FUSION OF DIVERSE FEATURES AND KERNELS USING LP-NORM BASED MULTIPLE KERNEL LEARNING IN HYPERSPECTRAL IMAGE PROCESSING

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ABSTRACT

Multiple kernel learning (MKL) is an elegant tool for a number of tasks such as heterogeneous data fusion, classification and input/feature selection. In the context of *support vector machine* (SVM) based classification, MK is specifically a homogenization transformation on different input spaces and it ultimately provides important flexibility in searching for high-quality linearly separable solutions in the *reproducing kernel Hilbert space* (RKHS). However, in general the performance of fusion often depends on diversity. Herein, we investigate a new way to extract diverse features from hyperspectral imagery based on band grouping and different proximity measures. The output of band group selection (a set of features) is subjected to ℓ_p-norm based MKL for feature-level fusion. Preliminary results on benchmark data indicates that MK-based fusion of diverse features and heterogeneous kernels leads to noticeable performance improvement.

Index Terms— Multiple kernel learning, feature fusion, hyperspectral image analysis, bandgrouping

1. INTRODUCTION

Hyperspectral imaging in remote sensing has a wide range of applications from mineralogy to weather forecasting, agriculture, surveillance, etc. A hyperspectral image can be described as a high dimensional data cube. Each sub-image in this data cube (usually on the order of hundreds) informs us about the radiance (or reflectance) properties of a scene at different narrowly spaced (and most often contiguous) bands in the *electromagnetic* (EM) spectrum. However, in order to truly achieve robust analysis of some geographical area, several remote sensing systems with sensors in different regions of the EM are required, e.g., visible, near IR and SWIR hyperspectral imagery, lidar and *synthetic aperture radar* (SAR), etc. The point is, numerous sensors are involved in remote sensing and ultimately theory is needed to fuse the information generated by these different sensors. Herein, we focus on the production and fusion of disparate information (specifically features) in hyperspectral imagery for advanced and robust classification. However, without loss of generality, the computational approach discussed in this article is equally applicable to multi-sensor fusion.

The last decade has seen a surge of interest in the development and use of *multiple kernel learning* (MKL) for tasks like heterogeneous data fusion, classification and input/feature selection in areas like machine learning, pattern recognition, signal processing, computer vision, etc. A good recent review of MKL mathematics and algorithms is [1]. In the context of *support vector machine* (SVM) based classification, it is often the case that a single kernel is not enough. In practice, a challenge is finding a quality kernel. This is where MK helps. Instead of being restricted to a limited set of known kernels, which may not solve a task, MK provides a solid foundation to combine (fuse) a set of known base kernels (those satisfying Mercer's conditions) to produce a new and more powerful tailored kernel. MK is both a homogenization transformation for different input spaces and it ultimately provides important flexibility for classification in terms of searching for quality linearly separable solutions in the *reproducing kernel Hilbert space* (RKHS). There are number of outstanding MKL problems, for example: how do we generate diverse inputs for MKL; what linear or nonlinear aggregation operators are needed to combine the base kernels; how are multiple kernels normalized; how do we mitigate overfitting in MKL; and what search algorithms are needed for fusion algorithm parameter estimation. To date, numerous algorithms have been put forth, e.g., MKLGL [2], ℓ_p-norm MKL [3], FIGA [4], $GAMKL_p$ [5], DeFIMKL [5], etc.

In terms of hyperspectral image processing, MKL has been used for tasks like classification [6], feature selection [7] and nonlinear unmixing [8]. For example, Gu *et al.* put forth a computationally efficient formulation of MKL, termed *representative MKL* (RMKL) [6]. A linear combination of *radial basis function* (RBF) kernels are used that yield the max-variance kernel with minimum F-norm error. In [7], simpleMKL [9] was used to help learn image features. Multiple RBF kernels are applied to a single feature, a group of features and features from heterogeneous sources. In [10], Honeine and Richard proved that the angular kernel is a valid reproducing kernel and it was explored for hyperspectral im-

age processing because spectral angle is a popular tool used in the literature due to its invariance to the spectral energy. In summary, while there has been interest in using MKL for hyperspectral image processing, work to date has primarily focused on the fusion of homogeneous kernels, e.g., multiple RBFs with varying sigma. However, it is extremely likely that different types of kernels are required, and as [3] shows, normalization of heterogeneous kernels is often a crucial step. In addition, to the best of our knowledge little-to-no work has focused on how to generate a diverse set of features for MKL via bandgrouping in hyperspectral image processing. Herein, we explore these two challenges. We show that diversity with respect to both features and kernels is important for MKL. Figure 1 is a high-level illustration of our approach.
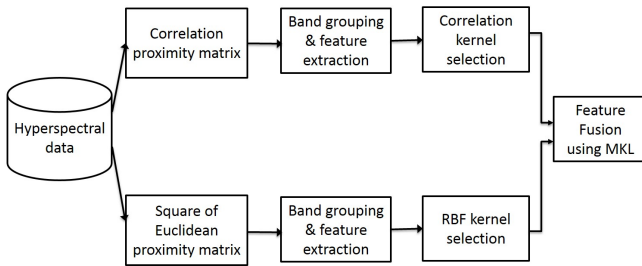

Fig. 1. High-level illustration of the proposed MKL approach. The hyperspectral data is subjected to different proximity measures, features are extracted using bandgrouping, different kernels are applied, and their results are fused using ℓ_p-norm based MKL (with kernel normalization when needed).

