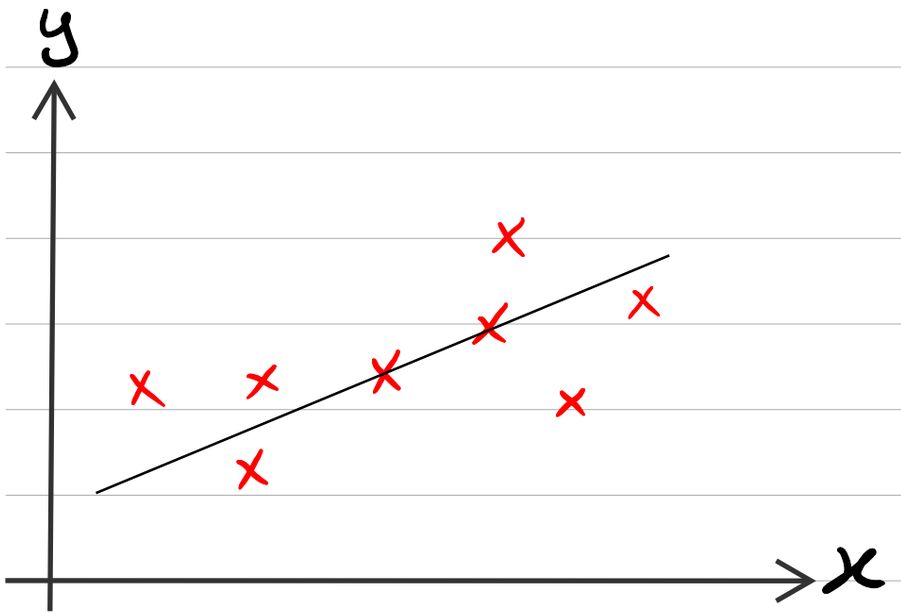
Session 3

Polynomial regression

Linear Regression:



$$f_{w,b}(x) = wx + b$$

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^{m} \left(\hat{y}^{(i)} - y^{(i)} \right)^2$$

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^{m} \left(\hat{y}^{(i)} - y^{(i)} \right)^2 \longrightarrow$$

$$J(w,b) = \frac{1}{2m} \sum_{i=1}^{m} \left(f_{w,b}(x^{(i)}) - y^{(i)} \right)^2$$

$$f_{w,b}(x) = wx + b$$

$$\Longrightarrow J(w,b) = \frac{1}{2m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right)^2$$

$$\frac{\partial J}{\partial b} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right) = 0$$

$$\frac{\partial J}{\partial w} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^{m} \left(wx^{(i)} + b - y^{(i)} \right) \times x^{(i)} = 0$$

$$w = \frac{m\Sigma xy - \Sigma x \Sigma y}{m\Sigma x^2 - (\Sigma x)^2}$$

$$b = \underbrace{\frac{\Sigma y}{m}}_{\bar{y}} - w \underbrace{\frac{\Sigma x}{m}}_{\bar{x}}$$

EX:

$m = 5$

	x	y	xy	x^2
	0	12	0	0
	1	19	19	1
	2	29	58	4
	3	37	111	9
	4	45	180	16
Σ	10	142	368	30

$$w = \frac{m\Sigma xy - \Sigma x \Sigma y}{m\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{\Sigma y}{m} - w\frac{\Sigma x}{m}$$

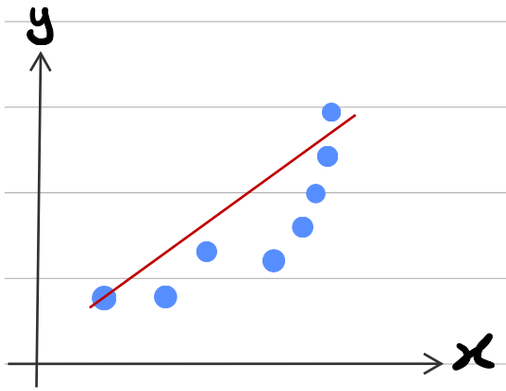
$$w = \frac{5 \cdot 368 - 10 \cdot 142}{5 \cdot 30 - 10^2} = \frac{5 \cdot 368 - 10 \cdot 142}{5 \cdot 30 - 100} = 8.4$$

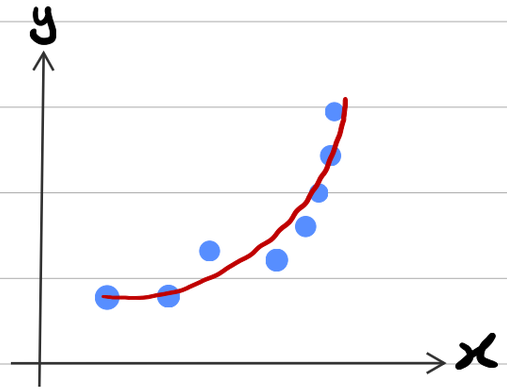
$$b = \frac{\Sigma y}{m} - w\frac{\Sigma x}{5m} = \frac{142}{5} - 8.4 \times \frac{10}{5} = 11.6$$

