Natural Language Processing
CSE 325/425

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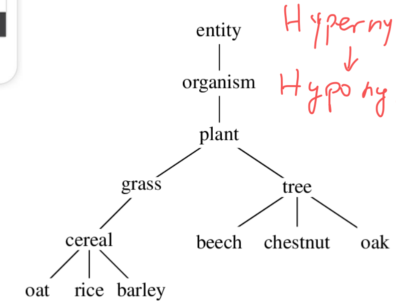
Lecture 4:
- Word vectors
- Word2Vec, Glove

How to represent the word meanings

Meaning <=> Semantics

Use a dictionary
- an entry is a lemma;
- can have multiple senses;
- each sense can have examples of using the lemma.



Hypernym
↓
Hyponym

Use WordNet
- Basic unit: word sense.
- Synsets: synset(“good-1”) = {“excellent”, “great”, …}
- Hypernyms: broader meanings
- Hyponyms: more specific meanings.

Advantages:

- represent human knowledge.
- sort of comprehensive.
- encode some relations.

Disadvantages:

- meanings become outdated easily.
- not so comprehensive.
- symbolic. → AI symbolic / statistical ML

ID / string → Concept

(Not probabilistic distribution)

Distributional representation

- Non-symbolic.
- Can be learned from the latest text data.

One-hot vectors

Index of "sam" in the V

The corpus = {
D1=[<s> I am Sam </s>]
D2=[<s> Sam I am </s>]
D3=[<s> I do not like green eggs and ham </s>]
}

Sam = [0 0 0 0 0 0 0 1 0 0]

I= [1 0 0 0 0 0 0 0 0 0]

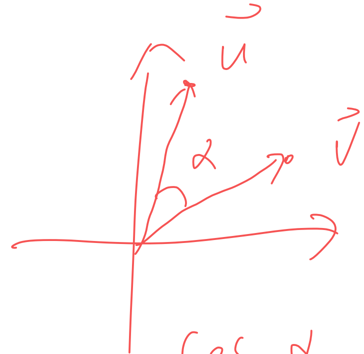
ham = [0 0 0 0 0 0 0 0 0 1]

Vector dimension = number of words in vocabulary (e.g., 500,000)

V = [<s>, I, am, Sam, do, not, - - - </s>]

0 1 2

In ML, stat, AI,
cosine similarity is used
to measure similarity of two vectors



$$\cos\alpha = \frac{\langle \vec{u}, \vec{v} \rangle}{\|\vec{u}\| \times \|\vec{v}\|}$$

Distributional representation

Issues with one-hot vectors

- Most document vectors are orthogonal to each other and have 0 similarity
  - find articles related to "ham", while "egg" won't show up.
- WordNet's synset can help, but it is not a good solution: incompleteness.
- Extremely high dimensional

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Vector dimension = number of words in vocabulary (e.g., 500,000)

D1 = [0 1 0 0 1 --- 1, 0 0 0 1 1 0]

D2 = [0 1 0 0 1 --- 1, 0 0 0 1 1 0]

cos (D1, D2) = 1

maximum similarity

Represent word semantics by their context

A very successful principle in NLP

"You shall know a word by the company it keeps" (J. R. Firth 1957: 11)

is dancing

walking

eating

Context of a word `w`:

- other words that appear closeby the word.
- reflect what `w` means.
- "closeby" can be defined by a window of 5 words, a sentence, a document.

The corpus = {
D1=[<s> I am Sam </s>]
D2=[<s> Sam I am </s>]
D3=[<s> I do not like green eggs and ham </s>]
}

a small window:

context("I") = {"am", "do"} <s>

a larger window:

context("I") = {"am", "Sam", "not"}

<s> I am, Sam </s>

2words ← ↑ → 2 words

Represent word semantics by their context

A dense word vector of 50 ~ 300 dimensions:

- each dimension represents some meaning: capital city, is a number, is a noun, etc.
- two vectors are similar if they appear in each other's context.
- also called: $\boxed{\text{word embedding,}}$ word representation.
- Why need more than two dimensions? Two words can be similar in different senses.
  - The following example shows a not-so-good word embedding.

