I. INTRODUCING THE PROBLEM

In this example, we are going to investigate a two-qubit system with the following Hamiltonian:

$$H_i = \epsilon(\sigma_z^i - \sigma_x^i), \tag{1}$$

in which σ_is are the Pauli matrices. The qubits are interacting through

$$H_{int} = \epsilon\Gamma(t)\sigma_z^1\sigma_z^2. \tag{2}$$

The system evolution is going to be controlled by a laser pulse to conduct the system from the initial state $|00\rangle$ to a final state $|11\rangle$ in a fixed time $T = \pi/\epsilon$ and the initial laser pulse is considered as $\Gamma(t) = \Gamma(0) = 1$. The goal is to minimize the distance of the system state after the evolution and the desired state $|11\rangle$ which is defined as

$$f_1 = 1 - |\left\langle \psi(T) \middle| 11 \right\rangle|^2. \tag{3}$$

This is indeed plotted in Fig.1 of the paper in terms of number of calls to optimization algorithm. To this end, we need to calculate the initial state $|00\rangle$ evolution

$$|\psi(t)\rangle = U(t)\,|00\rangle\,, \tag{4}$$

in which $U(t) = \mathbb{T}e^{-i\int H_{tot}(t)dt} = e^{-i[\epsilon(\sigma_z^1-\sigma_x^1+\sigma_z^2-\sigma_x^2)]T+\epsilon\sigma_z^1\sigma_z^2\int_0^T\Gamma(t)dt}$ is the unitary time evolution of the system.

In this relation $\Gamma(t) = \Gamma(o).g(t)$ and

$$g(t) = 1 + \frac{1}{\lambda(t)}\sum_{n=1}^{N_c} A_n \sin\omega_n t + B_n \cos\omega_n t \tag{5}$$

where $\omega_n = 2\pi k(1 + r_k)/T$, $k = 1..N_c$ and $r_k \in [-0.5, 0.5]$s are random numbers with flat distribution. In this relation $\lambda(t)$ is a shape function [2] (A shape function is the function which interpolates the solution between the discrete values). Thus, the following expression is going to be optimized:

$$f_1 = 1 - \langle 00|\,U(t)\,|11\rangle = 1 - \langle 00|\exp\left[i[\epsilon(\sigma_z^1 - \sigma_x^1 + \sigma_z^2 - \sigma_x^2)]T + \epsilon\sigma_z^1\sigma_z^2\int_0^T g(t)dt]\right]|11\rangle\,. \tag{6}$$

In plot 1 [1], truncation is done up to $N_c = 2$ and A_n and B_n are considered fixed and r_k is the parameter to be changed during simulations.

[1] Caneva, Tommaso and Calarco, Tommaso and Montangero, Simone, Physical Review A,84, 022326 (2011).
[2] Doria, Patrick and Calarco, Tommaso and Montangero, Simone, Physical review letters, 106, 190501 (2011).