Heuristic Creation: The goal is to get the 2x2 goal piece to the finishing position. Tiles can only move if there are adjacent spaces which can accommodate tiles of the necessary sizes. In my heuristic, I will get rid of the ability to only move tiles that are adjacent and allow tiles to overlap. This now leads to the puzzle having solutions which can be achieved by simply overlapping the tiles to move it to the goal position; similarly to manhattan distance. In my heuristic, I will use the manhattan distance to calculate the heuristic value but with a twist; i will add one to the cost if there is currently a piece in the goal position and zero otherwise.

Heuristic Definition: Let me define a function manhattan(x) which takes in a state x, and returns the manhattan distance of the goal location from the goal piece's current coordinates. Let f(x) be my heuristic function which takes in a state x and adheres to the following rules:

If x is a final state, $f(x) = 0$

Else if x has no pieces in the goal location, $f(x) = manhattan(x)$

Else if x has one/many pieces in the goal location, $f(x) = manhattan(x) + 1$

Proof of admissibility: Let $f^*(n)$ be the cheapest path from state n to the goal state

Let S be a set of all possible states

WTS - $0 \leq f(n) \leq f^*(n) \; \forall \; n \in S$

We know that $f(n)$ is always non negative as the minimum value is 0 at the goal. Since we allowed overlapping tiles (as part of problem relaxation) in this heuristic, $f(n) \leq f^*(n)$ will hold true; we will have two cases.

Case 1: $f(n) = f^*(n)$ This happens when the goal piece is adjacent to the empty goal location or is at the goal location. Then the cheapest cost would be one and zero respectively in both cases **Case 2**: $f(n) < f^*(n)$ This happens when the goal piece is not adjacent to the empty goal location and would therefore have to move other pieces before moving the goal piece. The moving of the other pieces increases the cost for $f^*(n)$ but not for $f(n)$. Therefore $0 \leq f(n) \leq f^*(n)$

Proof of domination: Case 1: A piece is in the goal position as in the board above:

Then we can see than $f(n) = 3 + 1$ while $manhattan(n) = 3$ This shows that $f(n) > manhattan(n)$ is true for atleast one state n.

Case 2: Nothing is in the goal position

Then $f(n) = manhattan(x)$ since there is no added cost. We can see that $f(n) = manhattan(x)$ which shows that $f(n) \geq manhattan(n)$ is true $\forall n \in S$.