Thermal Black-Oil Implementation for Multisegment Wells

Silviu Livescu

SUPRI-B/HW Industrial Consortia

Department of Petroleum Engineering

Stanford University

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1 Introduction

The isothermal black-oil model for multisegment wells described in Livescu and Shi[5] is extended to take into account the variation of temperature along the well. The flow is multiphase, multicomponent and slip between phases is allowed. Thus, the superficial velocity corresponding to phase p is $V_{sp} = \alpha_p V_p$. For more details regarding the isothermal drift-flux model in GPRS see Livescu and Shi.[5]

2 General equations

The wellbore is modeled as a system of one-dimensional segments. In the isothermal implementation, this system contains four primary variables corresponding to each segment: α_g, α_w, V_m and P^w, which are the free gas and water holdups, the mixture velocity and pressure, respectively. In the nonisothermal implementation, the temperature of the segment is added. The governing equations for each segment are the mass balance equation for each component, a pressure equation and an energy equation. The general mass balance for the component c:

$$\frac{\partial}{\partial t}\sum_p \rho_p \alpha_p \chi_{cp} - \frac{\partial}{\partial x}\sum_p \rho_p V_{sp}\chi_{cp} = \sum_p \tilde{m}_{cp} \tag{1}$$

where t is time, ρ_p is the phase density, α_p is free holdup of phase p and χ_p is the molar fraction of component c in phase p in the segment. The first term in equation 1 represents the mass accumulation of the component c over a time step and the second term represents the flux of the component c. More details regarding the source/sink term can be found elsewhere.[5]

The pressure equation, as used here, has three components, the hydrostatic, the frictional, and the acceleration components, respectively,

$$\Delta P = \Delta P_h + \Delta P_f + \Delta P_a \tag{2}$$

For more details regarding these terms see Livescu and Shi.[5] Note that the pressure drop relationship 2 has been obtained from the momentum conservation equation by assuming homogeneous flow. That is, the frictional term ΔP_f represents the pressure drop due to fluid friction with the wall, not due to friction, or slip, between phases. In order to be consistent, a full momentum conservation equation should be used instead of the pressure drop relationship, such that the phase slip would appear in all equations.

The energy equation can be written in many forms.[1–4,7,8] We use the folllowing form

$$\frac{\partial}{\partial t}\sum_p \rho_p \alpha_p \left(U_p + \frac{1}{2}V_p^2\right) = -\frac{\partial}{\partial x}\sum_p \rho_p \alpha_p V_p \left(H_p + \frac{1}{2}V_p^2\right) + \sum_p \rho_p \alpha_p V_p \bar{g} - \frac{\partial q}{\partial x} \tag{3}$$

where U_p is the phase internal energy, H_p is the phase enthalpy and $\bar{g} = g\cos\theta_i$ is the component of gravity along the well. θ_i is the inclination of segment i. The rate of work done on fluid by viscous forces is assumed to be negligible. The first term represents the rate of gain of energy per unit volume (accumulation); the second, the rate of energy input per unit volume by convection and work done by pressure forces; the third, the rate of work done on fluid per unit volume by gravitational forces; and the fourth represents the energy loss to surroundings.

This model uses the pressure drop equation and the momentum is not conserved. Instead, more generally, we can use the momentum conservation equation. Then, the energy equation can be written in terms of the internal energy. This is obtained by subtracting the mechanical energy equation from the total energy equation. Thus, the term representing the rate of work done on fluid by external forces (e.g., by gravity) disappears due to cancellation of terms in the subtracting process. If the rate of work done on fluid by viscous forces is assumed again to be negligible, the energy equation is

$$\frac{\partial}{\partial t}\sum_p \rho_p \alpha_p U_p = -\frac{\partial}{\partial x}\sum_p \rho_p U_p V_{sp} - \sum_p p_p \frac{\partial V_{sp}}{\partial x} - \frac{\partial q}{\partial x} \tag{4}$$

Here, the first term represents the rate of increase in internal energy (accumulation); the second term, the net rate of addition of internal energy by convective transport; the third term, the reversible rate of internal energy increase by compression; and the last term represents the heat loss to the surroundings.

