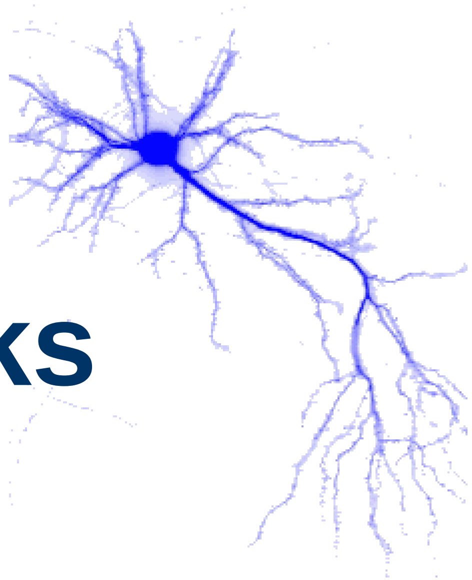

# Neural Networks

Ahmad Ali Abin

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- Learning Rules for Single-Layer Perceptron Networks
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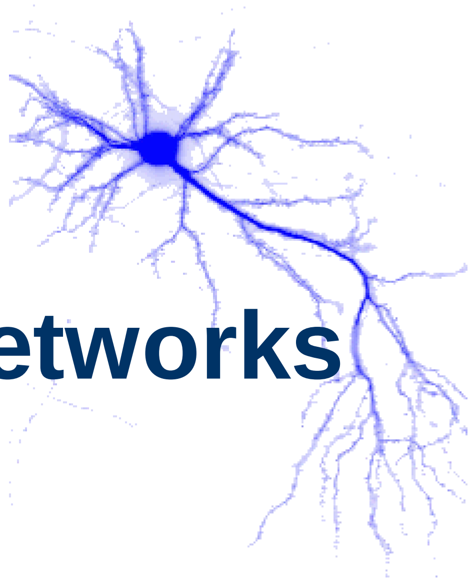
- Multilayer Perceptron
- Back Propagation Learning algorithm

# Feed-Forward Neural Networks



## Introduction

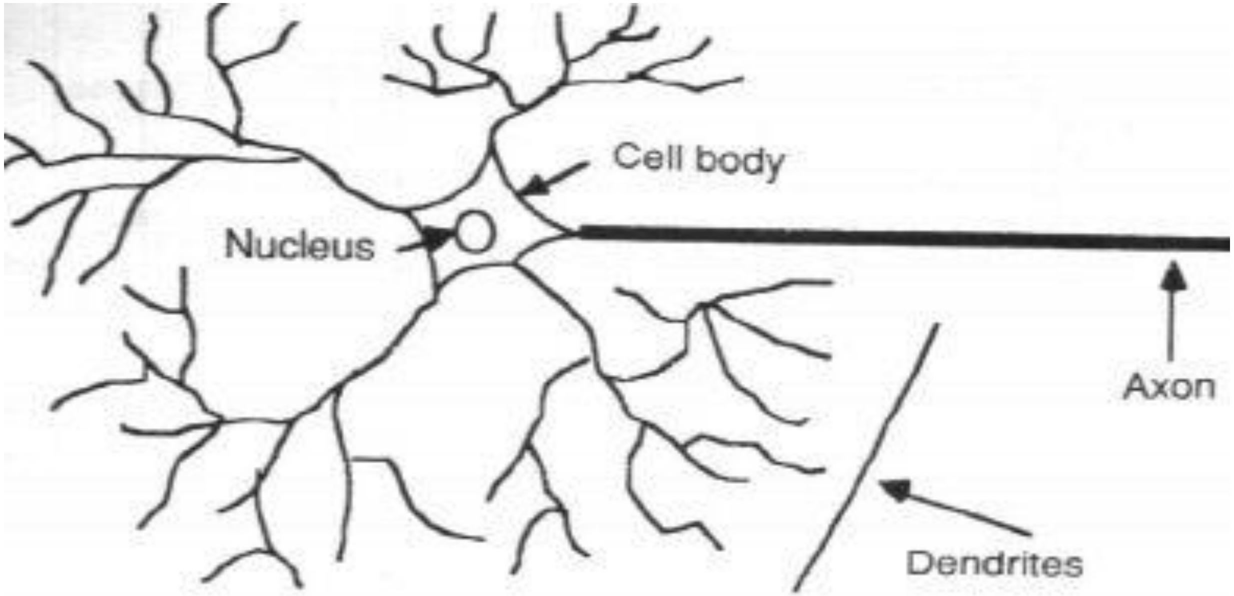
# Historical Background

- 1943 McCulloch and Pitts proposed the first computational models of neuron.
- 1949 Hebb proposed the first learning rule.
- 1958 Rosenblatt's work in perceptrons.
- 1969 Minsky and Papert's exposed limitation of the theory.
- 1970s Decade of dormancy for neural networks.
- 1980-90s Neural network return (self-organization, back-propagation algorithms, etc)

# Nervous Systems

- Human brain contains ~ $10^{11}$ neurons.
- Each neuron is connected ~ $10^4$ others.
- Some scientists compared the brain with a "complex, nonlinear, parallel computer".
- The largest modern neural networks achieve the complexity comparable to a nervous system of a fly.

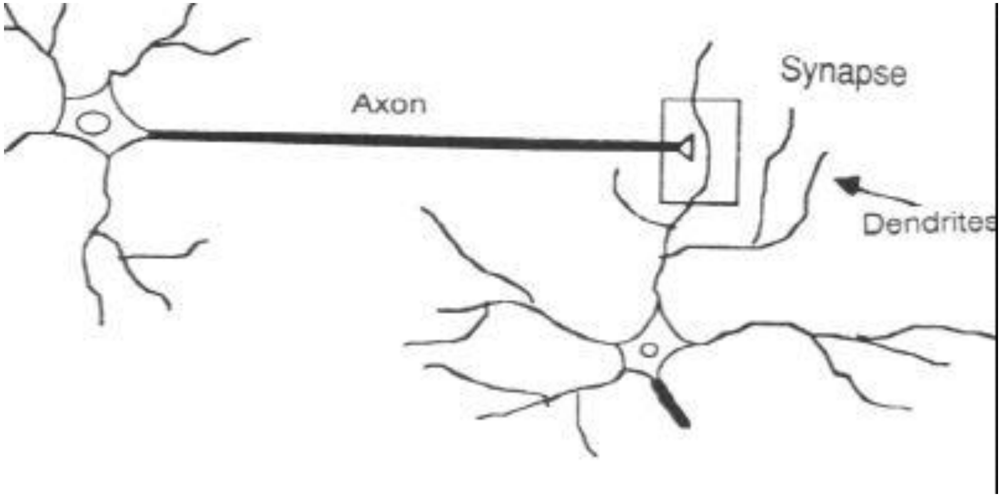
# Neurons



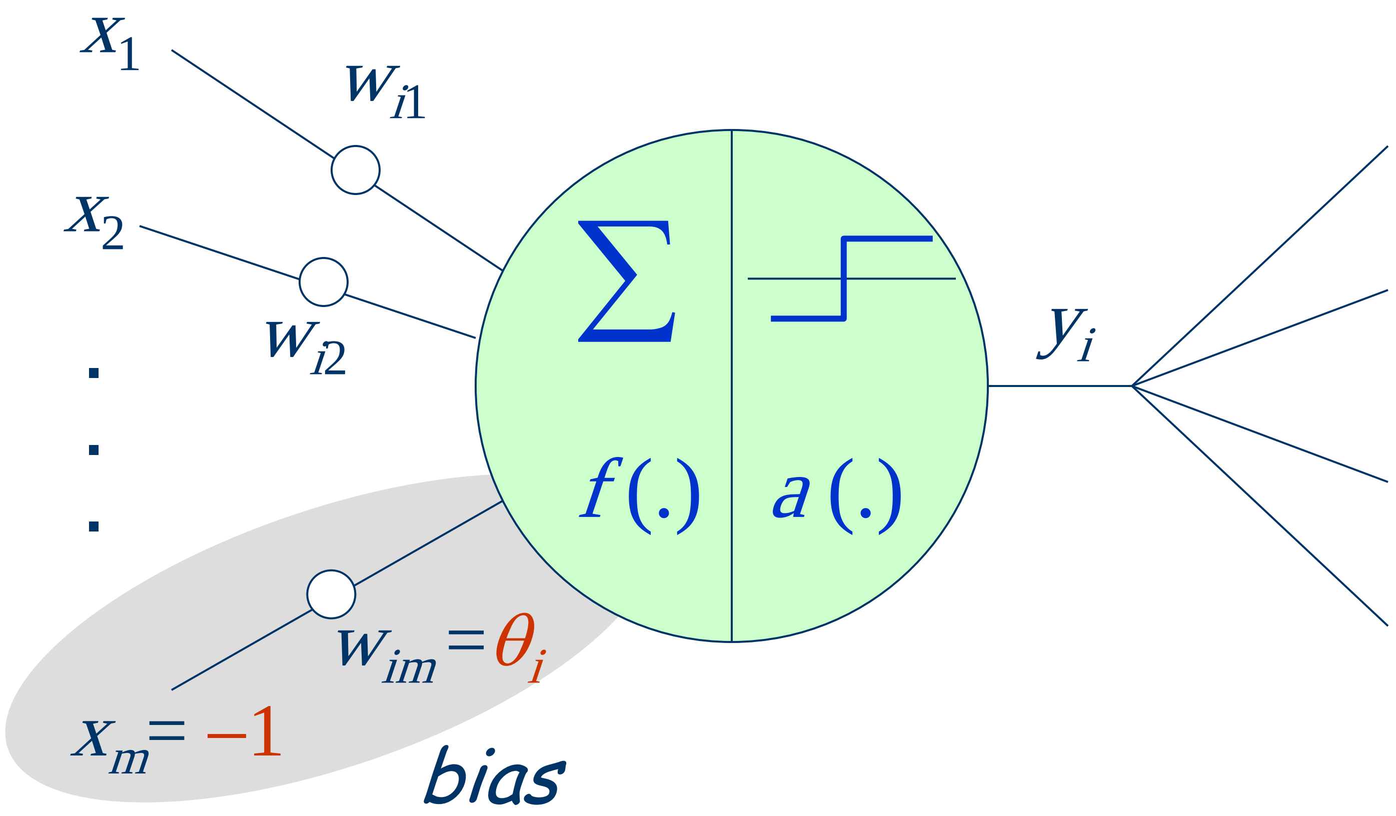

- The main purpose of neurons is to receive, analyze and transmit further the information in a form of signals (electric pulses).
- When a neuron sends the information we say that a neuron "fires".

# Neurons

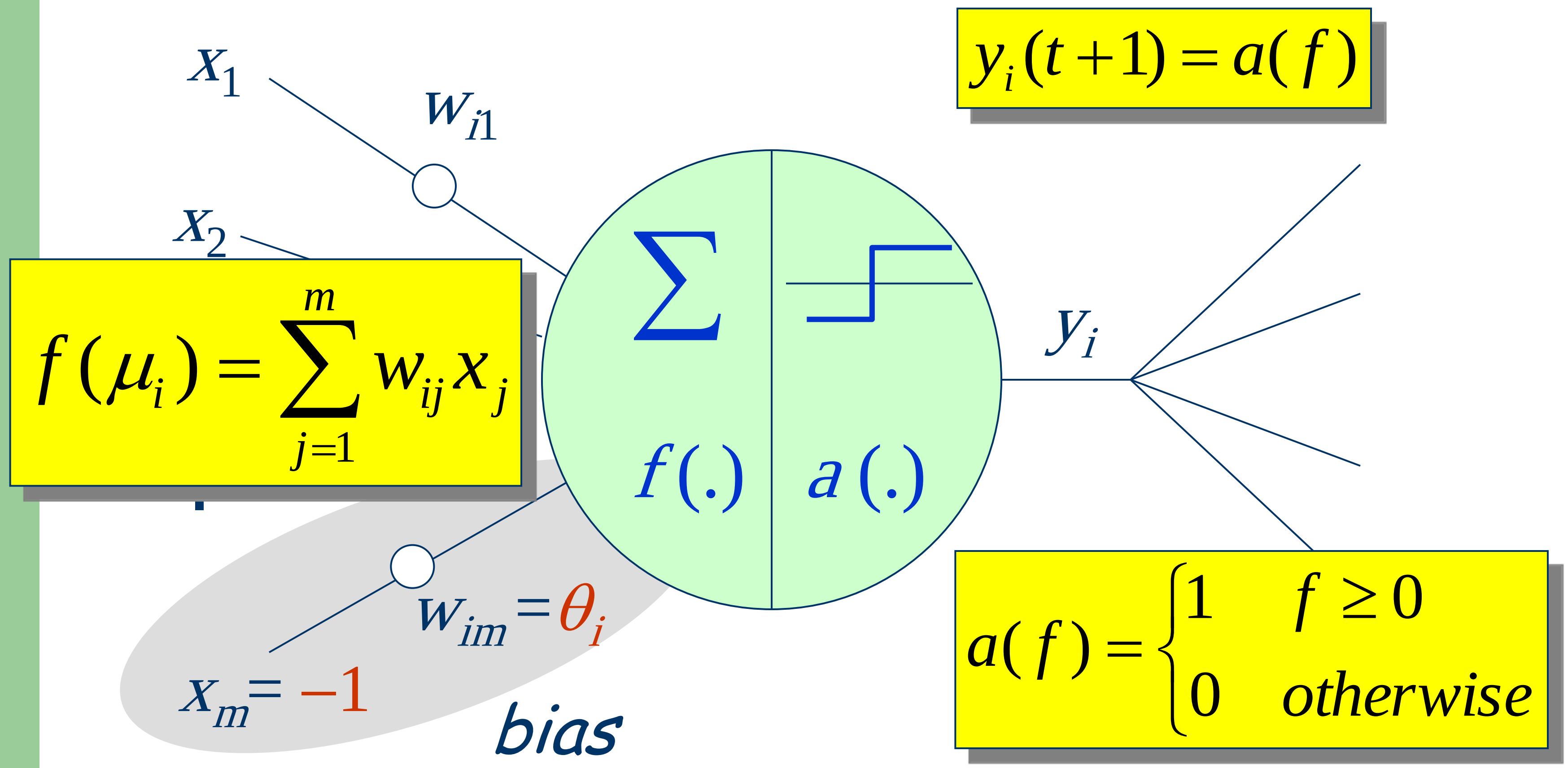
Acting through specialized projections known as dendrites and axons, neurons carry information throughout the neural network.



# A Model of Artificial Neuron



# A Model of Artificial Neuron




$$y_i(t+1) = a(f)$$

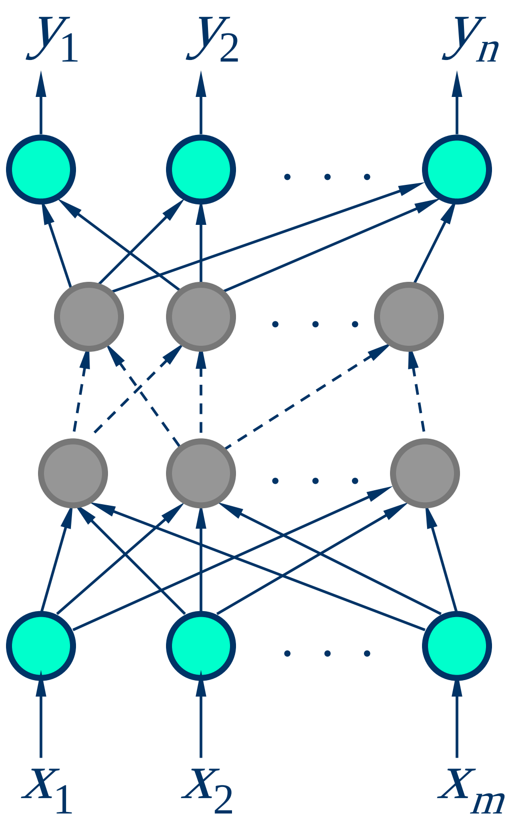
$$f(\mu_i) = \sum_{j=1}^{m} w_{ij} x_j$$

$$a(f) = \begin{cases} 1 & f \geq 0 \\ 0 & otherwise \end{cases}$$

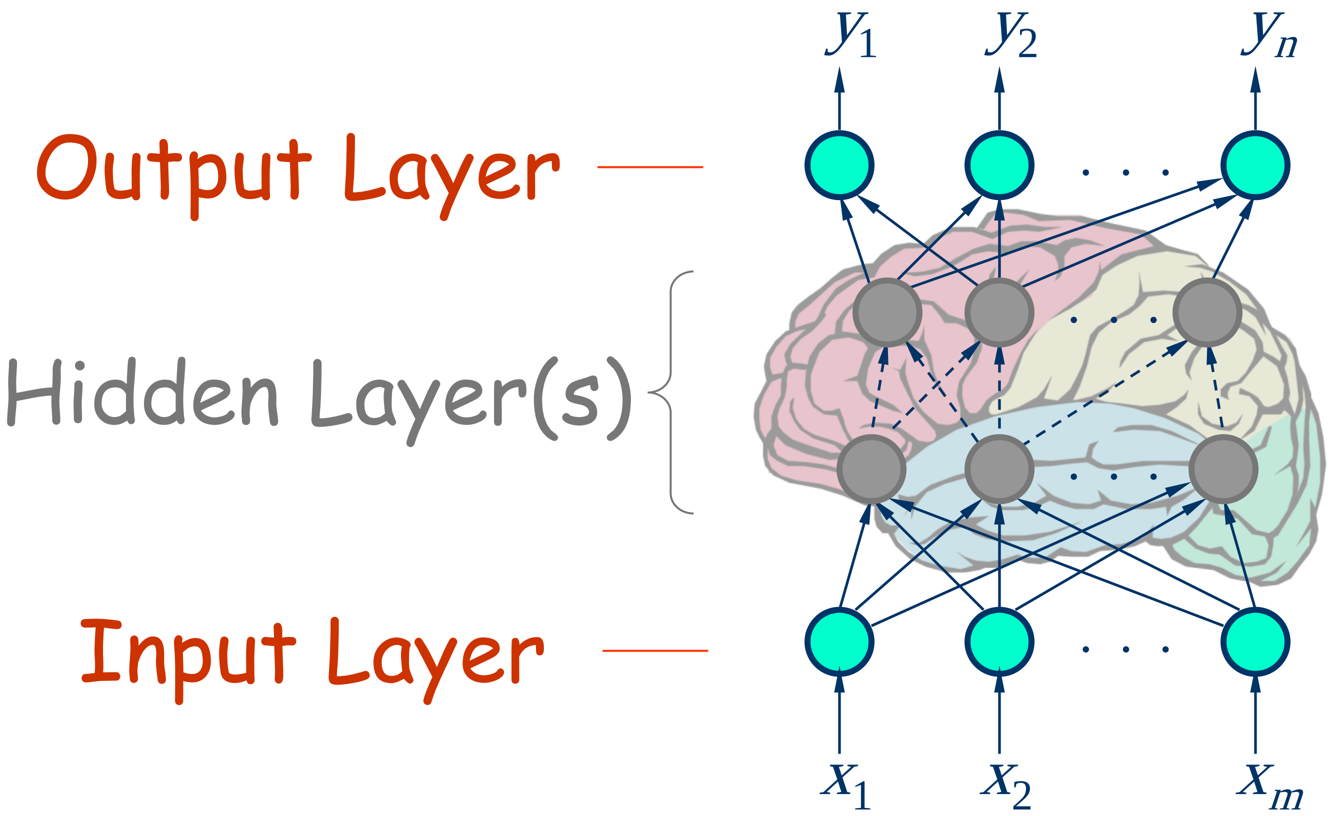
# Feed-Forward Neural Networks

- Graph representation:
  - *nodes*: neurons
  - *arrows*: signal flow directions

- A neural network that does *not* contain *cycles* (feedback loops) is called a feed-forward network (or perceptron).

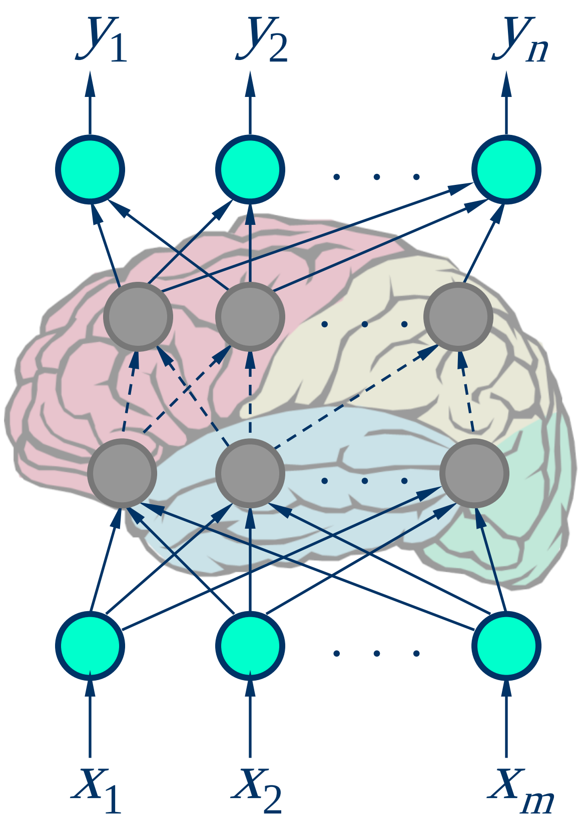


# Layered Structure



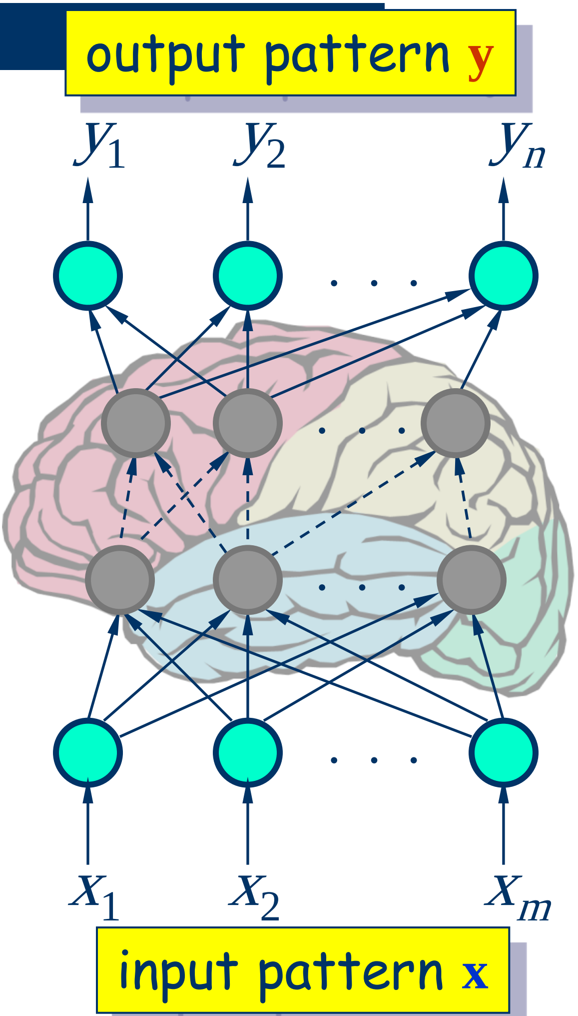
# Knowledge and Memory

- The output behavior of a network is determined by the weights.
- Weights – the memory of an NN.
- Knowledge – distributed across the network.
- Large number of nodes
  - increases the storage "capacity";
  - ensures that the knowledge is robust;
  - fault tolerance.
- Store new information by changing weights.



# Pattern Classification

- Function: $\mathbf{x} \rightarrow \mathbf{y}$
- The NN's output is used to distinguish between and recognize different input patterns.
- Different output patterns correspond to particular classes of input patterns.
- Networks with hidden layers can be used for solving more complex problems then just a linear pattern classification.



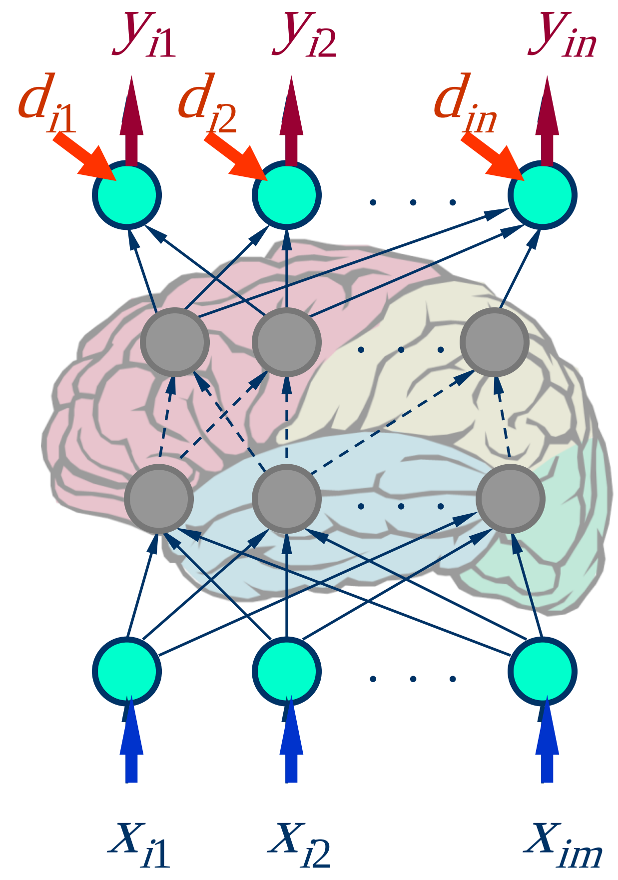
Training Set

$$\mathbf{T} = \left\{ (\mathbf{x}^{(1)}, \mathbf{d}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{d}^{(2)}), \ldots, (\mathbf{x}^{(k)}, \mathbf{d}^{(k)}), \ldots \right\}$$

# Training

$$\mathbf{x}^{(i)} = (x_{i1}, x_{i2}, \ldots, x_{im})$$

$$d^{(i)} = (d_{i1}, d_{i2}, \ldots, d_{in})$$



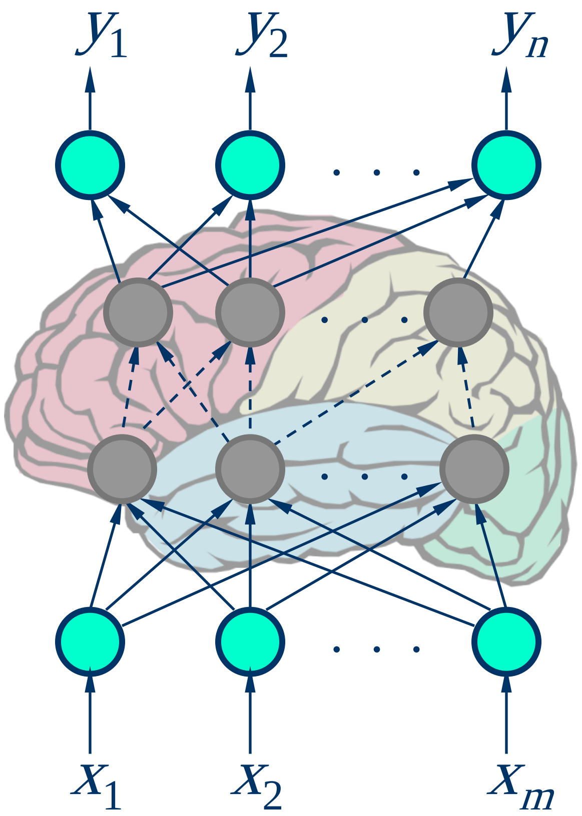

Goal:

$$\text{Min } E = \sum_i error(\mathbf{y}^{(i)} - \mathbf{d}^{(i)})$$

$$= \sum_i \left\| \mathbf{y}^{(i)} - \mathbf{d}^{(i)} \right\|^2$$

# Generalization

- By properly training a neural network may produce reasonable answers for input patterns not seen during training (generalization).
- Generalization is particularly useful for the analysis of a "noisy" data (e.g. time-series).



