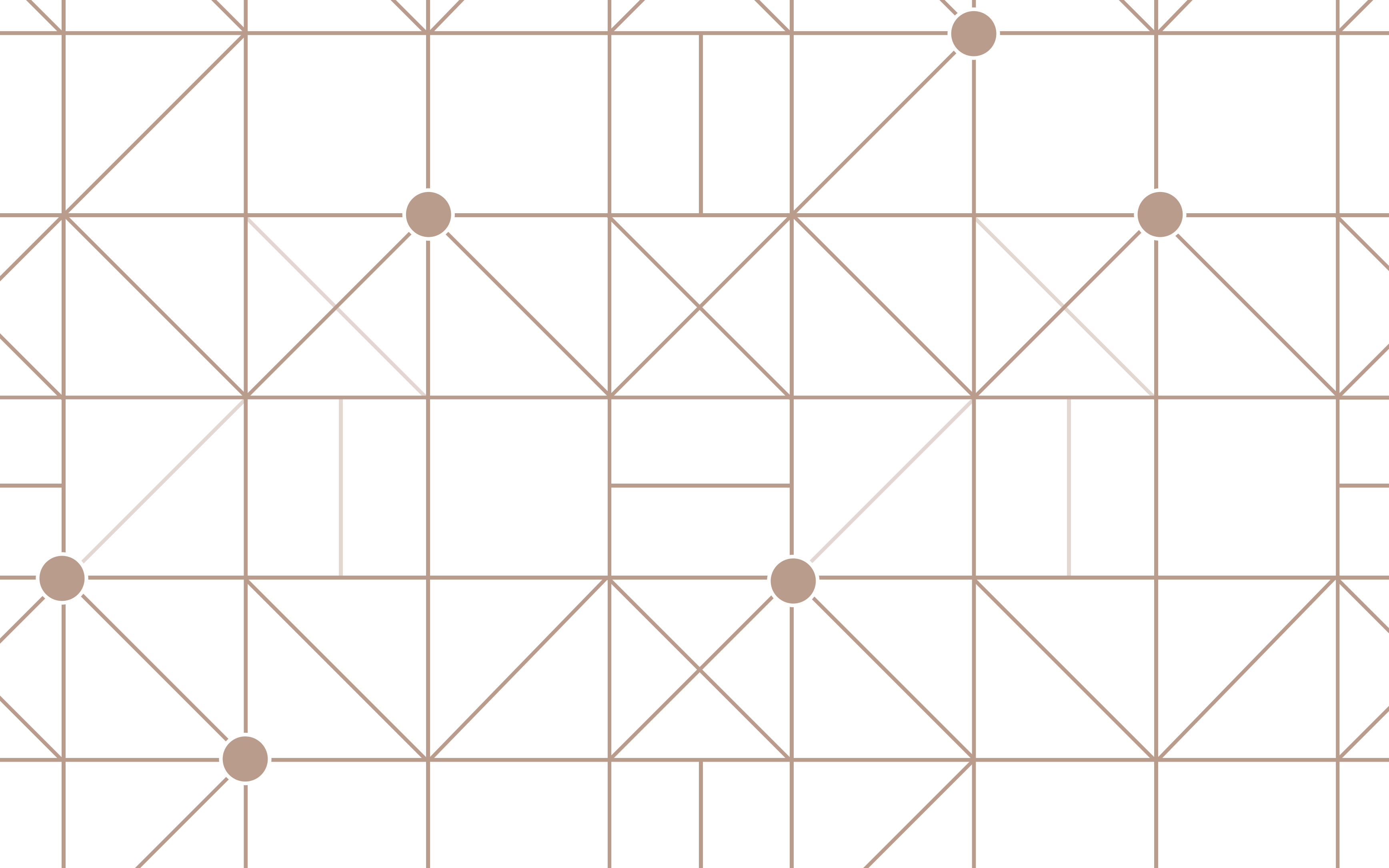


WEEK 1 :- INTRODUCTION TO LINEAR ALGEBRA

1) $2a + 3b = 8$
$10a + 1b = 13$ } THESE ARE 2 EQUATIONS

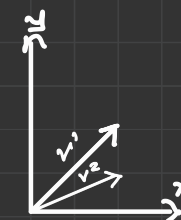
THIS CAN BE WRITTEN AS

$$\underbrace{\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}}_{\text{MATRICES}} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

VECTORS

✗ WHAT ARE VECTORS?

↳ PHYSICS PERSPECTIVE :
- ↳ VECTORS ARE ARROW POINTING IN SPACE
  - ↳ THESE ARROWS HAVE LENGTH & DIRECTION ($\longrightarrow$ $\uparrow$)
    - ↳ AS LONG AS LENGTH REMAINS THE SAME, IT IS THE SAME VECTOR EVEN IF DIRECTION IS DIFFERENT



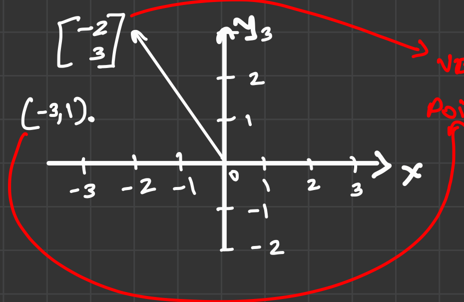
↳ COMPUTER SCIENCE PERSPECTIVE :
- ↳ VECTORS ARE ORDERED LIST OF NUMBERS

· Eg $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$ $\begin{bmatrix} 2.3 \\ -7.1 \end{bmatrix}$

↳ ORDER OF THESE NUMBERS MATTER

↳ MATH PERSPECTIVE:
- ↳ IT IS A MIX OF BOTH PERSPECTIVE
- ↳ ANY VECTOR SHOULD BE ABLE TO PERFORM MATH OPERATIONS LIKE ADDITION, MULTIPLICATION, SUBSTRACTION