3 The heat loss term

In equation 3 the heat loss term is the most difficult to be evaluated. For this, we present two models: the first one was developed by Fontanilla and Aziz[2,3] and the second one is briefly summarized in the ECLIPSE Technical Description Manual[1] (the original model was developed by Prats[6]). The main idea is to assume that the heat loss to the surroundings can be expressed by an overall heat transfer coefficient, U_{to}. This coefficient takes into account the conduction in tubing, casing and cement and natural convection and radiation in annulus. Since we assume pseudo steady state fluid flow in the wellbore, the heat loss to the overburden is expressed as

$$\frac{\partial q}{\partial x} = 2\pi r_{to} U_{to}(T_f - T_h) \tag{5}$$

In equation 5, the radius and the overall heat transfer coefficient can be based on any reference point. In this case, the outer tubing surface was chosen.

We follow now briefly the methodology for computing the overall coefficient U_{to}, dveloped by Fontanilla and Aziz[2,3] and by Willhite.[10]

The rate of heat conduction from the fluid to the cement-formation interface is given by equation 5. The rate of heat transfer between the fluid and the inside tubing is

$$\frac{\partial q}{\partial x} = 2\pi r_{ti} h_f (T_f - T_{ti}) \tag{6}$$

where h_f is the film coefficient for heat transfer based on the inside surface area of the tubing. Integrating Fourier's law of heat conduction over the thickness of the tubing wall we obtain

$$\frac{\partial q}{\partial x} = \frac{2\pi k_{tub}}{\ln(r_{to}/r_{ti})}(T_{ti} - T_{to}) \tag{7}$$

Integrating Fourier's law of heat conduction over the thickness of the insulation yields

$$\frac{\partial q}{\partial x} = \frac{2\pi k_{ins}}{\ln(r_{ins}/r_{to})}(T_{to} - T_{ins}) \tag{8}$$

The heat transfer through the annulus takes place due to conduction, natural convection and radiation. To account for these three modes, it is convenient to define the heat rate through the annulus in terms of the heat transfer coefficients h_c (natural convection and conduction) and h_r (radiation),

$$\frac{\partial q}{\partial x} = 2\pi r_{to}(h_c + h_r)(T_{ci} - T_{co}) \tag{9}$$

The heat conduction through the casing wall is

$$\frac{\partial q}{\partial x} = \frac{2\pi k_{cas}}{\ln(r_{co}/r_{ci})}(T_{ci} - T_{co}) \tag{10}$$

Finally, the heat conduction through the cement sheath is given by

$$\frac{\partial q}{\partial x} = \frac{2\pi k_{cem}}{\ln(r_h/r_{co})}(T_{co} - T_h) \tag{11}$$

Note that

$$T_f - T_h = (T_f - T_{ti}) + (T_{ti} - T_{to}) + (T_{to} - T_{ins}) + (T_{ins} - T_{ci}) + (T_{ci} - T_{co}) + (T_{co} - T_h) \tag{12}$$

Introducind equations 5- 11 into equation 12, after some simplifications and rearragements of the terms, the overall heat transfer coefficient is computed from

$$\frac{1}{U_{to}} = \frac{r_{to}}{r_{ti}h_f} + r_{to}\frac{\ln(r_{to}/r_{ti})}{k_{tub}} + r_{to}\frac{\ln(r_{ins}/r_{to})}{k_{ins}} + \frac{1}{h_c + h_r} + r_{to}\frac{\ln(r_{co}/r_{ci})}{k_{cas}} + r_{to}\frac{\ln(r_h/r_{co})}{k_{cem}} \tag{13}$$

Here, h_f is the film coefficient for heat transfer based on the inside surface area of the tubing, k_{tub} is the thermal conductivity for tubing, k_{ins} is the heat conductivity for insulation, h_c and h_r are the heat transfer coefficients for natural convection and conduction and for radiation in annulus, respectively, k_{cas} is the heat conductivity for casing, and k_{cem} is the thermal conductivity for cement. Further simplifications of equation 13 can be made. For example, the thermal conductivity of the tubing and casing steel is much larger than that of the other materials in the wellbore. Therefore, the second and fifth terms in the right-hand-side of equation 13 are neglected. Also, the film coefficient h_f for steam and water are high enough to justify the assumption of infinite film coefficient, $T_f = T_{ti}$. Thus, the equation 13 simplifies to

$$\frac{1}{U_{to}} = r_{to}\frac{\ln(r_{ins}/r_{to})}{k_{ins}} + \frac{1}{h_c + h_r} + r_{to}\frac{\ln(r_h/r_{co})}{k_{cem}} \tag{14}$$

In order to compute U_{to}, the conevction coefficient h_c and the radiation coefficient h_r heve to be evaluated. Let us follow again the derivation presented by Fontanilla and Aziz[2,3] for these coefficients.

