

Session 08

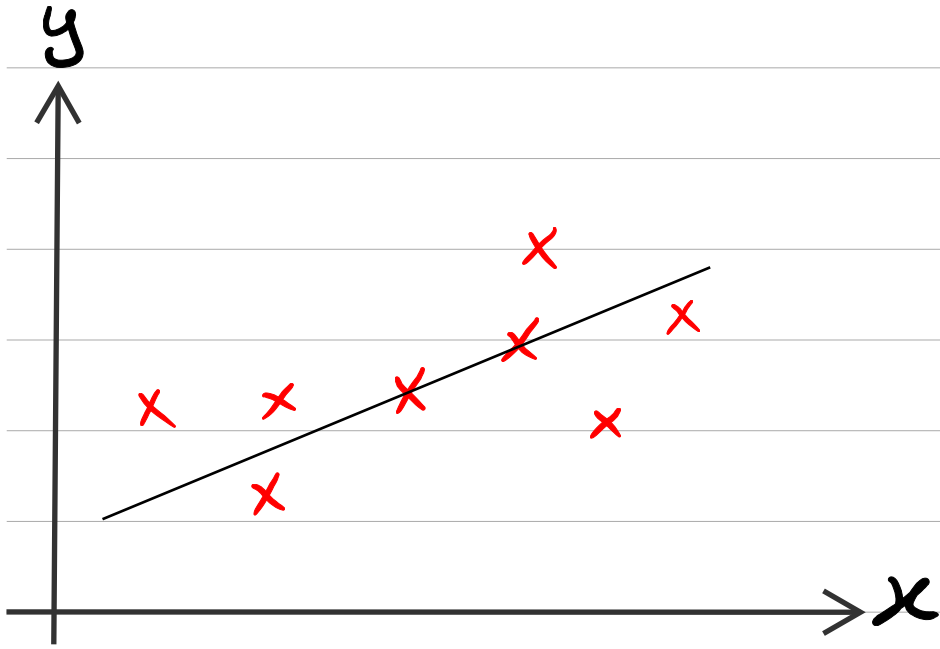
polynomial Regression

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Linear Regression:



$$f_{w,b}(x) = wx + b$$

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2 \rightarrow$$

$$\left. \begin{aligned} J(w, b) &= \frac{1}{2m} \sum_{i=1}^m (f_{w, b}(x^{(i)}) - y^{(i)})^2 \\ f_{w, b}(x) &= wx + b \end{aligned} \right\} \rightarrow J(w, b) = \frac{1}{2m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)})^2$$

$$\frac{\partial J}{\partial b} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)}) = 0$$

$$\frac{\partial J}{\partial w} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)}) \times x^{(i)} = 0$$

$$w = \frac{m \sum xy - \sum x \sum y}{m \sum x^2 - (\sum x)^2}$$

$$b = \underbrace{\frac{\sum y}{m}}_{\bar{y}} - w \underbrace{\frac{\sum x}{m}}_{\bar{x}}$$

Ex:

$$m = 5$$

	X	y	x y	x ²
	0	12	0	0
	1	19	19	1
	2	29	58	4
	3	37	111	9
	4	45	180	16
Σ	10	142	368	30

$$w = \frac{m \Sigma xy - \Sigma x \Sigma y}{m \Sigma x^2 - (\Sigma x)^2}$$

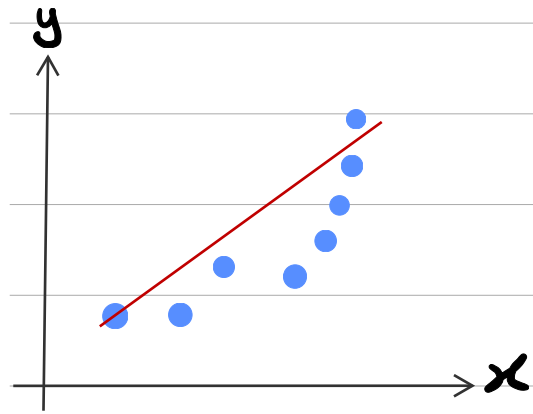
$$b = \frac{\Sigma y}{m} - w \frac{\Sigma x}{m}$$

$$w = \frac{\cancel{5} \cancel{368} - \cancel{10} \cancel{142}}{\cancel{5} \cancel{30} - (\cancel{10})^2} = \frac{5 \times 368 - 10 \times 142}{5 \times 30 - 100} = 8.4$$

$$b = \frac{\cancel{142}}{\cancel{5}} - \cancel{8.4} \frac{\cancel{100}}{\cancel{5} \cancel{m}} = \frac{142}{5} - 8.4 \times \frac{10}{5} = 11.6$$

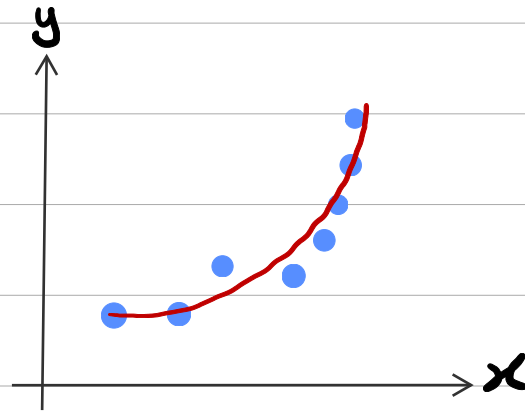
$$f_{w,b}(x) = wx + b \rightsquigarrow f_{w,b}(x) = 8.4x + 11.6$$