# Generalization

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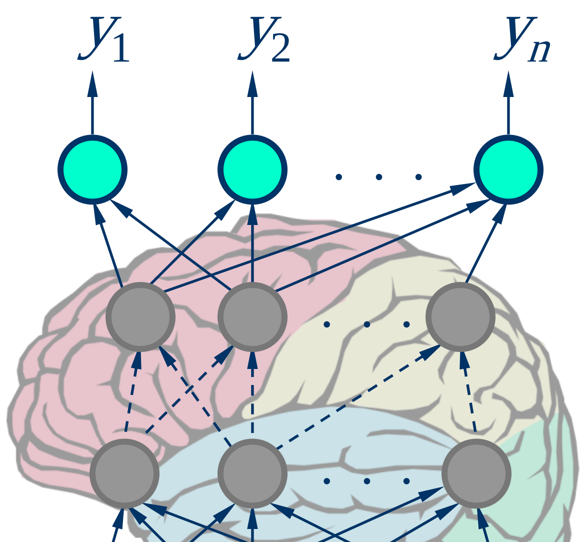
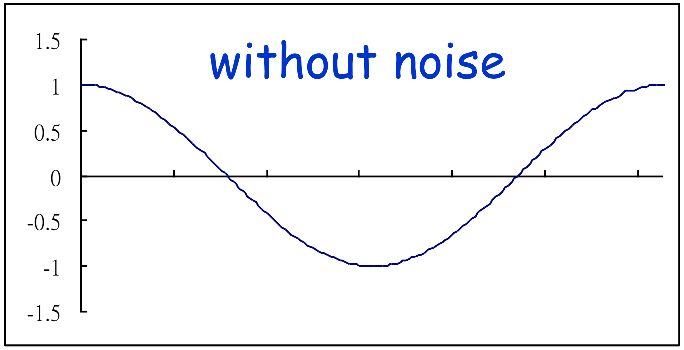
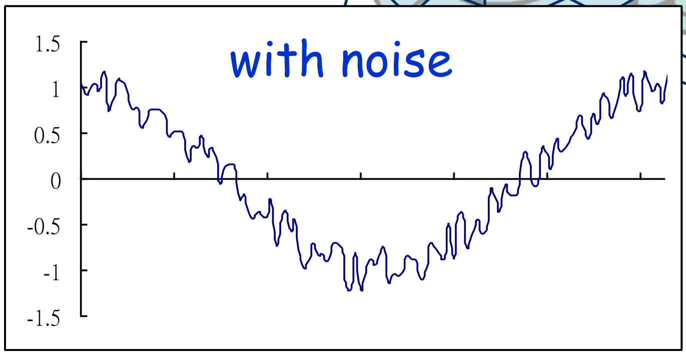
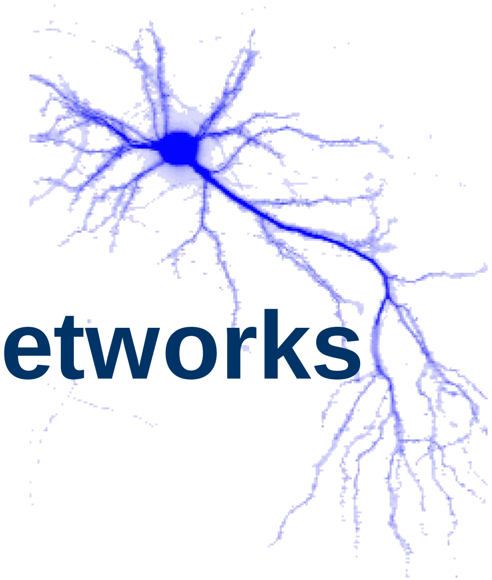


# Applications

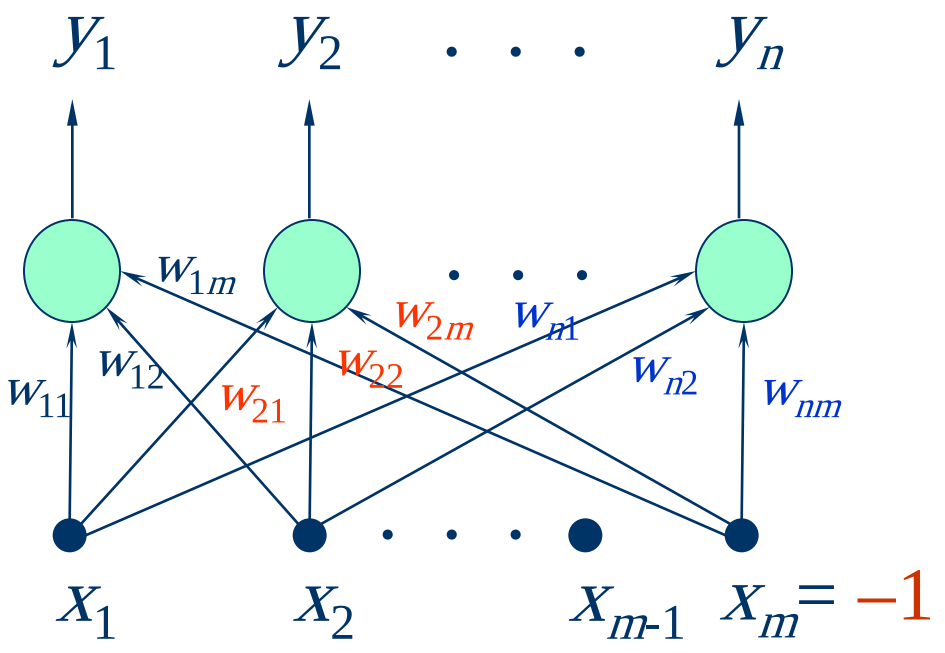
- Pattern classification
- Object recognition
- Function approximation
- Data compression
- Time series analysis and forecast
- . . .



# Feed-Forward Neural Networks

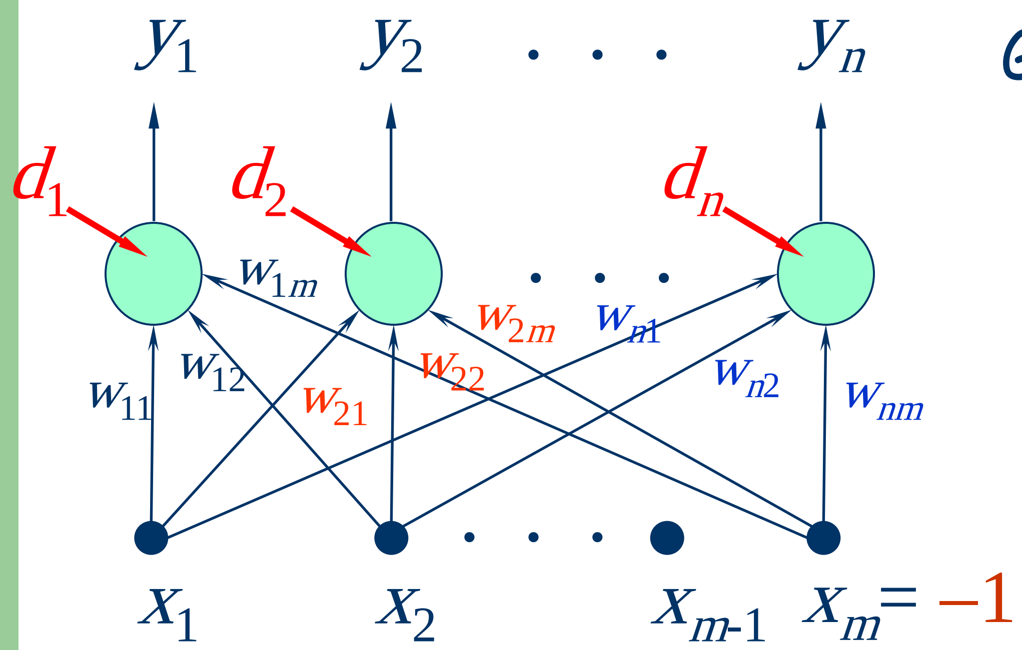
Single-Layer Perceptron Networks

# The Single-Layered Perceptron



# Training a Single-Layered Perceptron

Training Set $\mathbf{T} = \left\{ (\mathbf{x}^{(1)}, \mathbf{d}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{d}^{(2)}), \ldots, (\mathbf{x}^{(p)}, \mathbf{d}^{(p)}) \right\}$




Goal:

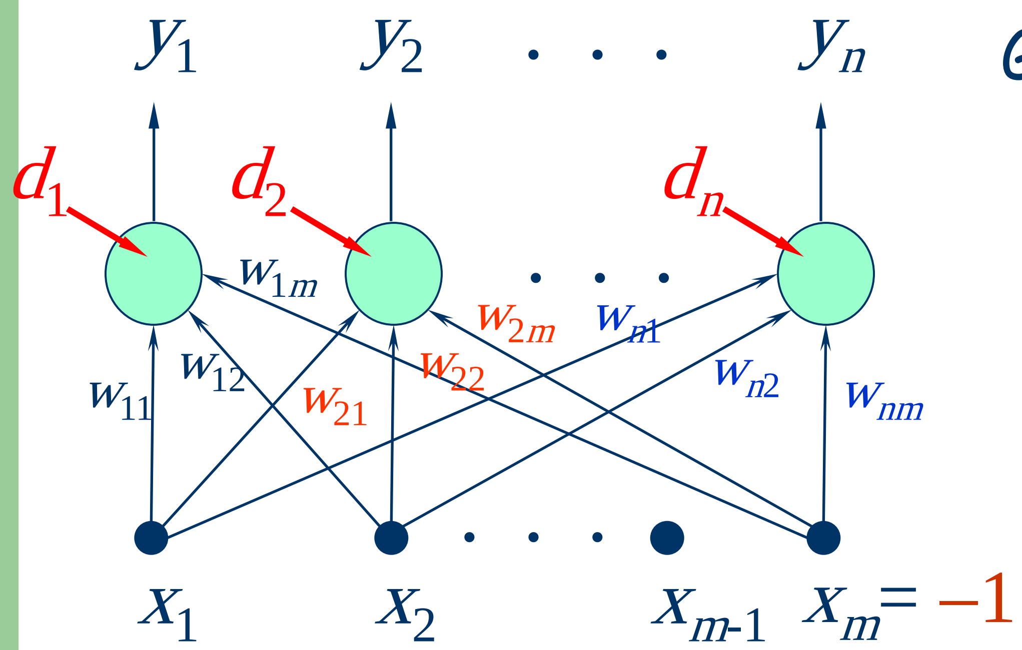
$$y_i^{(k)} = a(\mathbf{w}_i^T \mathbf{x}^{(k)})$$

$$= a\left( \sum_{l=1}^{m} w_{il} x_l^{(k)} \right) = d_i^{(k)}$$

$$\forall \begin{matrix} i = 1, 2, \ldots, n \\ k = 1, 2, \ldots, p \end{matrix}$$

# Learning Rules

- Linear Threshold Units (LTUs) : Perceptron Learning Rule
- Linearly Graded Units (LGUs) : Widrow-Hoff learning Rule

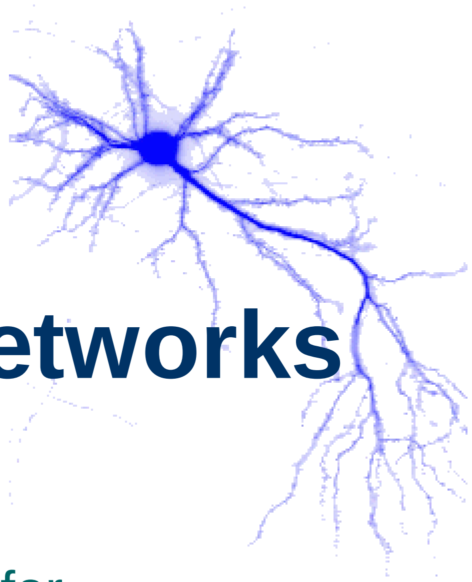



Goal:

$$y_i^{(k)} = a(\mathbf{w}_i^T \mathbf{x}^{(k)}) = a\left(\sum_{l=1}^{m} w_{il} x_l^{(k)}\right) = d_i^{(k)}$$

$$\forall \begin{array}{l} i = 1, 2, \ldots, n \\ k = 1, 2, \ldots, p \end{array}$$

# Feed-Forward Neural Networks

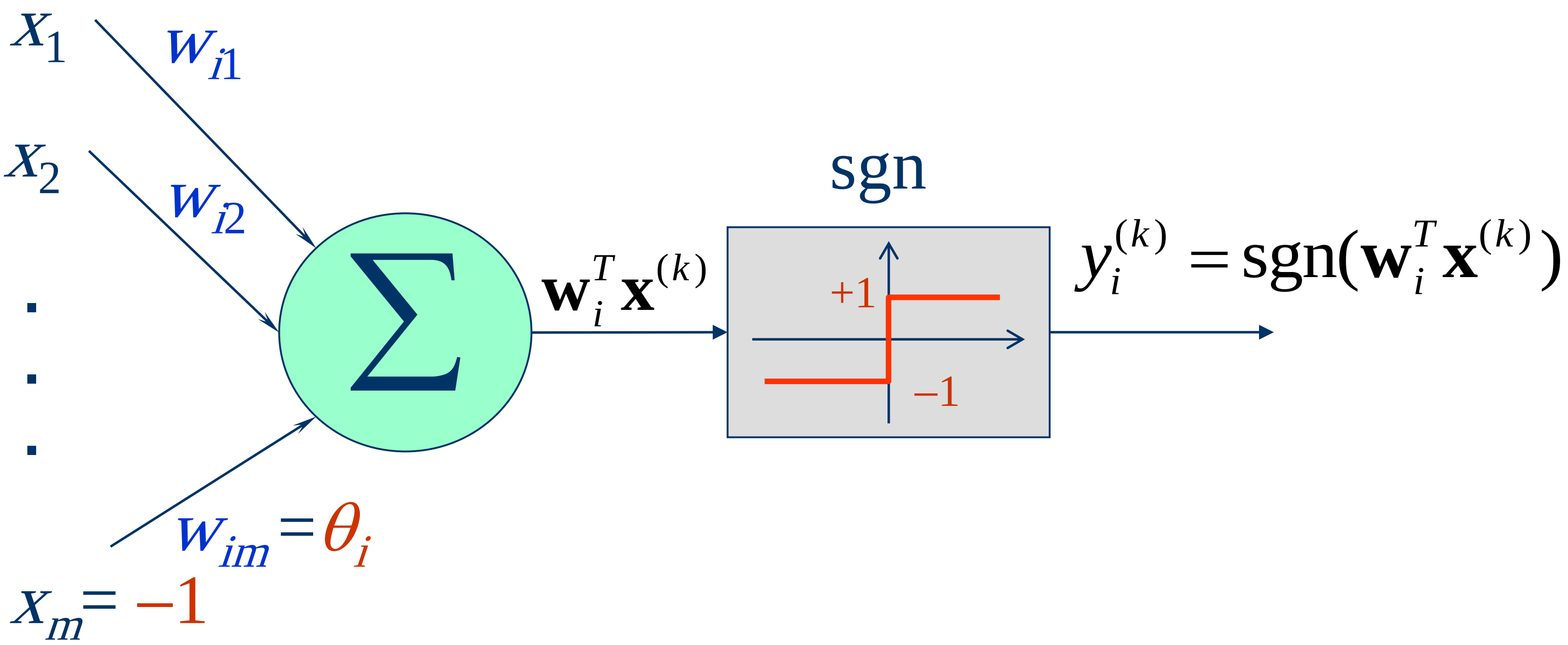


Learning Rules for
Single-Layered Perceptron Networks

- Perceptron Learning Rule
- Adline Leaning Rule
- $\delta$-Learning Rule

# Perceptron

Linear Threshold Unit



# Perceptron

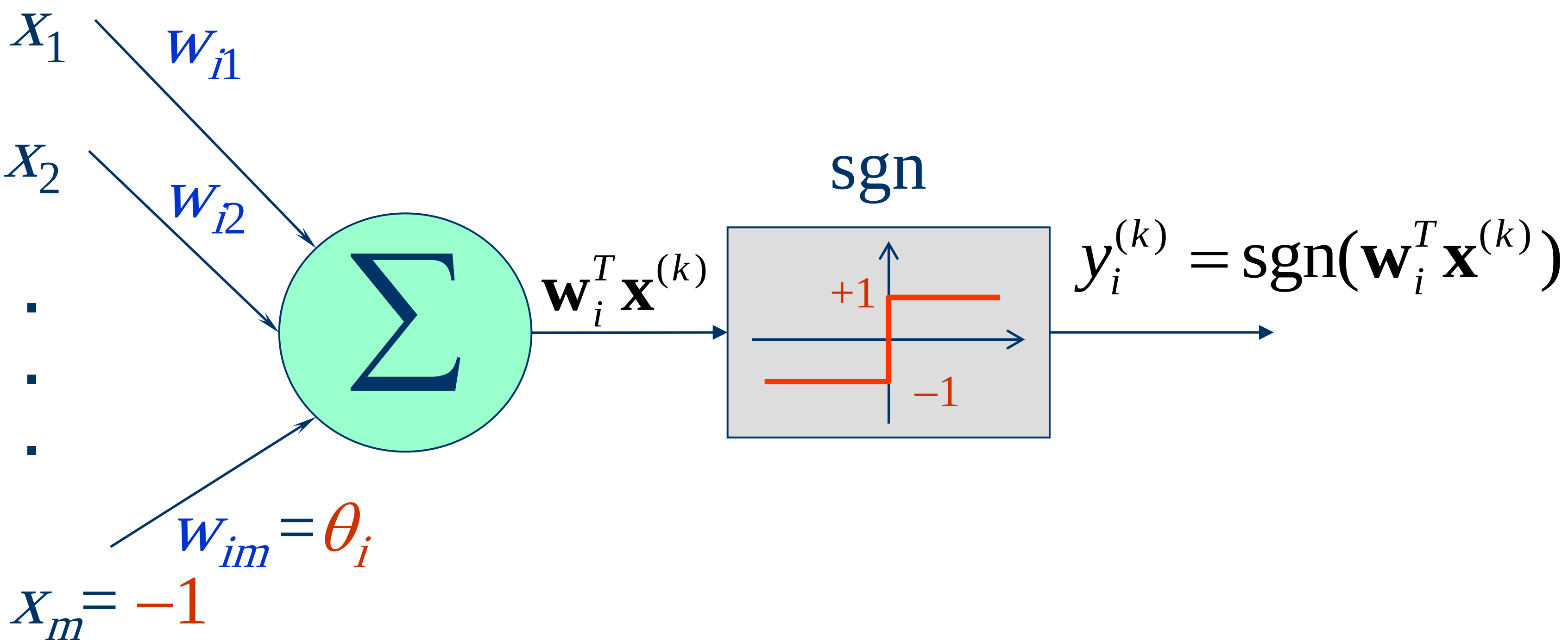
Goal:

$$y_i^{(k)} = \mathrm{sgn}(\mathbf{w}_i^T \mathbf{x}^{(k)}) = d_i^{(k)} \in \{1, -1\}$$

$$i = 1, 2, \ldots, n$$

$$k = 1, 2, \ldots, p$$

## Linear Threshold Unit



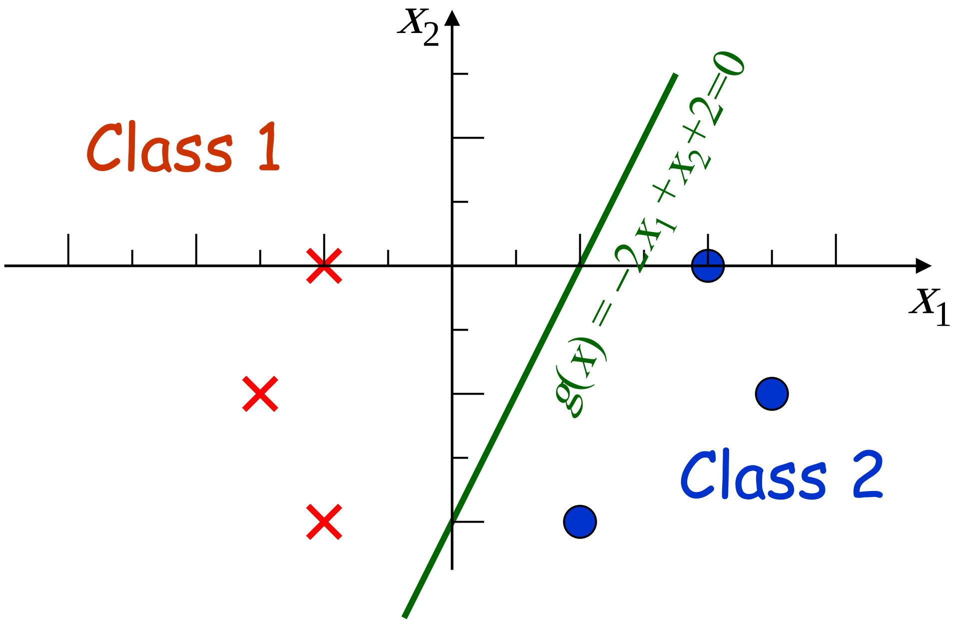
# Example

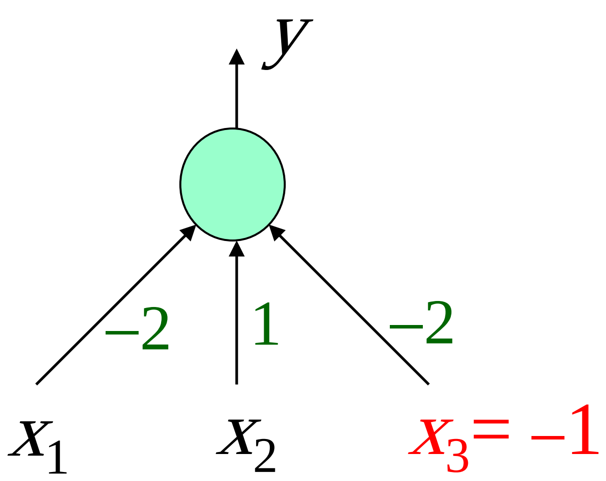
Goal:

$$y_i^{(k)} = \mathrm{sgn}(\mathbf{w}_i^T \mathbf{x}^{(k)}) = d_i^{(k)} \in \{1, -1\}$$
$$i = 1, 2, \ldots, n$$
$$k = 1, 2, \ldots, p$$

Class 1 (+1) — $\left\{[-1,0]^T, [-1.5,-1]^T, [-1,-2]^T\right\}$

Class 2 (–1) — $\left\{[2,0]^T, [2.5,-1]^T, [1,-2]^T\right\}$



Goal: $y^{(k)} = \mathrm{sgn}(\mathbf{w}^T \mathbf{x}^{(k)}) = d^{(k)}$

$\mathbf{w} = (w_1, w_2, w_3)^T$

# Augmented input vector

Class 1 (+1) — $\{[-1,0]^T, [-1.5,-1]^T, [-1,-2]^T\}$

Class 2 (–1) — $\{[2,0]^T, [2.5,-1]^T, [1,-2]^T\}$

Class 1 (+1)

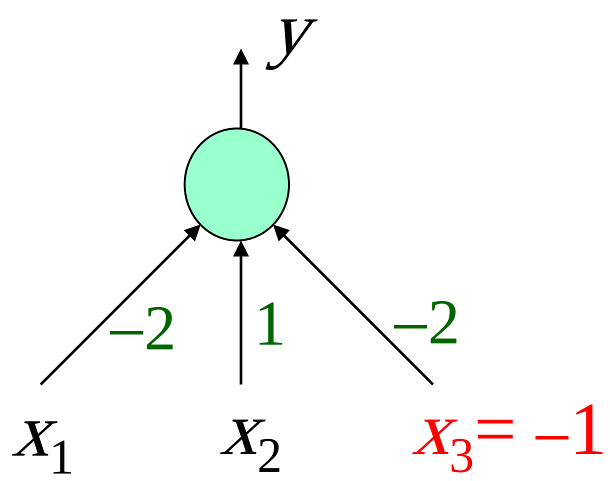
$$x^{(1)} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \quad x^{(2)} = \begin{bmatrix} -1.5 \\ -1 \\ -1 \end{bmatrix}, \quad x^{(3)} = \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$$

$$d^{(1)} = 1, \quad d^{(2)} = 1, \quad d^{(3)} = 1$$

Class 2 (–1)

$$x^{(4)} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \quad x^{(5)} = \begin{bmatrix} 2.5 \\ -1 \\ -1 \end{bmatrix}, \quad x^{(6)} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

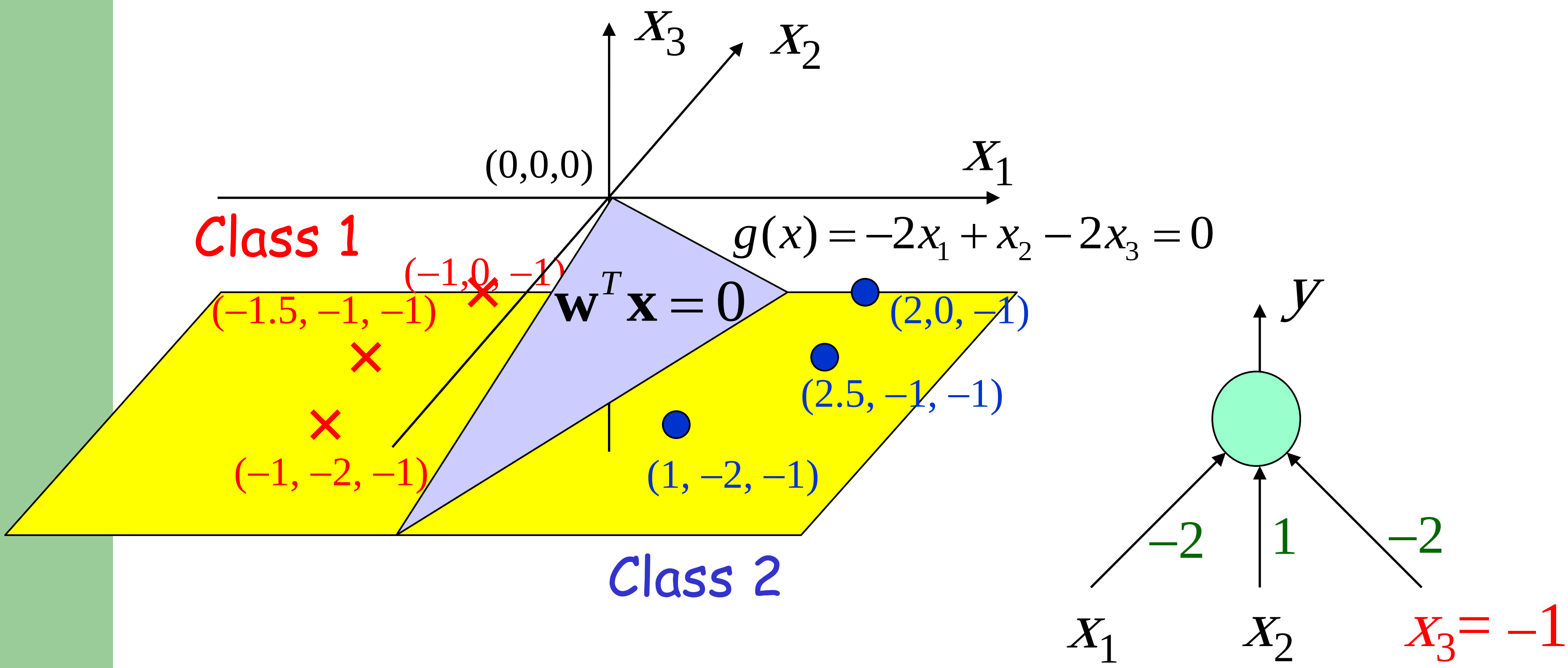
$$d^{(4)} = -1, \quad d^{(5)} = -1, \quad d^{(6)} = -1$$



Goal: $y^{(k)} = \mathrm{sgn}(\mathbf{w}^T \mathbf{x}^{(k)}) = d^{(k)}$

$\mathbf{w} = (w_1, w_2, w_3)^T$

# Augmented input vector

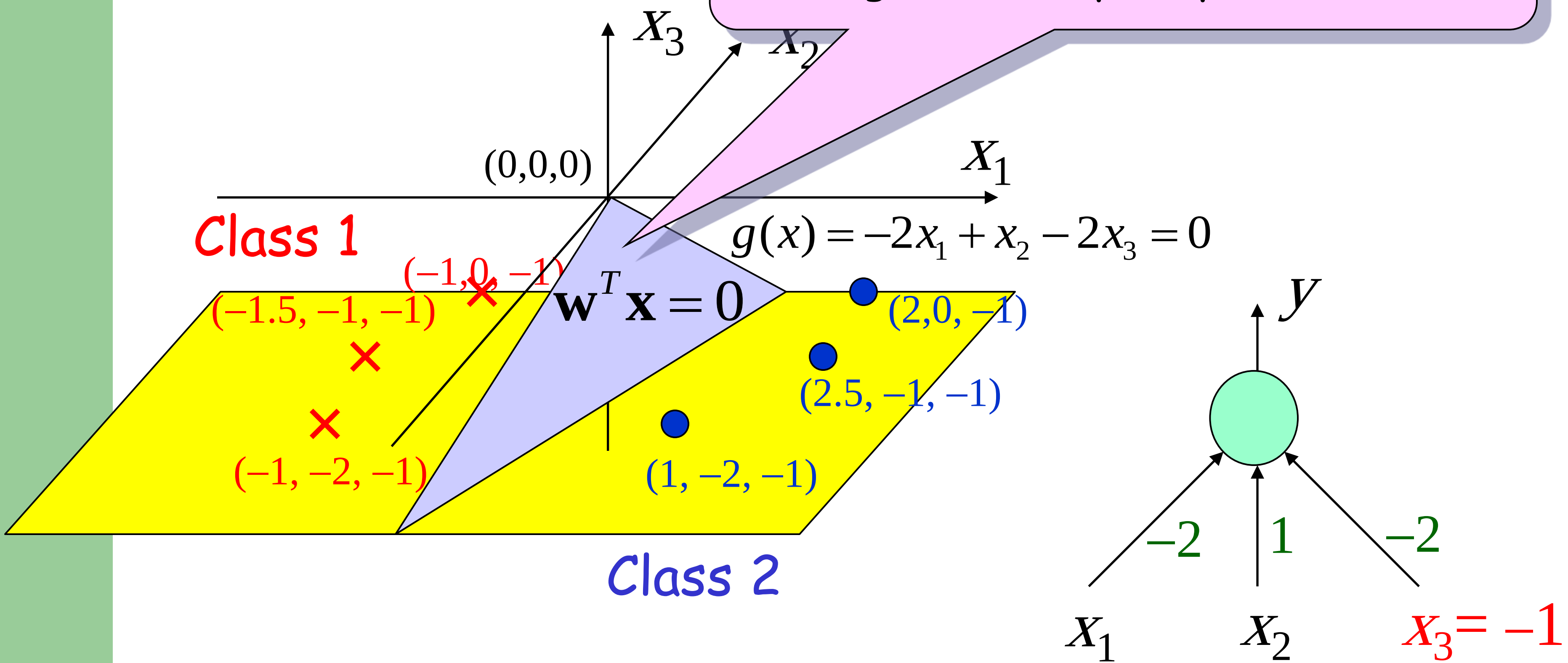


Goal: $y^{(k)} = \mathrm{sgn}(\mathbf{w}^T \mathbf{x}^{(k)}) = d^{(k)}$

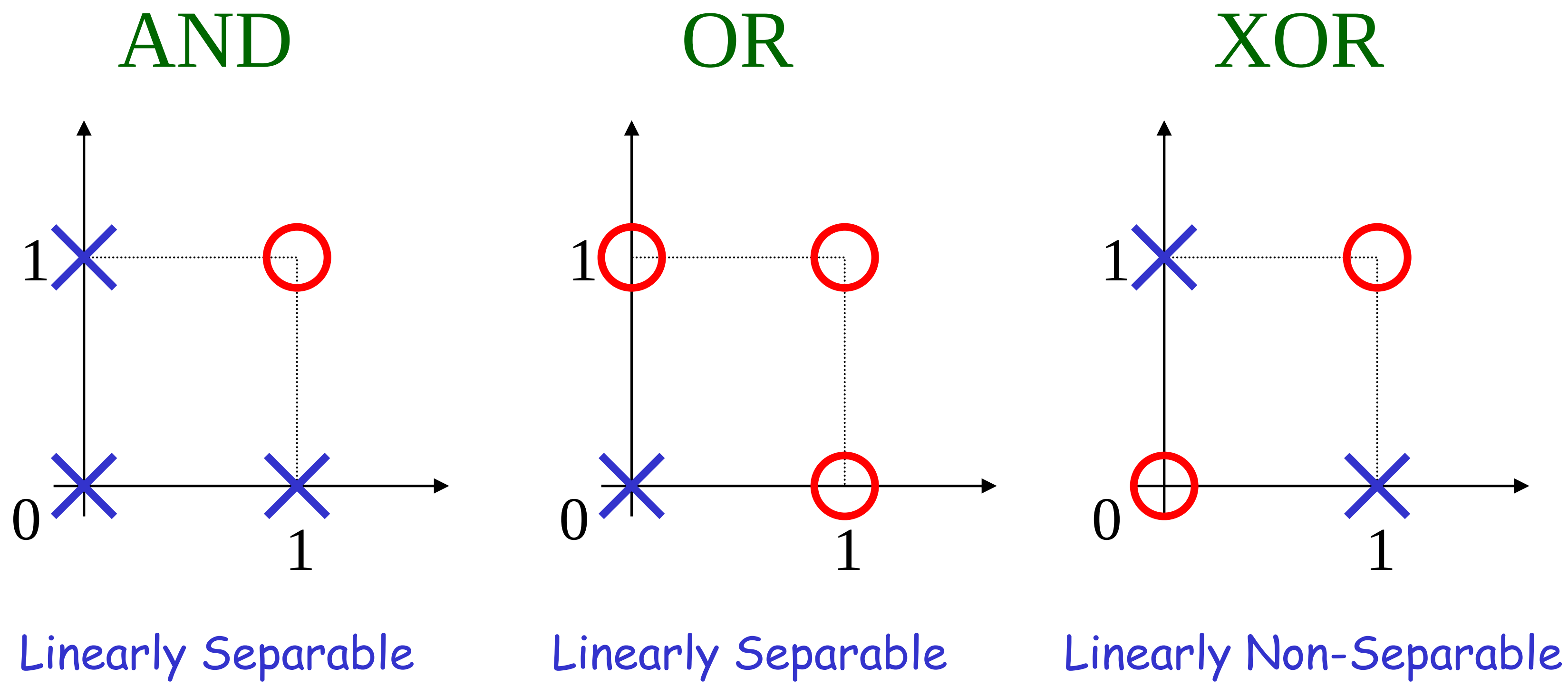
$\mathbf{w} = (w_1, w_2, w_3)^T$

# Augmented input vector

A plane passes through the origin in the augmented input space.