The heat transfer per unit length by conduction and free convection in the annulus is

$$\begin{aligned}\frac{dq_c}{dx} &= \frac{2\pi k_{hc}}{\ln(r_{ci}/r_{to})}(T_{to} - T_{ci}) \ \text{(no insulation)} \\ &= \frac{2\pi k_{hc}}{\ln(r_{ci}/r_{ins})}(T_{ins} - T_{ci}) \ \text{(insulation)} \end{aligned} \tag{15}$$

The same heat transfer can also be expressed as

$$\begin{aligned}\frac{dq_c}{dx} &= 2\pi r_{to} h_c (T_{to} - T_{ci}) \ \text{(no insulation)} \\ &= 2\pi r_{to} h_c (T_{ins} - T_{ci}) \ \text{(insulation)} \end{aligned} \tag{16}$$

Combining equations 15 and 16, we obtain

$$\begin{aligned} h_c &= \frac{k_{hc}}{r_{to}\ln(r_{ci}/r_{to})} \ \text{(no insulation)} \\ &= \frac{k_{hc}}{r_{ins}\ln(r_{ci}/r_{ins})} \ \text{(insulation)} \end{aligned} \tag{17}$$

The effective thermal conductivity of the annunlar fluid k_{hc} is related to the actual theral conductivity of the annular fluid k_{ha} as a function of the Grashoff number and the Prandtl number,

$$k_{hc} = 0.049 k_{ha} (Gr Pr)^{0.333} Pr^{0.074} \tag{18}$$

The Grashoff number is defined as

$$\begin{aligned} Gr &= \left(\frac{\rho_{an}}{\mu_{an}}\right)^2 (r_{ci} - r_{to})^3 \, g\beta (T_{to} - T_{ci}) \ \text{(no insulation)} \\ &= \left(\frac{\rho_{an}}{\mu_{an}}\right)^2 (r_{ci} - r_{ins})^3 \, g\beta (T_{ins} - T_{ci}) \ \text{(insulation)} \end{aligned} \tag{19}$$

and the Prandtl number is

$$Pr = \frac{C_{an}}{\mu_{an}} k_{ha} \tag{20}$$

Here, C_{an} and μ_{an} are the specific heat and viscosity of the annular fluid, respectively. β is the coefficient of volume expansion. For ideal gas, β is the reciprocical average absolute annulus temperature,

$$\beta = \frac{1}{T_{an} + 460} \tag{21}$$

and

$$\begin{aligned} T_{an} &= \frac{1}{2}(T_{to} + T_{ci}) \text{ (no insulation)} \\ &= \frac{1}{2}(T_{ins} + T_{ci}) \text{ (insulation)} \end{aligned} \tag{22}$$

For the radiation coefficient h_r, the tubing and the casing surfaces, which exchange heat with each other, may be modelled by resistances in series. More detailes are given by Fontanilla.[2] For two long concentric cylinders, the net heat transfer due to radiation is the overall potential difference divided by the sum of the resistances,

$$q_r = \frac{A_{to}\sigma(Ta_{to}^4 - Ta_{ci}^4)}{1 + \frac{1-\epsilon_{to}}{\epsilon_{to}} + \frac{A_{to}}{A_{ci}}\frac{1-\epsilon_{ci}}{\epsilon_{ci}}} \tag{23}$$

or, after some rearragements,

$$\frac{dq_r}{dx} = \frac{2\pi r_{to}\sigma(Ta_{to} - Ta_{ci})(Ta_{to} + Ta_{ci})(Ta_{to}^2 + Ta_{ci}^2)}{\frac{1}{\epsilon_{to}} + \frac{r_{to}}{r_{ci}}\left(\frac{1}{\epsilon_{ci}} - 1\right)} \tag{24}$$

Here, σ is the Stefan-Boltzmann constant, which is $0.1714 \cdot 10^{-8} BTU/(hr \cdot hr^2 \cdot^\circ R^4)$. ϵ_{to} and ϵ_{ci} are emissivities of outer tubing and inside casing, respectively (dimensionless). Equation 24 is equivalent to

$$\frac{dq_r}{dx} = 2\pi r_{to} h_r (T_{to} - T_{ci}) \tag{25}$$

From the last two equations, the radiation heat transfer through annulus is

$$\begin{aligned} h_r &= \frac{(Ta_{to} + Ta_{ci})(Ta_{to}^2 + Ta_{ci}^2)\sigma}{\frac{1}{\epsilon_{to}} + \frac{r_{to}}{r_{ci}}\left(\frac{1}{\epsilon_{ci}} - 1\right)} \text{ (no insulation)} \\ &= \frac{(Ta_{ins} + Ta_{ci})(Ta_{ins}^2 + Ta_{ci}^2)\sigma}{\frac{1}{\epsilon_{ins}} + \frac{r_{ins}}{r_{ci}}\left(\frac{1}{\epsilon_{ci}} - 1\right)} \text{ (insulation)} \end{aligned} \tag{26}$$