Polynomial Regression



$$y = w_1 x + b$$

؟: چه زمانی Linear Regression \neq Polynomial Regression



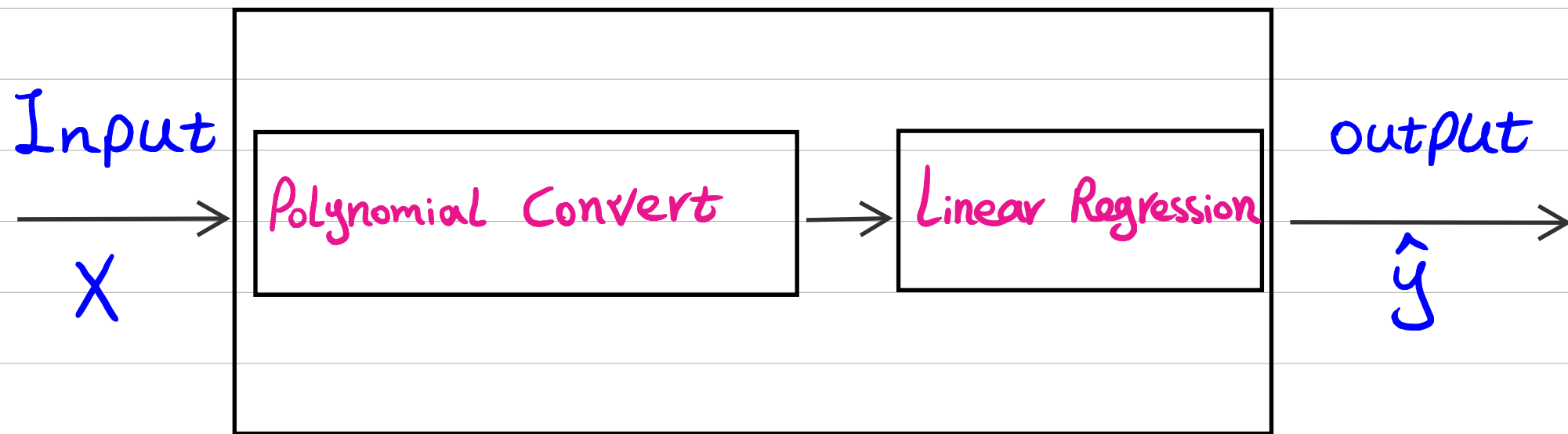
$$y = w_1 x + w_2 x^2 + b \rightarrow \text{degree} = 2$$

Polynomial Reg. شکلی از Linear Reg. است که رابطه‌ی غیر خطی بین متغیرهای وابسته و غیر وابسته را برقرار می‌کند.

Features \rightarrow Independent

y \rightarrow Dependent

$$y = w_1 x + w_2 x^2 + w_3 x^3 + b \rightarrow \text{degree} = 3$$



1. مشخص کردن درجه ی Polynomial

2. ایجاد X (ویژگی) های جدید

3. ساخت مدل \leftarrow Linear Regression

4. آموزش مدل

5. تست و ارزیابی مدل

degree

1 \leadsto

2 \leadsto

3 \leadsto

4 \leadsto

1

1 2 1

1 3 3 1

1 4 6 4 1

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$x_1, x_2 \leadsto (x_1+x_2)^0 + (x_1+x_2)^1 + (x_1+x_2)^2 = 1 + (x_1+x_2) + (x_1^2 + 2x_1x_2 + x_2^2)$$

degree=2

\leadsto Polynomial Features

$$[1, x_1, x_2, x_1^2, x_1x_2, x_2^2]$$

$$x_1, x_2 \leadsto (x_1+x_2)^0 + (x_1+x_2)^1 + (x_1+x_2)^2 + (x_1+x_2)^3$$

degree=3

$$= 1 + (x_1+x_2) + (x_1^2 + 2x_1x_2 + x_2^2) + (x_1^3 + 3x_2x_1^2 + 3x_1x_2^2 + x_2^3)$$

\leadsto Polynomial Features

$$[1, x_1, x_2, x_1^2, x_1x_2, x_2^2, x_1^3, x_2x_1^2, x_1x_2^2, x_2^3]$$

$X = [2, 3]$ $\xrightarrow{\text{degree}=3}$ Polynomial Features ?

$X = [2, 3]$ $\xrightarrow{\text{degree}=3}$ $[1, 2, 3, 4, 6, 9, 8, 12, 18, 27]$

$$X = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

degree = 3

$$\begin{matrix} x^0 & x^1 & x^2 & x^3 \\ \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix} \end{matrix}$$

$$X = \begin{bmatrix} 0 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$$

degree = 3

$$\begin{matrix} x_1 & x_2 & x_1^2 & x_1 x_2 & x_2^2 & x_1^3 & x_2 x_1 & x_1 x_2^2 & x_2^3 \\ \begin{bmatrix} 1 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 0 & 8 \\ 1 & 4 & 5 & 16 & 20 & 25 & 64 & 80 & 100 & 125 \\ 1 & 7 & 8 & 49 & 56 & 64 & 343 & 392 & 448 & 512 \end{bmatrix} \end{matrix}$$