# Linearly Separable vs. Linearly Non-Separable



# Goal

- Given training sets $T_1 \in C_1$ and $T_2 \in C_2$ with elements in form of $\mathbf{x}=(x_1, x_2, \ldots, x_{m-1}, x_m)^T$, where $x_1, x_2, \ldots, x_{m-1} \in R$ and $x_m = -1$.
- Assume $T_1$ and $T_2$ are linearly separable.
- Find $\mathbf{w}=(w_1, w_2, \ldots, w_m)^T$ such that

$$\boxed{\operatorname{sgn}(\mathbf{w}^T\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in T_1 \\ -1 & \mathbf{x} \in T_2 \end{cases}}$$

# Goal

$\mathbf{w}^T\mathbf{x} = 0$ is a hyperplain passes through the origin of *augmented input space.*

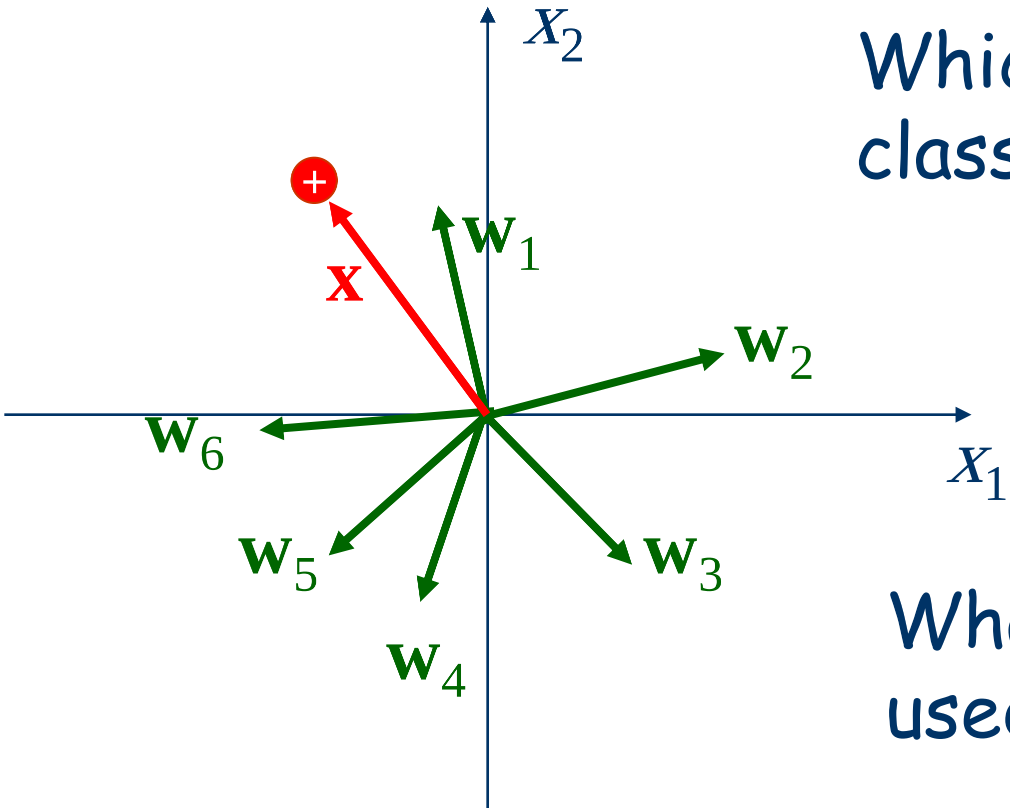
- Given training sets $T_1 \in C_1$ and $T_2 \in C_2$ with elements in form of $\mathbf{x}=(x_1, x_2, \ldots, x_{m-1}, x_m)^T$, where $x_1, x_2, \ldots, x_{m-1} \in R$ and $x_m = -1$.
- Assume $T_1$ and $T_2$ are linearly separable.
- Find $\mathbf{w}=(w_1, w_2, \ldots, w_m)^T$ such that

$$\operatorname{sgn}(\mathbf{w}^T\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \in T_1 \\ -1 & \mathbf{x} \in T_2 \end{cases}$$

# Observation

$d = +1$

$d = -1$



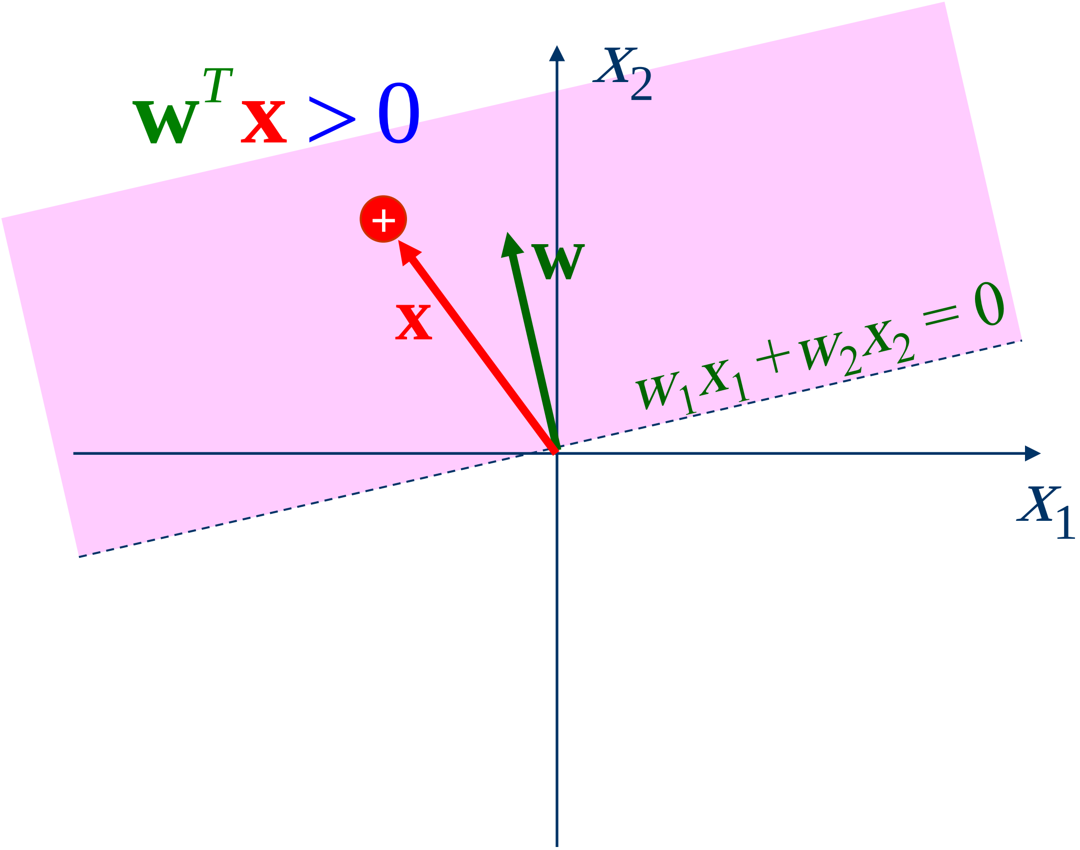

Which $\mathbf{w}$'s correctly classify $\mathbf{x}$?

What trick can be used?

# Observation

$\oplus \quad d = +1$

$\ominus \quad d = -1$

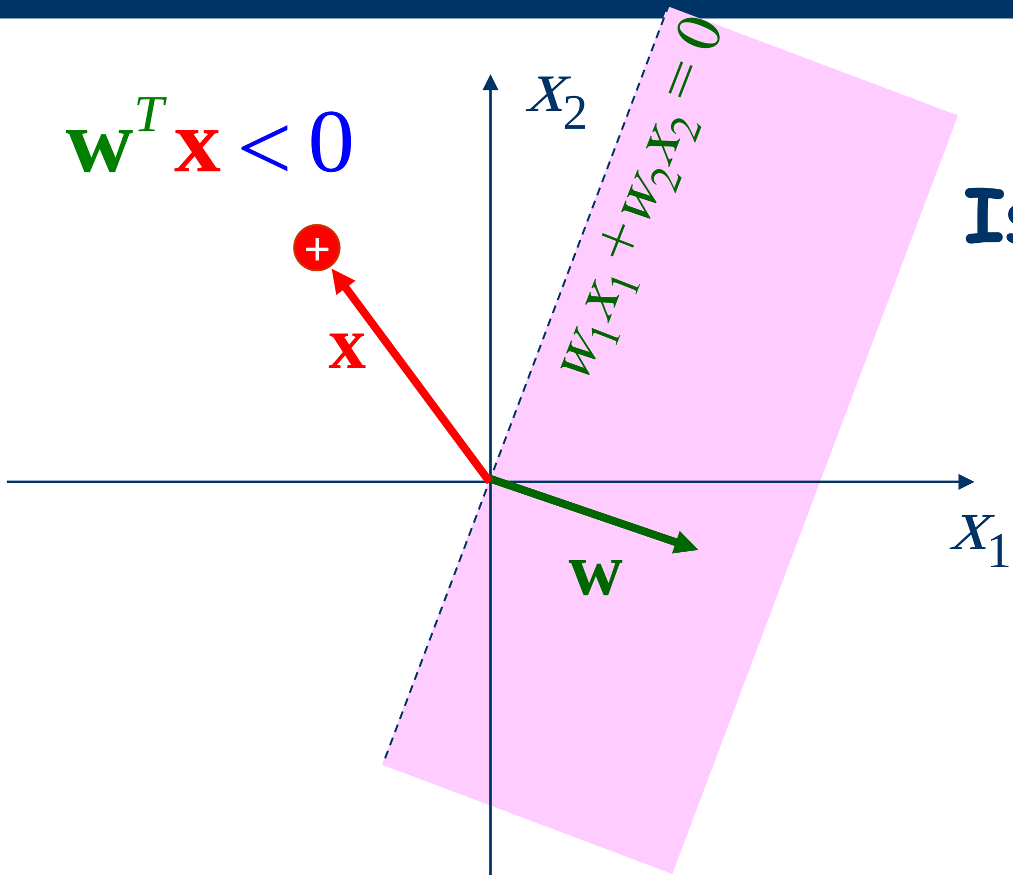



Is this $\mathbf{w}$ ok?

# Observation



$\mathbf{w}^T \mathbf{x} < 0$

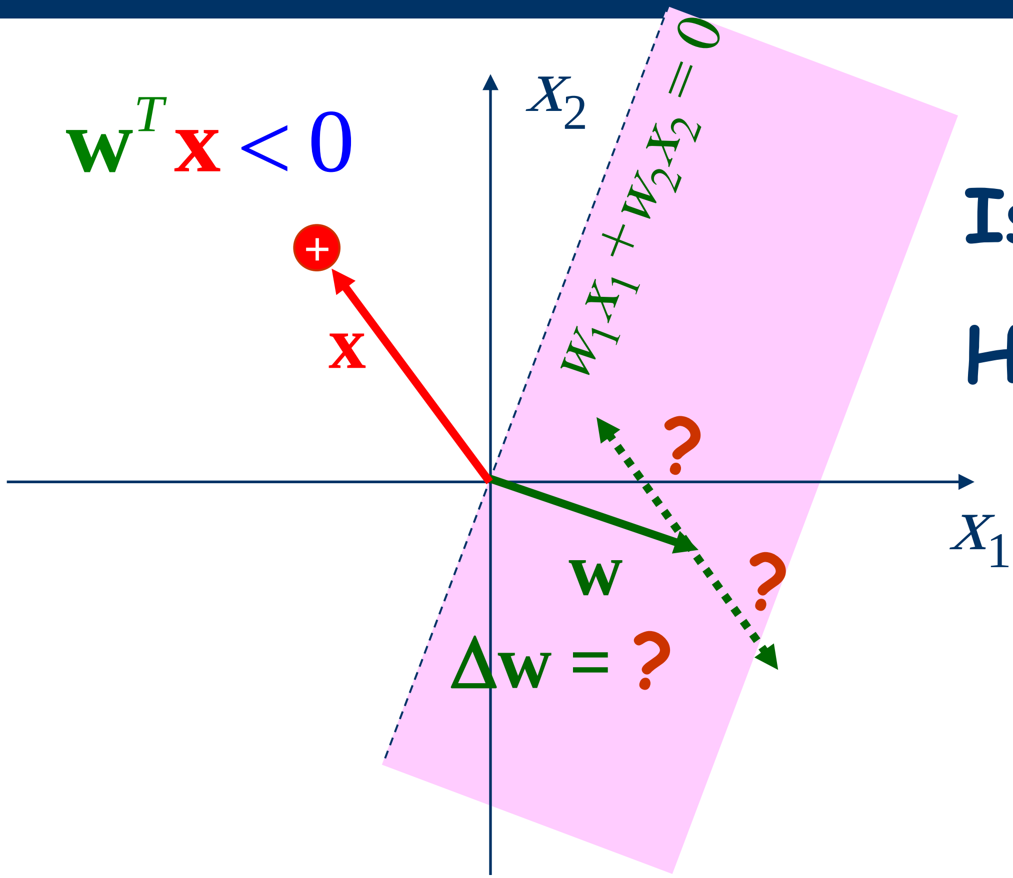



Is this $\mathbf{w}$ ok?

# Observation

$d = +1$

$d = -1$




Is this $\mathbf{w}$ ok?

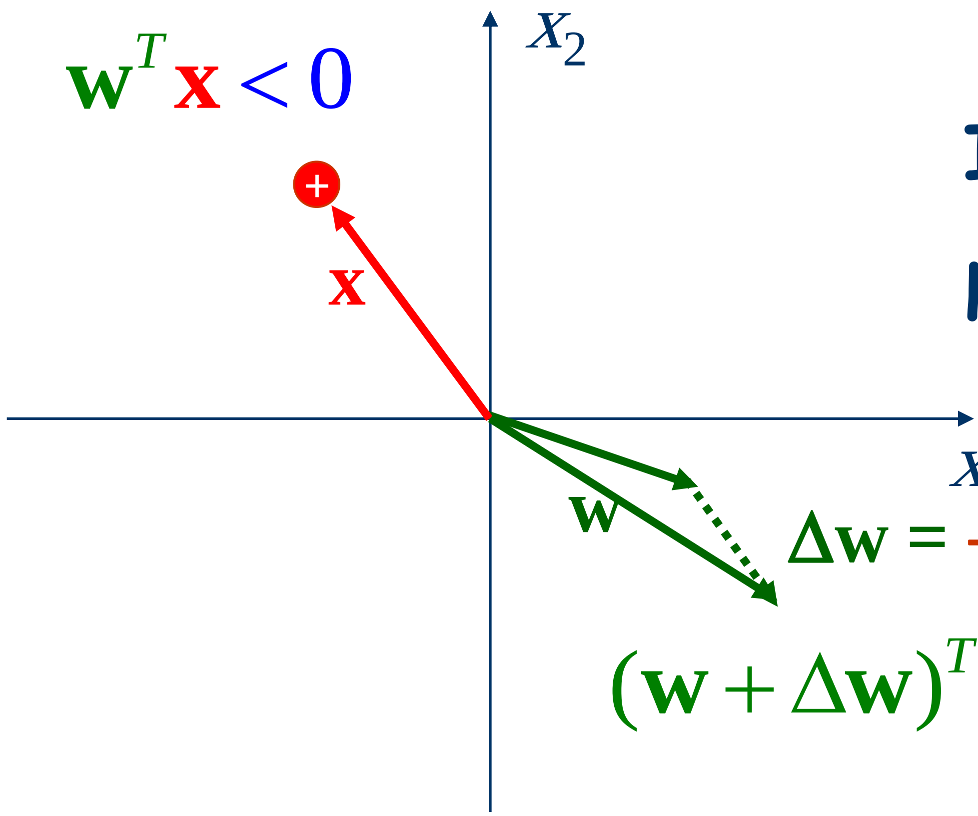
How to adjust $\mathbf{w}$?

$+ \quad d = +1$

$- \quad d = -1$

# Observation

$$\mathbf{w}^T\mathbf{x} < 0$$




Is this **w** ok?

How to adjust **w**?

$$\Delta\mathbf{w} = -\alpha\mathbf{x}$$

reasonable?

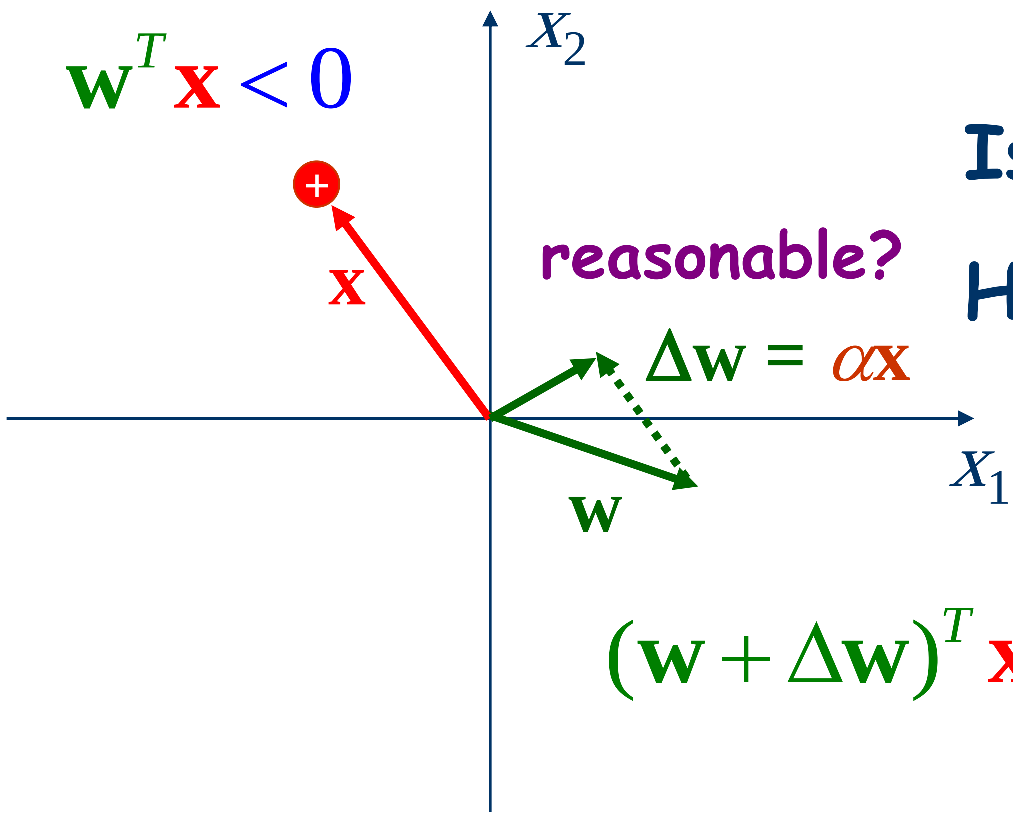
$$(\mathbf{w}+\Delta\mathbf{w})^T\mathbf{x} = \underbrace{\mathbf{w}^T\mathbf{x}}_{<0} - \alpha\underbrace{\mathbf{x}^T\mathbf{x}}_{>0}$$

# Observation

$\oplus$ $d = +1$

$\ominus$ $d = -1$

$$\mathbf{w}^T\mathbf{x} < 0$$




Is this $\mathbf{w}$ ok?

How to adjust $\mathbf{w}$?

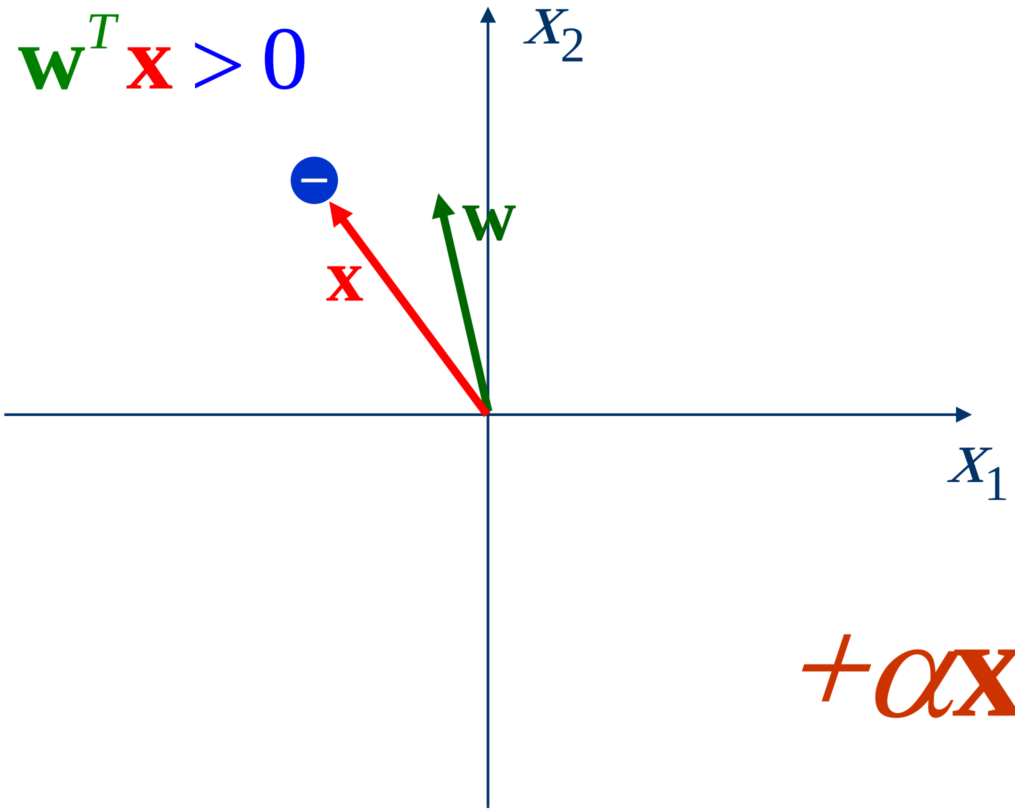
$$(\mathbf{w}+\Delta\mathbf{w})^T\mathbf{x} = \underbrace{\mathbf{w}^T\mathbf{x}}_{<0} + \underbrace{\alpha\mathbf{x}^T\mathbf{x}}_{>0}$$

# Observation

$d = +1$

$d = -1$

$\mathbf{w}^T \mathbf{x} > 0$




Is this $\mathbf{w}$ ok?

$$\Delta \mathbf{w} = ?$$

$+\alpha\mathbf{x}$ or $-\alpha\mathbf{x}$

$$\alpha > 0$$

# Perceptron Learning Rule

Upon misclassification on

$$d = +1 \qquad \Delta\mathbf{w} = \alpha\mathbf{x}$$

$$d = -1 \qquad \Delta\mathbf{w} = -\alpha\mathbf{x}$$

Define error

$$r = d - y = \begin{cases} +2 & \\ -2 & \\ 0 & \text{No error} \end{cases}$$

# Perceptron Learning Rule

$$\Delta \mathbf{w} = \eta r \mathbf{x}$$

Define error

$$r = d - y = \begin{cases} +2 & \oplus \rightarrow \ominus \\ -2 & \ominus \rightarrow \oplus \\ 0 & \text{No error} \end{cases}$$

# Perceptron Learning Rule

$$\Delta \mathbf{w} = \eta r \mathbf{x}$$

Learning Rate

Error $(d - y)$

Input

Based on the general weight learning rule.

# Summary – Perceptron Learning Rule

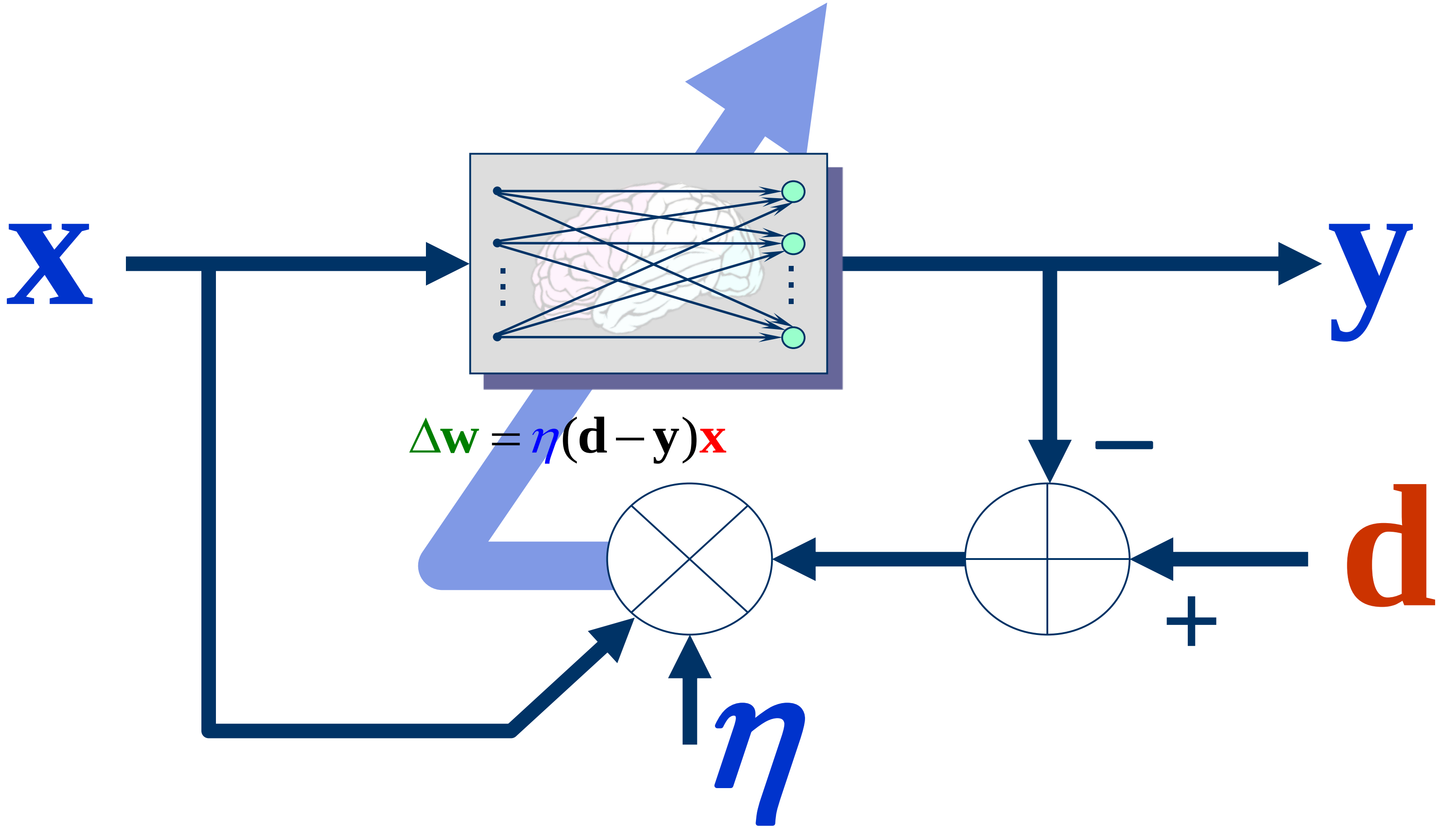
$$\Delta w_i(t) = \eta r_i x_i(t)$$

$$r_i = d_i - y_i = \begin{cases} 0 & d_i = y_i \quad \text{correct} \\ +2 & d_i = 1, y_i = -1 \\ -2 & d_i = -1, y_i = 1 \end{cases} \Bigg\} \text{incorrect}$$

$$\Delta w_i(t) = \eta (d_i - y_i) x_i(t)$$

# Summary – Perceptron Learning Rule

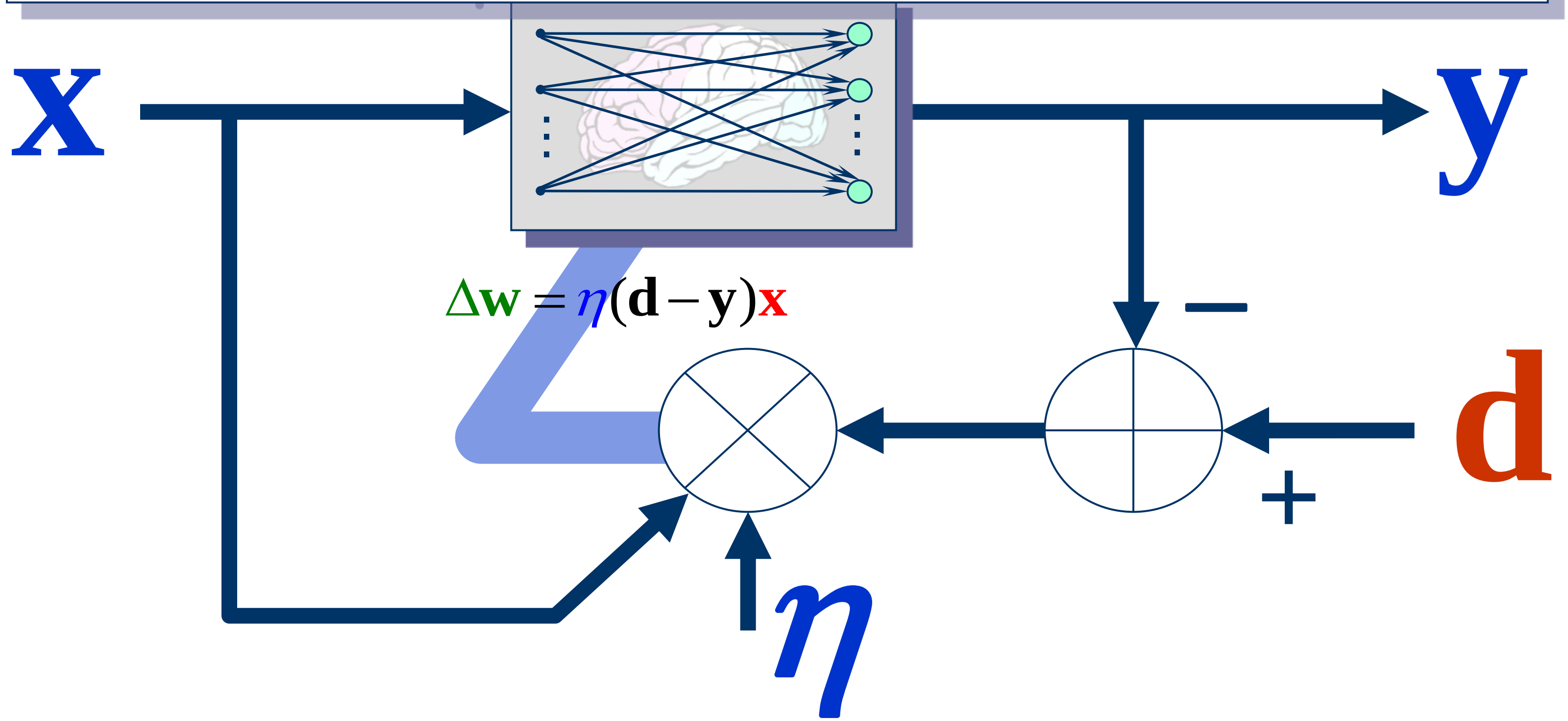
Converge?