The overall heat transfer coefficient used in ECLIPSE is

$$\begin{aligned} \frac{1}{U_{to}} &= \frac{r_{to}}{r_{ti}h_f} + \frac{r_{to}}{r_{ti}h_{Pi}} + r_{to}\frac{\ln(r_{to}/r_{ti})}{k_{tub}} + r_{to}\frac{\ln(r_{ins}/r_{to})}{k_{ins}} \\ &\quad + \frac{r_{to}}{h_{rc}r_{ins}} + r_{to}\frac{\ln(r_{co}/r_{ci})}{k_{cas}} + r_{to}\frac{\ln(r_h/r_{co})}{k_{cem}} + \frac{f(t_D)}{k_c} \end{aligned} \tag{27}$$

where h_{Pi} is the heat transfer coefficient between the fluid inside the pipe and the tubing, h_{rc} is the heat transfer coefficient due to radiation and convection in annulus, k_c is the thermal conductivity of the unaltered earth and $f(t_D)$ is the time function that reflects the thermal resistance of the earth. This model is better detailed by Prats.[6]

The model developed by Fontanilla and Aziz[2,3] is more rigorous and we will adopt it as our first choice. However, in the current GPRS implementation, a simple concept for the overall heat transfer coefficient s adopted. This coefficient is given as input, either as a constant, or as space-dependent. It is not specied in the input files though, but at the beginning of the file MSWellCont.cpp.

4 Discretization and variables

For the reservoir, the discretized mass and energy equations are

$$\begin{aligned} F_c &= 0 \\ F_e &= 0 \end{aligned} \tag{28}$$

For details see Voskov.[9] The primary variables for the reservoir are the pressure P, the saturations S_p and the temperature T.

For the well part, the discretized pressure drop, the mass and the energy equations are

$$\begin{aligned} R_{Pw} &= 0 \\ R_c &= 0, \;\; c = g, w, o \\ R_e &= 0 \end{aligned} \tag{29}$$

Detailes regarding the first two equations can be found for the isothermal case.[5] The discretized form of the energy equation 3 is

$$\begin{aligned} R_e \;=\; & \frac{V_i}{\Delta t}\left\{\left[\sum_p \rho_p\alpha_p\left(U_p+\frac{1}{2}V_p^2\right)\right]_i^{n+1} - \left[\sum_p \rho_p\alpha_p\left(U_p+\frac{1}{2}V_p^2\right)\right]_i^{n}\right\} \\ & +A_i\left\{\left[\sum_p \rho_p\alpha_p V_p\left(H_p+\frac{1}{2}V_p^2\right)\right]_{i+1}^{n+1} - \left[\sum_p \rho_p\alpha_p V_p\left(H_p+\frac{1}{2}V_p^2\right)\right]_i^{n+1}\right\} \\ & -V_i\left(\sum_p \rho_p\alpha_p V_p\bar{g}\right)_i^{n+1} + \Delta x\left(Q_{loss}\right)_i^{n+1} = 0 \end{aligned} \tag{30}$$

where the heat loss to the surroundings is defined as

$$Q_{loss,i} = 2\pi\left[rU_{to}(T^w - T)\right]_i \tag{31}$$

5 Derivatives and Jacobian matrix

The non-zero element in RRJ, RWJ, WRJ and WWJ are needed to form the Jacobian matrix corresponding to the reservoir and wells. For each of these, some of the derivatives were computed previously. Thus, the non-zero elements for RRJ are given by Voskov.[9] The rest of the derivates are listed here

$$\begin{aligned}\text{RWJ}: \quad \text{computed:} \quad & \frac{\partial F_c}{\partial P^w},\ \frac{\partial F_c}{\partial V_m},\ \frac{\partial F_c}{\partial \alpha_g},\ \frac{\partial F_c}{\partial \alpha_w} \\ \text{needed:} \quad & \frac{\partial F_c}{\partial T^w},\ \frac{\partial F_e}{\partial P^w},\ \frac{\partial F_e}{\partial V_m},\ \frac{\partial F_e}{\partial \alpha_g},\ \frac{\partial F_e}{\partial \alpha_w},\ \frac{\partial F_e}{\partial T^w}\end{aligned} \tag{32}$$

