

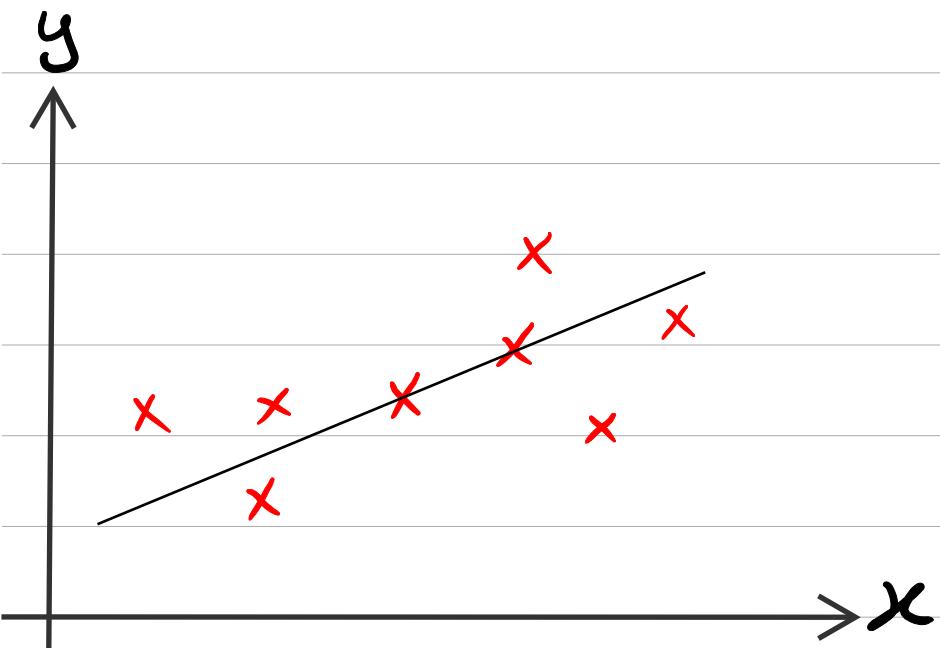
Session 04

Polynomial Regression

Machine Learning | Zahra Amini



Linear Regression:



$$f_{w,b}(x) = wx + b$$

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2$$

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (\hat{y}^{(i)} - y^{(i)})^2 \rightarrow$$

$$J(w, b) = \frac{1}{2m} \sum_{i=1}^m (f_{w,b}(x^{(i)}) - y^{(i)})^2 \quad \left. \right\} \rightarrow J(w, b) = \frac{1}{2m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)})$$

$$f_{w,b}(x) = wx + b$$

$$\cancel{\frac{\partial J}{\partial b}} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)}) = 0$$

$$\cancel{\frac{\partial J}{\partial w}} = 0 \rightsquigarrow \frac{1}{m} \sum_{i=1}^m (wx^{(i)} + b - y^{(i)}) \times x^{(i)} = 0$$

$$w = \frac{m \sum xy - \sum x \sum y}{m \sum x^2 - (\sum x)^2}$$

$$b = \underbrace{\frac{\sum y}{m}}_{\bar{y}} - w \underbrace{\frac{\sum x}{m}}_{\bar{x}}$$

EX:

$$m=5$$

X	y	xy	x ²
0	12	0	0
1	19	19	1
2	29	58	4
3	37	111	9
4	45	180	16
Σ	10	368	30

$$w = \frac{m \sum xy - \sum x \sum y}{m \sum x^2 - (\sum x)^2}$$

$$b = \frac{\sum y}{m} - w \frac{\sum x}{m}$$

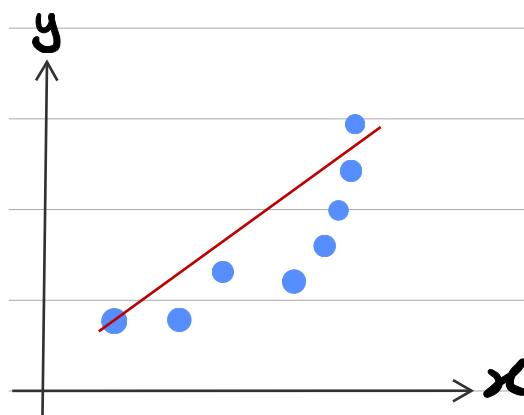
$$w = \frac{\cancel{m} \cancel{\sum xy} - \cancel{\sum x} \cancel{\sum y}}{\cancel{m} \cancel{\sum x^2} - (\cancel{\sum x})^2} = \frac{5 \times 368 - 10 \times 142}{5 \times 30 - 100} = 8.4$$

$$b = \frac{\sum y}{m} - w \frac{\sum x}{5m} = \frac{142}{5} - 8.4 \times \frac{10}{5} = 11.6$$

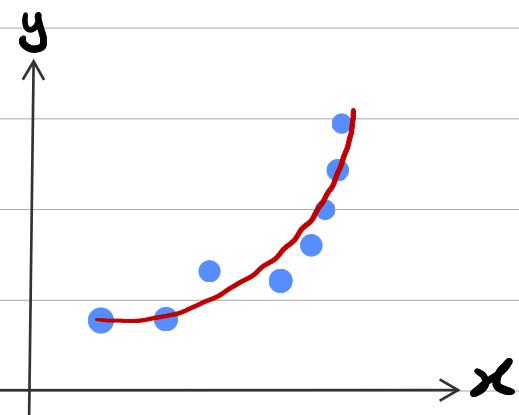
$$f_{w,b}(x) = wx + b \longrightarrow f_{w,b}(x) = 8.4x + 11.6$$