- Exercise: Reference some papers or textbooks to prove the theorem.

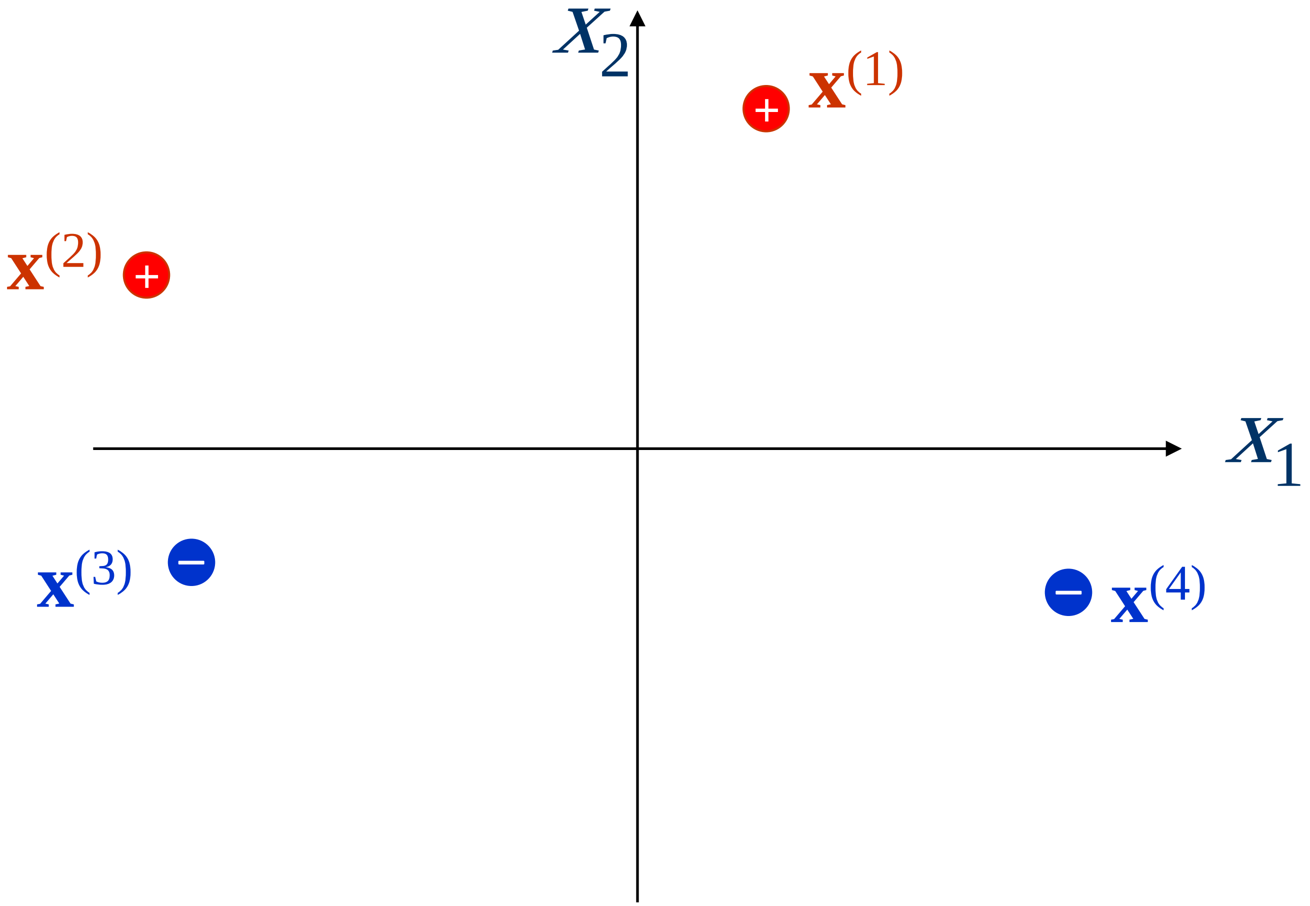
# Perceptron Convergence Theorem

**If the given training set is linearly separable, the learning process will converge in a finite number of steps.**

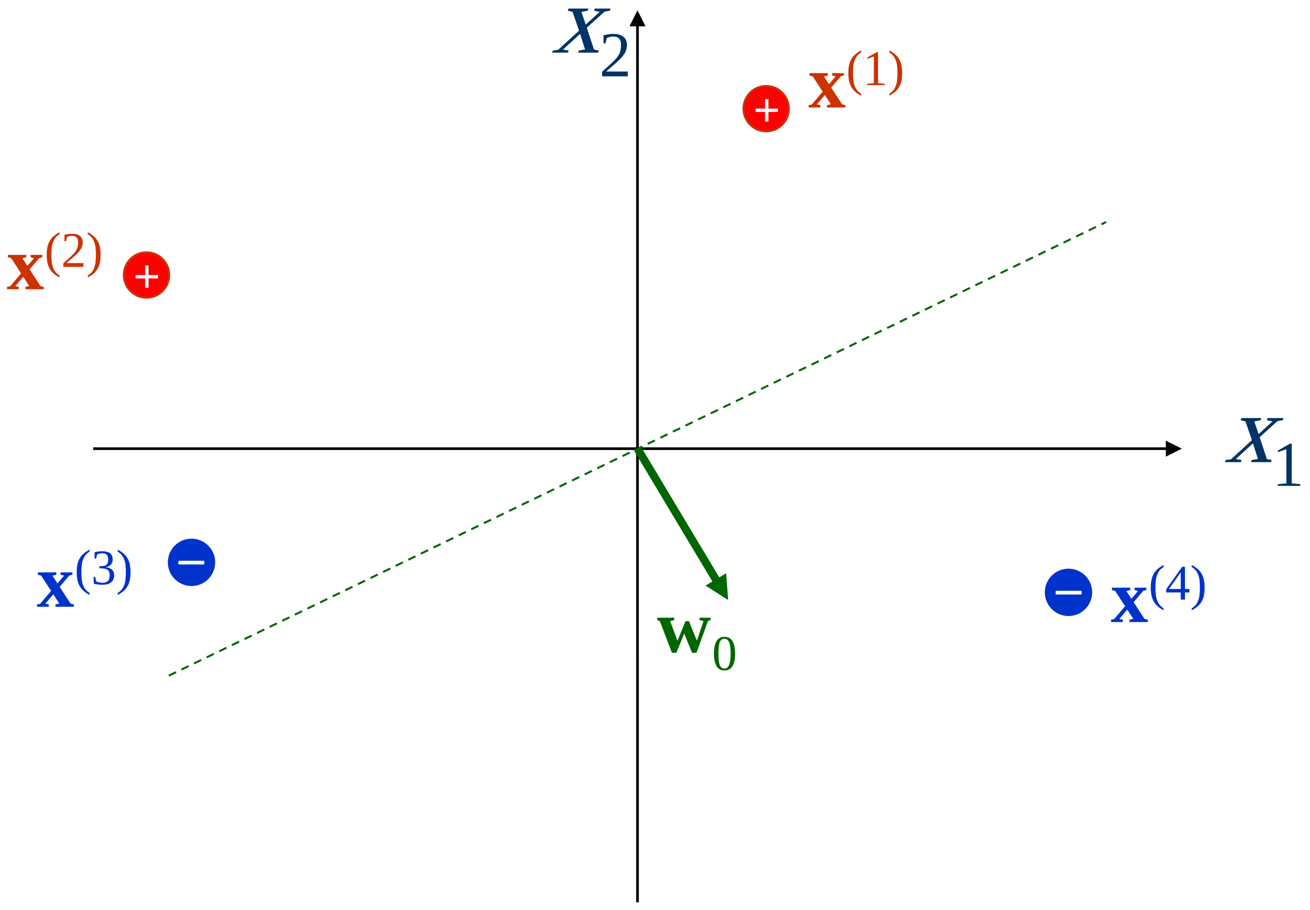


Linearly Separable.