$$\begin{aligned}\text{WRJ}: \quad \text{computed:} \quad & \frac{\partial R_{P^w}}{\partial P},\ \frac{\partial R_{P^w}}{\partial S_p},\ \frac{\partial R_p}{\partial P},\ \frac{\partial R_p}{\partial S_p} \\ \text{needed:} \quad & \frac{\partial R_{P^w}}{\partial T^w},\ \frac{\partial R_p}{\partial T^w},\ \frac{\partial R_e}{\partial P^w},\ \frac{\partial R_e}{\partial S_p},\ \frac{\partial R_e}{\partial T^w}\end{aligned} \tag{33}$$

$$\begin{aligned}\text{WWJ}: \quad \text{computed:} \quad & \frac{\partial R_{P^w}}{\partial P^w},\ \frac{\partial R_{P^w}}{\partial V_m},\ \frac{\partial R_{P^w}}{\partial \alpha_g},\ \frac{\partial R_{P^w}}{\partial \alpha_w},\ \frac{\partial R_p}{\partial P^w},\ \frac{\partial R_p}{\partial V_m},\ \frac{\partial R_p}{\partial \alpha_g},\ \frac{\partial R_p}{\partial \alpha_w} \\ \text{needed:} \quad & \frac{\partial R_{P^w}}{\partial T^w},\ \frac{\partial R_p}{\partial T^w},\ \frac{\partial R_e}{\partial P^w},\ \frac{\partial R_e}{\partial V_m},\ \frac{\partial R_e}{\partial \alpha_g},\ \frac{\partial R_e}{\partial \alpha_w},\ \frac{\partial R_e}{\partial T^w}\end{aligned} \tag{34}$$

5.1 Production wells

5.1.1 RWJ

$$\frac{\partial F_e}{\partial V_{m,i}} = \frac{\partial F_e}{\partial \alpha_{g,i}} = \frac{\partial F_e}{\partial \alpha_{w,i}} = 0 \tag{35}$$

$$\frac{\partial F_c}{\partial T_i^w} = 0 \tag{36}$$

$$\frac{\partial F_e}{\partial P_i^w} = -\left(WI \sum_p \lambda_p \rho_p H_p \right)_i \tag{37}$$

$$\frac{\partial F_e}{\partial T_i^w} = 0 \tag{38}$$

5.1.2 WRJ

$$\frac{\partial R_{P^w,i}}{\partial T_i} = -\left(\frac{2V_m}{A}\right)_i \left[\sum_p \frac{\partial(\rho_p \lambda_p)}{\partial T} (P - P^w) \right]_i \tag{39}$$

$$\frac{\partial R_{g,i}}{\partial T_i} = -\left[WI(P-P^w)\left(\chi_{gg}\frac{\partial(\rho_g\lambda_g)}{\partial T} + \chi_{go}\frac{\partial(\rho_o\lambda_o)}{\partial T}\right)\right]_i \tag{40}$$

$$\frac{\partial R_{c,i}}{\partial T_i} = -\left[WI(P-P^w)\left(\chi_{pp}\frac{\partial(\rho_p\lambda_p)}{\partial T}\right)\right]_i, \quad c = w, o \tag{41}$$

$$\frac{\partial R_{e,i}}{\partial P_i} = \left(\Delta x\frac{\partial Q_{loss}}{\partial P}\right)_i \tag{42}$$

$$\frac{\partial R_{e,i}}{\partial S_{p,i}} = \left(\Delta x\frac{\partial Q_{loss}}{\partial S_p}\right)_i = 0 \tag{43}$$

$$\frac{\partial R_{e,i}}{\partial T_i} = \left(\Delta x\frac{\partial Q_{loss}}{\partial T}\right)_i \tag{44}$$

5.1.3 WWJ

We divide WWJ in three parts: the diagonal part (derivatives with respect to the variables corresponding to the segment i), the upper off-diagonal part (dervatives with respect to the variables in the segment $i+1$) and the lower off-diagonal part (derivatives with respect to the variables in the segment $i-1$).

