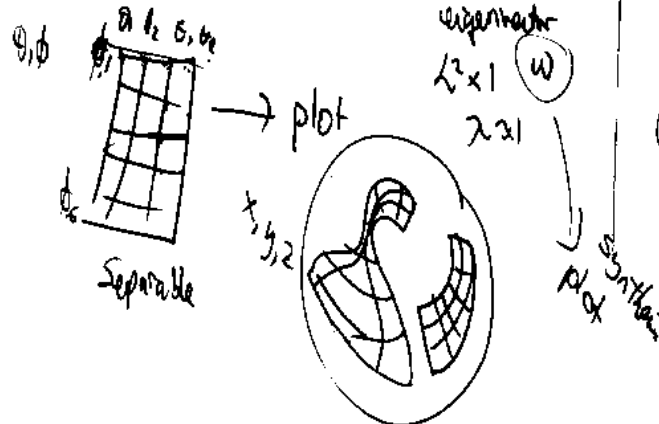
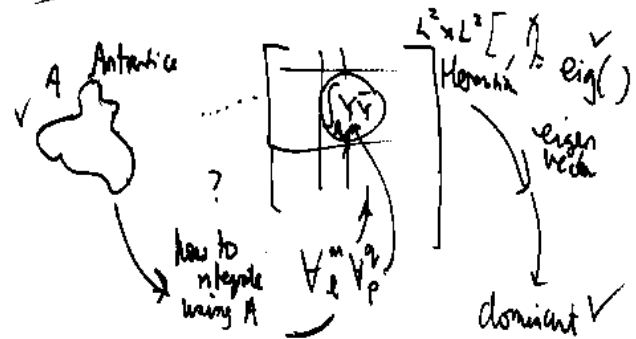


$$f(\theta, \phi) = \sum_{l=0}^{L-1} \sum_{m=-l}^l (f)_l^m Y_l^m(\theta, \phi)$$

$$f(\lambda) \xrightarrow{\text{IP}} (f)_l^m \text{ vector}$$

compute: $Y_l^m(\theta, \phi)$ plot over θ, ϕ

$$\text{Synth. } (f)_l^m \rightarrow f(x)$$



Complex SH
if f is real-valued
 $\Rightarrow (f)_l^m = (-1)^m \overline{(f)_l^{-m}}$

$$(f)_l^m = \frac{1}{\sqrt{2}} (C_l^m - i S_l^m)$$

$$(f)_l^{-m} = (-1)^m \overline{(f)_l^m}$$

$$(f)_l^0 = C_l^0 \quad \text{azimuthally symmetric}$$

$$N_l^{-m} P_l^{-m}(\cos \theta) = (-1)^m N_l^m P_l^m(\cos \theta)$$

real SH

$$Y_{l,m}(\theta, \phi) \triangleq \begin{cases} N_l^0 P_l^0(\cos \theta) & n=0 \\ \sqrt{2} N_l^m P_l^m(\cos \theta) \sin m\phi & m=1, 2, \dots \\ \sqrt{2} (-1)^m N_l^m P_l^m(\cos \theta) \cos m\phi & m=1, 2, \dots \end{cases}$$

[Flipped $-m \rightarrow m$]

$$\text{Then } (S_l^m) S_l^m(\theta, \phi) + (C_l^m) C_l^m(\theta, \phi) \quad C_l^0(\theta, \phi) = \sqrt{2} N_l^0 P_l^0(\cos \theta)$$

$$(7.68) \frac{1}{i\sqrt{2}} (Y_l^m(\theta, \phi) - (-1)^m Y_l^{-m}(\theta, \phi)) \quad \frac{1}{\sqrt{2}} (Y_l^m(\theta, \phi) + (-1)^m Y_l^{-m}(\theta, \phi)) \quad Y_l^0(\theta, \phi) = N_l^0 P_l^0(\cos \theta)$$

$$m > 0 \quad Y_l^m(\theta, \phi) \left[\frac{S_l^m}{i\sqrt{2}} + \frac{C_l^m}{\sqrt{2}} \right] = \frac{1}{\sqrt{2}} (C_l^m - i S_l^m) \triangleq (f)_l^m \quad m > 0$$

$$Y_l^{-m}(\theta, \phi) \left[\frac{-S_l^m}{i\sqrt{2}} (-1)^m + \frac{C_l^m}{\sqrt{2}} \right] = \frac{(-1)^m}{\sqrt{2}} [C_l^m + i S_l^m] = (-1)^m \overline{(f)_l^m}$$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \quad \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

meshgrid

CMB

$$P_l = \sum_{m=-l}^l |f_l^m|^2$$

degrees

$$(f)_l^m = \int Y_l^m(\theta, \phi) f(\theta, \phi) d\Omega$$

