Math (Absolute value and intervals)

1) $|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$

Absolute value and distance:

$OM = |x|$: the distance OM

represents the distance between the real number x and zero.

$MN = |x_M - x_N|$
$= |x_N - x_M|$

represents the distance between the real numbers x_M and x_N

If $x_M - x_N > 0$ then $MN = x_M - x_N$

If $x_M - x_N < 0$ then $MN = x_N - x_M$

Properties:

1) $|x-y| = \begin{cases} x-y & \text{if } x-y \geq 0 \\ -(x-y) & \text{if } x-y \leq 0 \end{cases}$

2) $\sqrt{x^2} = |x|$

3) $|x| = |-x|$

4) $|x^n| = |x|^n \quad n \in \mathbb{Z}$

5) $|x^2| = |x|^2 = x$

6) $|x-y| = |y-x|$

7) $|x \times y| = |x| \times |y|$

8) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ $\quad y \neq 0$

9) $x \leq |x|$

$|x+y| \leq |x| + |y|$

$|x-y| \leq |x| - |y|$ Triangular Inequality.

10) $|x| = |y|$ if and only if $x = y$ or $x = -y$

11) $|x| = r$ if and only if $x = r$ or $x = -r$

12) $|x| = -ve$ Impossible.

So, the equation admits no real solutions.

Intervals:

no equal ↳ dash ↳ open

equal ↳ lined ↳ closed

1) $x < a \Rightarrow$ Interval: $]-\infty; a[$

2) $x \leq a \Rightarrow$ Interval: $]-\infty; a]$

3) $x > a \Rightarrow$ Interval: $]a; +\infty[$

4) $x \geq a \Rightarrow$ Interval: $[a; +\infty[$

5) $a < x < b \Rightarrow$ Interval: $]a; b[$

6) $a \leq x < b \Rightarrow$ Interval: $[a; b[$

7) $a < x \leq b \Rightarrow$ Interval: $]a; b]$.

8) $a \leq x \leq b \Rightarrow$ Interval: $[a; b]$.

$\mathbb{R} =]-\infty; +\infty[$

Midpoint or Center:

$$c = \frac{a+b}{2}$$

Length or amplitude:

$$\alpha = b - a$$

Radius:

$$r = \frac{b-a}{2}$$

$\Rightarrow$ interval: $]c-r; c+r[$

or $[c-r; c+r]$.

$\Rightarrow$ distance: $|x - c| \leq r$

Absolute value and inequality

1) $|x| \leq r$

$\Downarrow$ $x \leq r$ or $-x \leq r$

$x \geq -r$

$\Rightarrow x \in [-r; r]$

2) $|x| \geq r$

$x \geq r$ or $-x \geq r$

$x \leq -r$ $\Rightarrow x \in]-\infty; r] \cup [r; +\infty[$

3) $|x - a| \leq r$

$x - a \leq r$ or $-x + a \leq r$

$x \leq r + a$

$-x \leq r - a$

$x \geq -r + a$

or $x \geq a - r$ $\Rightarrow x \in [a-r; a+r]$

4) $|x - a| \geq r$

$x - a \geq r$ or $-x + a \geq r$

$x \geq r + a$

$-x \geq r - a$

$x \cancel{\geq} \leq a - r$

or $x \leq -r + a$ $\Rightarrow x \in]-\infty; a-r] \cup [a+r; +\infty$

Remarks:

1) $]-\infty; a[\cup]a; +\infty[$

$\Downarrow$ "a" is the center.

2) $]-\infty; a[\cup]a; b[\cup]b; +\infty[$

$\Downarrow$

$\frac{a+b}{2}$ is the center.

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