- Diagonal part

  First, list the derivatives in the diagonal part:

$$\frac{\partial R_{P^w,i}}{\partial T_i^w} = -\left(gh\frac{\partial \rho_m}{\partial T^w}\right)_i - \left(\frac{2V_m^2\Delta x}{D}\right)_i\left(\frac{\partial(f_{tp}\rho_m)}{\partial T^w}\right)_i \tag{45}$$

$$\begin{aligned}\frac{\partial R_{g,i}}{\partial T_i^w} &= A_i\left(\frac{\Delta x}{\Delta t}\alpha_g + V_{sg}\right)_i\left(\frac{\partial(\rho_g\chi_{gg})}{\partial T^w}\right)_i + A_i\left(\frac{\Delta x}{\Delta t}\alpha_o + V_{so}\right)_i\left(\frac{\partial(\rho_o\chi_{go})}{\partial T^w}\right)_i \\ &\quad + A_i\left(\rho_g\chi_{gg}\frac{\partial V_{sg}}{\partial T^w} + \rho_o\chi_{go}\frac{\partial V_{so}}{\partial T^w}\right)_i\end{aligned} \tag{46}$$

$$\frac{\partial R_{c,i}}{\partial T_i^w} = A_i\left(\frac{\Delta x}{\Delta t}\alpha_p + V_{sp}\right)_i\left(\frac{\partial(\rho_p\chi_{pp})}{\partial T^w}\right)_i + A_i\left(\rho_p\chi_{pp}\frac{\partial V_{sp}}{\partial T^w}\right)_i, \quad c = w, o \tag{47}$$

$$
\begin{aligned}
\frac{\partial R_{e,i}}{\partial P_i^w} &= \frac{V_i}{\Delta t}\left[\sum_p \alpha_p \left(\frac{\partial(\rho_p U_p)}{\partial P^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^2)}{\partial P^w}\right)\right]_i \\
&\quad - A_i \left[\sum_p \alpha_p \left(\frac{\partial(\rho_p V_p H_p)}{\partial P^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^3)}{\partial P^w}\right)\right]_i \\
&\quad - V_i \left[\sum_p \alpha_p \frac{\partial \rho_p}{\partial P^w} V_p \bar{g}\right]_i + \left(\Delta x \frac{\partial Q_{loss}}{\partial P^w}\right)_i
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
\frac{\partial R_{e,i}}{\partial V_{m,i}} &= \frac{V_i}{\Delta t}\left(\sum_p \alpha_p \rho_p \frac{1}{2}\frac{\partial V_p^2}{\partial V_m}\right)_i - A_i \left[\sum_p \rho_p \alpha_p \left(H_p \frac{\partial V_p}{\partial V_m} + \frac{1}{2}\frac{\partial V_p^3}{\partial V_m}\right)\right]_i \\
&\quad - V_i \left(\sum_p \rho_p \alpha_p \frac{\partial V_p}{\partial V_m}\bar{g}\right)_i
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
\frac{\partial R_{e,i}}{\partial \alpha_{g,i}} &= \frac{V_i}{\Delta t}\left[\rho_g \left(U_g + \frac{1}{2}V_g^2\right) - \rho_o \left(U_o + \frac{1}{2}V_o^2\right)\right]_i \\
&\quad - A_i \left[\rho_g V_g \left(H_g + \frac{1}{2}V_g^2\right) - \rho_o V_o \left(H_o + \frac{1}{2}V_o^2\right)\right]_i \\
&\quad - V_i \left[\left(\rho_g V_g - \rho_o V_o\right)\bar{g}\right]_i
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
\frac{\partial R_{e,i}}{\partial \alpha_{w,i}} &= \frac{V_i}{\Delta t}\left[\rho_w \left(U_w + \frac{1}{2}V_w^2\right) - \rho_o \left(U_o + \frac{1}{2}V_o^2\right)\right]_i \\
&\quad - A_i \left[\rho_w V_w \left(H_w + \frac{1}{2}V_w^2\right) - \rho_o V_o \left(H_o + \frac{1}{2}V_o^2\right)\right]_i \\
&\quad - V_i \left[\left(\rho_w V_w - \rho_o V_o\right)\bar{g}\right]_i
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
\frac{\partial R_{e,i}}{\partial T_i^w} &= \frac{V_i}{\Delta t}\left[\sum_p \alpha_p \left(\frac{\partial(\rho_p U_p)}{\partial T^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^2)}{\partial T^w}\right)\right]_i \\
&\quad - A_i \left[\sum_p \alpha_p \left(\frac{\partial(\rho_p V_p H_p)}{\partial T^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^3)}{\partial T^w}\right)\right]_i \\
&\quad - V_i \left[\sum_p \alpha_p \frac{\partial \rho_p}{\partial T^w} V_p \bar{g}\right]_i + \left(\Delta x \frac{\partial Q_{loss}}{\partial T^w}\right)_i
\end{aligned} \tag{52}
$$

