

Riyazika | ریاضیکا

E02 | Continued Fraction



Let $f(x) = x + \frac{1}{2x + \frac{1}{2x + \frac{1}{2x + \dots \dots \infty}}}$ $\rightarrow g(x)$

Compute the value of $f(100)f'(100)$

$$f(x) = x + \frac{1}{g(x)} \Rightarrow f(x) - x = \frac{1}{g(x)} \xrightarrow{\text{cancel}} \frac{1}{f(x) - x} = g(x) \quad \textcircled{I}$$

if add $x \rightarrow f(x) + x = \frac{x+x}{2x} + \frac{1}{2x + \frac{1}{2x+\dots}} \rightarrow f(x) + x = 2x + \frac{1}{2x + \frac{1}{2x+\dots}}$ $\circlearrowleft g(x)$

$$\textcircled{I} \Rightarrow \frac{1}{f(x)-x} = f(x) + x \xrightarrow{\text{طريقتين وطبعاً}} (f(x)-x)(f(x)+x) = 1 \Rightarrow f^2(x) - x^2 = 1 \Rightarrow f^2(x) = 1 + x^2$$

$$\xrightarrow{\text{لما زاد}} 2f'(x)f(x) = 2x \xrightarrow{\div 2} f'(x)f(x) = x \xrightarrow{\text{if } x=100} f(100)f'(100) = 100$$

