Extracting rotational structure from motor cortical data

II-33

Cunningham JP*, Churchland MM*, Kaufman MT, and Shenoy KV

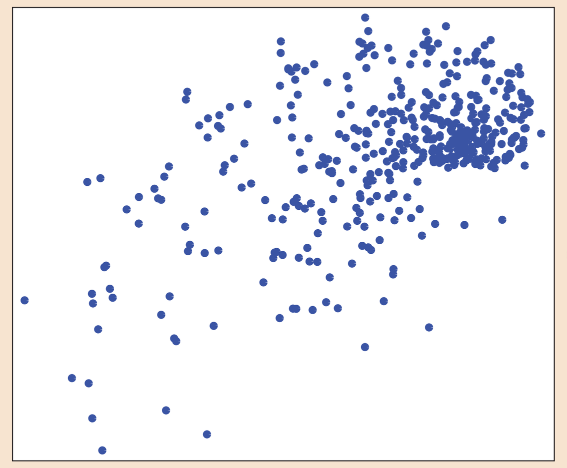
Department of Engineering, University of Cambridge, UK
Dept EE. and Neurosci. program, Stanford University, CA

Motivation

Goals:

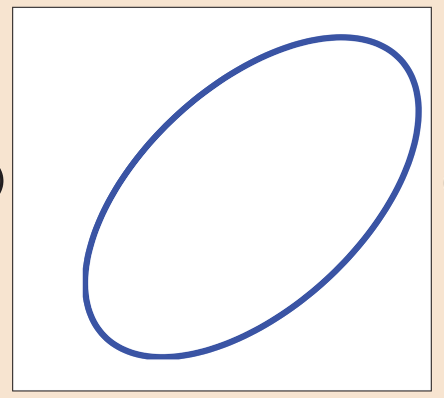
1) We want hypothesis-guided dimensionality reduction for summarizing and analyzing neural data.
2) Rotations/oscillations are common of neural circuitry. We want to see these fundamental features.
3) We want a method that will find this particular structure **IF AND ONLY IF** it is present in the data.

We have some data we want to summarize/analyze/project:



(Make summary matrix)

$$\Sigma = \mathrm{cov}(X)$$



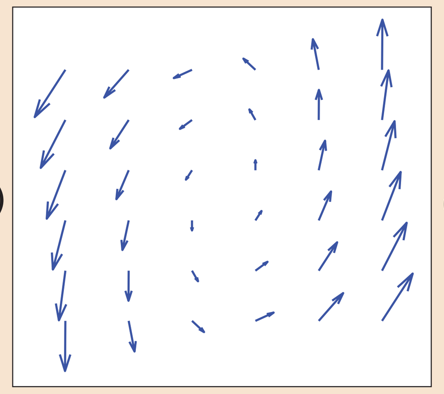
(Eigenvalue decomp)

PCA

But maybe we know that the (same) data is actually time series data:

(Make summary matrix)

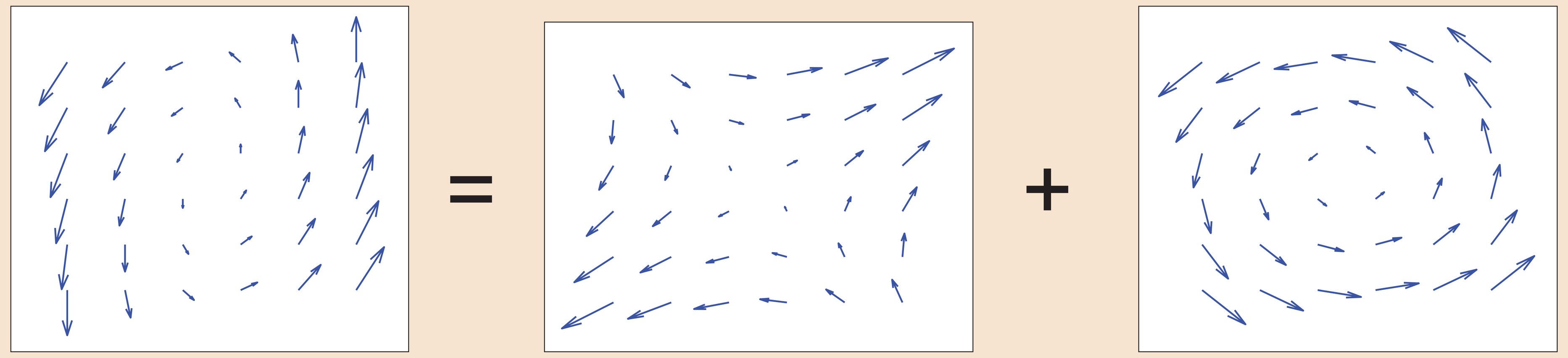
$$M^* = \mathrm{argmin}_{M \in \mathbb{R}^{n\times n}} ||\dot{X} - XM||_F$$