$\vec{\text{lion}}$ $\vec{\text{dog}}$

Word vectors	Dimensions			
dog	-0.4	0.37	0.02	-0.34
cat	-0.15	-0.02	-0.23	-0.23
lion	0.19	-0.4	0.35	-0.48
tiger	-0.08	0.31	0.56	0.07
elephant	-0.04	-0.09	0.11	-0.06
cheetah	0.27	-0.28	-0.2	-0.43
monkey	-0.02	-0.67	-0.21	-0.48
rabbit	-0.04	-0.3	-0.18	-0.47
mouse	0.09	-0.46	-0.35	-0.24
rat	0.21	-0.48	-0.56	-0.37

animal
domesticated
pet
fluffy

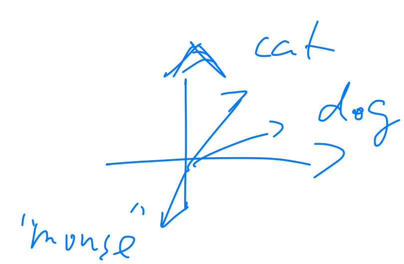
not prob

A single dimension will not describe a concept in full

image courtesy of https://medium.com/@jayeshbahire/introduction-to-word-vectors-ea1d4e4b84bf

Topic model (Bayesian Network)

$$\cos(\text{"dog"}, \text{"lion"}) = \frac{\langle \vec{\text{dog}}, \vec{\text{lion}} \rangle}{\|\vec{\text{dog}}\| \times \|\vec{\text{lion}}\|}$$

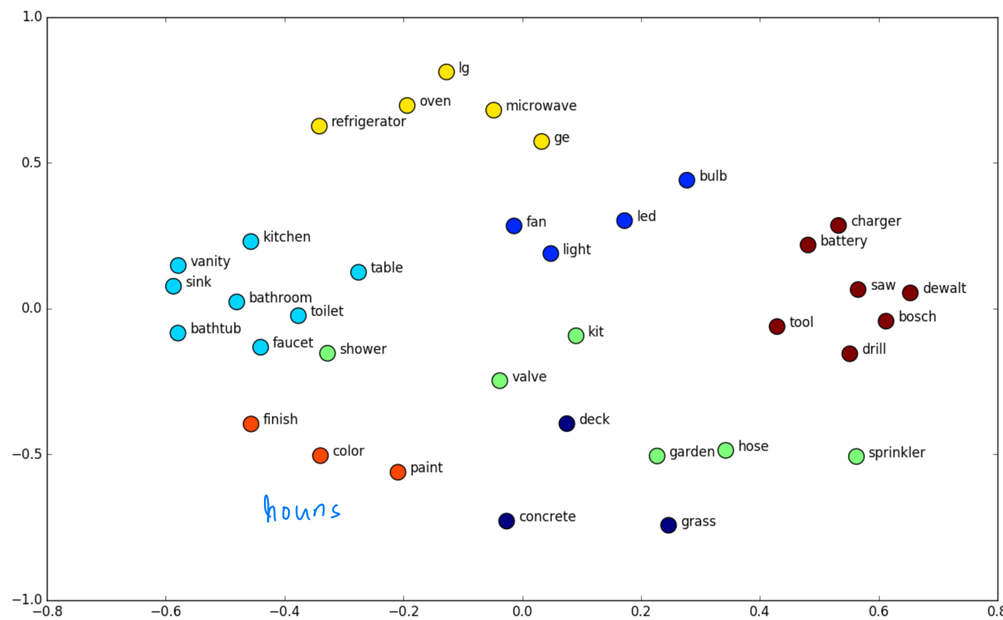


Visualization of word vectors

$$\text{fan} = \begin{bmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 0.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{bmatrix}$$

t-SNE

Further embedding



nouns

[image courtesy of https://www.shanelynn.ie/get-busy-with-word-embeddings-introduction/](https://www.shanelynn.ie/get-busy-with-word-embeddings-introduction/)

LM = { unigram, bi-gram, tri-gram, word2vec, HMM, ... }

$P(w)$ $P(w_t | w_{t-1})$

Word2Vec: overview

Tomas Mikolov, etc. **Efficient Estimation of Word Representations in Vector Space**, NIPS, 2013

Ideas:

- Learn vectors for many words (millions) from large corpus (billions).
- It is a neural language model: use neural networks to predict probabilities.
- Use center word to predict context word (Ski-gram) or the other way around (CBOW).
- Iterate through the corpus and adjust the word vectors to maximize the likelihood of word prediction.

