. Free falling object

Consider the particle:

Apply Newton's 2^{nd} Law:

$$\sum \vec{F}_{ext} = m\vec{a}$$

✓ $\vec{w} = m\vec{a}$

$m\vec{g} = m\vec{a}$

$\vec{g} = \vec{a}$

$\Rightarrow \vec{a} = \vec{g}$

$\vec{a} = -g\vec{j}\ (m/sec^2)$

✓ $\vec{a} = \frac{d\vec{V}}{dt}$

$\Rightarrow \vec{V} = \int \vec{a}.dt$

$\vec{V} = -gt\vec{j} + constant$

$\vec{V} = -gt\vec{j} + \vec{V_0}\ (m/sec)$

✓ $\vec{V} = \frac{d\vec{r}}{dt}$

$\Rightarrow \vec{r} = \int \vec{V}.dt$

$\vec{r} = \frac{-gt^2}{2}\vec{j} + constant$

$\vec{r} = \frac{-gt^2}{2}\vec{j} + \vec{r_o}\ (m)$

14. Projectile motion:

Consider the particle:

Apply Newton's 2^{nd} Law:

$$\sum \vec{F}_{ext} = m\vec{a}$$

✓ $\vec{w} = m\vec{a}$

$m\vec{g} = m\vec{a}$

✓ $\cos\alpha = \frac{V_0 x}{V_0}$

$\Rightarrow V_0 x = V_0 \cos\alpha$

✓ $\sin\alpha = \frac{V_0 y}{V_0}$

$\Rightarrow V_0 y = V_0 \sin\alpha$

$$\vec{g} = \vec{a}$$
$$\Rightarrow \vec{a} = \vec{g}$$
$$\boxed{\vec{a} = -g\vec{j}\ (m/sec^2)}$$

✓ $\vec{a} = \frac{d\vec{V}}{dt}$

$$\Rightarrow \vec{V} = \int \vec{a}.\,dt$$

$$\vec{V} = -gt\vec{j} + constant$$
$$\vec{V} = -gt\vec{j} + \vec{V_0}$$
$$\vec{V} = -gt\vec{j} + V_0 cos\alpha\vec{i} + V_0 sin\alpha\vec{j}$$
$$\boxed{\vec{V} = V_0 cos\alpha\vec{i} + (-gt + V_0 sin\alpha)\vec{j}\ (m/sec)}$$

✓ $\vec{V} = \frac{d\vec{r}}{dt}$

$$\Rightarrow \vec{r} = \int \vec{V}.\,dt$$

$$\boxed{\vec{r} = (V_0 cos\alpha)t\vec{i} + (\frac{-gt^2}{2} + V_0 sin\alpha t)\vec{j}\ (m)}$$

Conclusion:

$$\rightarrow \vec{a} = \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\rightarrow \vec{V} = \begin{cases} V_x = V_0 cos\alpha \\ V_y = -gt + V_o sin\alpha \end{cases}$$

$$\rightarrow \vec{r} = \begin{cases} x = (V_0 cos\alpha)t \\ y = \dfrac{-gt^2}{2} + V_0 sin\alpha t \end{cases}$$

15. Trajectory equation:

$$\vec{r} = (V_0 cos\alpha)t\vec{i} + (\frac{-gt^2}{2} + V_0 sin\alpha t)\vec{j}$$

$$\begin{cases} x = (V_0 cos\alpha)t \\ -gt^2 \end{cases} \longrightarrow t = \frac{x}{V_0 cos\alpha}$$

$$\begin{cases} x = (V_0\cos\alpha)t \\ y = \dfrac{-gt^2}{2} + V_0\sin\alpha t \end{cases}$$ → $t = \dfrac{x}{V_0\cos\alpha}$ ← Substitute in y

$$y = -\frac{g}{2}\left(\frac{x}{V_0\cos\alpha}\right)^2 + V_0\sin\alpha\left(\frac{x}{V_0\cos\alpha}\right)$$

$$\boxed{y = -\frac{g}{2V_0{}^2\cos^2\alpha}x^2 + \tan\alpha(x)}$$: equation of trajectory for a projectile motion

16. Maximum height (Elevation/ Altitude):

When reaching maximum height

$\Rightarrow V_y = 0$

$-gt + V_0\sin\alpha = 0$

$\Rightarrow t = \dfrac{V_0\sin\alpha}{g}$ → Substitute in y to get y_{max}

$$y = \frac{-gt^2}{2} + V_0\sin\alpha t$$

$$y = -\frac{g}{2}\left(\frac{V_0\sin\alpha}{g}\right)^2 + V_0\sin\alpha\left(\frac{V_0\sin\alpha}{g}\right)$$

$$y_{max} = -\frac{g}{2}\left(\frac{V_0{}^2\sin^2\alpha}{g^2}\right) + \frac{V_0{}^2\sin^2\alpha}{g}$$

$$y_{max} = -\frac{1}{2}\left(\frac{V_0{}^2\sin^2\alpha}{g}\right) + \frac{V_0{}^2\sin^2\alpha}{g}$$

$$y_{max} = \left(\frac{V_0{}^2\sin^2\alpha}{g}\right)\left(-\frac{1}{2} + 1\right)$$

$$\boxed{y_{max} = \frac{V_0{}^2\sin^2\alpha}{2g}}$$

17. Range : maximum distance traveled on the x-axis:

1^{st} method:

Y = 0

$$y = \frac{-gt^2}{2} + V_0\sin\alpha t$$

$$t\left(-\frac{gt}{2}+V_0 sin\alpha\right)=0$$

$$\Rightarrow t_1=0 \qquad -\frac{gt}{2}+V_0 sin\alpha=0$$

$$t_2=\frac{2V_0 sin\alpha}{g}$$ → substitute in x to determine the range

$$x_{max}=V_0 cos\alpha t$$

$$x_{max}=V_0 cos\alpha\left(\frac{2V_0 sin\alpha}{g}\right)$$

$$x_{max}=\frac{{V_0}^2 2sin\alpha cos\alpha}{g}$$

★ $2sin\alpha cos\alpha = sin2\alpha$

$$x_{max}=\frac{{V_0}^2 sin2\alpha}{g}$$

2nd method: Using equation of trajectory:

$$y=-\frac{g}{2{V_0}^2 cos^2\alpha}x^2+tan\alpha(x)$$

$$x\left(-\frac{g}{2{V_0}^2 cos^2\alpha}+tan\alpha\right)$$

$$\Rightarrow x_1=0 \qquad -\frac{g}{2{V_0}^2 cos^2\alpha}+tan\alpha=0$$

$$+\frac{g}{2{V_0}^2 cos^2\alpha}=+tan\alpha$$

$$x_{max}=\frac{2{V_0}^2 \cancel{cos^2}\alpha}{g}\times\frac{sin\alpha}{\cancel{cos\alpha}}$$

$$x_{max}=\frac{2{V_0}^2 sin\alpha cos\alpha}{g}$$