- ALWAYS THINK OF VECTORS IN A COORDINATE SYSTEM.
  - ↳ SITTING AT IS TAIL

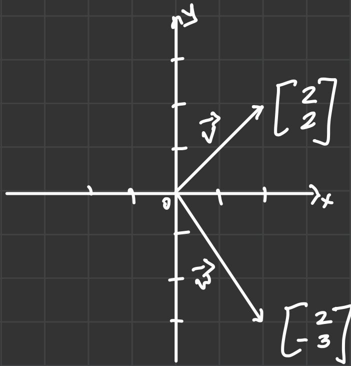


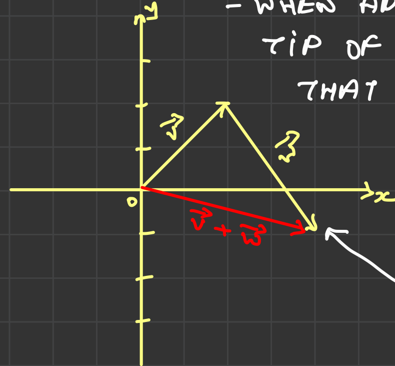
VECTOR WRITTEN IN SQUARE BRACKETS

POINTS WRITTEN IN ROUND BRACKETS

- Eg $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ THIS IS A 3 DIMENSIONAL VECTOR
  - 1 → X Axis
  - 2 → Y Axis
  - 3 → Z Axis

VECTOR ADDITION





- WHEN ADDING $\vec{v}$ WITH $\vec{w}$ JUST MOVE THE VECTOR $\vec{w}$ AT THE TIP OF VECTOR $\vec{v}$ AND JOIN TAIL OF $\vec{v}$ TO HEAD OF $\vec{w}$ THAT IS THE SUM OF TWO VECTORS

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2+2 \\ 2+(-3) \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

BASIS VECTOR

↳ $\hat{i}$ & $\hat{j}$ ARE UNIT VECTORS ON XY COORDINATES

↳ SO A VECTOR $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ CAN BE WRITTEN AS

↳ $2\hat{i} + (-3)\hat{j}$

LINEAR COMBINATIONS :

↳ $a\vec{v} + b\vec{w}$

SCALARS

↳ SET OF ALL LINEAR COMBINATION IS THE SPAN OF $\vec{v}$ & $\vec{w}$

* IF A VECTOR IS NOT ADDING NEW VECTOR IN THE SPAN, IT IS CALLED LINEARLY DEPENDANT

↳ $\vec{u} = a\vec{v} + b\vec{w}$

* IF A VECTOR IS ADDING NEW VECTOR IN THE SPAN, IT IS CALLED LINEARLY INDEPENDANT

↳ $\vec{u} \neq a\vec{v} + b\vec{w}$

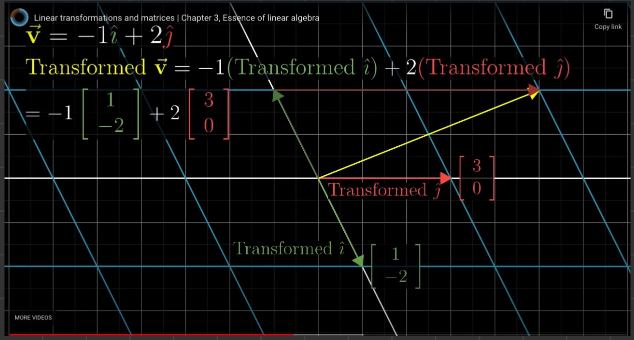
* LINEAR TRANSFORMATION:

↳ FUNCTION

↳ GREAT WAY TO UNDERSTAND FUNCTIONS OF VECTOR IS TO UNDERSTAND MOVEMENT

↳ FOR ALL LINEAR TRASFORMATION 2 THINGS SHOULD NOT CHANGE:

1) THE OUTPUT VECTOR SHOULD BE LINEAR
2) THE ORIGIN SHOULD NOT CHANGE



↳ IF WE KNOW WHERE $\hat{i}$ & $\hat{j}$ LANDS AFTER TRANSFORMATION, WE CAN IDENTIFY ANY TRANSFORMED VECTOR

↳ $\vec{v} = a\hat{i} + b\hat{j}$ → ORIGINAL

↳ $\vec{v}_1 = a\hat{i}_1 + b\hat{j}_1$ → TRANSFORMED

↳ HOW CAN WE DESCRIBE THIS TRANSFORMATION IN MATRICES!

↳ $\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ↳ $\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

↳ $a = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ ↳ $b = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

↳ $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

↳ $\vec{v}_1 = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_j \\ y_j \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$= x\hat{i} + y\hat{j}$

IN THIS BASICALLY IF WE CAN IDENTIFY NEW $\hat{i}$ & $\hat{j}$ WE CAN GET THE TRANSFORMED VECTOR

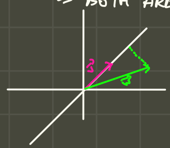
WEEK 2 :- VECTORS ARE OBJECTS THAT MOVE AROUND SPACE

* DOT PRODUCTS :

↳ IF $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ $\vec{w} = \begin{bmatrix} c \\ d \end{bmatrix}$ THEN $\boxed{\vec{v} \cdot \vec{w} = \begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = a \cdot c + b \cdot d}$

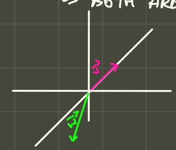
1) $\vec{v} \cdot \vec{w} > 0$

↳ BOTH ARE POINTING IN SAME DIRECTION



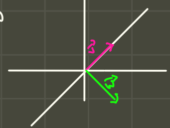
2) $\vec{v} \cdot \vec{w} < 0$

↳ BOTH ARE POINTING IN OPPOSITE DIRECTION



3) $\vec{v} \cdot \vec{w} = 0$

↳ BOTH ARE PERPENDICULAR



↳ THIS CAN ALSO BE EXPLAINED AS PROJECTION OF ONE VECTOR ON ANOTHER AND THEN MULTIPLYING THE LENGTHS

1) DUALITY:

$$\underbrace{\begin{bmatrix} 2 & 1 \end{bmatrix}}_{\text{TRANSFORM}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{v}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

2) COMMUTATIVE :

↳ $\boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$

3) DISTRIBUTIVE OVER ADDITION:

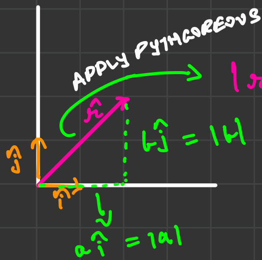
↳ $\boxed{\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}}$

4) ASSOCIATIVE OVER SCALAR MULTIPLICATION:

↳ $\boxed{\vec{A} \cdot (a \vec{B}) = a(\vec{A} \cdot \vec{B})}$

↳ WHERE a IS A SCALAR NUMBER

✱ LENGTH OF A VECTOR:



$$|r| = \sqrt{a^2 + b^2}$$

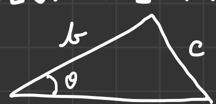
✱ SIZE OF THE VECTOR:

↳ WE CAN IDENTIFY THE SIZE OF THE VECTOR BY TAKING THE DOT PRODUCT OF THE VECTOR WITH ITSELF

↳
$$\boxed{\begin{aligned}\vec{A}\cdot\vec{A} &= A_1A_1 + A_2A_2 \\ &= A_1^2 + A_2^2 \\ &= \left(\sqrt{A_1^2 + A_2^2}\right)^2 \\ &= |A|^2\end{aligned}}$$

✱ COSINE & DOT PRODUCT:

↳ FROM GEOMETRY THE COSINE RULE SAY FOR A TRIANGLE WITH SIDES a, b & c:

↳ 

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos(\theta)}$$

↳ NOW CONVERT THE SIDES TO VECTORS AND APPLY THE COSINE RULE:

$$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos(\theta)$$

↳ $|r-s|^2 = (r-s)\cdot(r-s) = |r|^2 + |s|^2 - 2r\cdot s$

$\therefore \cancel{|r|^2} + \cancel{|s|^2} - 2r\cdot s = \cancel{|r|^2} + \cancel{|s|^2} - 2|r||s|\cos(\theta)$

↳ $-2\, r \cdot s = -2\, |r|\,|s| \cos(\theta)$

↳ $\boxed{r \cdot s = |r|\,|s| \cos(\theta)}$

✗ PROJECTIONS:

↳ SCALAR PROJECTION

$$\cos(\theta) = \frac{\text{SCALAR PROJECTION}}{\text{LENGTH OF } s}$$

$$\boxed{|s| \cos(\theta) = \text{SCALAR PROJECTION}}$$

WE KNOW THAT

↳ $r \cdot s = |r|\,|s| \cos(\theta)$

↳ $\boxed{|s| \cos(\theta) = \dfrac{r \cdot s}{|r|}}$

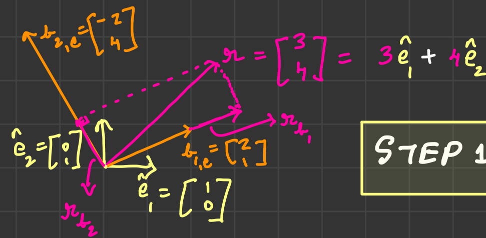
$$\boxed{\text{VECTOR PROJECTION} = \left(\frac{r \cdot s}{|r|}\right) \frac{r}{|r|} = r \left(\frac{r \cdot s}{|r|\,|r|}\right)}$$

HOW TO IDENTIFY A VECTOR IN NEW CO-ORDINATE SYSTEM ?

↳ GIVEN VECTOR r IN CO-ORDINATE SYSTEM WITH BASIS e_1 & e_2

↳ IDENTIFY THE VECTOR r IN CO-ORDINATE SYSTEM WITH BASIS b_1 & b_2

↳



$\boxed{\text{STEP 1:}}$ IDENTIFY THE VALUE OF b_1 & b_2 IN THE CO-ORDINATE SYSTEM OF e_1 & e_2

↳ $b_{1,e} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ & $b_{2,e} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$\boxed{\text{STEP 2:}}$ VERIFY THAT b_1 & b_2 ARE AT 90°.

↳ $b_1 \cdot b_2 = 0$

$$b_1 \cdot b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$= 2(-2) + 1(4)$$

$$= -4 + 4$$

$$= 0$$

$\boxed{\text{STEP 3:}}$ CALCULATE SCALAR PROJECTION OF r ON b_1 & b_2

$$r_{b_1} = \frac{r_e \cdot b_1}{|b_1|^2} = \frac{\begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}}{\left(\sqrt{2^2 + 1^2}\right)^2} = 2$$

$$r_{b_2} = \frac{r_e \cdot b_2}{|b_2|^2} = \frac{\begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix}}{\left(\sqrt{(-2)^2 + 4^2}\right)^2} = \frac{1}{2}$$

STEP 5: CALCULATE THE VECTOR PROJECTIONS OF r ON b_1 & b_2

↳ $\vec{r}_{b_1} = b_1$ (SCALAR PROJECTION) $= 2b_1$

↳ $\vec{r}_{b_2} = b_2$ (SCALAR PROJECTION) $= \frac{1}{2} b_2$

↳ $\vec{r}_b = 2b_1 + \frac{1}{2} b_2$

* IF THE DETERMINANT OF VECTORS IS NON-ZERO THEY ARE LINEARLY INDEPENDANT

WEEK 3 : MATRICES - OBJECTS THAT OPERATE ON VECTORS

✗ MATRICES, VECTORS & SOLVING SIMULTANEOUS EQUATION PROBLEM :

1) IF WE HAVE 2 EQUATIONS :

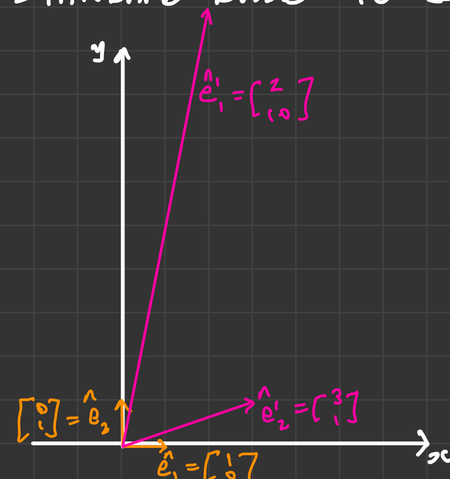
$$2a + 3b = 8$$
$$10a + 1b = 13$$

2) WE CAN WRITE THEM IN A MATRIX FORM:

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 8 \\ 13 \end{bmatrix}}_{\vec{r}}$$

3) THIS MEANS VECTOR $\vec{x}$ IS TRANSFORMED TO VECTOR $\vec{r}$ AFTER APPLYING THE A TRANSFORMATION

4) WE CAN APPLY THE A TRANSFORMATION ON STANDARD BASIS TO IDENTIFY NEW BASIS



$$\hat{e}'_1 = \begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \times 1 + 3 \times 0 \\ 10 \times 1 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

$$\hat{e}'_2 = \begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \times 0 + 3 \times 1 \\ 10 \times 0 + 1 \times 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Types of Matrix Transformation:

1) IDENTITY MATRIX:
- IT DOES NO TRANSFORMATION TO ANY VECTOR.
- IT CONTAINS STANDARD BASIS VECTORS.
  - $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2) SCALED MATRIX
- $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$
  - THIS MEANS $\hat{e}_1$ IS SCALED BY 3 TIMES AND $\hat{e}_2$ IS SCALED BY 2 TIMES.

2) INVERSE MATRIX:
- $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

4) 90° ROTATION MATRIX:
- $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Composition of Matrix Transformation:

- APPLY MULTIPLE TRANSFORMATIONS
  1) ROTATION:
     - IN ROTATION WE WANT TO ROTATE THE PLANE BY 90° LEFT
     - SO, THE NEW $\hat{j}$ WILL BE AT $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ FROM $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2) SHEER:

- In sheer we make the plane a bit slant such that the $\hat{i}$ & $\hat{j}$ are stretched
- The new values for $\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\hat{j} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- APPLYING BOTH TOGETHER:

READ RIGHT TO LEFT

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{SHEER}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{ROTATION}} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

MATRIX MULTIPLICATION

- In other words this can be repsented as

$$\boxed{\underbrace{f(\underbrace{g(x)}_{\text{ROTATION}})}_{\text{SHEER}}}$$

* MATRIX MULTIPLICATION:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \left[\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} \right]$$

$$= \left[e \begin{bmatrix} a \\ c \end{bmatrix} + g \begin{bmatrix} b \\ d \end{bmatrix} \quad f \begin{bmatrix} a \\ c \end{bmatrix} + h \begin{bmatrix} b \\ d \end{bmatrix} \right]$$

$$= \begin{bmatrix} ea + gb & fa + hb \\ ec + gd & fc + hd \end{bmatrix}$$

↳ ORDER OF MULTIPLICATION MATTERS:

$$\boxed{M_1 M_2 \neq M_2 M_1}$$

↳ ASSOCIATIVE:

$$\boxed{(M_1 M_2) M_3 = M_1 (M_2 M_3)}$$

↳ BECAUSE THE ORDER OF MULTIPLICATION IS NOT CHANGED

✱ LINEAR TRANSFORMATION IN THREE DIMENSIONS:

↳ ALL THE CONCEPTS OF ADDITION & MULTIPLICATION REMAINS THE SAME IN 3D.

GAUSSIAN ELIMINATION:

LINEAR SYSTEM OF EQUATIONS:

$$\begin{aligned} 2x + 5y + 3z &= -3 \\ 4x + 0y + 8z &= 0 \\ 1x + 3y + 0z &= 2 \end{aligned} \implies \underbrace{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}}$$

$$A\vec{x} = \vec{v}$$

THIS MEANS VECTOR $\vec{x}$ IS TRANSFORMED TO VECTOR $\vec{v}$ AFTER APPLYING THE LINEAR TRANSFORMATION A

INVERSE MATRIX

IF $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ THEN $A^{-1} = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}^{-1}$

$$A^{-1} \cdot A = \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

↳ IN ABOVE EXAMPLE TO FIND $\vec{x}$ APPLY A^{-1} ON BOTH SIDES

↳ $A^{-1} \cdot A \vec{x} = A^{-1} \vec{v}$

↳ $\boxed{\vec{x} = A^{-1} \vec{v}}$

↳ THE ABOVE ALL POINTS FOR INVERSE MATRIX ONLY APPLY WHEN $\det(A) \neq 0$

* IN GAUSSIAN ELIMINATION WE TRY TO ELIMINATE EACH ROW OF THE GIVEN MATRIX TO REACH TO IDENTITY MATRIX :

↳ Eg:

$$\begin{array}{c} 1: \\ 2: \\ 3: \end{array} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

1) ELIMINATE ROW 2 FROM 1

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

2) THEN ELIMINATE ROW 3 FROM ROW 1 TWICE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ $a = -2$

$b = 1$

$c = 1$

FIND INVERSE MATRIX:

$$A A^{-1} = I$$

LET'S SAY WE HAVE $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}$ AND $A^{-1} = B$ THEN

WE CAN SAY

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NOW SOLVE THIS USING ELIMINATION TO GET THE VALUES FOR B

STEP1 : Rows

RIGHT

$$\begin{array}{c c} 1. & \\ 2. & \\ 3. & \end{array} \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

LEFT

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

STEP 2: ELIMINATE ROW 1 FROM ROW 2 & 3

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

STEP 3: MULTIPLY ROW 3 BY (-1)

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

STEP 4: ELIMINATE ROW 3 FROM ROW 2 & 3 TIMES FROM ROW 1

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 3 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

STEP 5: ELIMINATE ROW 2 FROM ROW 1

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

STEP 6:

$$\boxed{B = \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}}$$

* RESEARCH ABOUT DECOMPOSITION METHOD FOR MATRIX COMPUTATION TO IDENTIFY AN INVERSE

THE DETERMINANT ?