The remainder of this paper is structured as follows. Section 2 describes the proposed algorithms and Section 3 contains preliminary results and discussion for a well-known benchmark data set, Indian Pines.

2. METHOD

In this section, we describe the three major parts of our approach—proximity measure calculation, band grouping for feature extraction, and feature space fusion using MKL. For notational purposes, the 3D hyperspectral data cube is remapped into a 2D space such that each row represents a pixel and each column is a band. Let the re-shaped 2D data set be $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \cdots\ \mathbf{x}_n]^T \in \mathbb{R}^{n \times b}$, where n is the number of pixels in the image and b, the number of bands.

2.1. Proximity Measure Calculation

Hyperspectral sensors are wonderful because each pixel has a wealth of information and tells a story, versus traditional single channel or "RGB" imagery. However, hyperspectral imagery also suffers from the curse of dimensionality, spatially, spectral and sometimes temporal. Instead of using all bands, or features, it is often the case that selecting individual bands, feature projection (e.g., *Principle Component Analysis* (PCA), random projection, etc.), or grouping bands leads to a better solution (higher accuracy and more robust). While numerous approaches have been proposed, it has been shown that band group partitioning is of utility and it can be derived, in a supervised or unsupervised fashion, from a proximity measure [11]. For example, one can compute the correlation matrix between the different bands (unsupervised approach). Structure in this matrix can be used to derive a band group partitioning. However, there are a number of unsupervised proximity measures, such as Euclidean, correlation, Jeffrey K. Matusita, Bhattacharyya, *spectral angle mapper* (SAM), etc. In general, it has been demonstrated that selection of proximity measure depends in part on the data set and task, meaning there does not appear to be a global best. Herein, we explore the square of Euclidean, which measures the distance between a pair of vectors, and correlation, which measures angular similarity. We selected these two proximity measures for demonstration as they capture different aspect of the data via its features. However, in future work this is likely a parameter that needs to be included in our algorithm.

Proximity Measure 1: Square of Euclidean The square of Euclidean distance between vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ is

$$d_{ij} = d(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{L}(x_{ik} - x_{jk})^2.$$

Note, the square of Euclidean distance is always positive and depends in large on L, e.g., the length of the vectors or the number of pixels in the training data set.

Proximity Measure 2: Correlation Correlation is a similarity measure between two signatures. The Pearson's correlation coefficient is the co-variance of two vectors normalized by the product of the standard deviation of two distributions,

$$s(\mathbf{x}_i, \mathbf{x}_j) = corr(\mathbf{x}_i, \mathbf{x}_j) = \frac{cov(\mathbf{x}_i, \mathbf{x}_j)}{\sigma_{\mathbf{x}_i}\sigma_{\mathbf{x}_j}}.$$

The correlation coefficient is in $[0, 1]$. Distance, or the dissimilarity measure, $d(\mathbf{x}_i, \mathbf{x}_j)$ is obtained herein by simply subtracting the $s(\mathbf{x}_i, \mathbf{x}_j)$ from 1.

Figure 2 illustrates the $b \times b$ square of Euclidean and correlation proximity matrices for Indian Pines. As the reader can see, each proximity measure has different structure and, hence, leads to different distinct sets of features.

2.2. Feature Extraction

In this step, we first partition similar bands from a given proximity measure into groups and we then apply a feature extraction or reduction technique to each band group to extract a single feature from that group. Herein, we use the algorithm
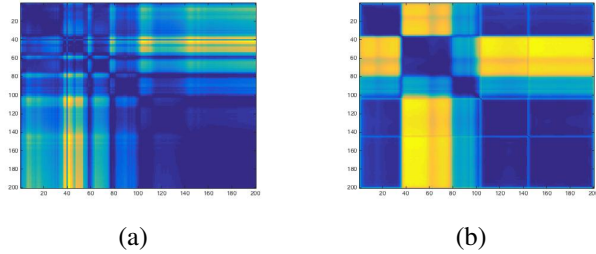
Fig. 2. $b \times b$ dissimilarity matrices for (a) square of Euclidean and (b) correlation on Indian Pines. Note, "block" structure indicates similarity or relatedness between different bands.

proposed by Ball *et al.* that performs unsupervised grouping of contiguous similar bands with respect to a provided proximity measure (see [11] for full algorithm details). After band grouping, we can apply a number of feature extraction techniques such as *stepwise linear discriminant analysis* (SLDA) [12], mean or weight, to get features equal to the number of band groups. In this paper, we calculate the mean of all bands in a groups as the feature. While mean might not be the most sophisticated technique, its advantage is that it is simple to realize in hardware and gives rise to a simpler multispectral versus hyperspectral sensor.