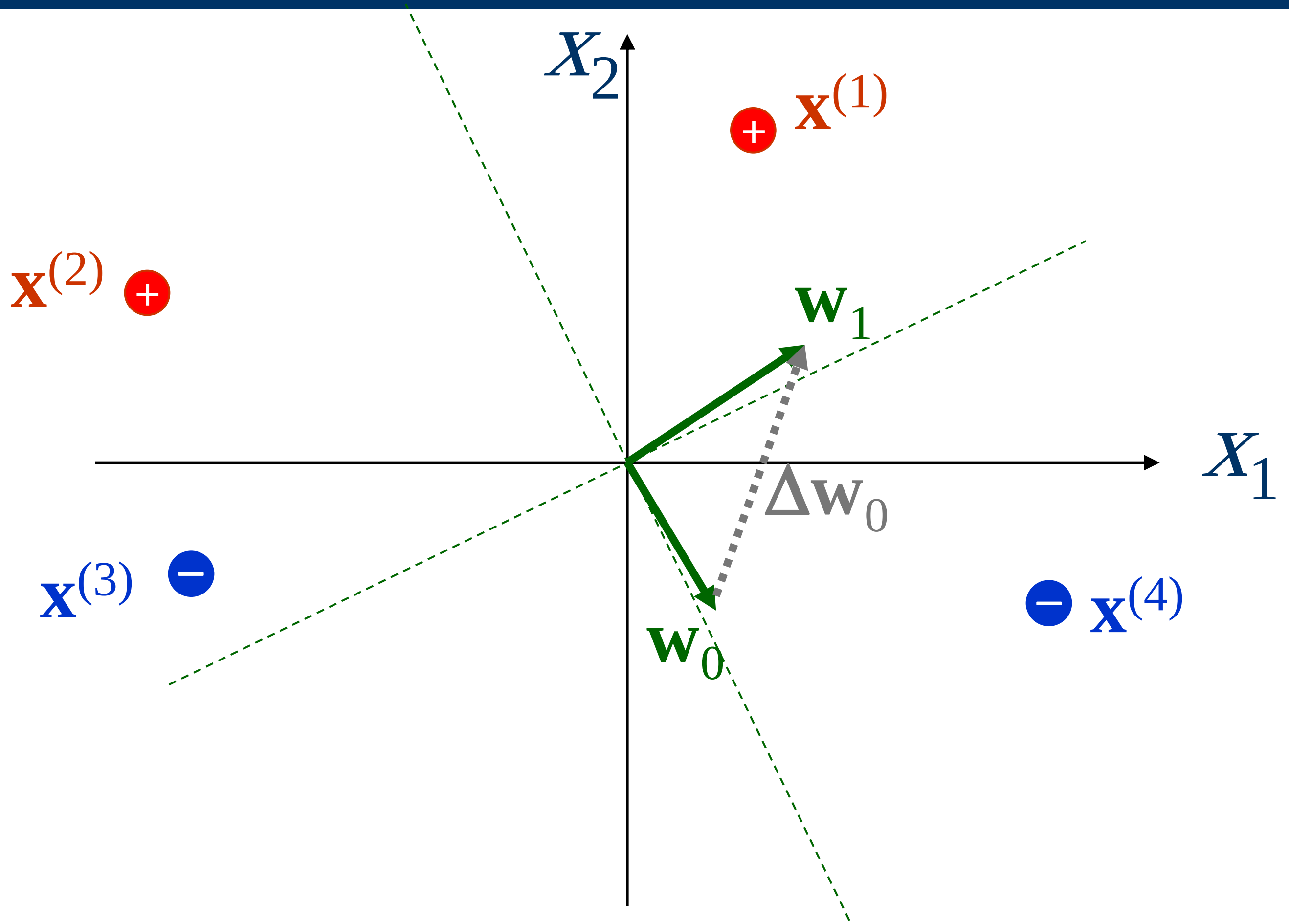
# The Learning Scenario



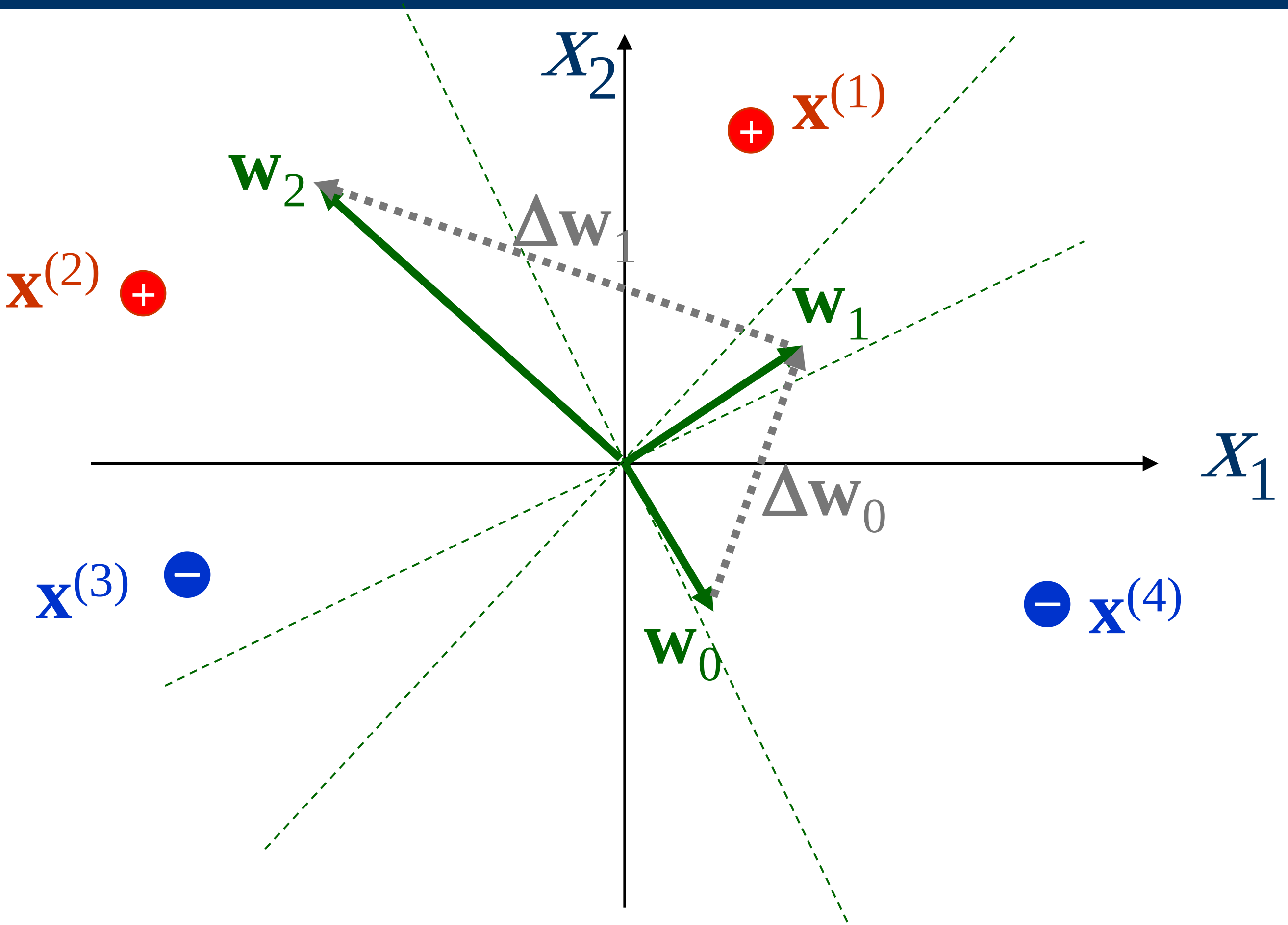
# The Learning Scenario



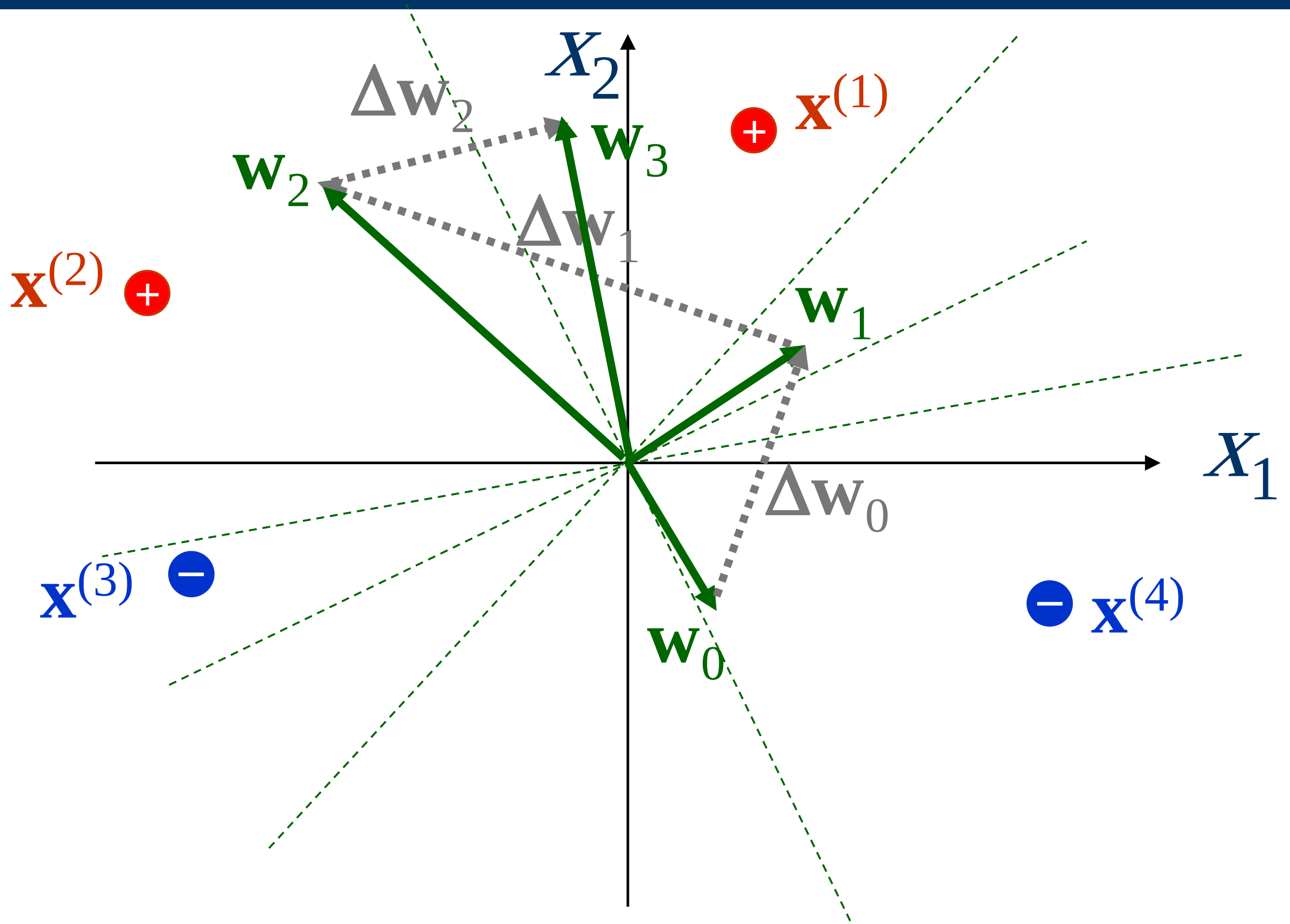
# The Learning Scenario



# The Learning Scenario

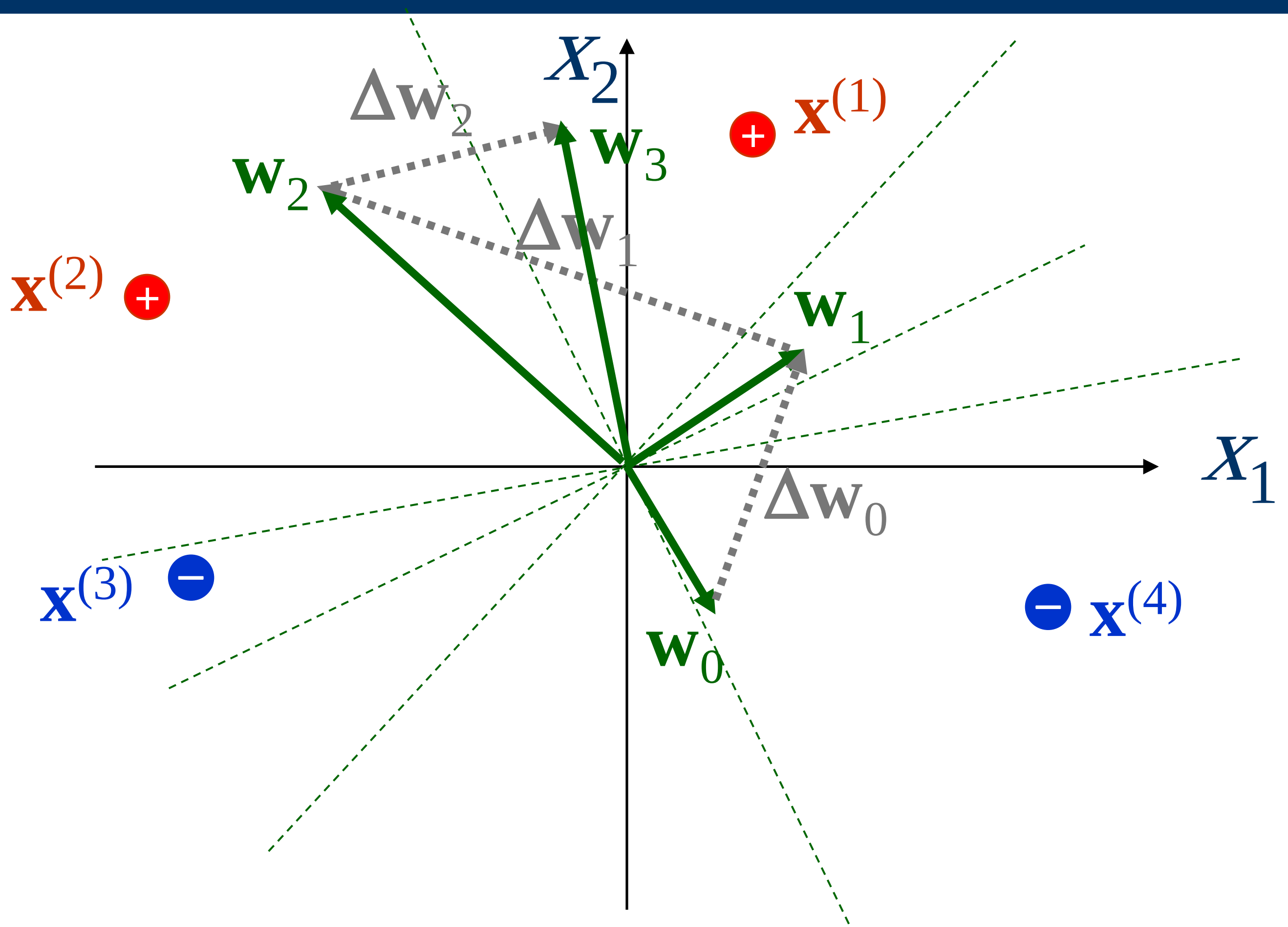


# The Learning Scenario

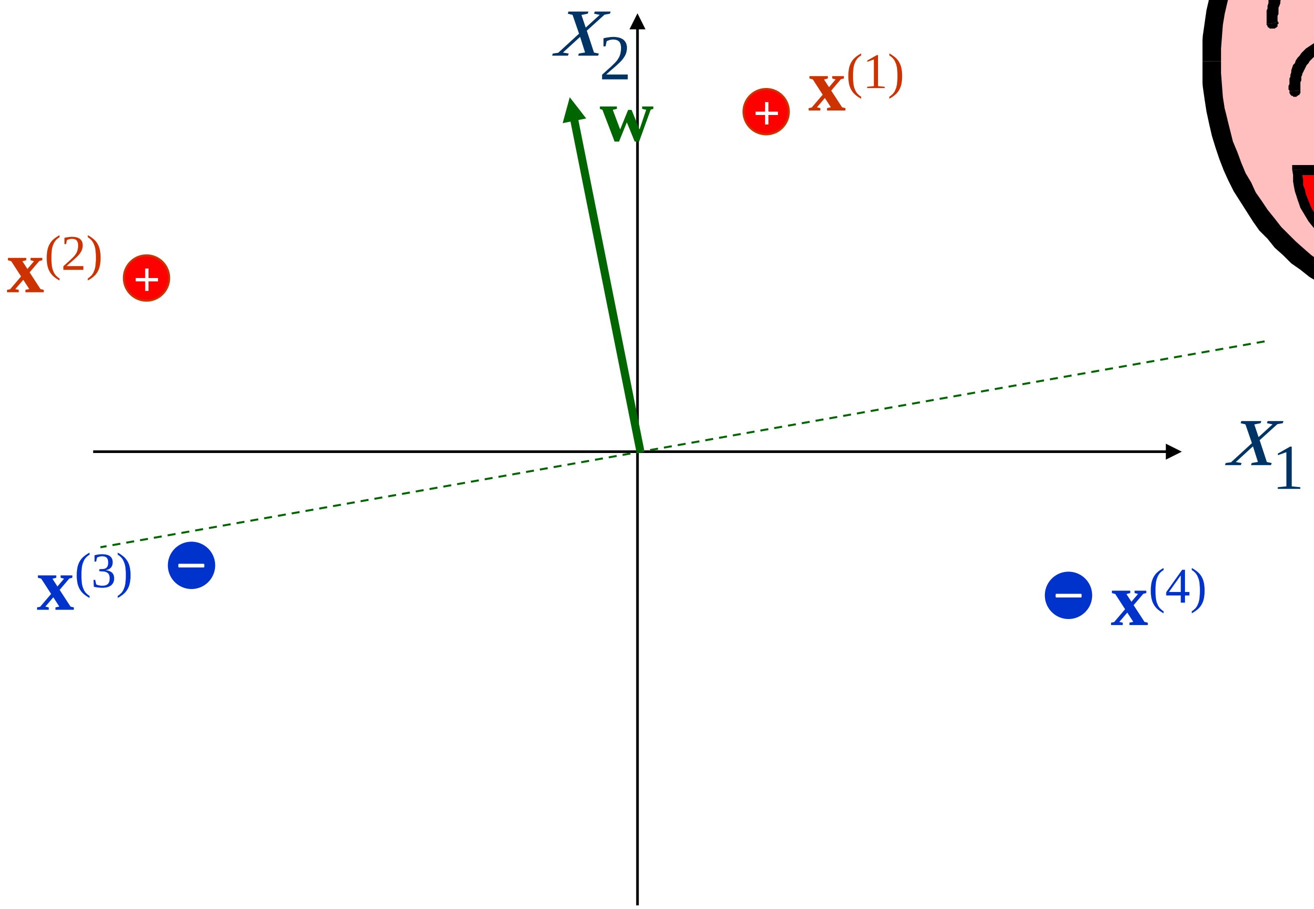


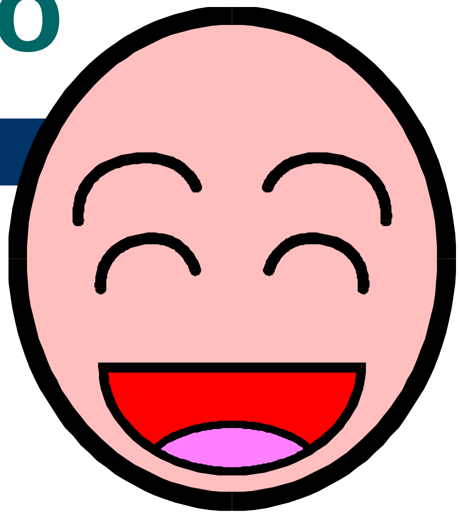
$$\mathbf{w}_4 = \mathbf{w}_3$$

# The Learning Scenario



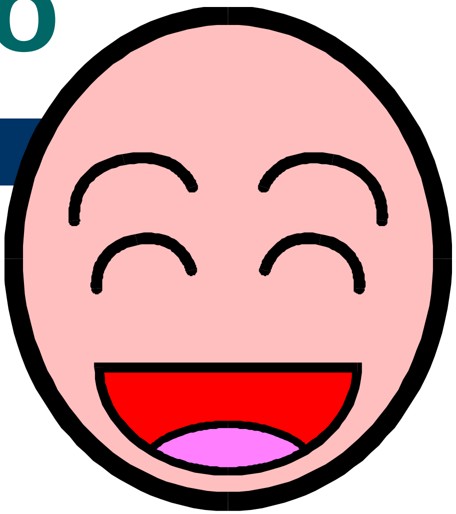
# The Learning Scenario

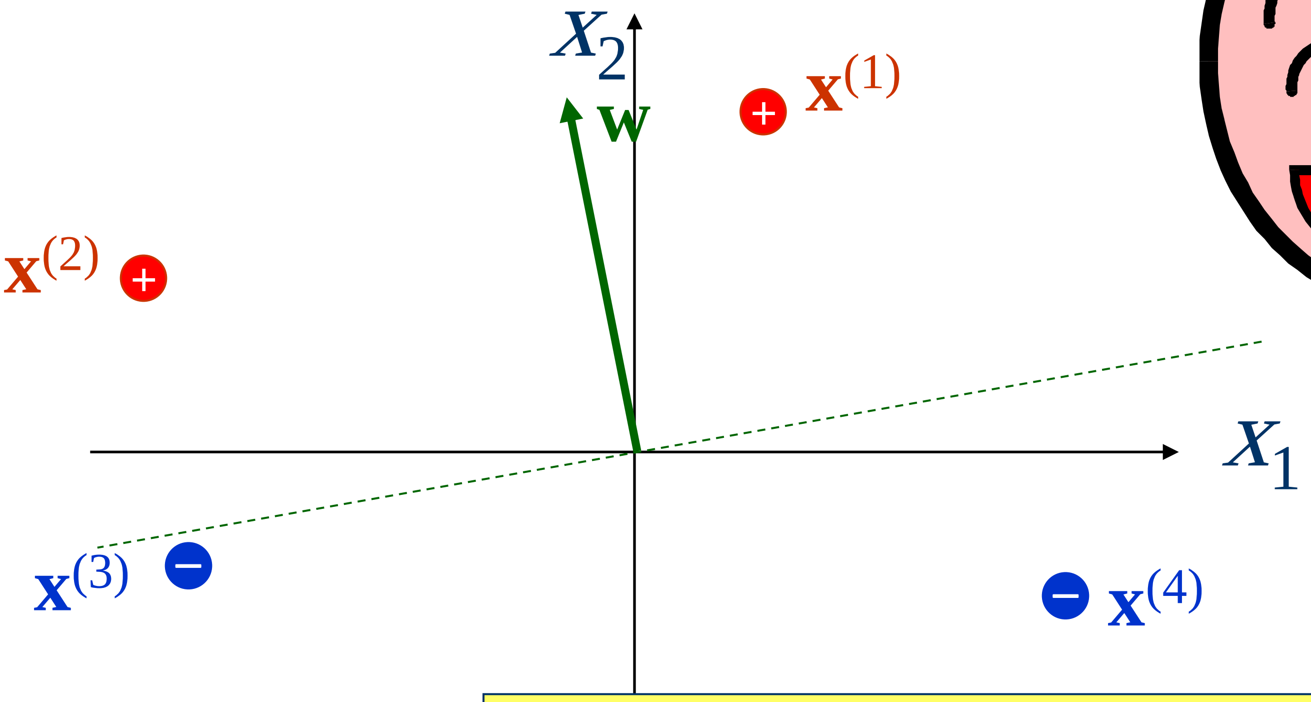


The demonstration is in augmented space.

# The Learning Scenario

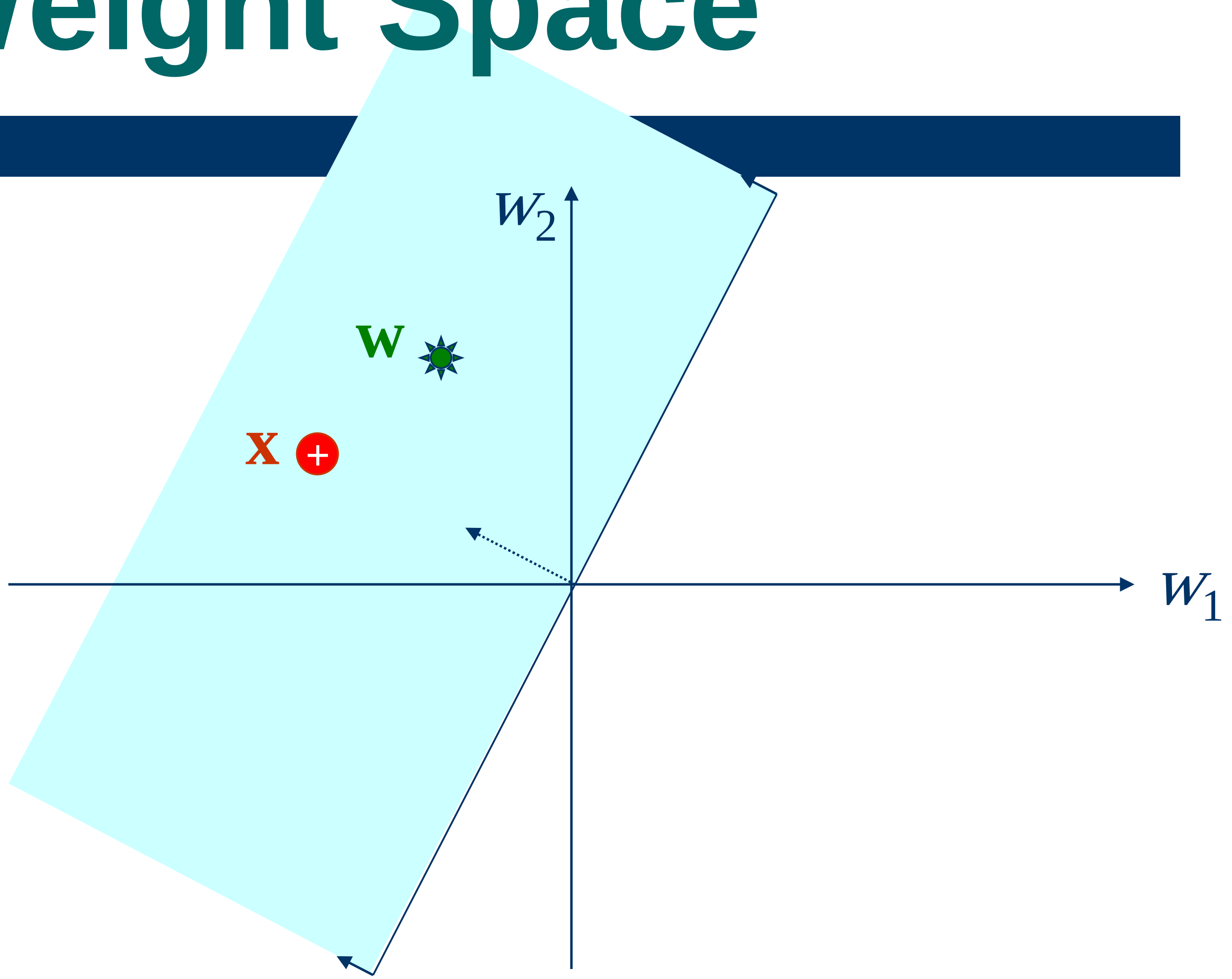





Conceptually, in augmented space, we adjust the weight vector to fit the data.

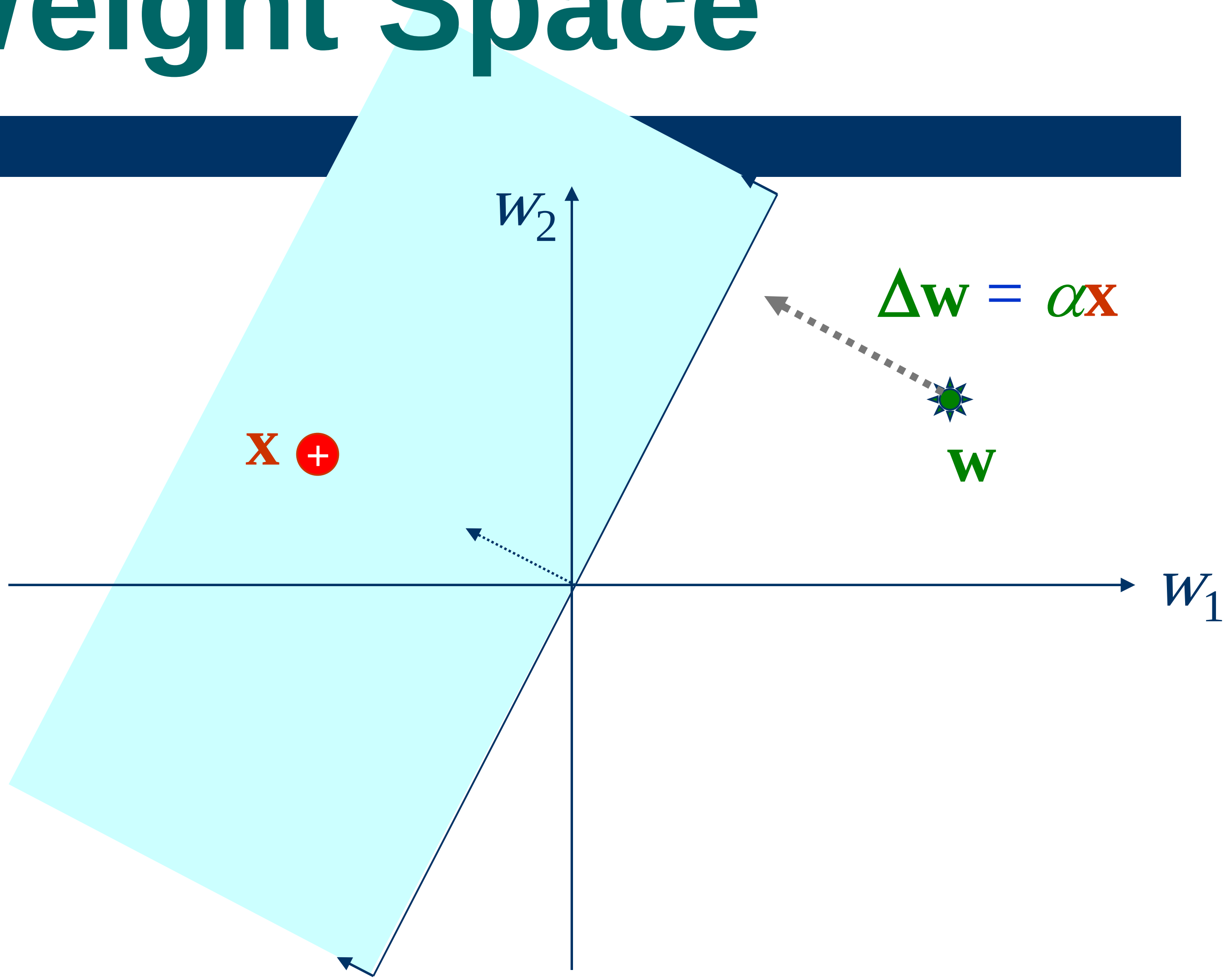
A weight in the shaded area will give correct classification for the positive example.

# Weight Space



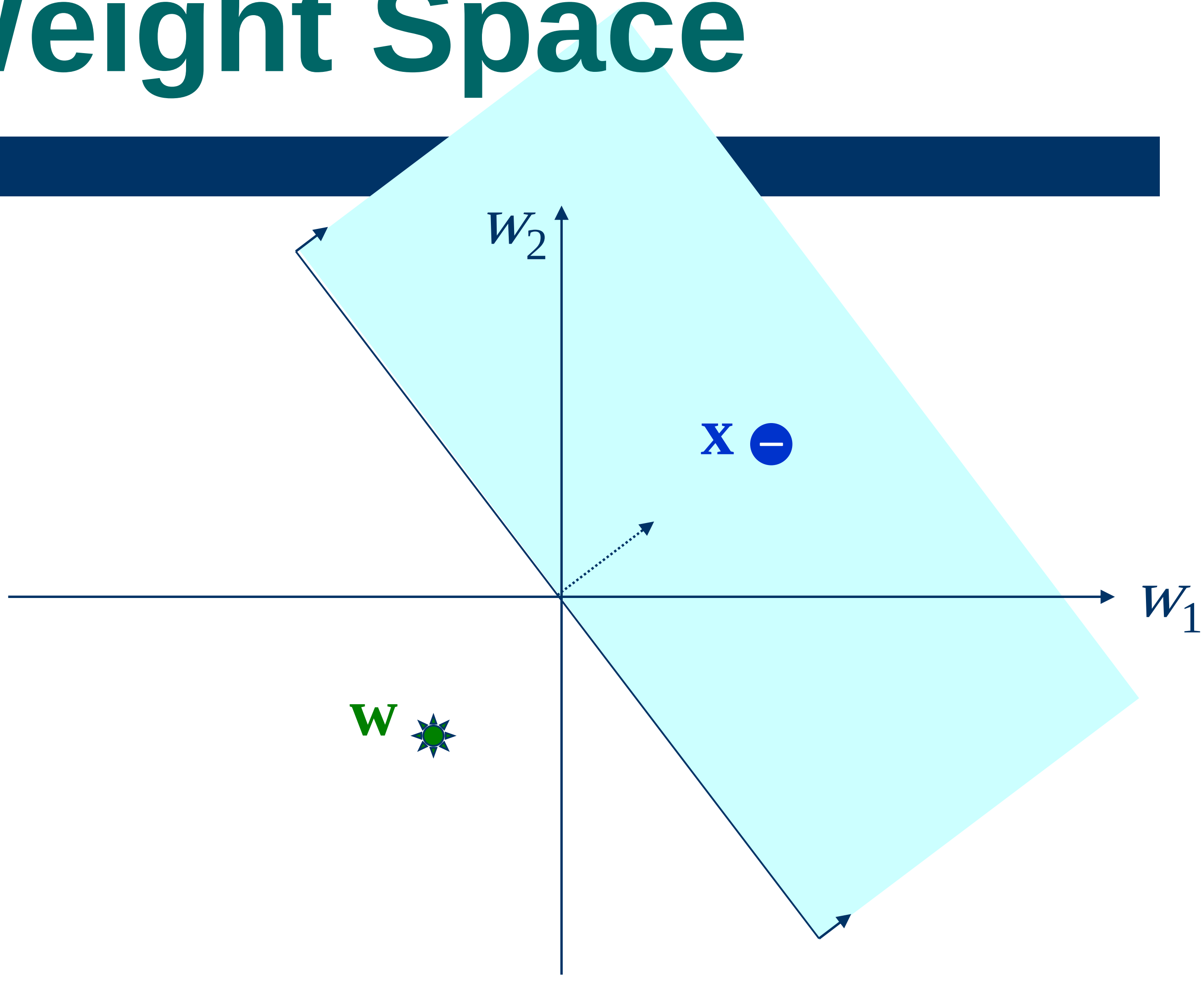
A weight in the shaded area will give correct classification for the positive example.

# Weight Space



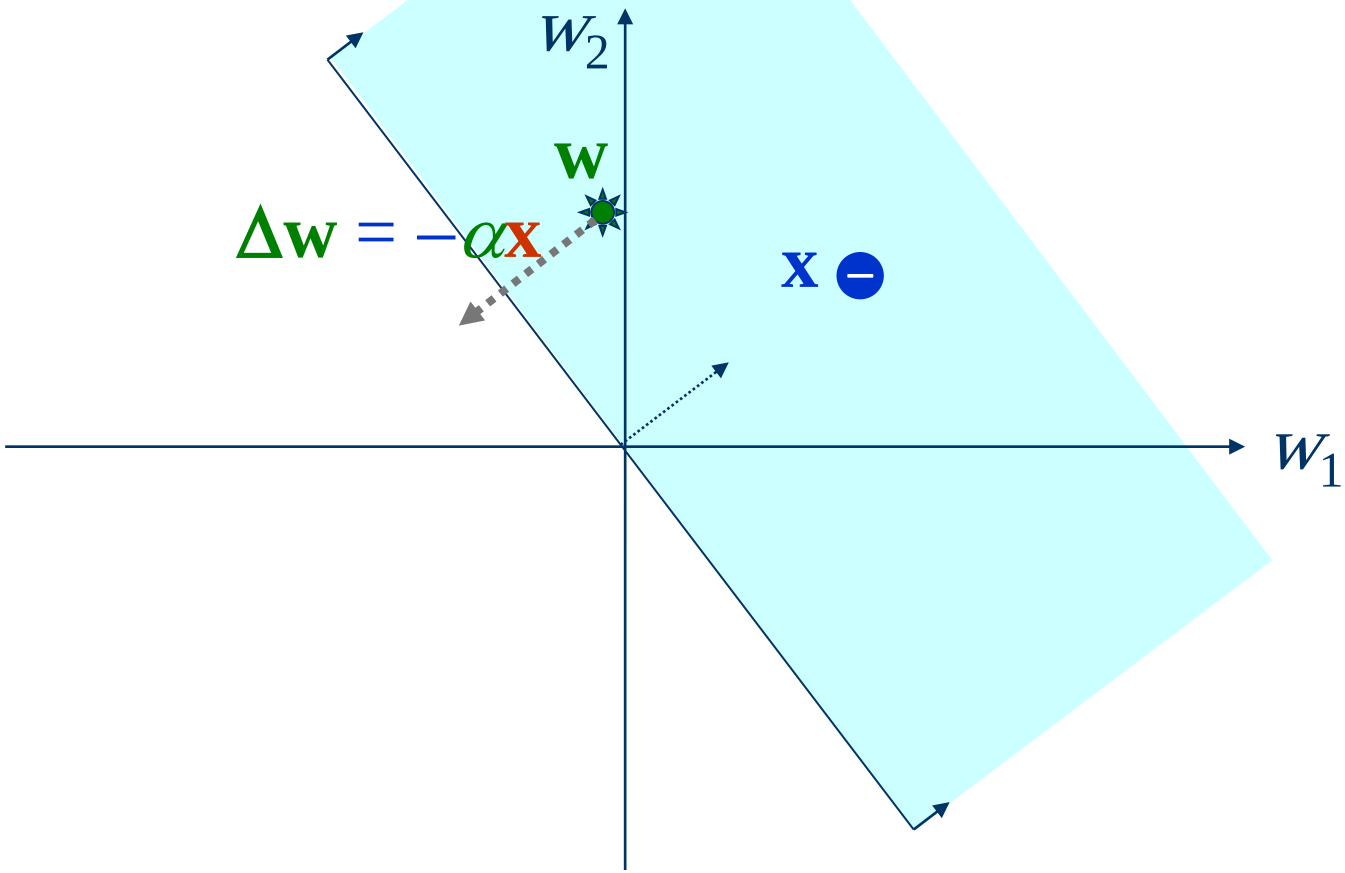
A weight *not* in the shaded area will give correct classification for the negative example.

# Weight Space

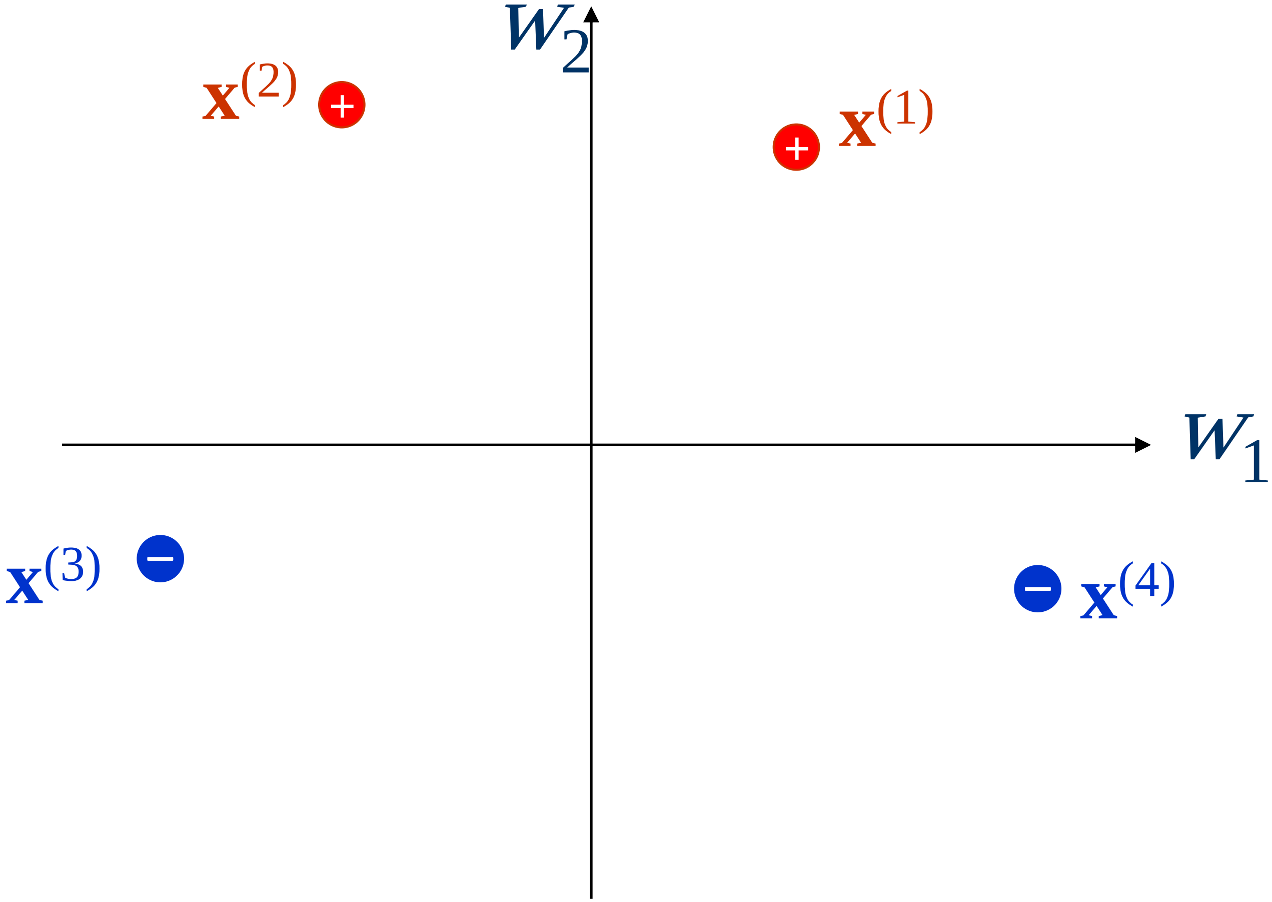


A weight *not* in the shaded area will give correct classification for the negative example.

# Weight Space

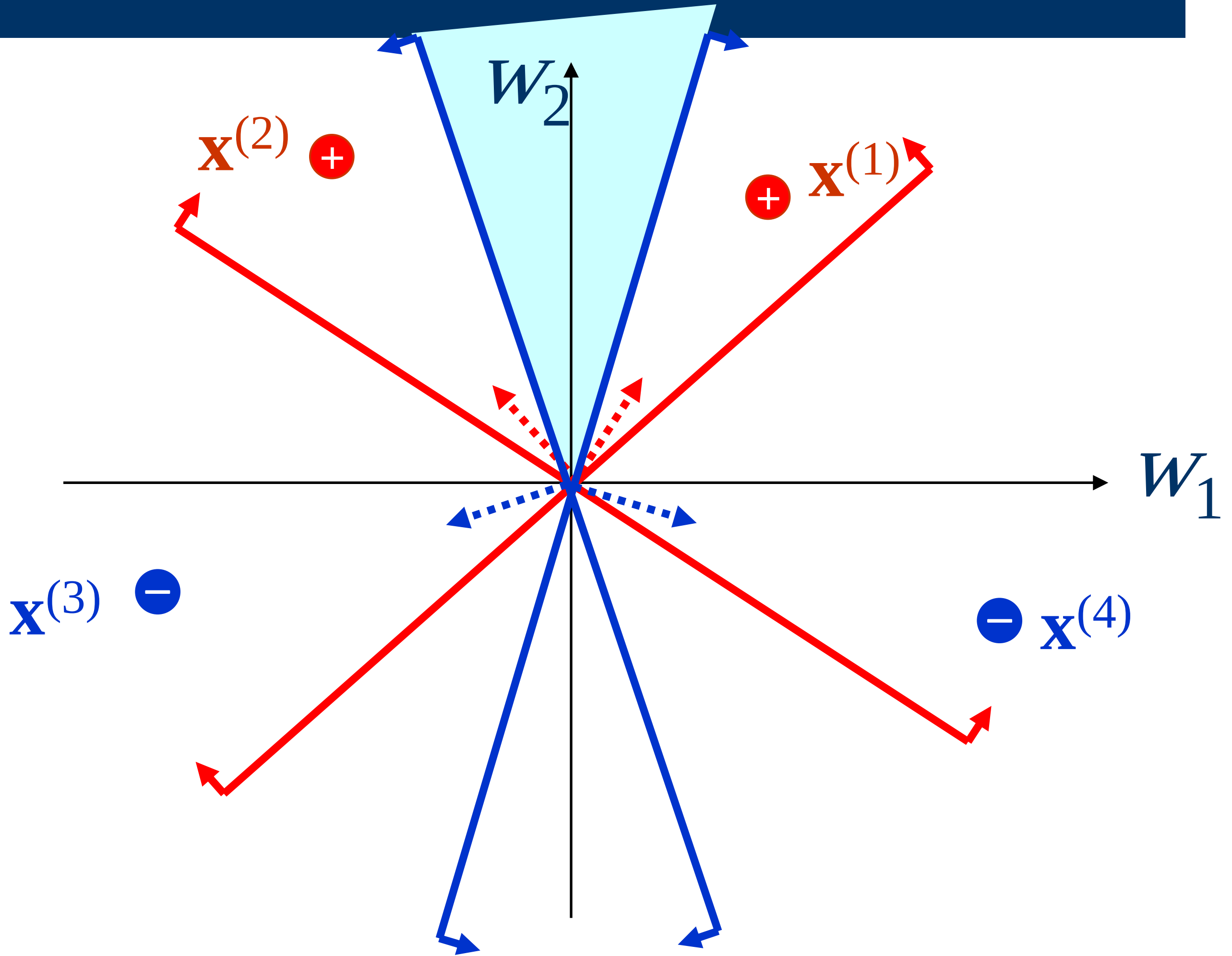


# The Learning Scenario in Weight Space



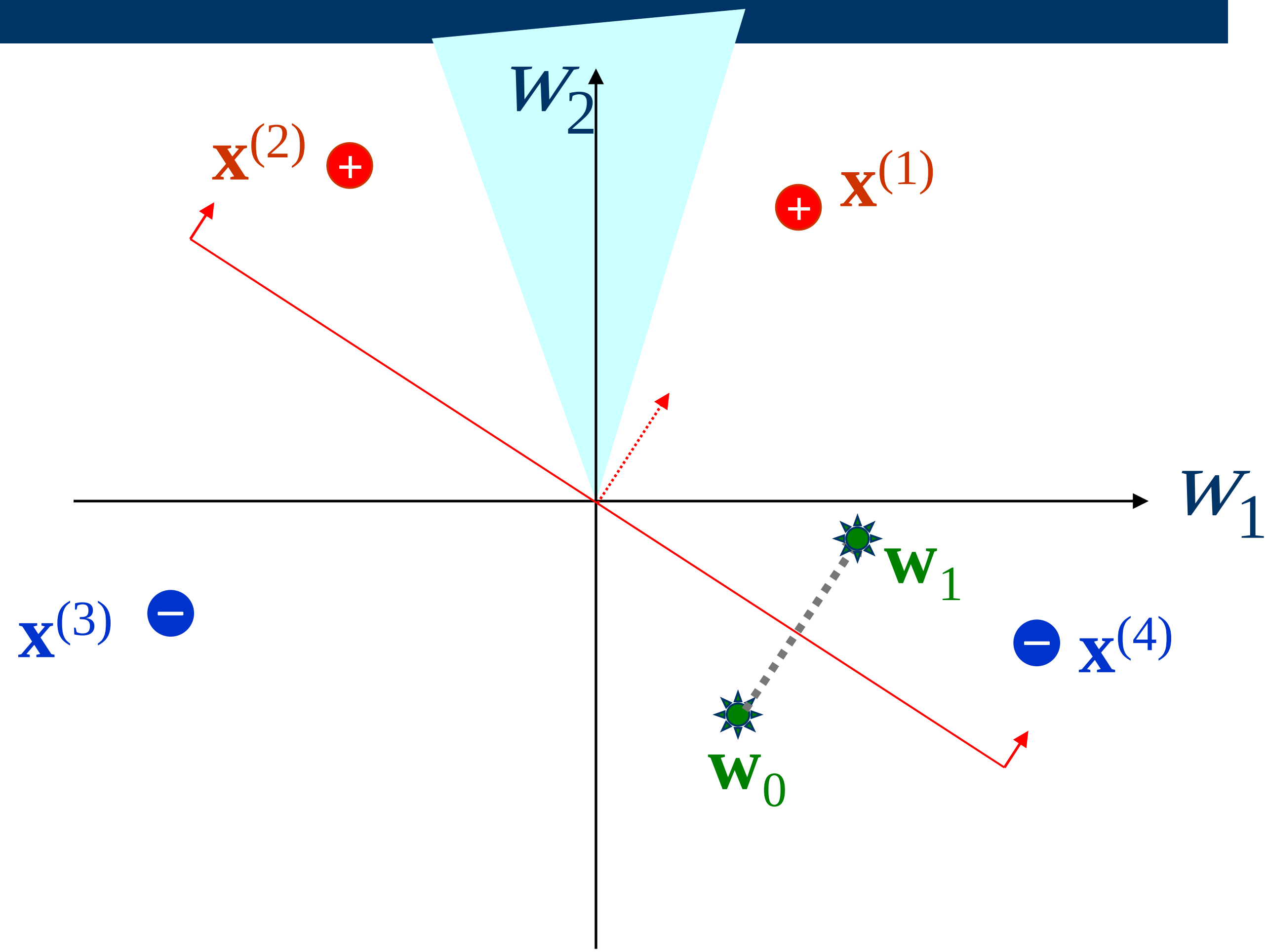
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