- WHEN VECTOR TRANSFORMATION HAPPENS IT IS IMP TO UNDERSTAND, HOW MUCH THE AREA HAS SCALED

- THE FACTOR BY WHICH THE LINEAR TRANSFORMATION HAS CHANGED IS CALLED THE DETERMINANT OF THAT TRANSFORMATION

- FORMULA:

↳ $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$

↳ $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a \det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b \det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c \det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$

↳ $\det(M_1 M_2) = \det(M_1)\det(M_2)$

IF DET IS 0 THIS MEANS THE BASIS VECTORS ARE LINEARLY INDEPENDENT:

↳ WHICH MEANS THE INVERSE DOESN'T EXIST

WEEK 4 - MATRICES MAKE LINEAR MAPPINGS

* EINSTEIN SUMMATION CONVENTION

↳ LET US GENERALIZE MATRIX MULTIPLICATION:

1) SAME SIZE MATRIX: A_{nn} & B_{nn} WHEN MULTIPLIED GIVES MATRIX C_{nn}

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{12} & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ b_{n1} & \cdots & \cdots & b_{nn} \end{bmatrix} = \overset{C}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}$$

ROW
COLUMN

↳ FROM C IF WE WANT TO IDENTIFY VALUE OF ELEMENT 23

↳ $(C)_{23}$ = (ROW 2 OF A) (COLUMN 3 OF B)

$$= a_{21}b_{13} + a_{22}b_{23} + \ldots + a_{2n}b_{n3}$$

$$= \sum_j a_{ij}\, b_{jk}$$

→ ACCORDING TO EINSTEIN CONVENTION
→ WE SAY, THIS IS THE SUM OVER SOME ELEMENT j OF a_{ij} WITH b_{jk}

$$\boxed{C_{ik} = a_{ij}\, b_{jk}}$$

2) DIFFERENT SIZE MATRIX:

↳ IF THE MATRICES HAVE THE SAME NUMBER OF ENTERIES IN J WE CAN MULTIPLY THEM TOGETHER EVEN IF THEY ARE NOT THE SAME SHAPE.

↳ Eg: MULTIPLY 2×3 MATRIX WITH 3×4 MATRIX

$$2\times\underbrace{3}_{J} \qquad \underbrace{3}_{J}\times 4$$

↳
$$\begin{array}{cc} & j=3 \\ i=2 & \begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \end{array}
\begin{array}{cc} & k=4 \\ j=3 & \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} \end{array}
= \begin{array}{cc} & k=4 \\ i=2 & \begin{bmatrix} - & - & - & - \\ - & - & - & - \end{bmatrix} \end{array}$$

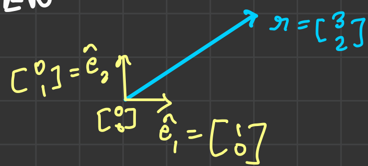
$$A_{ij} \qquad B_{jk} \qquad C_{ik}$$

✗ DOT PRODUCT IN CONTEXT OF SUMMATION CONVENTION:

↳ IF WE HAVE 2 VECTORS u AND v HAVING ELEMENTS u_i AND v_i

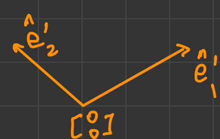
↳
$$\begin{pmatrix} u_i \\ \\ \end{pmatrix} \cdot \begin{pmatrix} v_i \\ \\ \end{pmatrix} = \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = u_i\, v_i$$

* CHANGES OF BASIS & TRANSFORMATION:

- IF WE HAVE A VECTOR r WE CAN DESCRIBE THIS VECTOR AS A COMBINATION OF SCALAR EXPANSION OF STANDARD BASIS VECTOR $\hat{e}_1$ & $\hat{e}_2$
  - GIVEN
    - 

$$\rightarrow \quad r = 3\hat{e}_1 + 2\hat{e}_2 = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- NOW IF YOU HAVE A DIFFERENT CO-ORDINATE SYSTEM WE WOULD ALSO HAVE A NEW SET OF BASIS VECTORS $\hat{e}'_1$ & $\hat{e}'_2$



- WHERE $\hat{e}'_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & $\hat{e}'_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ IN THE NEW CO-ORDINATE SYSTEM

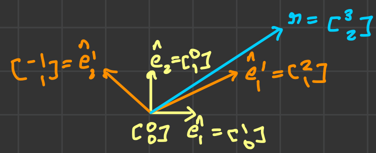
    - BASICALLY WE ARE LOOK AT THE SAME VECTORS IN THE SPACE BUT DIFFERENTLY

↳ HOW CAN WE TRASLATE THE VECTOR r FROM STANDARD CO-ORDINATE SYSTEM TO THIS NEW CO-ORDINATE SYSTEM?

↓

STEP 1: $\boxed{\text{IDENTIFY } \hat{e}'_1 \text{ \& } \hat{e}'_2 \text{ IN THE STANDARD CO-ORDINATE SYSTEM.}}$

↳ THIS IS CALLED CHANGE OF BASIS MATRIX



$\therefore \quad \hat{e}'_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \& \quad \hat{e}'_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

↳ SO, WHEN WE SAY $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ IN STANDARD CO-ORDINATE SYSTEM THAT MEANS $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ IN THAT OTHER CO-ORDINATE SYSTEM.

STEP 2: TAKE THE INVERSE OF THE 2^{nd} CO-ORDINATE SYSTEMS BASIS VECTOR REPRESENTED IN THE STANDARD CO-ORDINATE SYSTEM

↳ $$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

STEP 3 : NOW WE MULTIPLY THE VECTOR r WHICH IS IN STANDARD CO-ORDINATE SYSTEM WITH THE INVERSE FROM STEP 2

$$\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}$$

$$\boxed{[\hat{e}']^{-1}[r]}$$

THIS IS THE SAME VECTOR r BUT IN AN ANOTHER CO-ORDINATE SYSTEM

HOW CAN WE TRASLATE THE VECTOR r' FROM ANOTHER CO-ORDINATE SYSTEM TO STANDARD CO-ORDINATE SYSTEM ?

