Trigonometry

α: measures in degrees.
β: measures in radians.

$$\frac{\alpha}{180°} = \frac{\beta}{\pi}$$

Remark: - π rad → 180 degrees.
- 2π rad → 360 degrees.
- $\frac{\pi}{2}$ rad → 90 degrees.

Length of an arc in a circle:

meas. $\widehat{AB} = \alpha° = \beta_{rad}$.

$l(\widehat{AB}) = r|\alpha|$
$= r|\beta|$

Remark: - central angle in degrees

⇓ $l = \frac{2\pi r \times \text{central angle}}{360°}$

- central angle is radians

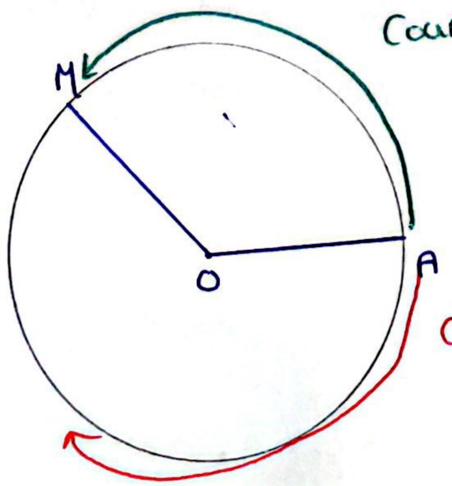
⇓ $l = r|\alpha|$

or $l = r|\beta|$

Orientation of a circle:

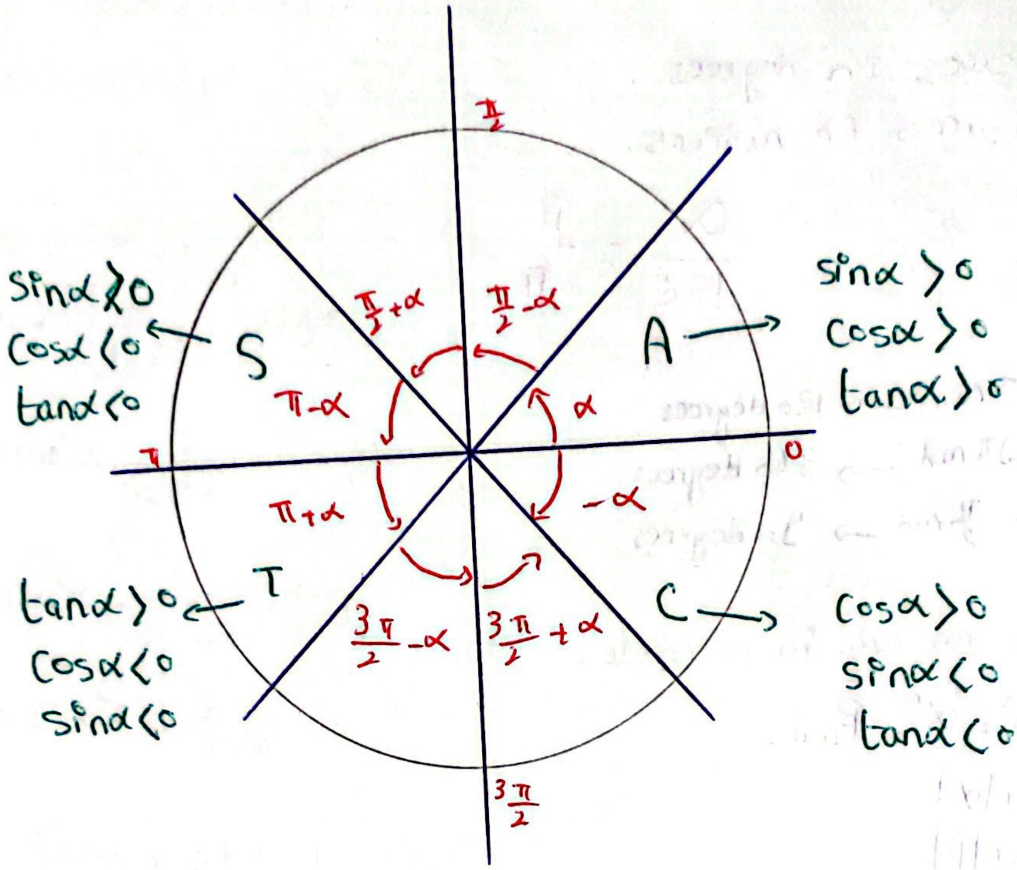
- length of trigonometric circle is 2π.
- $\widehat{AM} = \beta + 2K\pi$

number of turns completed till the end of the arc.



Counter clockwise
⇓ +ve sense
⇓ direct sense

Clock wise
⇓ -ve sense
⇓ indirect sense



Remark:

$\sin\alpha = \frac{adj.}{hyp.}$

$\cos\alpha = \frac{opp.}{hyp.}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{opp.}{adj.}$

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha} = \frac{adj.}{opp.}$

$0 < \sin\alpha < 1$

$0 < \cos\alpha < 1$

If 'a and b are two complemetary angles, then $\cos a = \sin b$.

Fundemental Relations:

$\cos^2\alpha + \sin^2\alpha = 1$ rule

$\cos^2\alpha = 1 - \sin^2\alpha$ result

$\sin^2\alpha = 1 - \cos^2\alpha$ result

$\cos^2\alpha = \dfrac{1}{1+\tan^2\alpha}$ rule

$1+\tan^2\alpha = \dfrac{1}{\cos^2\alpha}$ result

$\sin^2\alpha = \dfrac{1}{1+\cot^2\alpha}$ rule

$1+\cot^2\alpha = \dfrac{1}{\sin^2\alpha}$ result.

$\tan\alpha \times \cot\alpha = 1$ rule.

Remark:

- The cosine and sine are periodic of period 2π.

$\cos(\alpha + 2K\pi) = \cos\alpha \quad K \in \mathbb{Z}$

$\sin(\alpha + 2K\pi) = \sin\alpha \quad K \in \mathbb{Z}$

The tangent and cotangent are periodic of period π.

$\tan(\alpha + K\pi) = \tan\alpha \quad \alpha \in \mathbb{Z}$

$\cot(\alpha + K\pi) = \cot\alpha \quad \alpha \in \mathbb{Z}$

- $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ reverse the trigonometric lines

$\cos\alpha \to \sin\alpha$

$\sin\alpha \to \cos\alpha$

$\tan\alpha \to \cot\alpha$

$\cot\alpha \to \tan\alpha$.

- diametricaly opposite $\Rightarrow$ sum of the angles $= 180° \Rightarrow \pi$ rad.
- confunded $\Rightarrow$ same principal measure.

$$\begin{array}{l} \\ \sin \\ \cos \end{array} \frac{\sqrt{\begin{array}{ccccc} 0 & \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} & \frac{\pi}{2} \\ \hline 0 & 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 & 0 \end{array}}}{2}$$