2.3. Feature Space Fusion Using ℓ_p-Norm MKL

In the kernel approach, inputs are ideally projected into a high, possibly infinite, dimensional RHKS space, where the patterns for different classes are now linearly separable. The trick is that we can do this all via a "kernel function" in the original low(er) dimensional space and we never have to do the actual lifting. However, in reality we do not know what kernel to select and in general the choice of kernel is task specific. There is currently no straightforward way to select a kernel for a given set of data. As already mentioned, MKL provides one such path to help search for the idea kernel by the simple concept of combining simple known (base) kernels to form custom (tailored) kernels. However, we must search for this kernel and the space is both extremely large and if we are not careful, MKL tends to succumb to overfitting (can learn the training data perfectly but not generalize well to new test data). Herein, we restrict our analysis to a *linear convex sum* (LCS) of kernels. This is by far the predominant MKL approach. While a few nonlinear approaches have been put forth, e.g., FIGA, for various reasons (such as proving that a given aggregation operator always yields a valid Mercer kernel) nonlinear MKL is still an unsolved problem.

For a function to be a kernel, it need to satisfy the Mercer's kernel properties such as continuity, symmetry, and positive semi-definiteness. There are numerous kernel functions in use, e.g., *radial basis function* (RBF), polynomial, etc. In this paper, we use RBF and correlation kernel. The RBF func-

tion is

$$k_r(\mathbf{x}_i, \mathbf{x}_j) = exp\left(-\frac{||\mathbf{x}_i^2 - \mathbf{x}_j^2||}{2\sigma^2}\right),$$

where σ is the so-called width parameter of the RBF kernel. The correlation kernel is

$$k_c(\mathbf{x}_i, \mathbf{x}_j) = exp\left(-\frac{1 - corr(\mathbf{x}_i, \mathbf{x}_j)}{2\sigma^2}\right)$$

where $corr(\mathbf{x}_i, \mathbf{x}_j)$ is the Pearson's correlation coefficient for $\mathbf{x}_i$ and $\mathbf{x}_j$. In [13], the authors have shown that the correlation kernel satisfies the Mercer's kernel properties. Note, our two kernels are already more-or-less to scale by design. However, if one is using heterogeneous kernels that produce very different scales, then the zero mean and unit variance RHKS approach in [3] can be used.

The convex sum of M kernels is also a Mercer's kernel. This is because both the sum and multiplication by positive constant are *positive semidefinite* (PSD) preserving operators (on M different Gram matrices). The combined kernel with ℓ_p-regularized weight w_m is

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sum_{m=1}^{M} w_m k_m(\mathbf{x}_i, \mathbf{x}_j)$$

subject to $||\mathbf{w}||_p \leq 1$ and $w_m \in \mathbb{R}^+$, where $||\mathbf{w}||_p$ is the p-norm of $\mathbf{w}$. Though the above expression is notationally for M kernels on the same set of features, it is trivially generalized to multiple features, e.g., different kernels on different subsets of features [5]. Optimization-based MKL solutions, versus fixed rule or heuristic approaches, optimize (using alternating optimization typically) the weights of the kernels and the SVM criteria function. Again, we use ℓ_p-norm MKL [6, 7] to derive the LCS weights. However, we could use a number of other search algorithms for feature level fusion, such as MKLGL$_p$ or GAMKL$_p$, or decision-level MKL, e.g., DeFIMKL$_p$. The ℓ_p-norm condition is more-or-less the same across a number of solvers. In general, the different approaches represent variations in search, e.g., Group Lasso (MKLGL), genetic algorithm based (GAMKLp), and nonlinear decision-level fusion via DeFIMKL.

3. PRELIMINARY RESULTS AND DISCUSSION

The Indian Pines data set consists of 145×145 pixels with a spatial resolution of 20 meters and 220 spectral channels (bands). During data pre-processing of the data, 20 water absorption bands, $104 - 108$, $150 - 163$ and 220 were removed. We considered the following 9 classes for classification — Corn-notill, Corn-min, Grass-pasture, Grass-trees, Hay-windowed, Soybean-notill, Soybean-mintill, Soybean-clean and woods. Random jack-knife partitioning is used, where 20% is training and the remainder is testing. Hereafter, the squared Euclidean is denoted as 'SqE' and correlation

Table 1. Producer's accuracies for ℓ_p-norm MKL based fusion.