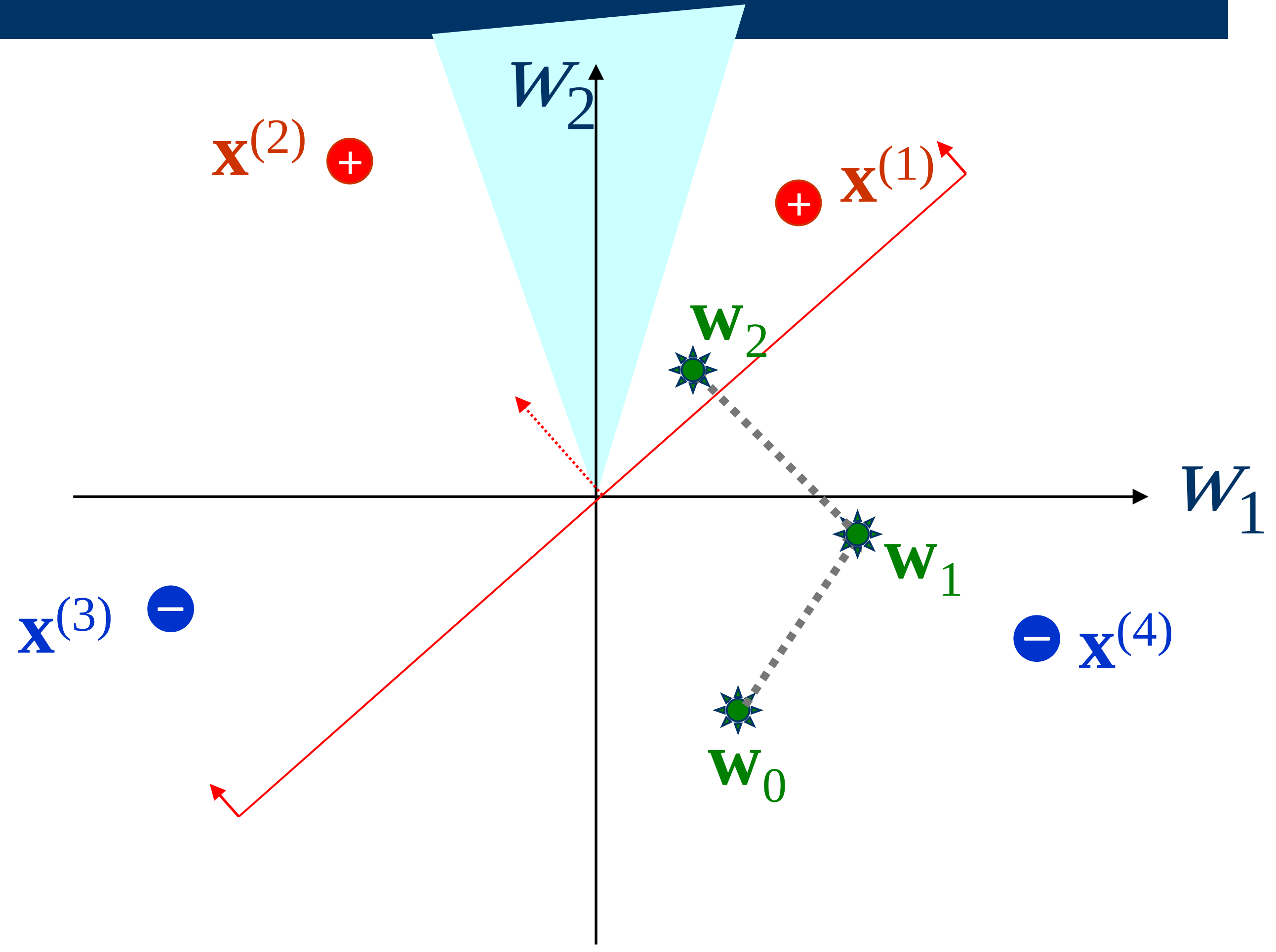
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



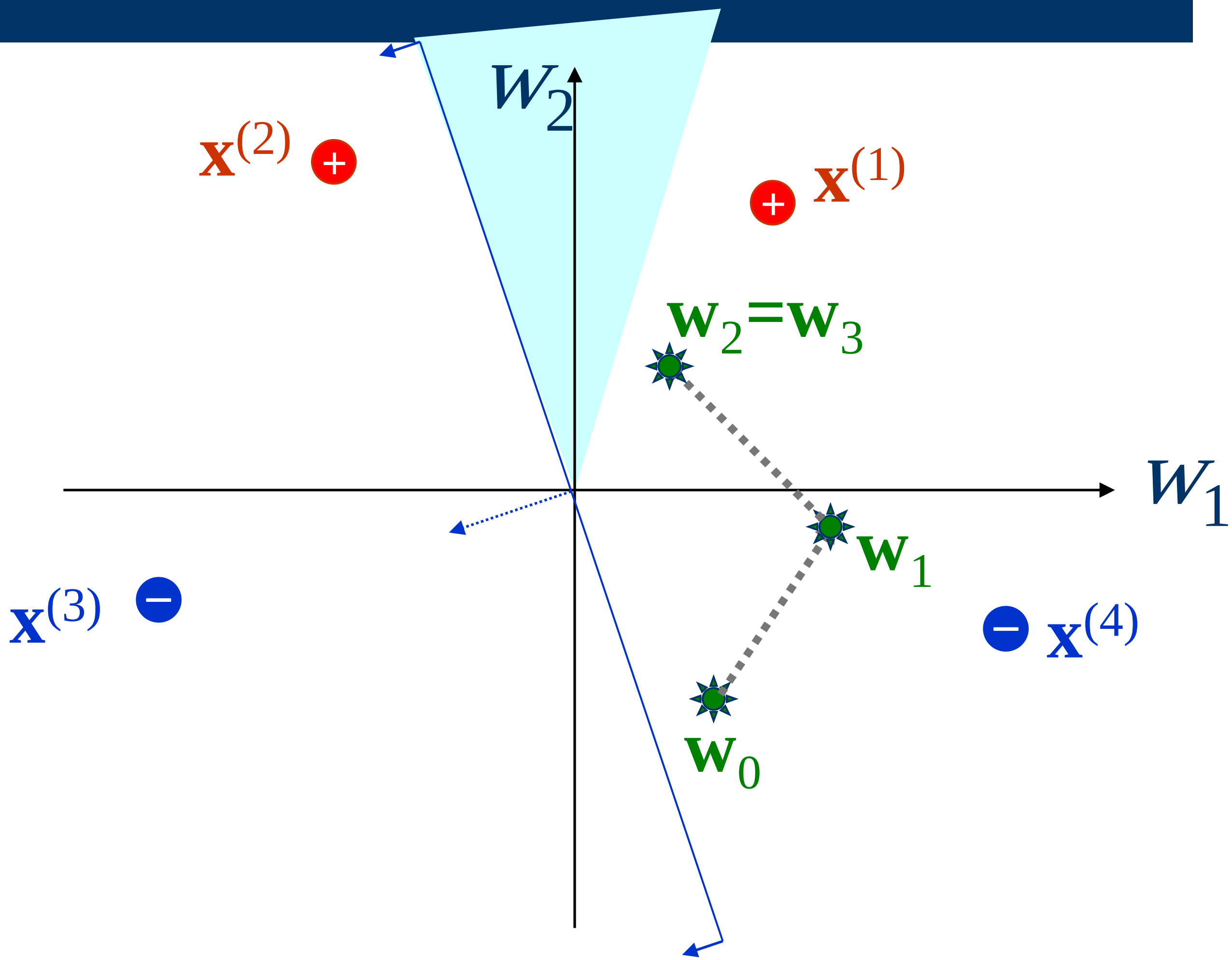
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



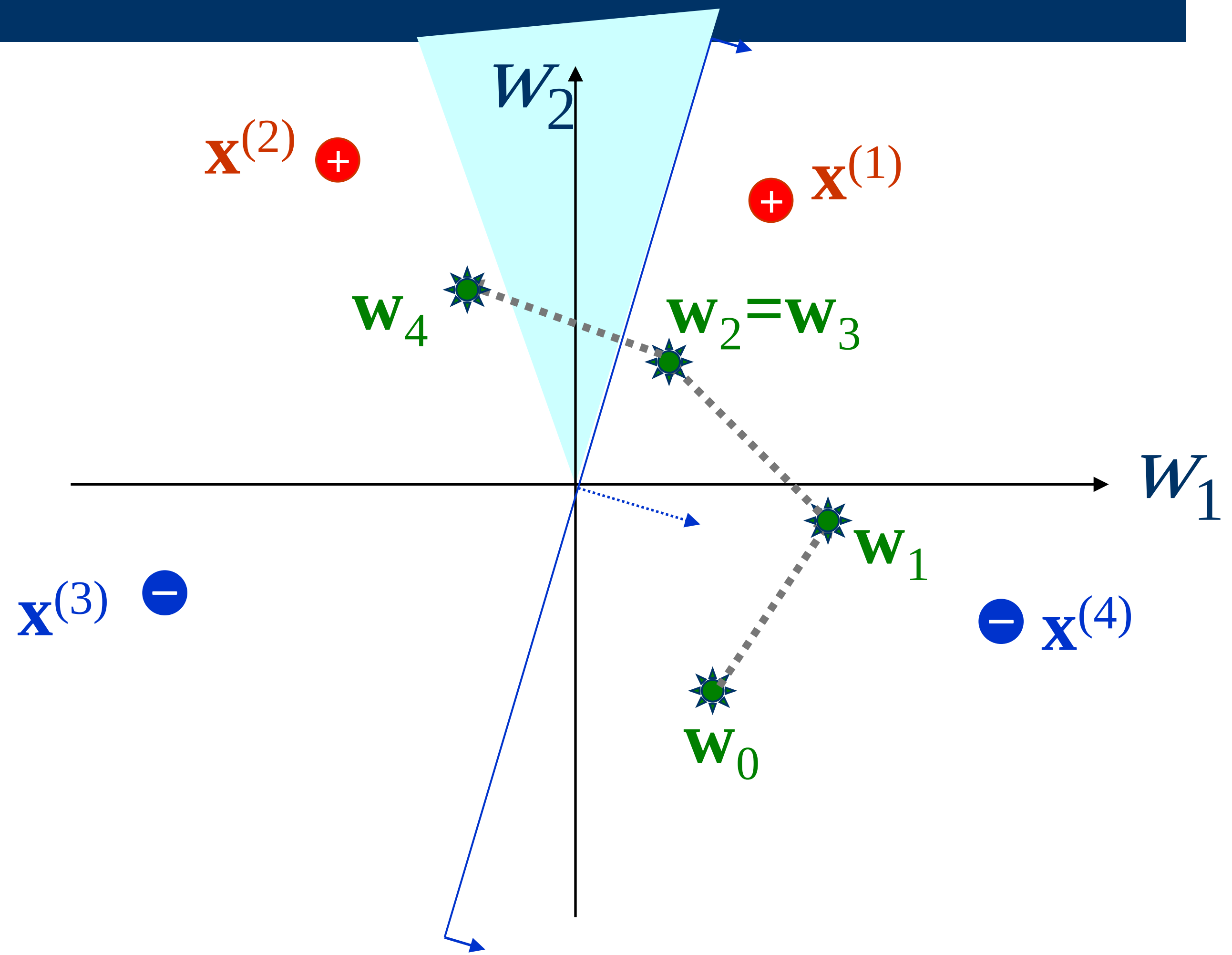
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



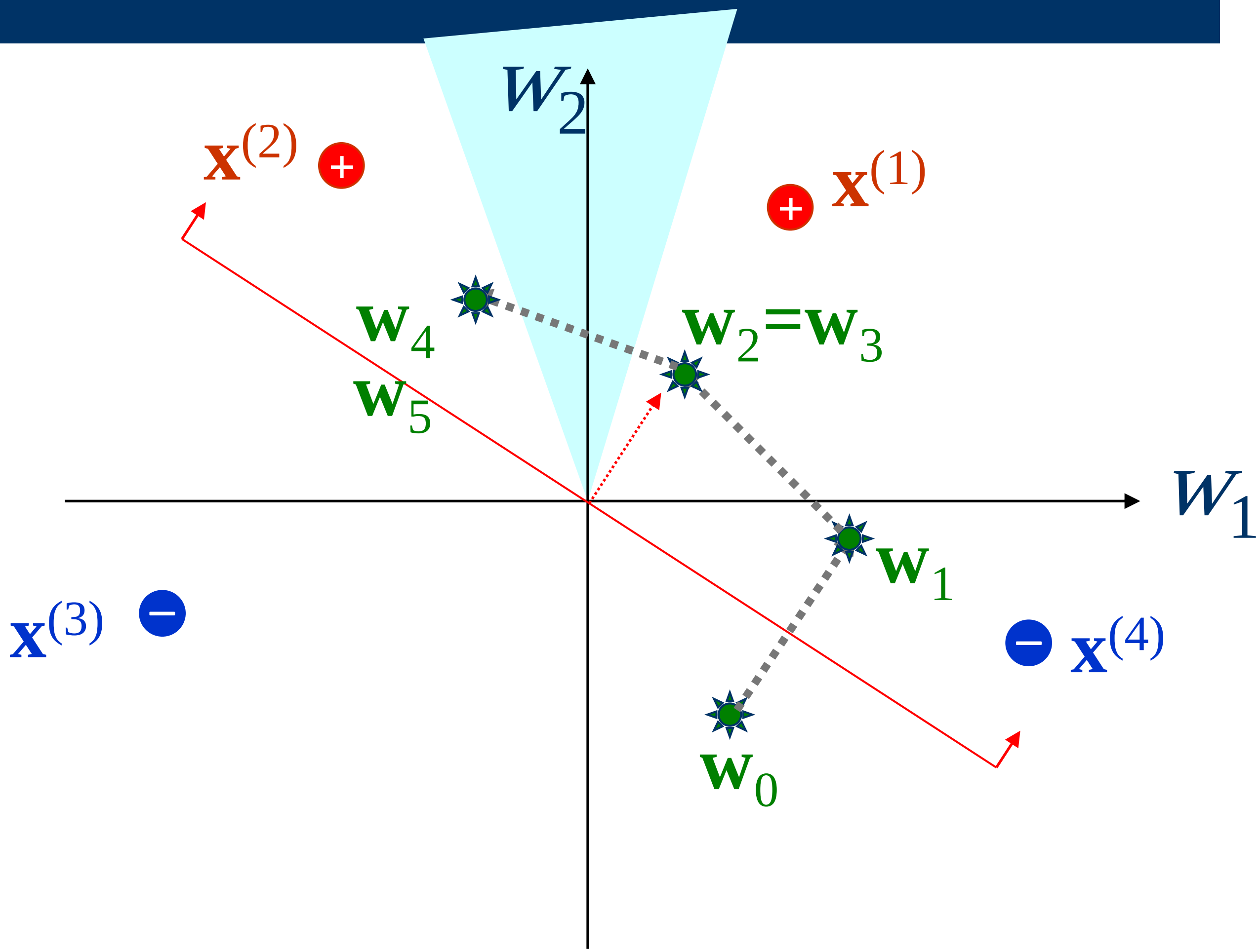
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



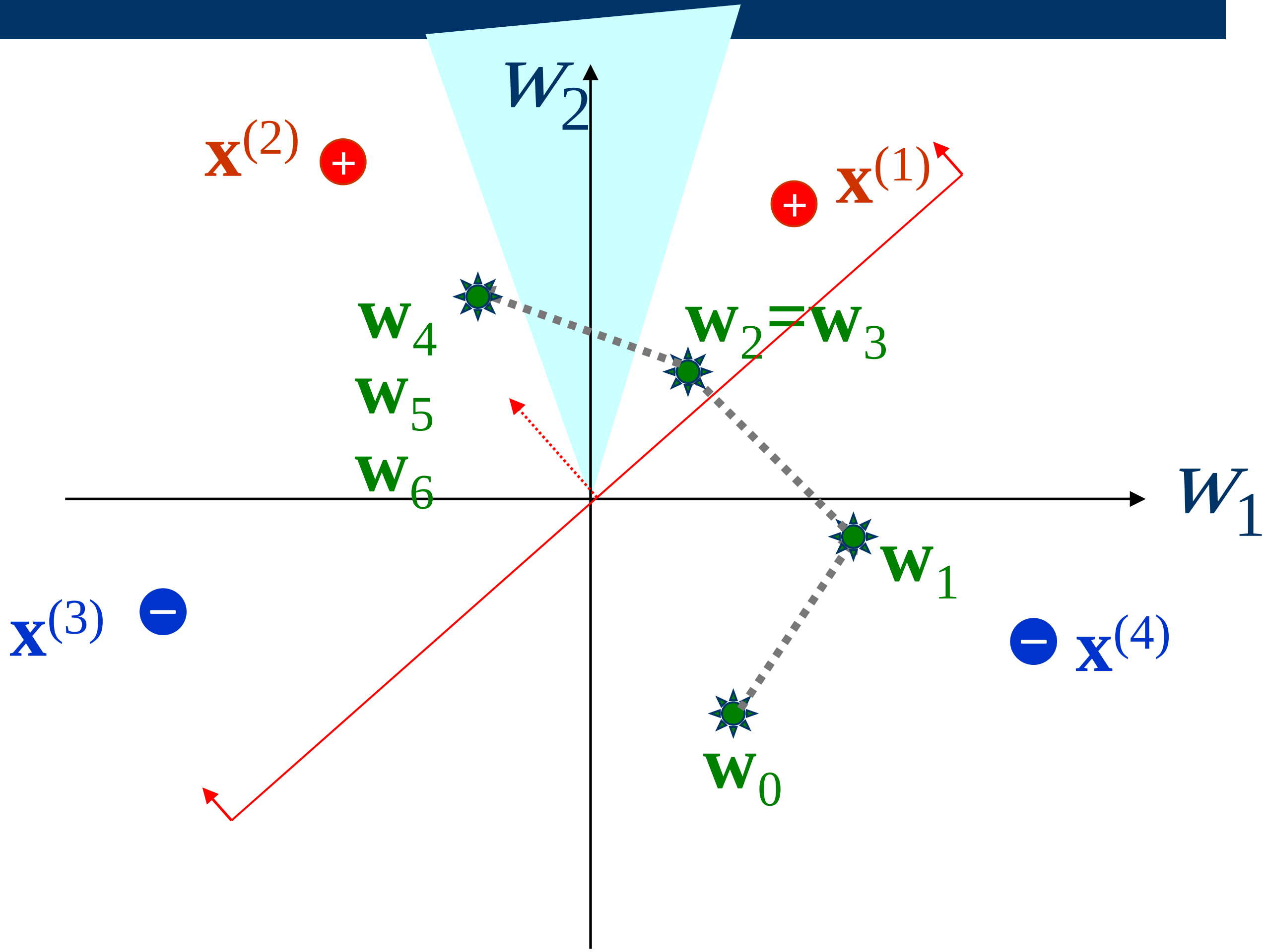
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



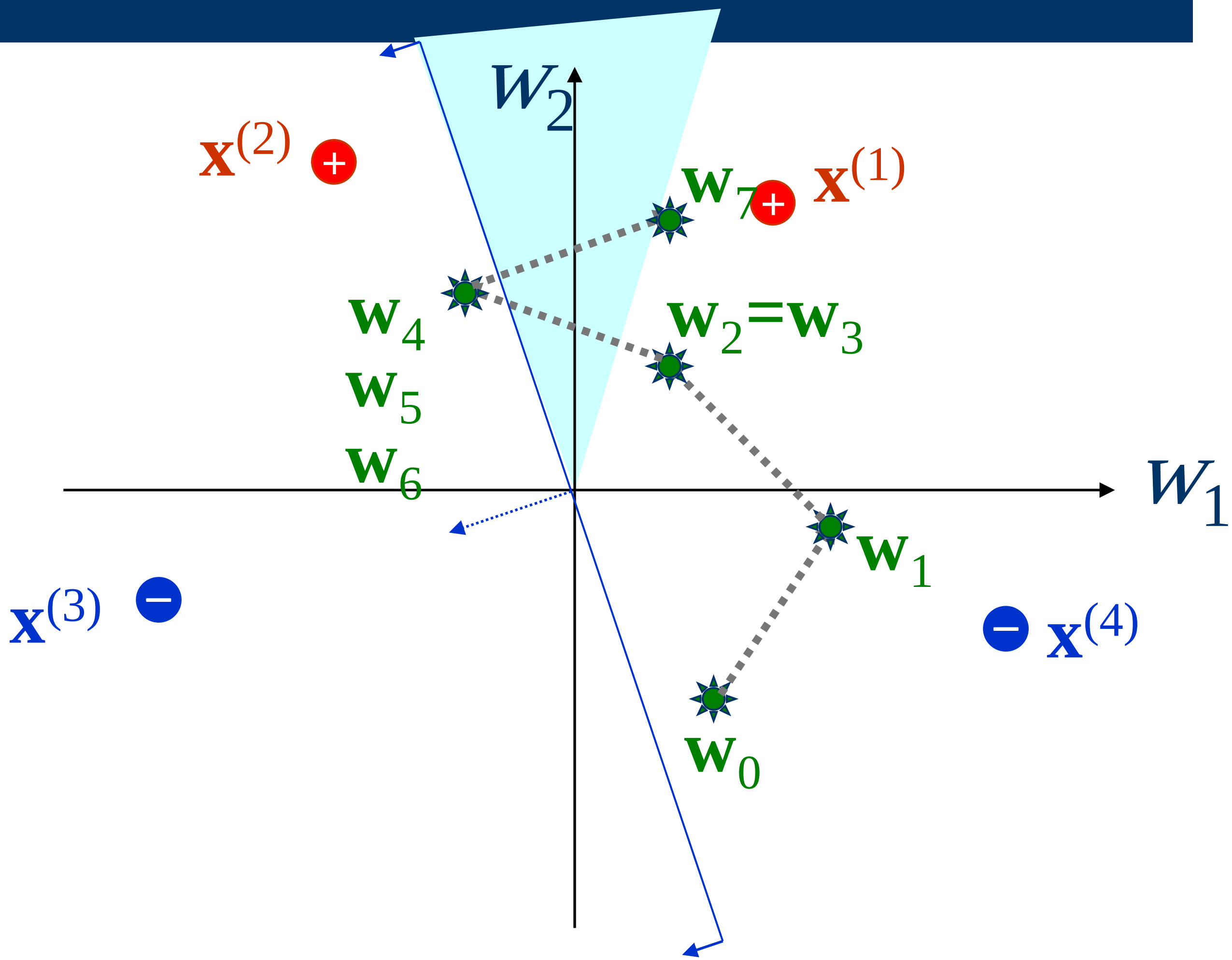
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



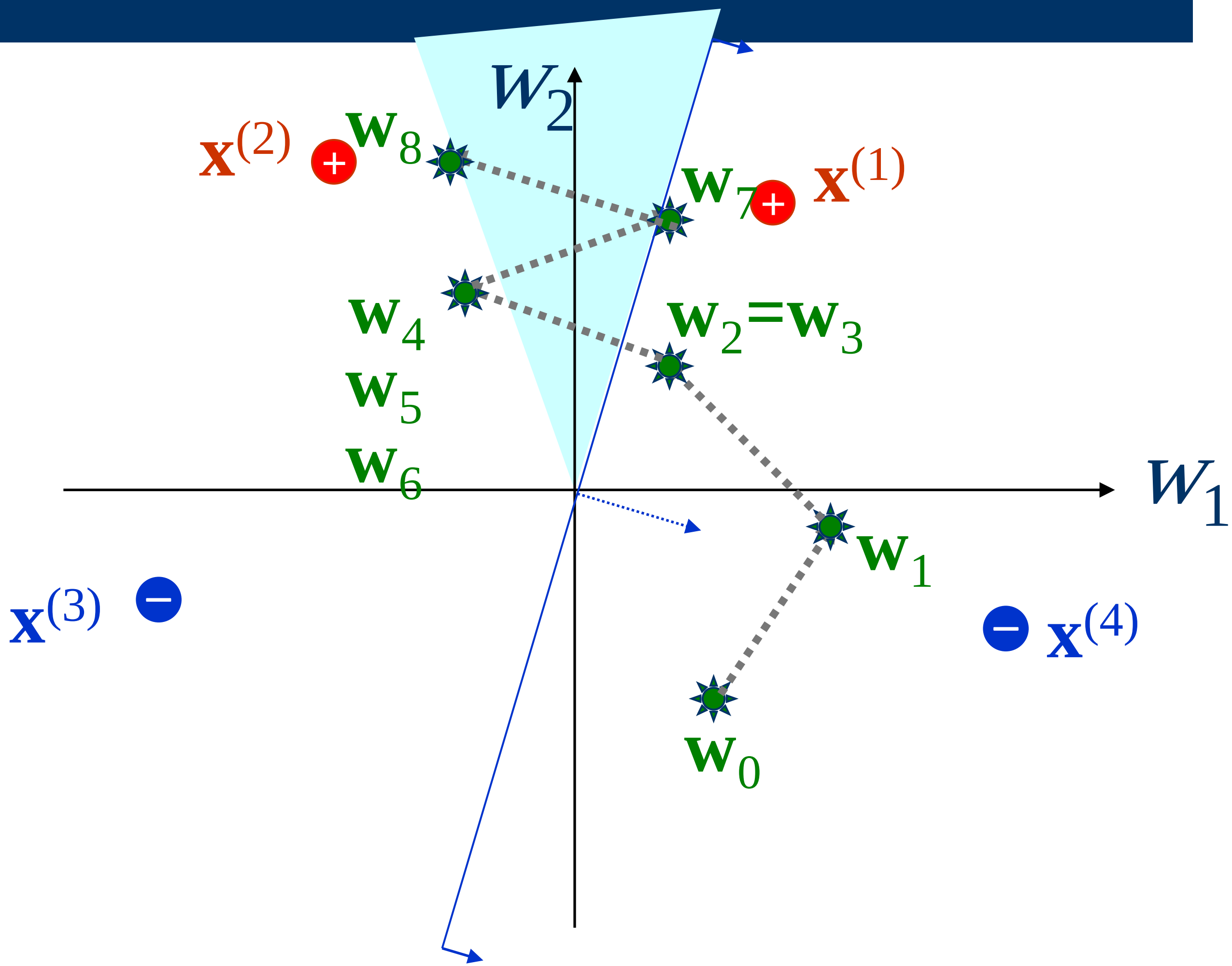
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



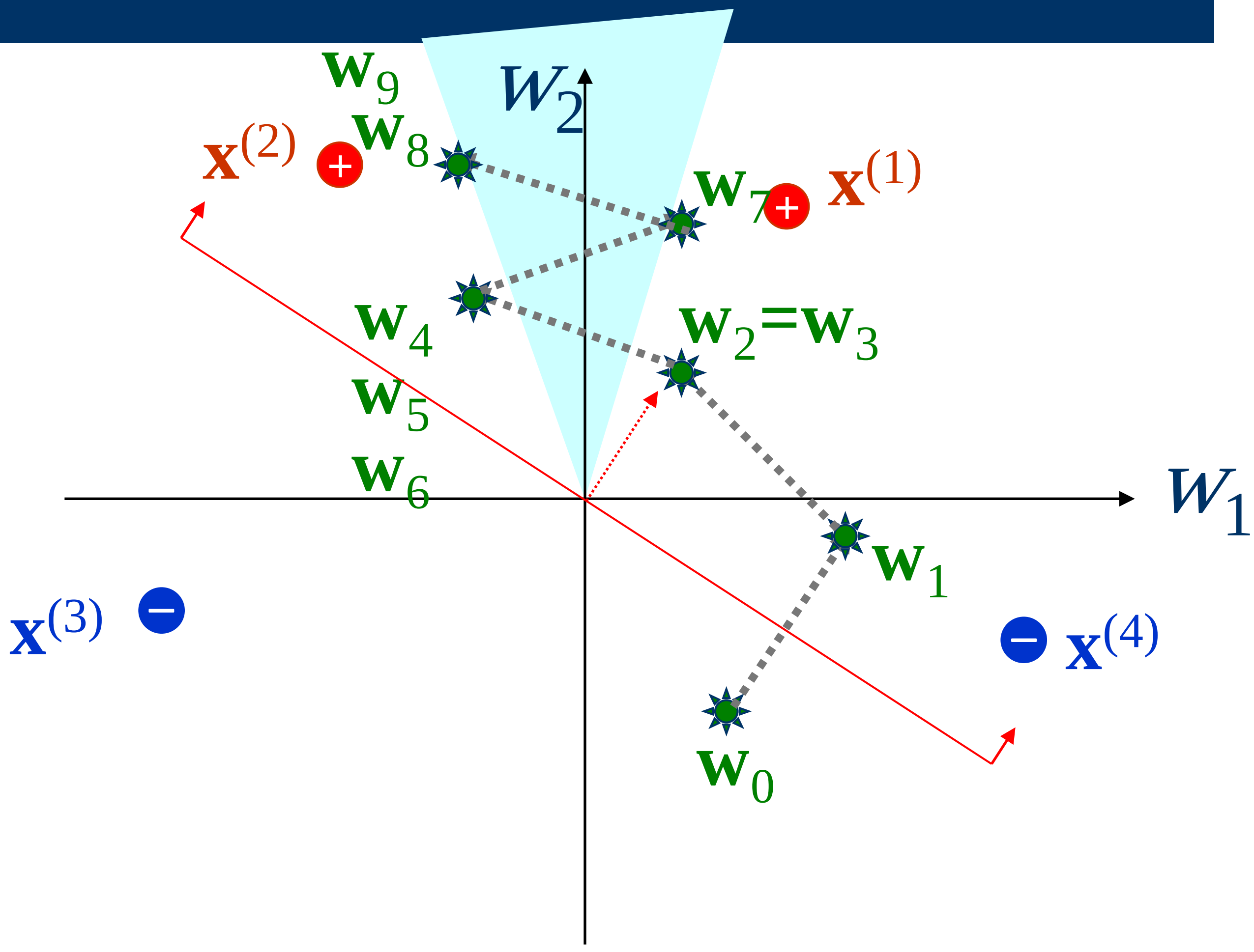
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