STEP 1 : JUST MULTIPLY THE VECTOR r' WITH THE BASIS VECTOR CONVERTED FROM ANOTHER CO-ORDINATE SYSTEM TO STANDARD CO-ORDINATE SYSTEM

IN THIS CASE $[\hat{e}'][r']$

↳ HOW DOES THE TRANSLATION OF VECTOR HAPPEN B/W 2 CO-ORDINATE SYSTEMS ALONG WITH ROTATION (AKA TRANSFORMATION)

↓

STEP 1: START WITH THE VECTOR IN ANOTHER CO-ORDINATE SYSTEM

STEP 2: THEN TRANSLATE IT INTO STANDARD CO-ORDINATE SYSTEM USING CHANGE OF BASIS MATRIX

STEP 3: THEN APPLY THE TRANSFORMED BASIS VECTOR OF STANDARD CO-ORDINATE SYSTEM TO THE OUTPUT OF STEP 1 & 2

STEP 4: THEN APPLY THE INVERSE CHANGE OF BASIS MATRIX AND THAT WILL GIVE THE FINAL TRANSFORMED VECTOR IN ANOTHER CO-ORDINATE SYSTEM

$$\underbrace{[\hat{e}']^{-1}\underbrace{[M]\underbrace{[\hat{e}']\underbrace{[x']}_{\text{STEP 1}}}_{\text{STEP 2}}}_{\text{STEP 3}}}_{\text{STEP 4}}$$

✗ OTHOGONAL MATRICES :

↳ TRANSPOSE OF A MATRIX:

↳ $\boxed{A^T_{ij} = A_{ji}}$

↳ $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

↳ IF WE HAVE A MATRIX A_{ij} WHERE

↳ $a_i \cdot a_j = 0$ FOR $i \neq j$

↳ $a_i \cdot a_j = 1$ FOR $i = j$

THIS IS CALLED ORTHONORMAL BASIS

THEN $\boxed{A^T A = I}$ → $A^T = A^{-1}$

↳ SUCH MATRIX A IS CALLED ORTHOGONAL MATRIX

NOTE :-

1) IN DATA SCIENCE WE WANT TO USE ORTHONORMAL BASIS VECTOR SET WHEN WE TRANSFORM OUR DATA

↳ i.e WE WANT OUR TRANSFORMATION MATRIX TO BE AN ORTHOGONAL MATRIX

2) SO THAT COMPUTING INVERSE IS EASY.

3) TRANSMISSION IS REVERSIBLE BECAUSE IT DOESN'T COLLAPSE SPACE.

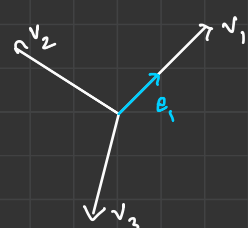
4) PROJECTION IS JUST A DOT PRODUCT.

GRAM-SCHMIDT PROCESS
(HOW TO CONSTRUCT ORTHONORMAL BASIS)

↳ LET'S SAY WE HAVE N NUMBER OF VECTORS $V = \{v_1, v_2, v_3 \ldots \ldots v_n\}$ WHICH ARE ALL ORTHOGONAL

↳ ALL OF THEM ARE LINEARLY INDEPENDENT. IF WE WANT TO CHECK THEIR LINEAR INDEPENDANCY, WE CAN TAKE ALL THE COLUMNS OF THE VECTOR AND CALCULATE THE DETERMINENT (IT SHOULD COME TO ZERO)

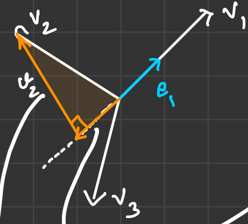
↳ LET'S TAKE 3 VECTORS FROM V TO UNDESTAND HOW TO CALCULATE ORTHONORMAL BASIS



STEP 1: CALCULATE THE BASIS VECTOR FOR V_1

$$\boxed{e_1 = \frac{v_1}{|v_1|}}$$

STEP 2: WE CAN CONSIDER v_2 AS THE COMBINATION OF A COMPONENT IN THE DIRECTION OF v_1 AND A COMPONENT PERPENDICULAR TO v_1



WE CAN FIND THIS COMPONENT BY TAKING THE v_2 PROJECTION ON e_1 . AND SINCE

↳ $v_2 = (v_2 \cdot e_1)$

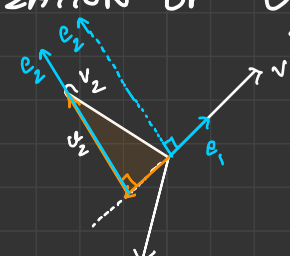
↳ $v_2 \text{ (VECTOR)} = (v_2 \cdot e_1) \dfrac{e_1}{|e_1|}$

TO CALCULATE THIS WE CAN SUBSTRACT THE v_2 PROJECTION ON e_1 FROM v_2

↳ $$\boxed{u_2 = v_2 - (v_2 \cdot e_1) e_1}$$

STEP 3: NOW TAKE THE NORMALIZATION OF u_2 TO GET THE BASIS VECTOR

↳ $$\boxed{e_2 = \dfrac{u_2}{|u_2|}}$$



STEP 4: V_3 IS NOT IN THE PLANE OF V_1 & V_2. SO IT IS NEITHER IN THE PLANE OF e_1 & e_2.

↳ SO WE WOULD HAVE TO PROJECT V_3 ON THE PLANE OF V_1 & V_2 AND THAT PROJECTION THEN CAN BE WRITTEN IN THE COMBINATION OF e_1 & e_2.

↳ $$u_3 = V_3 - \underline{(V_3 \cdot e_1)e_1} - \underline{(V_3 \cdot e_2)e_2}$$

u_3 ↓ PERPENDICULAR COMPONENT ON THE PLANE

$(V_3 \cdot e_1)e_1$ ↓ COMPONENT OF V_3 PROJECTION MADE UP OF e_1

$(V_3 \cdot e_2)e_2$ ↳ COMPONENT OF V_3 PROJECTIONS MADE UP OF e_2

↳ NOW WE WILL NORMALIZE u_3 TO GET BASIS VECTOR FOR u_3

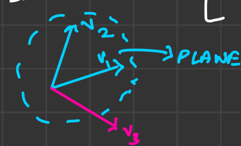
↳ $$\boxed{e_3 = \frac{u_3}{|u_3|}}$$

REFLECTING IN A PLANE :

↳ HOW DOES A VECTOR LOOK WHEN REFLECTED IN A PLANE ?