- Off-diagonal part A

$$\frac{\partial R_{P^w,i}}{\partial T^w_{i+1}} = 0 \tag{53}$$

$$\frac{\partial R_{g,i}}{\partial T^w_{i+1}} = -A_i \left[\frac{\partial(\rho_g \chi_{gg} V_{sg})}{\partial T^w} + \frac{\partial(\rho_o \chi_{go} V_{so})}{\partial T^w}\right]_{i+1} \tag{54}$$

$$\frac{\partial R_{c,i}}{\partial T^w_{i+1}} = -A_i \left[\frac{\partial(\rho_p \chi_{pp} V_{sp})}{\partial T^w}\right]_{i+1}, \ \ c = w, o \tag{55}$$

$$\frac{\partial R_{e,i}}{\partial P^w_{i+1}} \ \ = \ \ A_i \left[\sum_p \alpha_p \left(\frac{\partial(\rho_p V_p H_p)}{\partial P^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^3)}{\partial P^w}\right)\right]_{i+1} \tag{56}$$

$$\frac{\partial R_{e,i}}{\partial V_{m,i+1}} \ \ = \ \ A_i \left[\sum_p \alpha_p \left(\rho_p H_p \frac{\partial V_p}{\partial V_m} + \frac{1}{2}\rho_p \frac{\partial V_p^3}{\partial V_m}\right)\right]_{i+1} \tag{57}$$

$$\frac{\partial R_{e,i}}{\partial \alpha_{g,i+1}} \ \ = \ \ A_i \left[\rho_g V_g \left(H_g + \frac{1}{2}V_g^2\right) - \rho_o V_o \left(H_o + \frac{1}{2}V_o^2\right)\right]_{i+1} \tag{58}$$

$$\frac{\partial R_{e,i}}{\partial \alpha_{w,i+1}} \ \ = \ \ A_i \left[\rho_w V_w \left(H_w + \frac{1}{2}V_w^2\right) - \rho_o V_o \left(H_o + \frac{1}{2}V_o^2\right)\right]_{i+1} \tag{59}$$

$$\frac{\partial R_{e,i}}{\partial T^w_{i+1}} \ \ = \ \ A_i \left[\sum_p \alpha_p \left(\frac{\partial(\rho_p V_p H_p)}{\partial T^w} + \frac{1}{2}\frac{\partial(\rho_p V_p^3)}{\partial T^w}\right)\right]_{i+1} \tag{60}$$

- Off-diagonal part B

  All of the derivatives needed in 34 are zero,

$$\frac{\partial R_{P^w,i}}{\partial T^w_{i-1}} = \frac{\partial R_{c,i}}{\partial T^w_{i-1}} = \frac{\partial R_{e,i}}{\partial P^w_{i-1}} = \frac{\partial R_{e,i}}{\partial V_{m,i-1}} = \frac{\partial R_{e,i}}{\partial \alpha_{g,i-1}} = \frac{\partial R_{e,i}}{\partial \alpha_{w,i-1}} = \frac{\partial R_{e,i}}{\partial T^w_{i-1}} = 0 \tag{61}$$

5.2 Injection wells

Here we present only the derivatives that are different from th eprevious case (production). They will replace the corresponding derivatives for the production wells. All other derivatives remain unchanged.

5.2.1 RWJ

$$\frac{\partial F_{c,i}}{\partial T_i^w} = \left(WI(P - P^w) \sum_p \lambda_p \frac{\partial(\rho_p \chi_{pp})}{\partial T^w} \right)_i \tag{62}$$

$$\frac{\partial F_{e,i}}{\partial P_i^w} = \left[WI(P - P^w) \sum_p \lambda_p \left(\frac{\partial \rho_p}{\partial P^w} H_p + \rho_p \frac{\partial H_p}{\partial P^w} \right) \right]_i - \left(WI \sum_p \lambda_p \rho_p H_p \right)_i \tag{63}$$

$$\frac{\partial F_{e,i}}{\partial T_i^w} = \left[WI(P - P^w) \sum_p \lambda_p \left(\frac{\partial \rho_p}{\partial T^w} H_p + \rho_p \frac{\partial H_p}{\partial T^w} \right) \right]_i \tag{64}$$