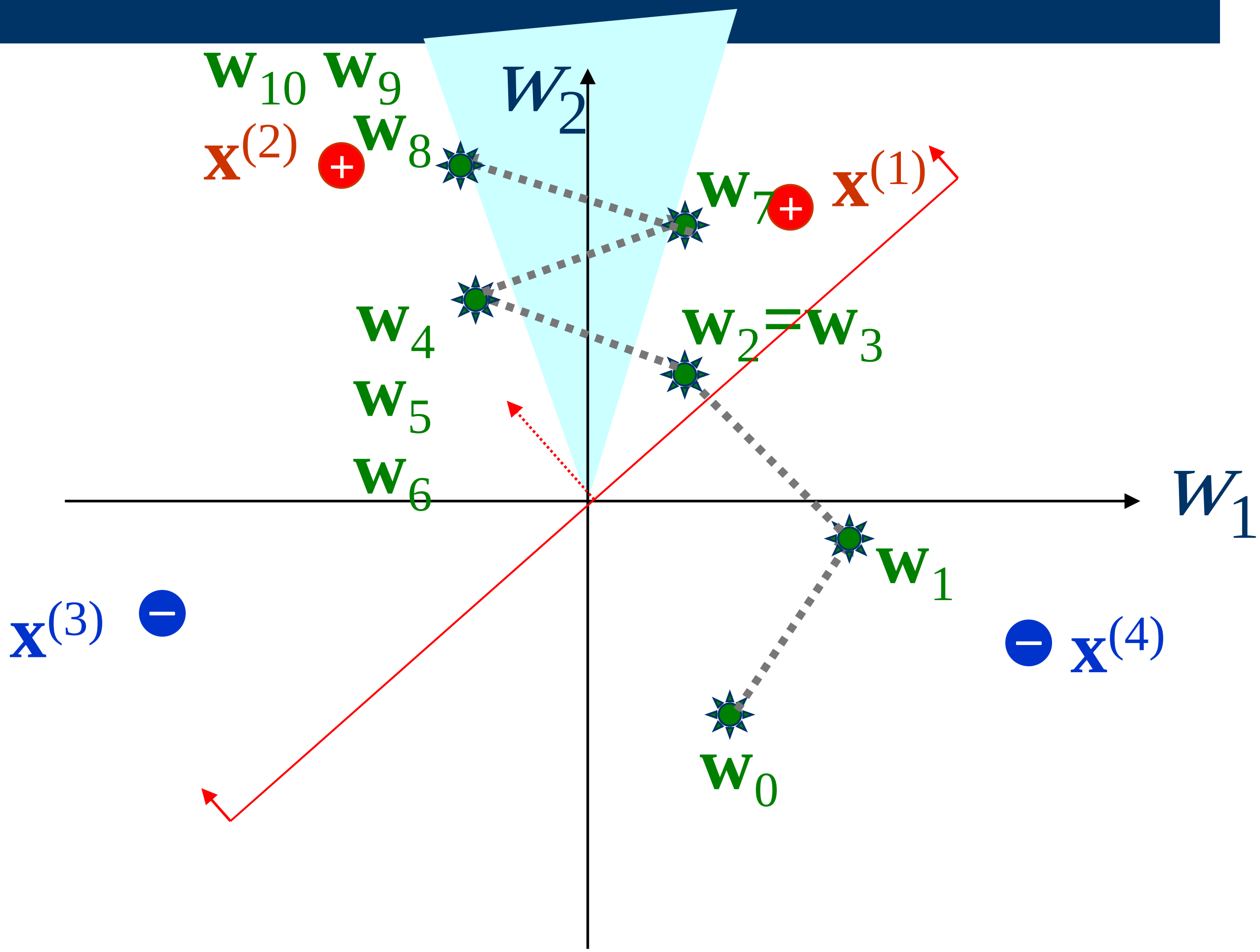
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



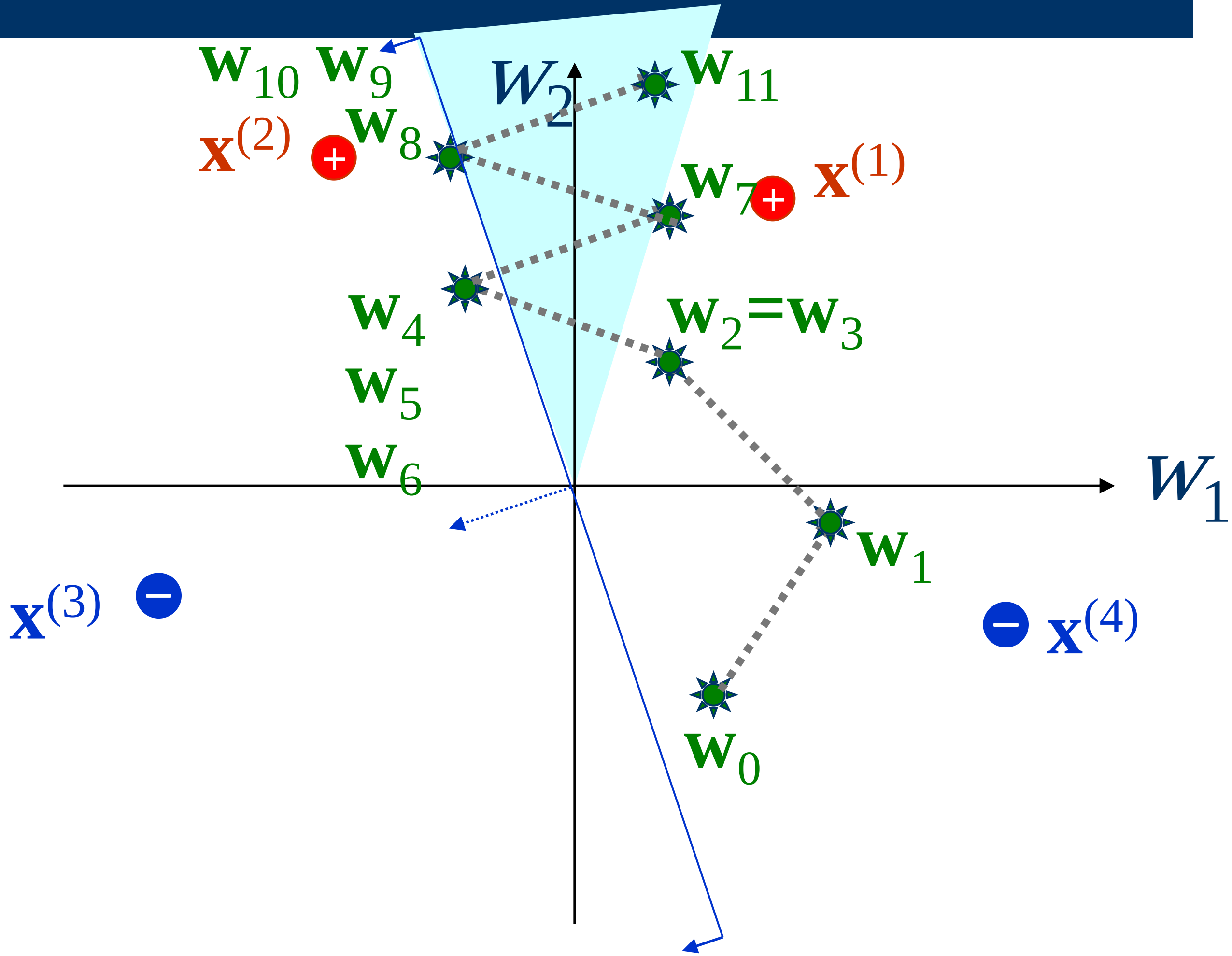
To correctly classify the training set, the weight must move into the shaded area.

# The Learning Scenario in Weight Space



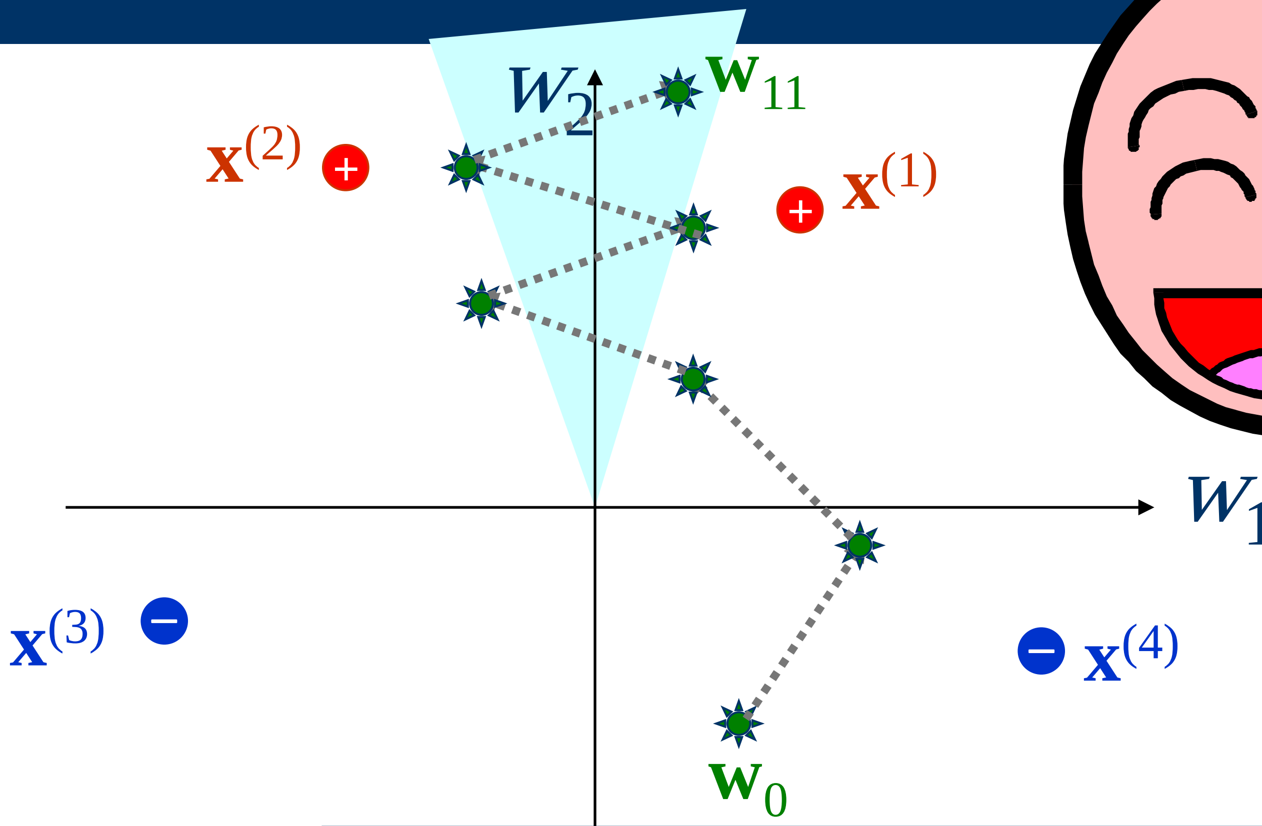
To correctly classify the training set, the weight must move into the shaded area.

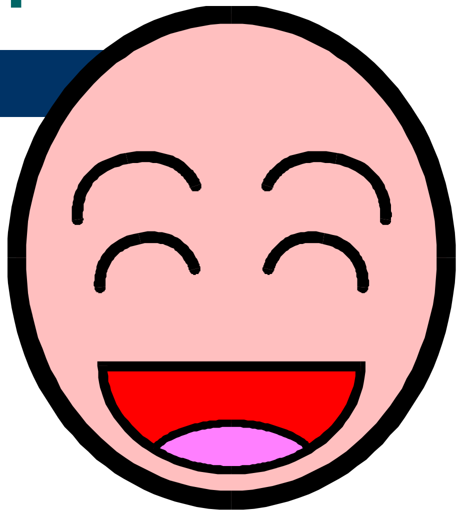
# The Learning Scenario in Weight Space



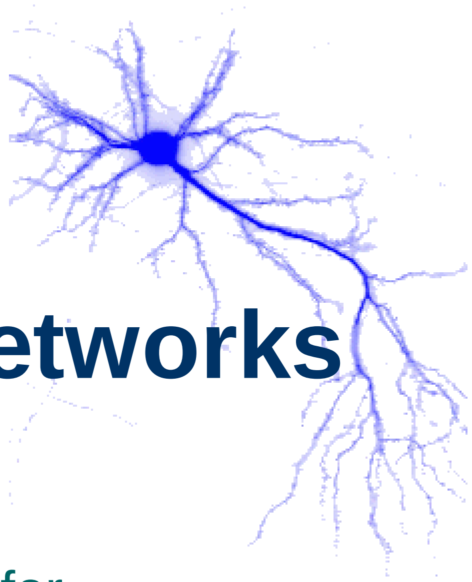
To correctly classify the training set, the weight must move into the shaded area.

## The Learning Scenario in Weight Space



Conceptually, in weight space, we move the weight into the feasible region.



# Feed-Forward Neural Networks

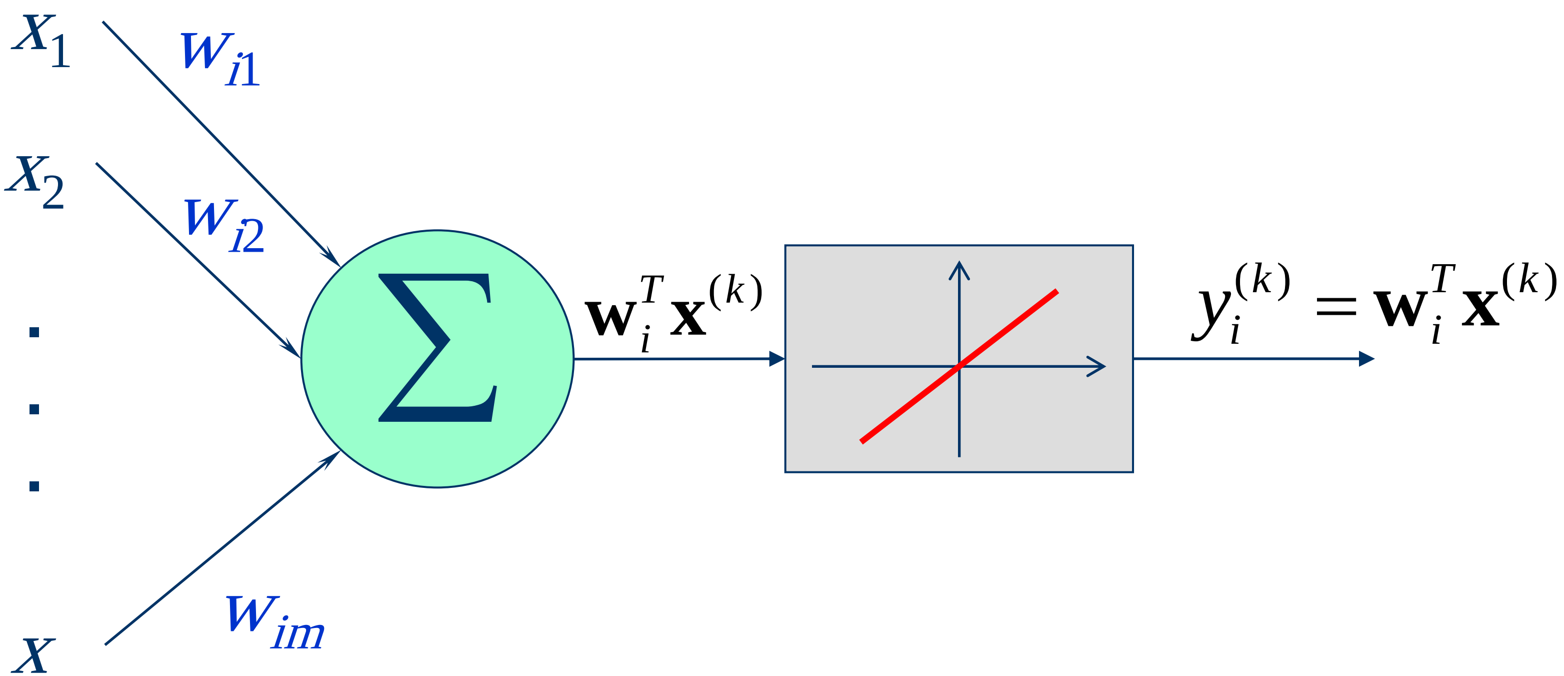
Learning Rules for

Single-Layered Perceptron Networks

- Perceptron Learning Rule
- Adaline Leaning Rule
- $\delta$-Learning Rule

# Adaline (Adaptive Linear Element)

Widrow [1962]



## In what condition, the goal is reachable?

# Adaline (Adaptive Linear Element)

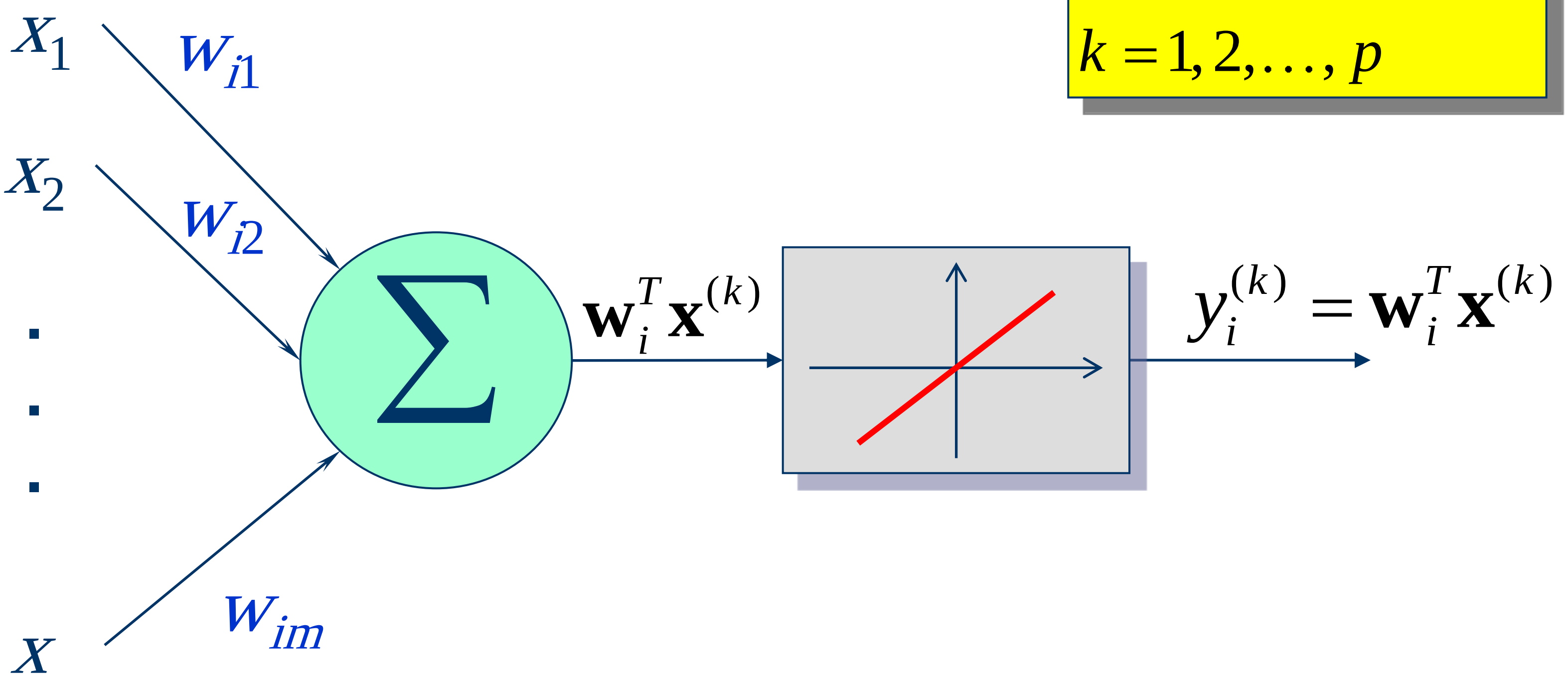
Widrow [1962]

**Goal:**

$$y_i^{(k)} = \mathbf{w}_i^T \mathbf{x}^{(k)} = d_i^{(k)}$$

$$i = 1, 2, \ldots, n$$

$$k = 1, 2, \ldots, p$$



# LMS (Least Mean Square)

Minimize the cost function (error function):

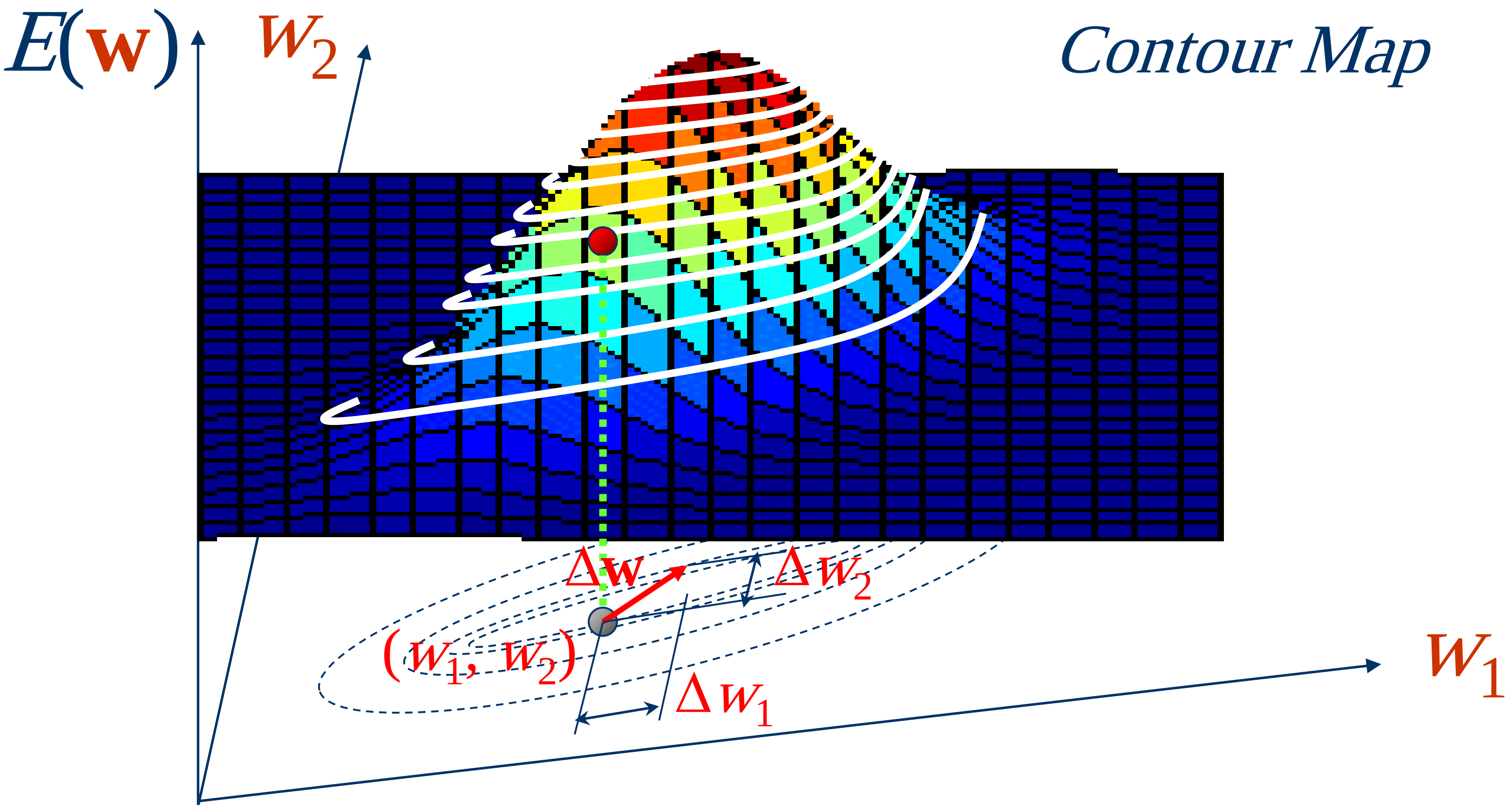
$$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}(d^{(k)} - y^{(k)})^2$$

$$= \frac{1}{2}\sum_{k=1}^{p}(d^{(k)} - \mathbf{w}^T\mathbf{x}^{(k)})^2$$

$$= \frac{1}{2}\sum_{k=1}^{p}\left(d^{(k)} - \sum_{l=1}^{m} w_l x_l^{(k)}\right)^2$$

Our goal is to go *downhill*.

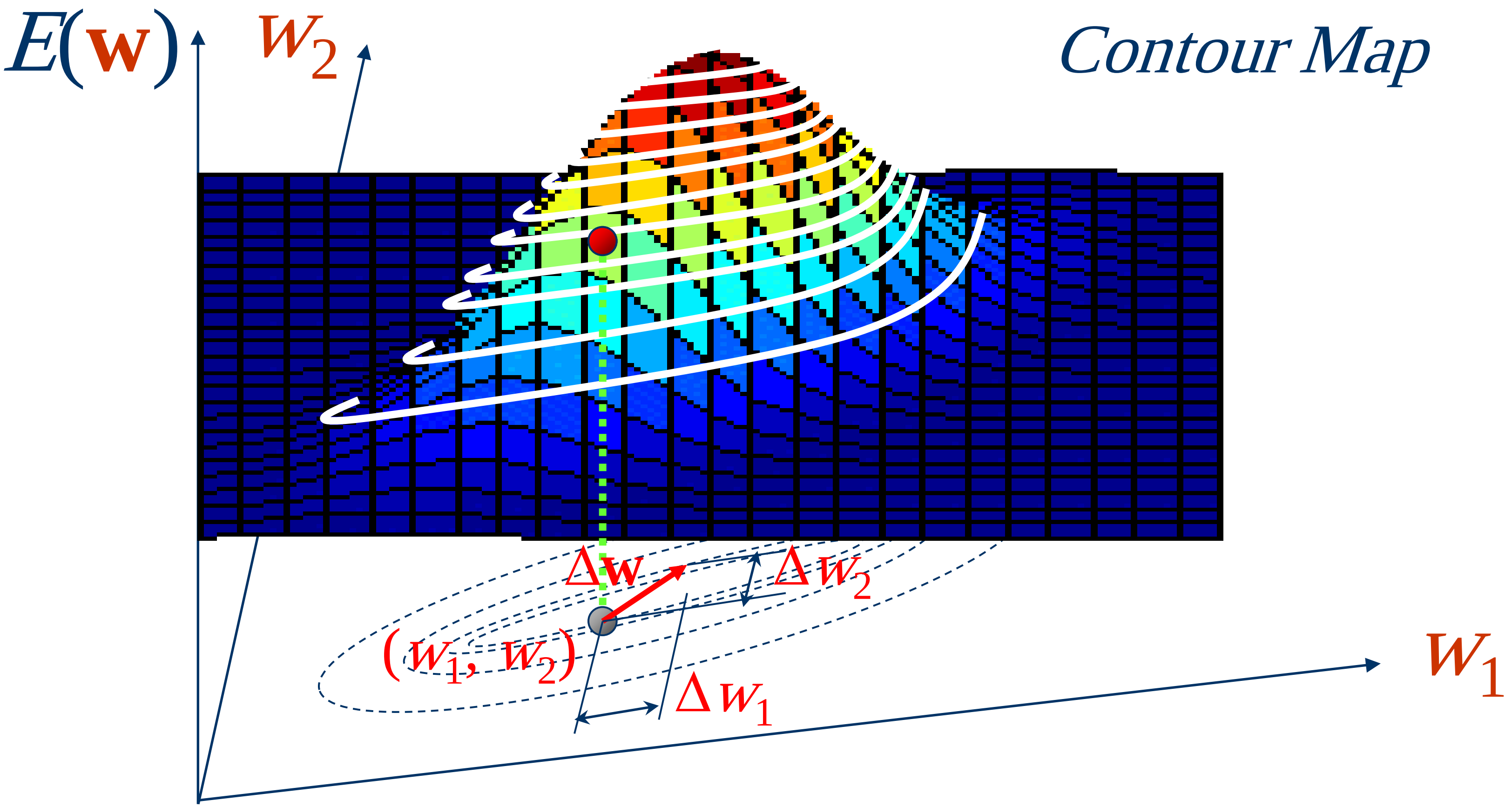
# Gradient Decent Algorithm



Our goal is to go *downhill*.

# Gradient Decent Algorithm

How to find the steepest decent direction?



# Gradient Operator

Let $f(\mathbf{w}) = f(w_1, w_2, \ldots, w_m)$ be a function over $R^m$.