$$f_{w,b}(x) = wx + b \rightsquigarrow f_{w,b}(x) = 8.4x + 11.6$$

Polynomial Regression

چه زمانی از Polynomial Regression یا Linear Regression استفاده کنیم؟





$$y = w_1 x + b$$

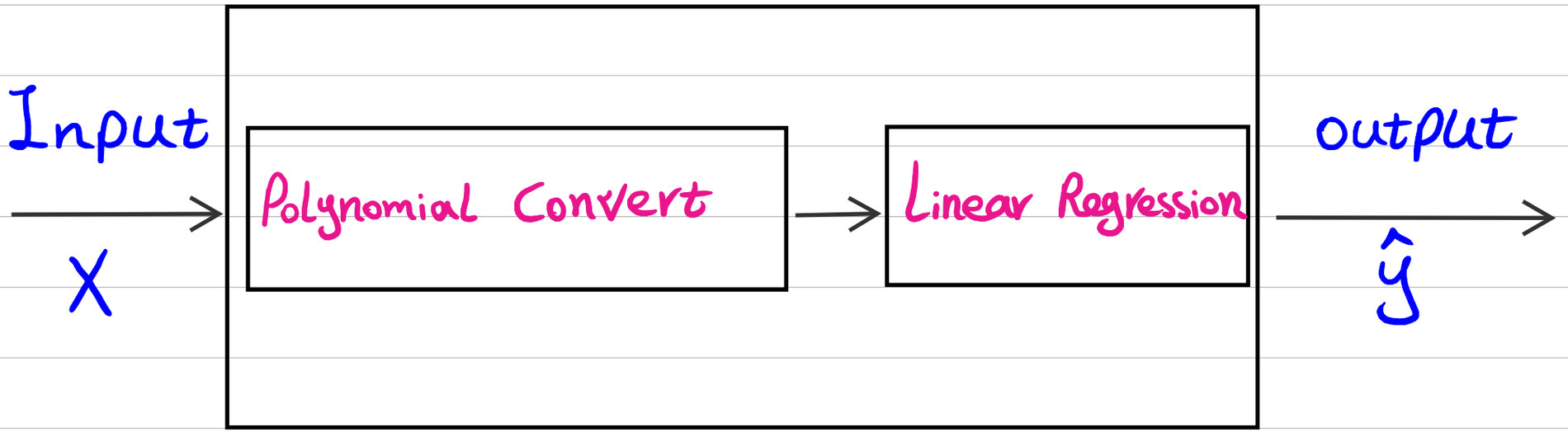
$$y = w_1 x_1 + w_2 x_1^2 + b \longrightarrow \text{degree} = 2$$

شکلی از Linear Reg. است که رابطه‌ی غیر خطی بین متغیرهای وابسته و غیروابسته را برقرار می‌کند. Polynomial Reg.

Features ⟶ Independent

y ⟶ Dependent

$$y = w_1 x_1 + w_2 x_1^2 + w_3 x_1^3 + b \longrightarrow \text{degree} = 3$$



1. مشخص کردن درجه ی Polynomial

2. ایجاد X (ویژگی) های جدید

3. ساخت مدل ← Linear Regression

4. آموزش مدل

5. تست و ارزیابی مدل

degree

1 → 1

2 → 1 2 1

3 → 1 3 3 1

4 → 1 4 6 4 1

$$(x+y)^2 = x^2 + 2xy + y^2$$

$x_1, x_2 \longrightarrow (x_1+x_2)^0 + (x_1+x_2)^1 + (x_1+x_2)^2 = 1 + (x_1+x_2) + (x_1^2 + 2x_1x_2 + x_2^2)$

degree = 2

Polynomial Features

$$[1, x_1, x_2, x_1^2, x_1x_2, x_2^2]$$

$x_1, x_2 \longrightarrow (x_1+x_2)^0 + (x_1+x_2)^1 + (x_1+x_2)^2 + (x_1+x_2)^3$

degree = 3

$$= 1 + (x_1+x_2) + (x_1^2 + 2x_1x_2 + x_2^2) + (x_1^3 + 3x_2x_1^2 + 3x_1x_2^2 + x_2^3)$$

Polynomial Features

$$[1, x_1, x_2, x_1^2, x_1x_2, x_2^2, x_1^3, x_2x_1^2, x_1x_2^2, x_2^3]$$

$x = [2, 3]$ $\xrightarrow{\text{degree}=3}$ Polynomial Features ?

$x = [2, 3]$ $\xrightarrow{\text{degree}=3}$ $[1, 2, 3, 4, 6, 9, 8, 12, 18, 27]$

$$X = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \xrightarrow{\text{degree}=3}$$

x^0	x^1	x^2	x^3
1	2	4	8
1	3	9	27
1	4	16	64

$$X = \begin{bmatrix} 0 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \xrightarrow{\text{degree}=3}$$

	x_1	x_2	x_1^2	x_1x_2	x_2^2	x_1^3	x_2x_1	$x_1x_2^2$	x_2^3
1	0	2	0	0	4	0	0	0	8
1	4	5	16	20	25	64	80	100	125
1	7	8	49	56	64	343	392	448	512