1) I DON'T KNOW THE PLANE OF THE MIRROR BUT I KNOW 2 VECTORS IN THE PLANE v_1 & v_2 AND 3^{rd} VECTOR OUTSIDE THE PLANE

$$\underbrace{v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}_{\text{SAME PLANE}} \qquad v_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$



2) NOW DO THE GRAM - SCHMIDT PROCESS TO IDENTIFY ORTHONORMAL VECTORS DESCRIBING THE PLANE AND THE NORMAL v_3.

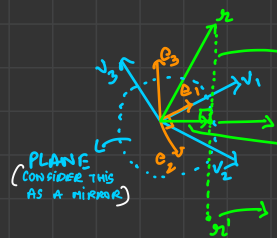
↳ CALCULATE e_1, e_2 & e_3 USING GRAM-SCHMIDT PROCESS

↳ TRANSFORMATION MATRIX E IS DESCRIBED USING e_1, e_2 & e_3

↳ $$E = \Big[\underbrace{(e_1)\,(e_2)}_{\text{PLANE}}\,\underbrace{(e_3)}_{\text{NORMAL TO THE PLANE}}\Big] = \left(\frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right)$$

3) NOW LET'S SAY WE HAVE A VECTOR $r = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ WHICH WE WANT TO REFLECT THROUGH THIS PLANE.

- THAT MEANS 1* THE REFLECTION OF r WILL INTERSECT WITH THE PLANE PERPENDICULAR
- THEN IT WILL PROJECT THE REFLECTION ON THE OTHER SIDE.



THIS CAN BE DEFINED AS SOME COMBINATION OF e_3

INTERSECTING PERPENDICULAR WITH PLANE

THIS CAN BE DEFINED AS A COMBINATION OF e_1 AND e_2 AS IT IS IN THE PLANE

THE OTHER SIDE

- WE CAN WRITE DOWN r' AS A TRANSFORMATION MATRIX IN BASIS E AS:

THIS IS NOT IN OUR PLANE BUT IN THE BASIS OF E

$$T_E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

KEEP THE E_1 & E_2 AS SAME AND E_3 BECOMES THE REFLECTION

4) NOW WE WANT TO CONVERT r TO r'

$$\begin{tikzcd} \boxed{r} \arrow[r, "\text{HARD}"'] \arrow[d, "E^{-1}"'] & \boxed{r'} \\ \boxed{r_E} \arrow[r, "T_E"'] & \boxed{r'_E} \arrow[u, "E"'] \end{tikzcd}$$

STEP 1: TRANSFORMING r IN THE BASIS OF E USING E^{-1}

STEP 2: TRANSFORM r_E AS A REFLECTION IN THE BASIS OF E USING T_E

STEP 3: TRANSFORM BACK INTO THE BASIS OF THE PLANE USING THE TRANSFORMATION OF E

$\rightarrow$ $E \, T_E \, E^{-1} r = r'$

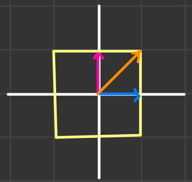
WEEK 5:- EIGENVALUES & EIGENVECTORS

✱ WHAT ARE EIGEN-THINGS?

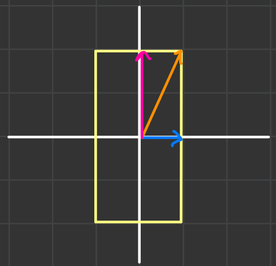
↳ EIGENVECTORS ARE THE VECTORS THAT DO NOT CHANGE THEIR DIRECTIONS EVEN AFTER ALL TRANSFORMATIONS ARE APPLIED.

↳ EIGENVALUES ARE LENGTH OF THE EIGENVECTORS.

↳ Eg:- CONSIDER THIS SQUARE CENTERED AT ORIGIN AS A GROUP OF MULTIPLE VECTORS



↳ NOW IF WE SCALE VERTICALY BY A FACTOR OF 2:



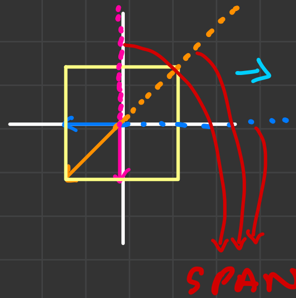
→ IT HAS 2 EIGENVECTORS BLUE & PINK AS THEIR DIRECTIONS DID NOT CHANGE.

→ EIGENVALUES ARE

↳ 1 → BLUE VECTOR

↳ 2 → PINK VECTOR

↳ NOW IF WE APPLY SOME TRANSFORMATION TO THE 3 ORIGINAL VECTORS SUCH THAT THE NEW VECTOR ARE :



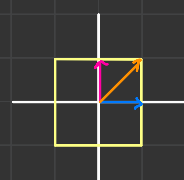
→ ALL THE 3 ORIGINAL VECTORS ARE ALSO EIGENVECTORS FOR THIS TRANSFORMATION. BECAUSE EVEN IF THEY ARE POINTING IN AN OPPOSITE DIRECTION THEY HAVE NOT CHANGED THEIR SPAN

↳ SPAN IS REPRENSTED AS A LINE THAT CROSSES THE VECTOR

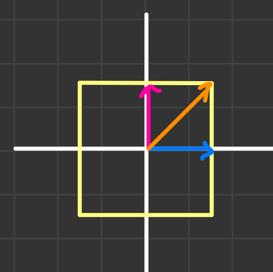
✗ SPECIAL EIGEN-CASES :

▷ FOR A UNIFORM SCALING ALL THE ORIGINAL VECTORS ARE EIGENVECTORS

↳ Eg :-



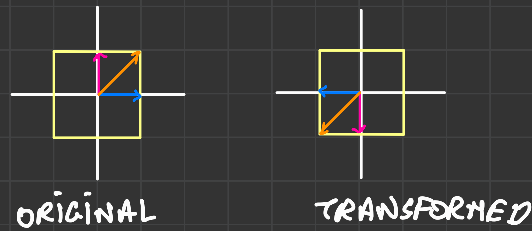
ORIGININAL



TRANSFORMED

2) WHEN WE DO THE ROTATION BY 180°, WE WILL GET SOME EIGENVECTORS.

↳ eg:-



NOTE:- IF WE CAN IDENTIFY AN EIGENVECTOR AFTER THE ROTATION IN 3D, THAT EIGENVECTOR IS ALSO THE AXIS OF ROTATION.