(Eigenvalue decomp)

"Dynamical PCA"

A bit of linear algebra:



General (square) matrix
- $M \in \mathbb{R}^{n\times n}$
- **complex eigenvalues**
- **(function)**
- **(linear vector field)**

Symmetric matrix
- $M_{sym} \in \mathbb{S}^{n\times n}$
- $M_{sym} = (M + M^T)/2 = M_{sym}^T$
- **real eigenvalues**
- **(even part of function)**
- **(divergence, no curl)**

Skew-symmetric matrix
- $M_{skew} \in \mathbb{S}\!\!\!/^{n\times n}$
- $M_{skew} = (M - M^T)/2 = -M_{skew}^T$
- **imaginary eigenvalues**
- **(odd part of function)**
- **(curl, no divergence)**

The rest of this poster:

1) We derive jPCA: a method that finds the best rotational dynamical system describing the data.
2) We use jPCA on toy data, real leech data, and real monkey data to demonstrate its properties.
3) We point to the neuroscientific implications of these findings (see M Churchland poster tomorrow).

Math

Time series data:

$$X \in \mathbb{R}^{T\times n} \qquad \mathbf{x}(t) = [x_1(t), ..., x_n(t)]$$

Fitting a linear dynamical system (matrix notation):

$$M^* = \mathrm{argmin}_{M \in \mathbb{R}^{n\times n}} ||\dot{X} - XM||_F$$

Awkward but helpful vector notation (same problem):

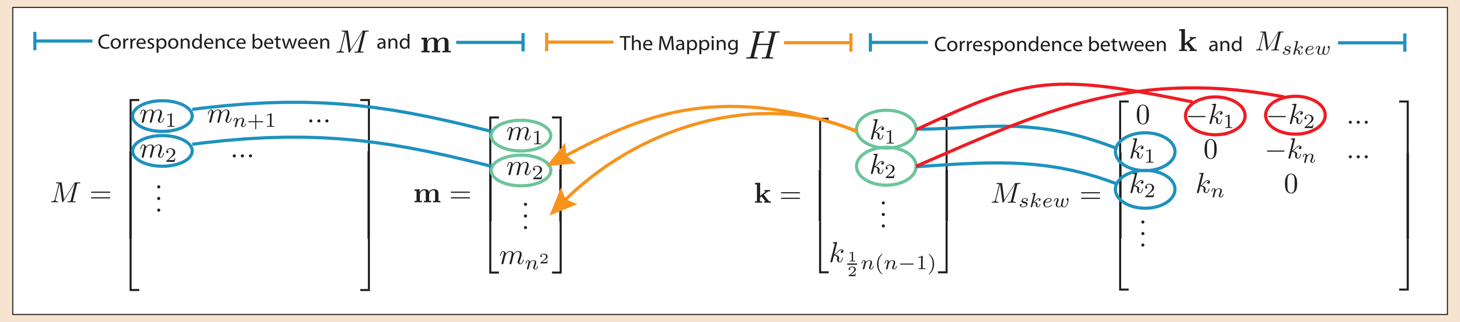
$$\mathbf{m}^* = \mathrm{argmin}_{\mathbf{m} \in \mathbb{R}^{n^2}} ||\dot{\mathbf{x}} - \tilde{X}\mathbf{m}||_2$$

where $\mathbf{m} = M(:) \in \mathbb{R}^{n^2}$ $\quad \dot{\mathbf{x}} = \dot{X}(:)$

$$\tilde{X} = \mathrm{blkdiag}(X)$$

Defining skew-symmetric matrices in vector notation:

$$\mathbf{m} = H\mathbf{k} \qquad \text{where} \qquad \mathbf{k} \in \mathbb{R}^{n(n-1)/2}$$



Fitting a rotational dynamical system:

$$M^* = \mathrm{argmin}_{M \in \mathbb{S}\!\!\!/^{n\times n}} ||\dot{X} - XM||_F \iff$$

$$\mathbf{k}^* = \mathrm{argmin}_{\mathbf{k} \in \mathbb{R}^{\frac{1}{2}n(n-1)}} ||\dot{\mathbf{x}} - \tilde{X}H\mathbf{k}||_2$$

jPCA:

(1) Make summary matrix

$$M^* = \mathrm{argmin}_{M \in \mathbb{S}\!\!\!/^{n\times n}} ||\dot{X} - XM||_F$$

(2) Eigenvalue decomposition and projection

Implementation notes:

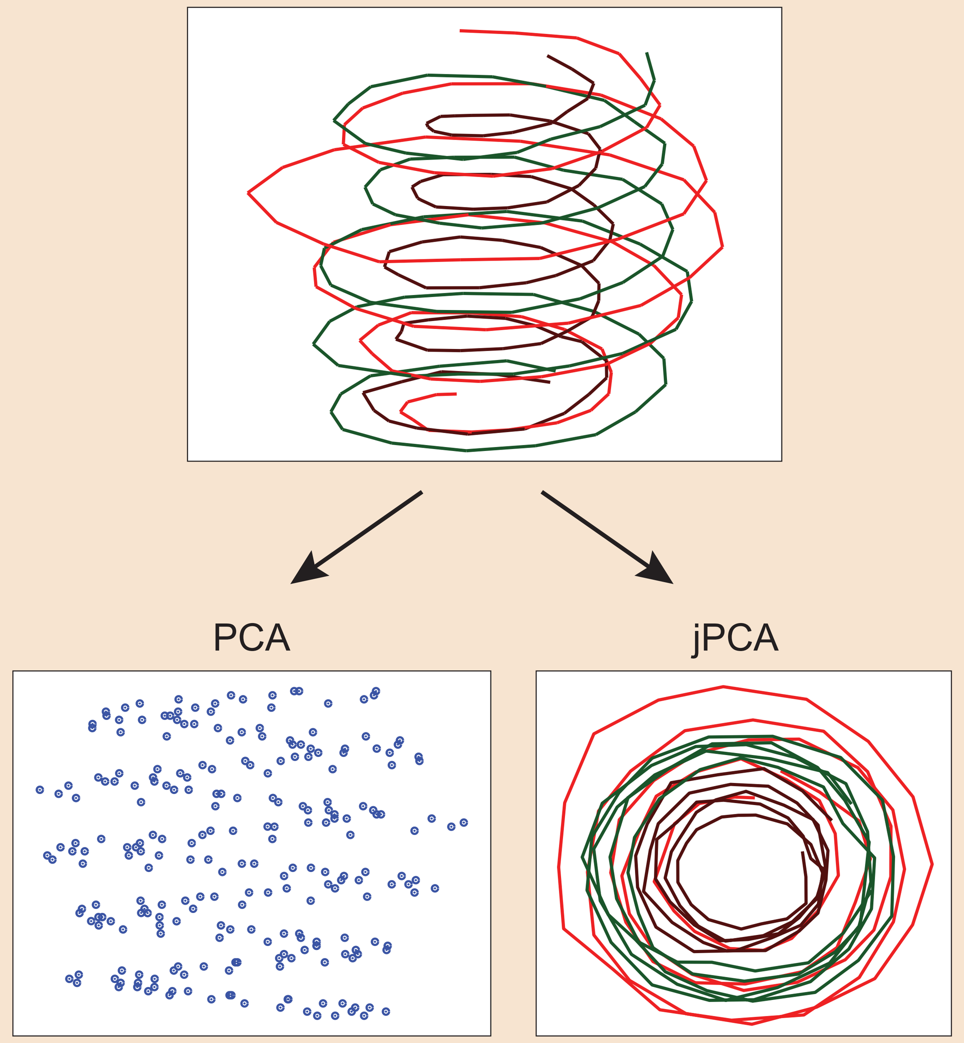
1) Eigenvectors in conjugate pairs, hence:

$$\mathbf{u}_{i,1} = \mathbf{v}_{i,1} + \mathbf{v}_{i,2}^*$$
$$\mathbf{u}_{i,2} = j(\mathbf{v}_{i,1} - \mathbf{v}_{i,2}^*)$$

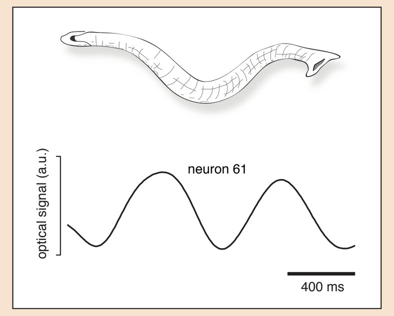
2) Creating big matrices can be burdensome, so we avoid explicit representation using a gradient method: very fast, global optimizer.