Polynomial Regression

پولینومیال ریگریشن چیزیست : ؟



$$y = w_1 x + b$$



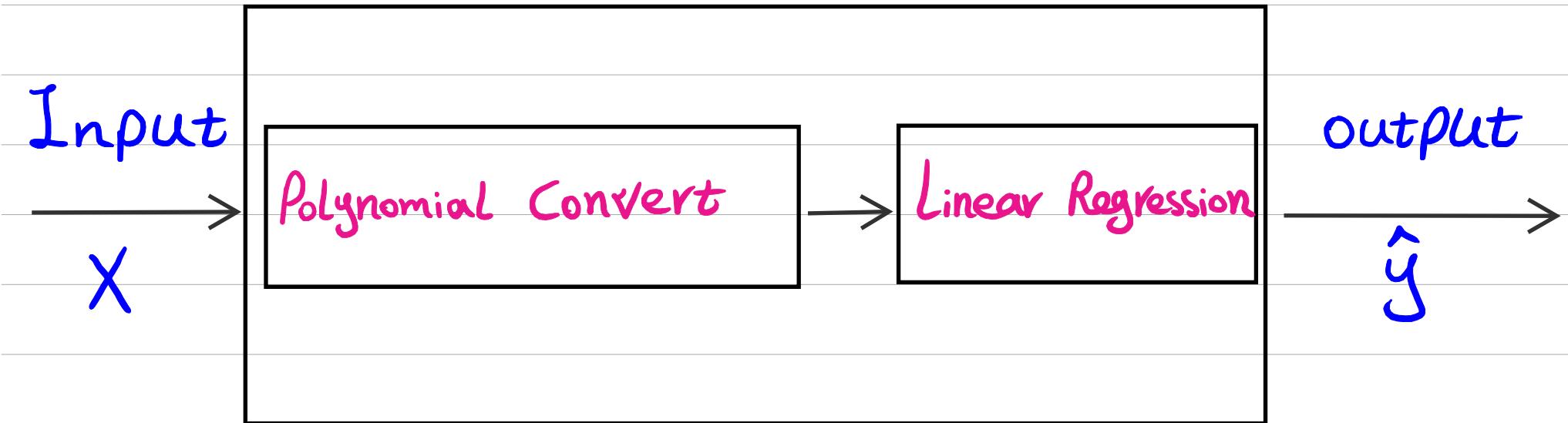
$$y = w_1 x_1 + w_2 x_1^2 + b \rightarrow \text{degree} = 2$$

این رابطه ایست که روابطی غیرخطی بین متغیرهای دابته و غیردابته را برقرار می‌کند.
کلی از Linear Reg. کمتر است Polynomial Reg.

Features \rightarrow Independent

$y \rightarrow$ Dependent

$$y = w_1 x_1 + w_2 x_1^2 + w_3 x_1^3 + b \rightarrow \text{degree} = 3$$



1. شناسی کردن درجه کی Polynoimal

2. ایجاد X (دیگری) عوای جدید

3. ساخت مدل Linear Regression

4. آموزش مدل

5. تست دارزیابی مدل

degree	1	1	2	1
1 →				
2 →				
3 →	1	3	3	1
4 →	1	4	6	4

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$x_1, x_2 \rightarrow (x_1 + x_2)^0 + (x_1 + x_2)^1 + (x_1 + x_2)^2 = 1 + (x_1 + x_2) + (x_1^2 + 2x_1x_2 + x_2^2)$$

degree=2

↓ { Polynomial Features }

$$\left[1, x_1, x_2, x_1^2, x_1x_2, x_2^2 \right]$$

$$x_1, x_2 \rightarrow (x_1 + x_2)^0 + (x_1 + x_2)^1 + (x_1 + x_2)^2 + (x_1 + x_2)^3$$

degree = 3

$$= 1 + (x_1 + x_2) + (x_1^2 + 2x_1x_2 + x_2^2) + (x_1^3 + 3x_2x_1^2 + 3x_1x_2^2 + x_2^3)$$

{ Polynomial Features

$$\left[1, x_1, x_2, x_1^2, x_1x_2, x_2^2, x_1^3, x_2x_1^2, x_1x_2^2, x_2^3 \right]$$

$x = [2, 3] \xrightarrow{\text{degree}=3}$ Polynomial Features ?

$x = [2, 3] \xrightarrow{\text{degree}=3} [1, 2, 3, 4, 6, 9, 8, 12, 18, 27]$

$$X = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

degree = 3

$$\begin{array}{cccc} x^0 & x^1 & x^2 & x^3 \\ \hline 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{array}$$

$$X = \begin{bmatrix} 0 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$$

degree = 3

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$$\begin{array}{ccccccccc} x_1 & x_2 & x_1^2 & x_1x_2 & x_2^2 & x_1^3 & x_2x_1 & x_1x_2^2 & x_2^3 \\ \hline 1 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 8 \\ 1 & 4 & 5 & 16 & 20 & 25 & 64 & 80 & 100 \\ 1 & 7 & 8 & 49 & 56 & 64 & 343 & 392 & 448 \\ & & & & & & 512 & & \end{array}$$