CALCULATE EIGENVECTORS:

- WHEN WE APPLY A TRANSFORMATION THE EIGENVECTORS DON'T CHANGE THEIR SPAN BUT CAN CHANGE ITS LENGTH.
  - $\therefore$ $A x = \lambda x$

    TRANSFORMATION: A — VECTOR: x — SCALAR: λ — VECTOR: x

    Ax: THIS IS A VECTOR MULTIPLICATION

    λx: THIS A SCALAR MULTIPLICATION

  - CONVERT RIGHT SIDE TO VECTOR MULTIPLICATION

    $$A x = (\lambda I) x$$

    I: THIS IS AN IDENTITY MATRIX OF SIZE A

  - SIMPLIFYING FURTHER

    $\rightarrow A x - (\lambda I) x = 0$

    $\rightarrow (A - \lambda I) x = 0$

  - THERE ARE ONLY 2 POSSIBILITIES FOR ABOVE EQUATIONS TO BE ZERO
    - $x = 0$
    - $A - \lambda I = 0$

↳ TO CALCULATE ANY MATRIX CALCULATION TO BE ZERO WE USE DETERMINANT.

$$\det(A - \lambda I) = 0$$

↳ ONCE YOU SOLVE THE DETERMINANT AND GET POSSIBLE ANSWERS FOR λ PUT THOSE VALUES BACK INTO THE ORIGINAL EQUATION TO IDENTIFY DIFFERENT x

↳ $(A - \lambda I)x = 0$

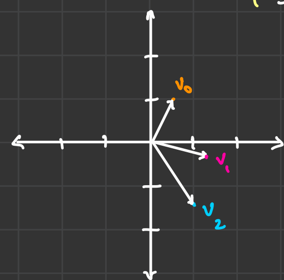
CHANGE TO EIGENBASIS

↳ THIS CHANGE TO EIGENBASIS HELPS IN MATRIX OPERATION OF DIAGONALISATION.

↳ SOMETIMES WE HAVE TO APPLY SAME MATRIX MULTIPLICATION MANY TIMES.

↳ Eg:- THIS TRANSFORMATION T REPRESENTS CHANGE IN LOCATION OF A PARTICLE IN A SINGLE TIMESTAMP

$$T = \begin{bmatrix} 0.9 & 0.8 \\ -1 & 0.35 \end{bmatrix} \qquad v_0 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$



$v_1 = T v_0$ → CHANGE IN POSITION OF v_0 AFTER APPLYING TRANSFORMATION T

$v_2 = T v_1$ → PARTICLE LOCATION AFTER 2 TIMESTAMP

$v_2 = T(T v_0)$

$v_2 = T^2 v_0$

$$\boxed{v_n = T^n v_0}$$

→ N NUMBER OF TRANSFORMATION AFTER n^{th} TIMESTAMP

↳ SO, IF WE HAVE TO DO LARGE TIMESTAMP CALCULATION ON A HIGHER DIMENSIONAL MATRIX. IT IS VERY TOUGH, BUT INSTEAD IF IT IS DONE ON AN IDENTITY MATRIX IT IS VERY EASY

↳ $T^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$

↳ WHAT IF T IS NOT A DIAGONAL MATRIX ?

STEP 1: WE HAVE TO FIND A BASIS WHERE T IS DIAGONAL.

↳ THIS IS CALLED EIGEN-BASIS.

↳ $C = \begin{bmatrix} e_1 & e_2 & e_3 \\ \vdots & \vdots & \vdots \end{bmatrix}$

→ EIGEN BASIS CONVERSION MATRIX.

→ e_1, e_2, e_3 ARE 3 EIGENVECTORS.

STEP 2: APPLY THE POWER OF n ON THE DIAGONALIZED FORM.

↳ $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

→ D IS A DIAGONAL MATRIX.

→ $\lambda_1, \lambda_2, \lambda_3$ ARE THE EIGEN VALUES.

STEP 3: FINALLY TRANSFORM THE RESULTING MATRIX BACK AGAIN.

↳ $\boxed{T = CDC^{-1}}$

→ APPLYING TRANSFORMATION T IS EQUAL TO:

↳ CHANGE IN BASIS

↳ DIAGONALIZATION

↳ CONVERTING BACK

$$\hookrightarrow \quad T^2 = CD\underbrace{C^{-1}C}DC^{-1}$$

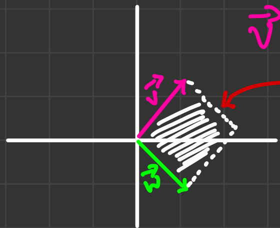
$\hookrightarrow I$ = DOING NOTHING. SO REMOVE IT

$$T^2 = CDDC^{-1} = CD^2C^{-1}$$

$$\boxed{T^n = CD^nC^{-1}}$$

$$\begin{tikzcd}
\boxed{v} \arrow[r, "T^n"] \arrow[d, "C"'] & \boxed{T^n v} \\
\boxed{[v]_E} \arrow[r, "D^n"'] & \boxed{[T^n v]_E} \arrow[u, "C^{-1}"']
\end{tikzcd}$$

CROSS PRODUCTS :



$\vec{v} \times \vec{w} =$ AREA OF PARALLELOGRAM

$$\boxed{\vec{v} \times \vec{w} = \det\left(\begin{bmatrix} \vec{v} & \vec{w} \end{bmatrix}\right)}$$

1) $\vec{v} \times \vec{w}$ IS NEGATIVE WHEN $\vec{v}$ IS ORIENTED ON THE LEFT OF $\vec{w}$.
2) $\vec{v} \times \vec{w}$ IS POSITIVE WHEN $\vec{v}$ IS ORIENTED ON THE RIGHT OF $\vec{w}$.

↳ IN 3D WORLD:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det\left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right)$$

↳ IF $\vec{v} \times \vec{w} = \vec{p}$

↳ THEN LENGTH OF $\vec{p}$ = AREA OF PARALLELOGRAM

↳ AND $\vec{p}$ IS PERPENDICULAR TO $\vec{v}$ & $\vec{w}$