ℓ_p-norm	Method (SqE = Square of Euclidean, Corr = Correlation)	# of kernels	corn (notill)	corn (min)	grass (pasture)	grass (trees)	hay (windowed)	soybeans (notill)	soybeans (min)	soybeans (clean)	woods
NA	SqE	1	64.52	46.18	75.57	92.80	97.19	63.82	78.57	28.31	97.78
	Corr	1	62.42	34.93	81.11	85.09	96.16	62.02	66.46	30.55	97.49
$p = 1.1$	Fusion of SqE & Corr	$1+1$	68.35	46.48	79.60	90.79	96.16	64.99	79.08	45.42	**97.87**
	SqE	2	65.91	52.62	87.41	92.29	97.44	64.34	80.09	37.27	97.78
	Corr	2	62.77	36.13	82.12	87.27	96.16	62.02	66.41	30.55	97.49
	Fusion of SqE & Corr	$2+2$	68.70	52.02	88.41	93.13	96.42	64.99	80.45	46.64	97.87
$p = 2$	Fusion of SqE & Corr	$1+1$	69.92	53.37	82.87	91.96	96.16	69.77	80.14	52.55	97.20
	SqE	2	69.14	57.12	88.66	92.80	97.70	69.51	81.81	46.64	97.78
	Corr	2	65.82	39.28	87.15	88.11	97.19	63.70	66.67	37.07	97.49
	Fusion of SqE & Corr	$2+2$	73.50	62.82	91.18	93.97	97.70	72.22	83.23	60.08	97.10
$p = 100$	Fusion of SqE & Corr	$1+1$	71.49	60.57	83.63	93.63	96.93	72.87	81.16	60.29	96.14
	SqE	2	72.97	64.32	89.67	94.47	97.70	73.00	83.13	55.80	97.29
	Corr	2	68.88	46.03	89.92	89.61	97.44	66.93	67.68	45.42	97.39
	Fusion of SqE & Corr	$2+2$	**77.24**	**69.42**	**92.44**	**94.97**	**97.95**	**76.49**	**85.11**	**66.40**	95.75

is 'Corr'. Proximity matrices are computed on the training data based on SqE and Corr. The number of band groups and thus features extracted was 11 for SqE and 16 for Corr (forming two feature vectors). The training data is standardized for each feature to have zero mean and unit variance and the testing data is standardized using the mean and standard deviation of the training data. For SqE and Corr, we used 10 kernels each with $\sigma = \left\{ 2^{-3}, 2^{-2}, \cdots, 2^6 \right\}$. Top performing kernels for each feature were selected using SVM accuracy and fused using ℓ_p-norm MKL. 'SVMLight' and 'MKL' implementations in the Shogun toolbox [14] were used. For the ℓ_p-norm, we tried $p = 1.1$ (approximately city block distance), $p = 2$ (Euclidean) and $p = 100$.

Table 1 shows that inter-method fusion, i.e., fusion of SqE and Corr, is the top performer for all classes. Also, a larger p produces the best results for all classes except 'woods'. Inter-method fusion has an improvement of approximately 2% to 10% relative to intra-method for corn-notill, corn-min, grass-pasture, soybeans-notill, soybeans-min and soybeans-clean. It has almost the same performance for grass-tress and hay-windowed. The behavior of wood is different from all other classes. It has the best result at $p = 1.1$, and it continues to degrade with increasing p. Note, 'wood' is easily classifiable with a single kernel. $\ell_{1.1}$ MKL, which promotes sparse solutions, is more suitable for this task. In [3], Kloft et al. showed

that when kernels are diverse, higher norm MKL is more appropriate and yields better results. In our case, results improve for 8 out of 9 classes as ℓ_p-norm increases, which supports our claim that diversity in features and kernels is useful for MKL-based hyperspectral classification.

4. FUTURE WORK

As stated above, our results are preliminary but promising—ran on a single well-known benchmark data set that was not trivial to solve using a single kernel. In future work, we will apply our technique to additional data sets. We will also investigate a search procedure for MKL parameter selection, including kernel type and associated parameters (a critical aspect of MKL that is typically overlooked due to complexity). Here, we explored, for demonstration, one kernel based on Euclidean distance and another based on angle for diversity. We believe it is also of interest to explore different, or a combination of different band group selection algorithms and what particular proximity measures are fed to these techniques to ultimately generate diverse features for MKL. Last, we are currently using all features produced by band grouping. However, sometimes some bands (or band groups) are not useful for a task at hand and performance can be raised if we do a feature selection step before fusion.

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