Word2Vec: overview

Finding pairs of (center, context) words:

center word → green

context word in the window → eggs

window size = 5

*I do [not like **green** eggs and] ham*

Use the center word to predict the context words:

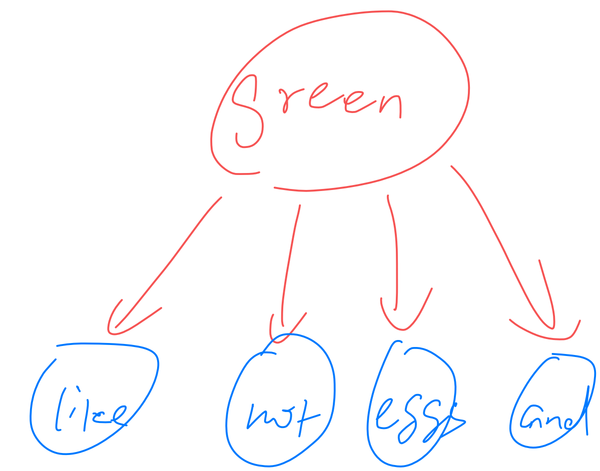
- maximize the conditional probabilities.
- context words are assumed to be conditional independent.

observe "like" given "green"

P(*like* | green)   P(*eggs* | green)

P(*not* | green)   P(*and* | green)

sufficient statistic



Word2Vec: overview

Finding pairs of (center, context) words:

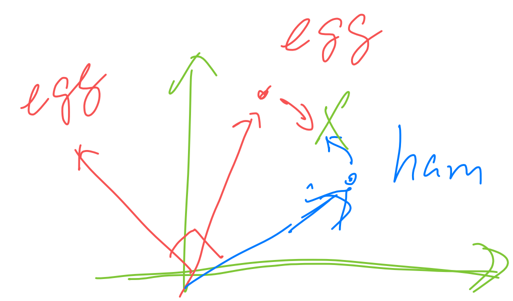
center word *context word in the window*

I do not [*like green eggs and ham*]

? $\begin{bmatrix} \vdots \end{bmatrix}$ $\begin{bmatrix} \vdots \end{bmatrix}$?

P(*like* | egg) P(*ham* | egg)

P(*green* | egg) P(*and* | egg)



$$v_{\text{"egg"}} = \begin{bmatrix} \vdots \end{bmatrix}$$

$$\max_{u_{ham},\, v_{egg}} P(ham \mid egg) = \frac{\exp(u_{\text{"ham"}}\, v_{\text{"egg"}})}{\sum \exp(u_w\, v_{\text{"egg"}})}$$

any w in vocab

$$u_{\text{"ham"}} = \begin{bmatrix} \vdots \end{bmatrix}$$

θ to be optimized

Alg MLE

Input: $\mathcal{D}$ observed sufficient statistics

$P(\mathcal{D} \mid \theta)$

output: $\theta^* = \arg\max_\theta P(D \mid \theta)$

Maximum Likelihood Est (MLE)

Word2Vec: loss function

For each position $t = 1, \ldots, T$

predict context words w_{t+j} within a window of fixed size *m*, given center word w_t

Likelihood: $$L(\theta) = \prod_{t=1}^{T} \prod_{\substack{j=-m \\ j \neq 0}}^{m} P(w_{t+j} \mid w_t)$$

Indexing the center word

Indexing the context

context

$$[\underbrace{w_{t-2} \; w_{t-1}}_{m=2} \; w_t \; \underbrace{w_{t+1} \; w_{t+2}}_{m=2}]$$

center

Loss function (negative log-likelihood, or cross entropy loss):

$$-\log L(\theta) = -\sum_{t=1}^{T} \sum_{j=-m, j \neq 0}^{m} \log P(w_{t+j} \mid w_t)$$

T in the scale of billions

with millions of word vectors

Word2Vec: parametrization

How to represent the probability using some parameters?

- In bi-gram model, we use a single number to represent a probability.
- Neural networks allow more complex parametrization of common probability distributions.
  - Logistic regression parametrized a binomial distribution.
  - Multi-layered Perceptron (MLP) can output a multinomial distribution.