Results

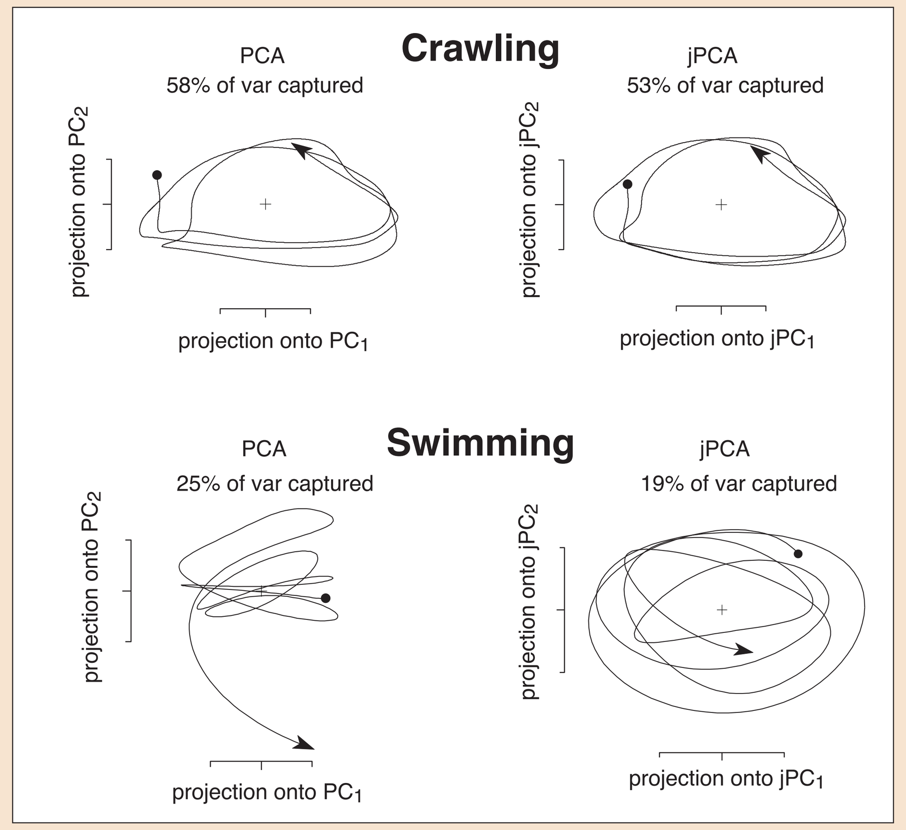
Helical data (toy):



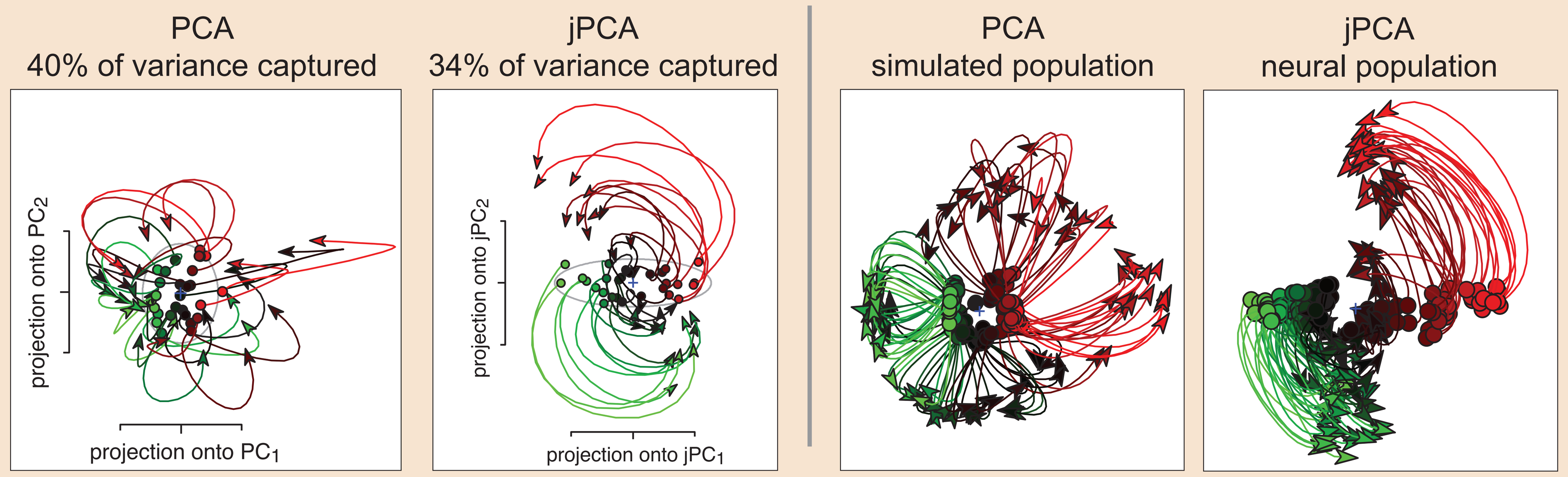
Leech data (real):



Data from K. Briggman and W. Kristan



Monkey data (real):



Conclusions

1) jPCA: global optimizer that finds the best rotational dynamical system describing the data.

2) jPCA yields simple projection vectors - so the interpretation of results is identical to PCA.

3) Many more results and their neuroscientific implications: M Churchland poster tomorrow.

4) (choice of skew-sym is not critical... could also create "symmetric PCA" based on M_{sym}.)

Acknowledgments: K. Briggman and W. Kristan for leech data, M. Risch for animal care.
Funding: UK EPSRC EP/H019472/1, NIH Pioneer, Burroughs Wellcome, NIH CRCNS R01.