5.2.2 WRJ

$$\frac{\partial R_{P^w,i}}{\partial T_i} = -\left(\frac{2V_m}{A} \right)_i \left[\sum_p \rho_p \frac{\partial \lambda_p}{\partial T} (P - P^w) \right]_i \tag{65}$$

$$\frac{\partial R_{g,i}}{\partial T_i} = -\left[WI(P - P^w) \left(\rho_g \chi_{gg} \frac{\partial \lambda_g}{\partial T} + \rho_o \chi_{go} \frac{\partial \lambda_o}{\partial T} \right) \right]_i \tag{66}$$

$$\frac{\partial R_{c,i}}{\partial T_i} = -\left[WI(P - P^w) \left(\chi_{pp} \rho_p \frac{\partial \lambda_p}{\partial T} \right) \right]_i, \quad c = w, o \tag{67}$$

$$\frac{\partial R_{e,i}}{\partial P_i} = \left(\Delta x \frac{\partial Q_{loss}}{\partial P} \right)_i \tag{68}$$

$$\frac{\partial R_{e,i}}{\partial S_{p,i}} = \left(\Delta x \frac{\partial Q_{loss}}{\partial S_p} \right)_i = 0 \tag{69}$$

$$\frac{\partial R_{e,i}}{\partial T_i} = \left(\Delta x \frac{\partial Q_{loss}}{\partial T} \right)_i \tag{70}$$

5.2.3 WWJ

- Diagonal part

$$\frac{\partial R_{P^w,i}}{\partial T_i^w} = -\left(gh\frac{\partial \rho_m}{\partial T^w}\right)_i - \left(\frac{2V_m^2 \Delta x}{D}\right)_i \left(\frac{\partial (f_{tp}\rho_m)}{\partial T^w}\right)_i - \left(\frac{2V_m}{A}\right)_i \left(WI\sum_p \lambda_p \frac{\partial \rho_p}{\partial T^w}\right)_i \quad (71)$$

$$\begin{aligned}\frac{\partial R_{g,i}}{\partial T_i^w} &= A_i\left(\frac{\Delta x}{\Delta t}\alpha_g + V_{sg}\right)_i \left(\frac{\partial(\rho_g \chi_{gg})}{\partial T^w}\right)_i + A_i\left(\frac{\Delta x}{\Delta t}\alpha_o + V_{so}\right)_i \left(\frac{\partial(\rho_o \chi_{go})}{\partial T^w}\right)_i \\ &+ A_i\left(\rho_g\chi_{gg}\frac{\partial V_{sg}}{\partial T^w} + \rho_o\chi_{go}\frac{\partial V_{so}}{\partial T^w}\right)_i \\ &- \left[WI\left(\lambda_g\frac{\partial(\rho_g\chi_{gg})}{\partial T^w} + \lambda_o\frac{\partial(\rho_o\chi_{go})}{\partial T^w}\right)(P - P^w)\right]_i \end{aligned} \quad (72)$$

$$\begin{aligned}\frac{\partial R_{c,i}}{\partial T_i^w} &= A_i\left(\frac{\Delta x}{\Delta t}\alpha_p + V_{sp}\right)_i \left(\frac{\partial(\rho_p \chi_{pp})}{\partial T^w}\right)_i + A_i\left(\rho_p\chi_{pp}\frac{\partial V_{sp}}{\partial T^w}\right)_i \\ &- \left[WI\lambda_p\frac{\partial(\rho_p\chi_{pp})}{\partial T^w}(P - P^w)\right]_i, \quad c = w, o \end{aligned} \quad (73)$$

6 Input Files

The thermal black-oil input files have two parts, each describing the reservoir or the well. The reservoir part is explained in detail by Voskov.[9] That part is actually defining the thermal inputs and all other reservoir and fluid properties. The well part is similar to that for isothermal examples, with one modification. The isothermal MSWell input is described elsewhere.[1,5] In the thermal mode, each segment is allowed to have one connection with the corresponding reservoir cell:

- completion (COMPDAT)

number of connections 80

LOC(i,j,k) WI Seg No.

100 0 1

200 10000 2

The connections are two types, as shown in the exanple above. If the well index WI is 0 (first line in the example above), then no flow is allowed between the well and the reservoir, but only heat loss, defined through the overal heat coefficent U_{to}. If the well index is different from 0 (second line in the example above), then there is flow between the well and the reservoir, thermal inflow between the well and the reservoir (the term $\tilde{m}_{cp}$ in the energy conservation equation) and the heat loss defined by U_{to}.

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[5] Livescu, S. and Shi, H., *GPRS Technical Report: Isothermal multisegment well implementation*, Stanford University, October 2006.

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[9] Voskov, D., *GPRS Technical Report: Thermal reservoir implementation*, Stanford University, May 2006.

[10] Willhite, G.P., *Overall heat transfer coefficients in steam and hot water injection wells*, J. Pet. Tech. (May 1967), 607-615.