Softmax as distribution over the vocab:

Let each word have two parameter vectors:

- $\mathbf{v}_w$ word vector of any w when used as a center word.
- $\mathbf{u}_w$ word vector of any w when used as a context word.

$$P(w_o|w_c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\underbrace{\sum_w}_{\in \text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)}$$

$$\sum_{\substack{w_o \\ \in \text{Vocab}}} P(w_o | w_c) = 1$$

Word2Vec: parametrization

Softmax as distribution over the vocab:

$$P(w_o|w_c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_w \exp(\mathbf{u}_w^\top \mathbf{v}_c)}$$

- It is indeed a probability distribution: non-negative and sum-to-one.
- It is “max” since the exp enlarge the gap between different inner products.
- It is “soft” since it is does not assign all probability to the maximum.

Word2Vec: optimization

Want to minimize the loss

$$-\log L(\theta) = -\sum_{t=1}^{T} \sum_{j=-m, j\neq 0}^{m} \log P(w_{t+j}|w_t) \qquad P(w_o|w_c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_w \exp(\mathbf{u}_w^\top \mathbf{v}_c)}$$

- The parameter θ to be optimized is the (2 x V) d-dimensional word vectors.
- Stochastic gradient descent
  - sample a batch of (center, context) pairs.
  - compute the gradient of the loss with respect to the relevant vectors.

Alg "SGD"

Init randomly word vectors

for $i = 1, \cdots \infty$ // an epoch

sample a mini-batch of B observed (w_c, w_o) pairs

eval $-\log L(\theta)$ using the mini-batch, and find grad to update the involved word vectors pushes u_o towards v_c making them closer $\Leftarrow$

Word2Vec: optimization

gradient descent on u_o :

$$u_o \leftarrow u_o - \eta \frac{\partial J}{\partial u_o}$$

$$= u_o + \underbrace{\eta}_{\substack{\text{learning rate} \\ > 0}} \underbrace{[1 - P(w_o|w_c)]}_{> 0} v_c$$

Want to minimize the loss

$$-\log L(\theta) = -\sum_{t=1}^{T} \sum_{j=-m, j\neq 0}^{m} \log P(w_{t+j}|w_t) \qquad P(w_o|w_c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_w \exp(\mathbf{u}_w^\top \mathbf{v}_c)}$$

Let's focus on one (center, context) pair. The gradient w.r.t. the center word vector is:

$$J = -\log P(w_o|w_c) = -\log \exp(u_o^T v_c) + \log \sum_w \exp(u_w^T v_c)$$

$$\frac{\partial J}{\partial (u_o^T v_c)} = -1 + \frac{1}{\sum_w \exp(u_w^T v_c)} \cdot \exp(u_o^T v_c) = P(w_o|w_c) - 1$$

$$\frac{\partial J}{\partial u_o} = \frac{\partial J}{\partial (u_o^T v_c)} \cdot \frac{\partial (u_o^T v_c)}{\partial u_o} = [P(w_o|w_c) - 1] \cdot v_c$$

see above

Negative sampling :

$p(w_o | w_c) \approx \frac{\exp(u_o^T v_c)}{\sum \exp(u_w^T v_c)}$

sample

Just 10 random w

Word2Vec: optimization

$$P(w_o|w_c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_w \exp(\mathbf{u}_w^\top \mathbf{v}_c)}$$

Optimizing the original loss is too expensive:

- Due to the denominator in the softmax.
- Negative sampling: reduce the number of summands in the denominator.

Stochastic gradient descent is more preferred:

- With billions of tokens and millions of words, computing the gradient using all training (center, context) pairs can be slow.
- Same a small batch (mini-batch) of training pairs and just optimized the vectors of the selected words => a very sparse gradient.

Word2Vec: negative sampling

Turn the problem into a binary classification problem

- Classify the context word from the words outside the window centered at w_c

$k = 10$

$$J(w_o, w_c, \theta) = -\log \sigma(\mathbf{v}_o|\mathbf{v}_c) - \sum_{k=1}^{K} \log(1 - \sigma(\mathbf{v}_k|\mathbf{v}_c))$$

- How to sample the non-context words? $\tilde{P}(w) = P(w)^{3/4}$
- How many to sample? $\boxed{\text{K=10}}$ is good enough.

$P(w)$: relative frequency