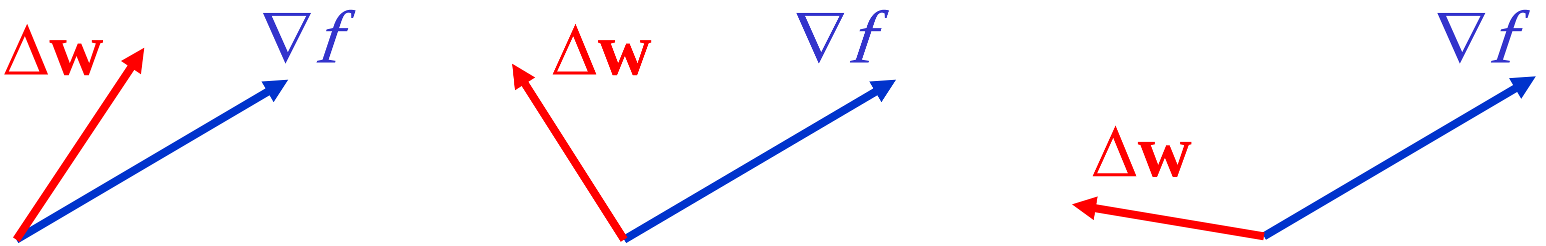
$$df = \frac{\partial f}{\partial w_1} dw_1 + \frac{\partial f}{\partial w_2} dw_2 + \cdots + \frac{\partial f}{\partial w_m} dw_m$$

Define $$\nabla f = \left( \frac{\partial f}{\partial w_1}, \frac{\partial f}{\partial w_2}, \cdots, \frac{\partial f}{\partial w_m} \right)^T$$

$$\Delta \mathbf{w} = \left( dw_1, dw_2, \cdots, dw_m \right)^T$$

$$df = \langle \nabla f, \Delta \mathbf{w} \rangle = \nabla f \bullet \Delta \mathbf{w}$$

# Gradient Operator




$df$: positive — Go uphill

$df$: zero — Plain

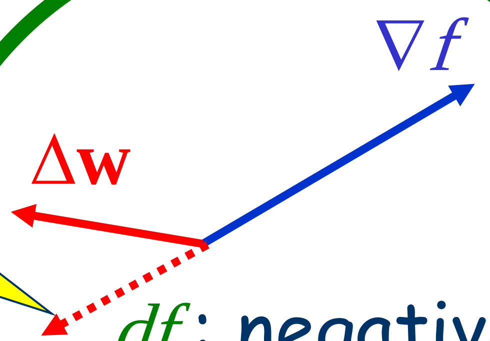
$df$: negative — Go downhill

$$df = \langle \nabla f, \Delta\mathbf{w} \rangle = \nabla f \bullet \Delta\mathbf{w}$$

# The Steepest Decent Direction

To minimize $f$, we choose

$$\Delta\mathbf{w} = -\eta \nabla f$$




| $df$: positive | $df$: zero | $df$: negative |
|---|---|---|
| Go uphill | Plain | Go downhill |

$$df = \langle \nabla f, \Delta\mathbf{w} \rangle = \nabla f \bullet \Delta\mathbf{w}$$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p} \delta^{(k)} x_j^{(k)} \qquad \delta^{(k)} = d^{(k)} - y^{(k)}$$

# LMS (Least Mean Square)

Minimize the cost function (error function):

$$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}\left(d^{(k)} - \sum_{l=1}^{m} w_l x_l^{(k)}\right)^2$$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p}\left(d^{(k)} - \sum_{l=1}^{m} w_l x_l^{(k)}\right)x_j^{(k)}$$

$$= -\sum_{k=1}^{p}\left(d^{(k)} - \mathbf{w}^T\mathbf{x}^{(k)}\right)x_j^{(k)} = -\sum_{k=1}^{p}\overbrace{\left(d^{(k)} - y^{(k)}\right)}^{\delta^{(k)}}x_j^{(k)}$$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p} \delta^{(k)} x_j^{(k)} \qquad \delta^{(k)} = d^{(k)} - y^{(k)}$$

# Adaline Learning Rule

Minimize the cost function (error function):

$$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}\left(d^{(k)} - \sum_{l=1}^{m} w_l x_l^{(k)}\right)^2$$

$$\nabla_w E(\mathbf{w}) = \left(\frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial E(\mathbf{w})}{\partial w_m}\right)^T$$

$$\Delta \mathbf{w} = -\eta \nabla_{\mathbf{w}} E(\mathbf{w})$$ — Weight Modification Rule

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p} \delta^{(k)} x_j^{(k)} \qquad \delta^{(k)} = d^{(k)} - y^{(k)}$$

# Learning Modes

- Batch Learning Mode:

$$\Delta w_j = \eta \sum_{k=1}^{p} \delta^{(k)} x_j^{(k)}$$

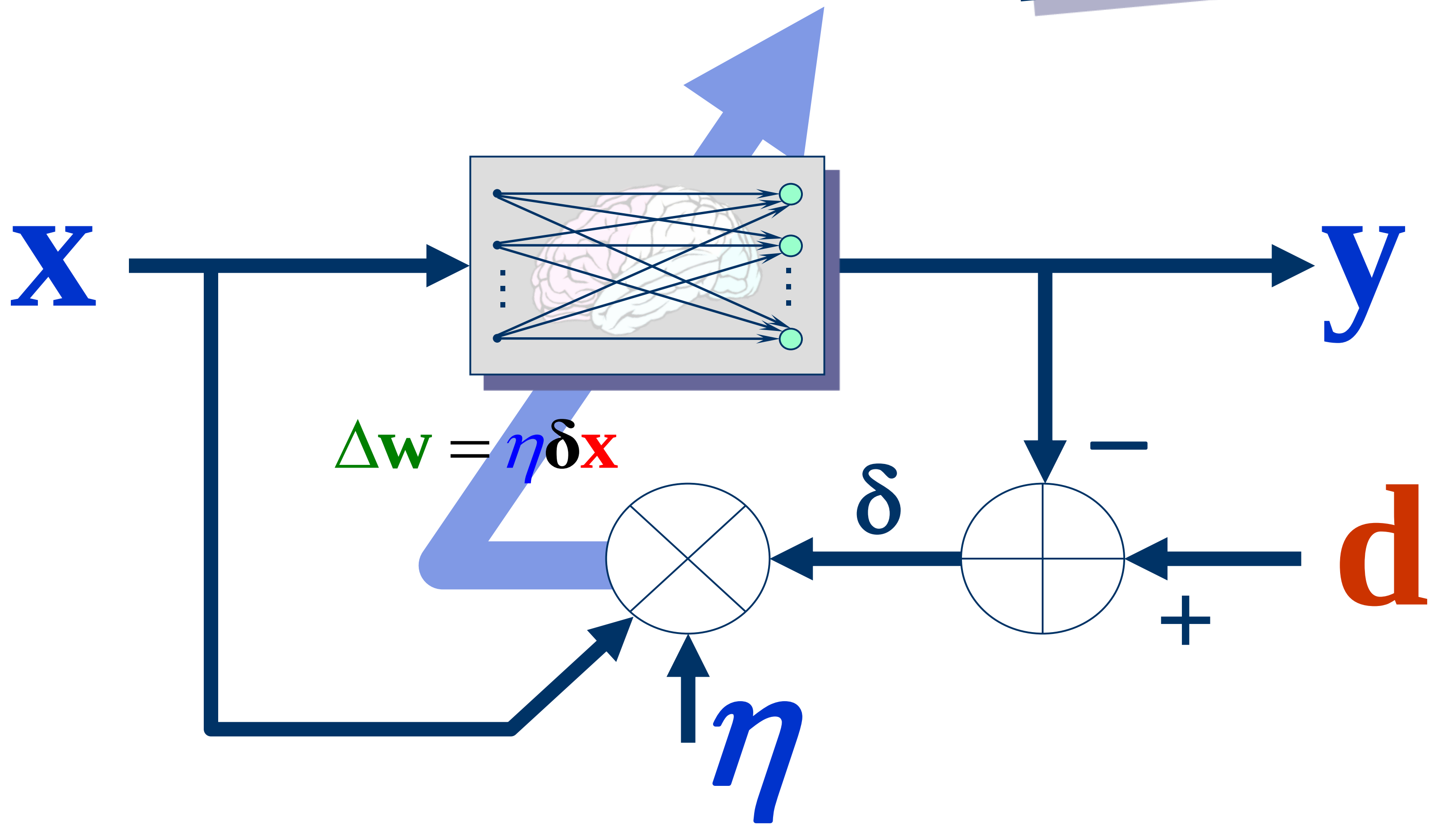
- Incremental Learning Mode:

$$\Delta w_j = \eta \delta^{(k)} x_j^{(k)}$$

δ-Learning Rule
LMS Algorithm
Widrow-Hoff Learning Rule

# Summary – Adaline Learning Rule

Converge?



# LMS Convergence

Based on the independence theory (Widrow, 1976).

1. The successive input vectors are statistically independent.
2. At time $t$, the input vector $\mathbf{x}(t)$ is statistically independent of all previous samples of the desired response, namely $d(1)$, $d(2)$, …, $d(t-1)$.
3. At time $t$, the desired response $d(t)$ is dependent on $\mathbf{x}(t)$, but statistically independent of all previous values of the desired response.
4. The input vector $\mathbf{x}(t)$ and desired response $d(t)$ are drawn from Gaussian distributed populations.

# LMS Convergence

It can be shown that LMS is convergent if

$$0 < \eta < \frac{2}{\lambda_{\max}}$$

where $\lambda_{max}$ is the largest eigenvalue of the correlation matrix $\mathbf{R}_{\mathbf{x}}$ for the inputs.

$$\mathbf{R}_{\mathbf{x}} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{\infty} \mathbf{x}_i \mathbf{x}_i^T$$

Since $\lambda_{max}$ is hardly available, we commonly use

$$0 < \eta < \frac{2}{tr(\mathbf{R_x})}$$

# LMS Convergence

It can be shown that LMS is convergent if

$$0 < \eta < \frac{2}{\lambda_{\max}}$$

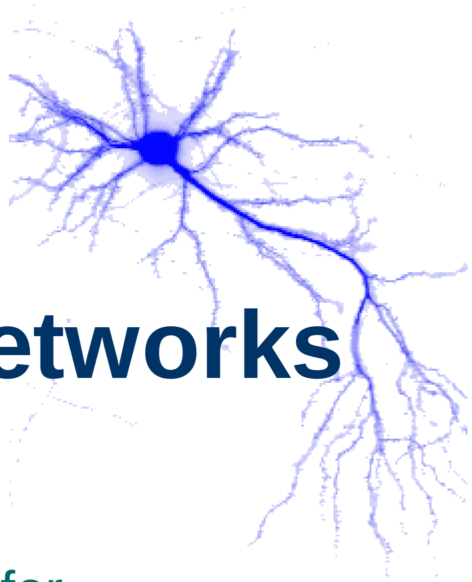
where $\boxed{\lambda_{max} \text{ is the largest eigenvalue}}$ of the correlation matrix $\mathbf{R_x}$ for the inputs.

$$\mathbf{R_x} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{\infty} \mathbf{x}_i \mathbf{x}_i^T$$

# Comparisons

| | Perceptron Learning Rule | Adaline Learning Rule (Widrow-Hoff) |
|---|---|---|
| Fundamental | Hebbian Assumption | Gradient Decent |
| Convergence | In finite steps | Converge Asymptotically |
| Constraint | Linearly Separable | Linear Independence |

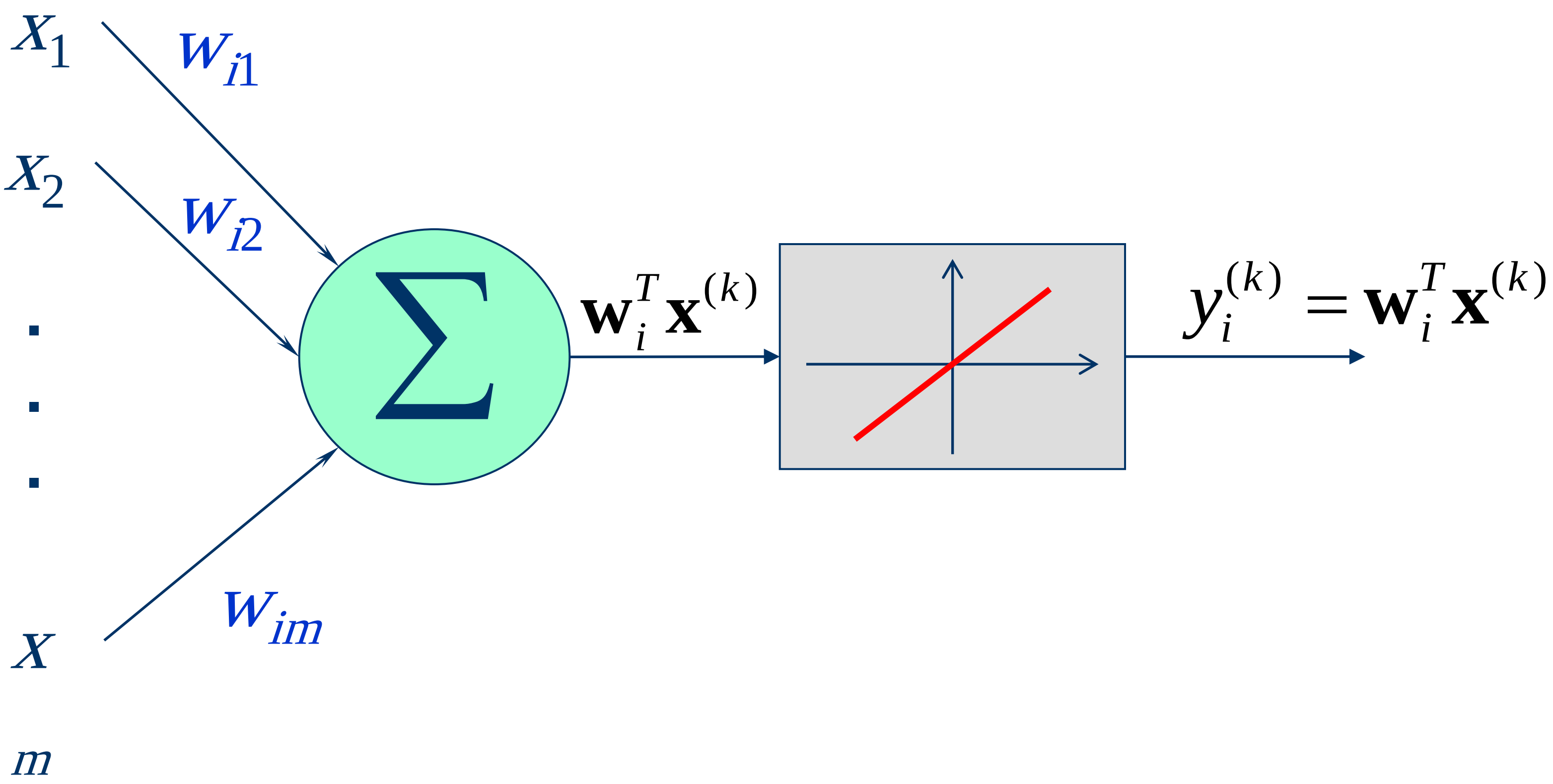
# Feed-Forward Neural Networks



Learning Rules for
Single-Layered Perceptron Networks

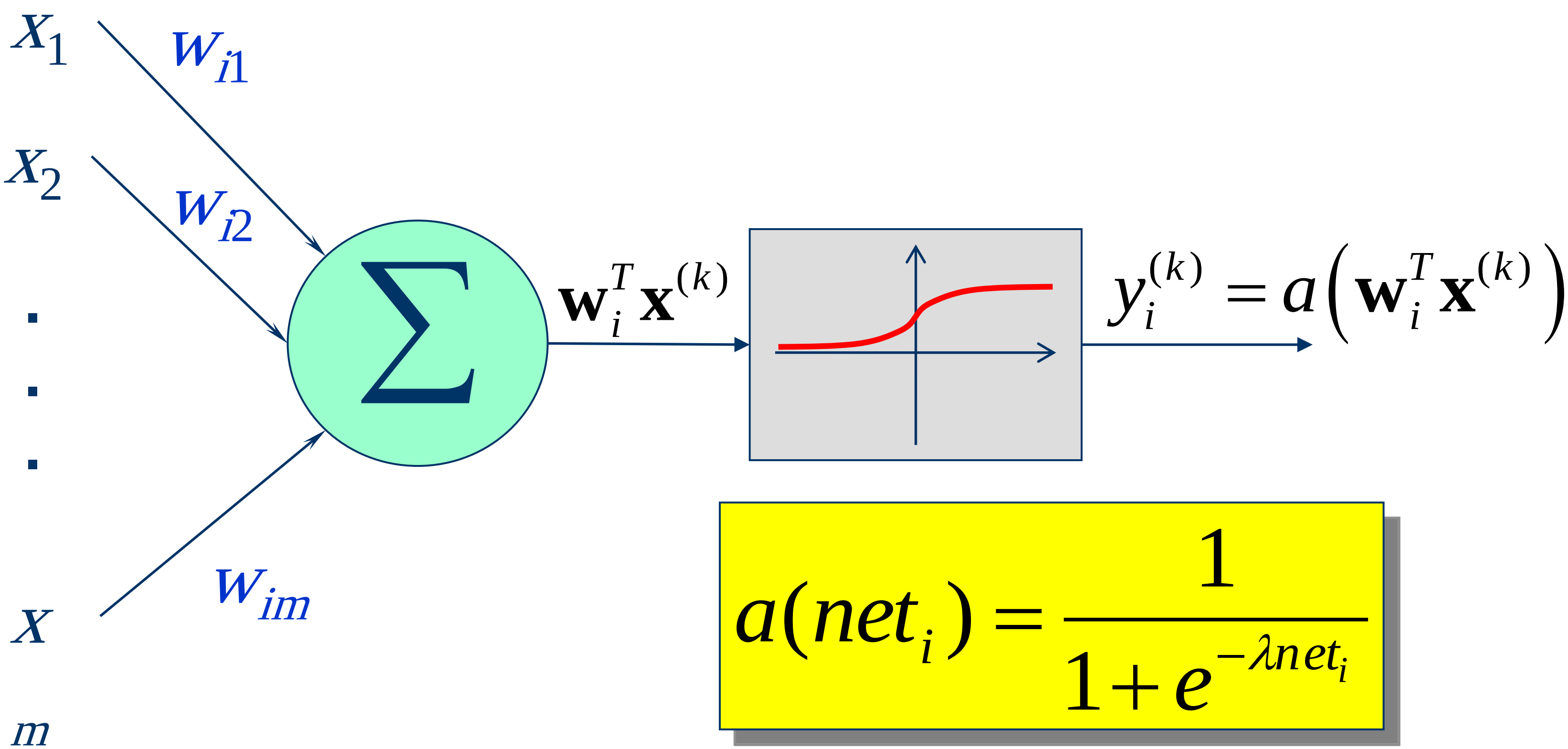
- Perceptron Learning Rule
- Adaline Leaning Rule
- $\delta$-Learning Rule

# Adaline



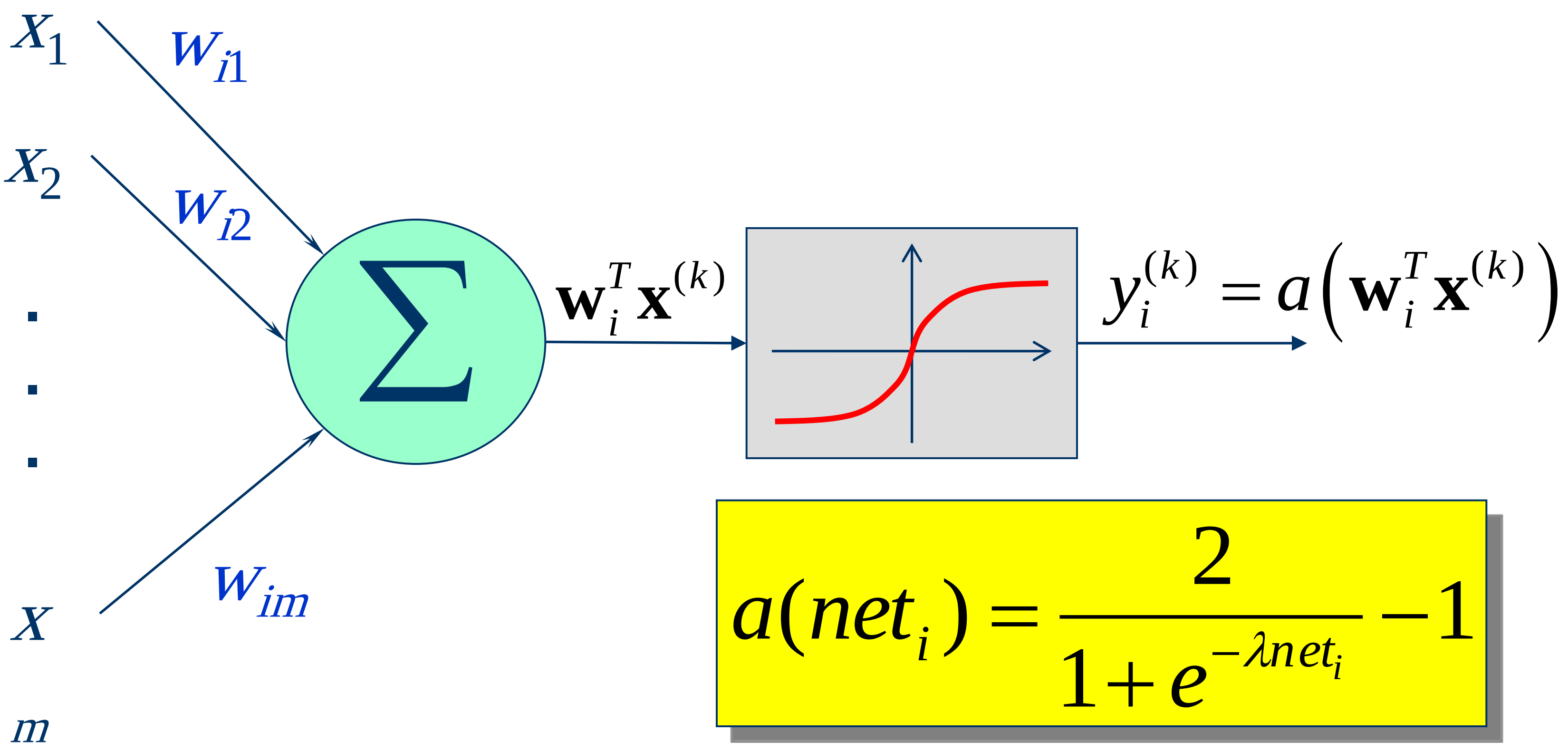
$x_1$
$w_{i1}$
$x_2$
$w_{i2}$
.
.
.
$x_m$
$w_{im}$
$\Sigma$
$\mathbf{w}_i^T \mathbf{x}^{(k)}$
$y_i^{(k)} = \mathbf{w}_i^T \mathbf{x}^{(k)}$

# Unipolar Sigmoid




$$a(net_i) = \frac{1}{1 + e^{-\lambda net_i}}$$

# Bipolar Sigmoid

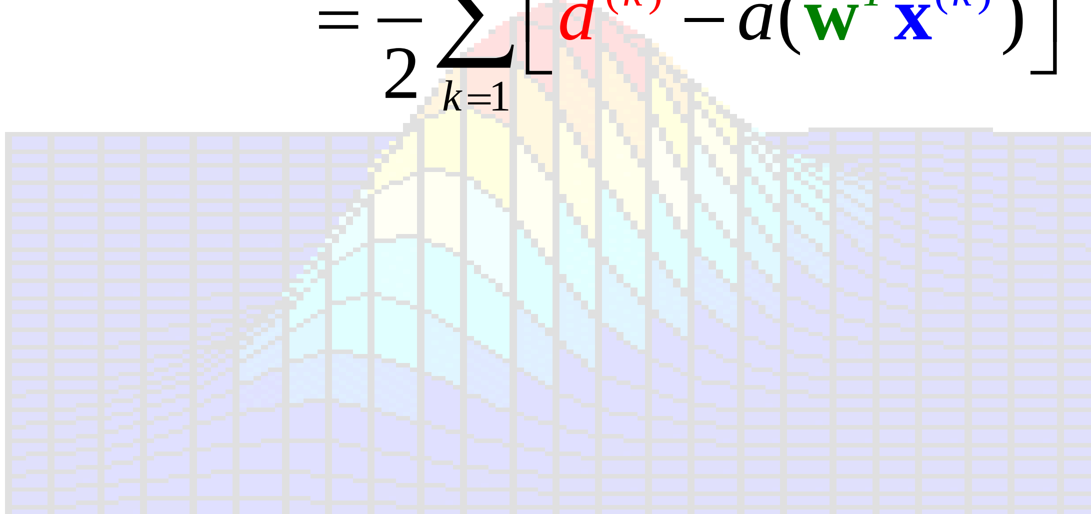



$$a(net_i) = \frac{2}{1+e^{-\lambda net_i}} - 1$$

# Goal

Minimize $$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}\left(d^{(k)} - y^{(k)}\right)^2$$

$$= \frac{1}{2}\sum_{k=1}^{p}\left[d^{(k)} - a\left(\mathbf{w}^T\mathbf{x}^{(k)}\right)\right]^2$$

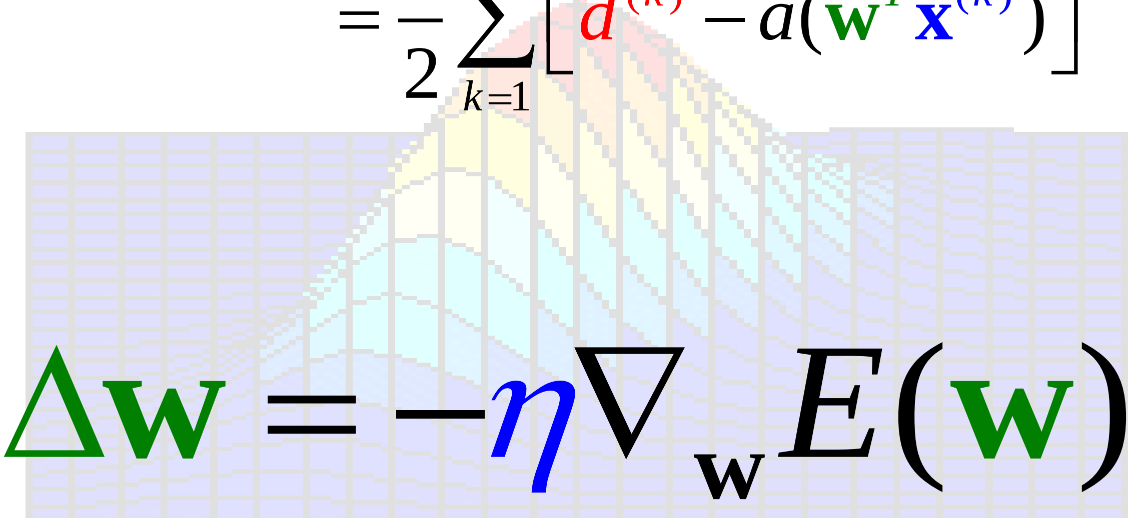


$$\nabla_w E(\mathbf{w}) = \left( \frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial E(\mathbf{w})}{\partial w_m} \right)^T$$

# Gradient Decent Algorithm

Minimize $$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}\left(d^{(k)} - y^{(k)}\right)^2$$

$$= \frac{1}{2}\sum_{k=1}^{p}\left[d^{(k)} - a\left(\mathbf{w}^T\mathbf{x}^{(k)}\right)\right]^2$$



$$\Delta\mathbf{w} = -\eta\nabla_{\mathbf{w}} E(\mathbf{w})$$

$$\nabla_w E(\mathbf{w}) = \left( \frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial E(\mathbf{w})}{\partial w_m} \right)^T$$

# The Gradient

$$y^{(k)} = a\left(\mathbf{w}^T \mathbf{x}^{(k)}\right)$$

Minimize $E(\mathbf{w}) = \frac{1}{2} \sum_{k=1}^{p} (d^{(k)} - y^{(k)})^2$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p} (d^{(k)} - y^{(k)}) \frac{\partial y^{(k)}}{\partial w_j}$$

$$= -\sum_{k=1}^{p} (d^{(k)} - y^{(k)}) \underbrace{\frac{\partial a\left(net^{(k)}\right)}{\partial net^{(k)}}}_{?} \underbrace{\frac{\partial net^{(k)}}{\partial w_j}}_{?}$$

Depends on the activation function used.

$$net^{(k)} = \mathbf{w}^T \mathbf{x}^{(k)} = \sum_{i=1}^{m} w_i x_i^{(k)} \Rightarrow \frac{\partial net^{(k)}}{\partial w_j} = x_j^{(k)}$$

$$\nabla_w E(\mathbf{w}) = \left( \frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \dots, \frac{\partial E(\mathbf{w})}{\partial w_m} \right)^T$$

# Weight Modification Rule

$$y^{(k)} = a\left(net^{(k)}\right)$$

$$\delta^{(k)} = d^{(k)} - y^{(k)}$$

Minimize $$E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}(d^{(k)} - y^{(k)})^2$$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p}(d^{(k)} - y^{(k)})x_j^{(k)} \frac{\partial a\left(net^{(k)}\right)}{\partial net^{(k)}}$$

Learning Rule

Batch $$\Delta w_j = \eta \sum_{k=1}^{p} \delta^{(k)} x_j^{(k)} \frac{\partial a\left(net^{(k)}\right)}{\partial net^{(k)}}$$

Incremental $$\Delta w_j = \eta \delta^{(k)} x_j^{(k)} \frac{\partial a\left(net^{(k)}\right)}{\partial net^{(k)}}$$

$$\nabla_w E(\mathbf{w}) = \left( \frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial E(\mathbf{w})}{\partial w_m} \right)^T$$

# The Learning Efficacy

$$y^{(k)} = a\left(net^{(k)}\right)$$

Minimize $E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}(d^{(k)} - y^{(k)})^2$

$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p}(d^{(k)} - y^{(k)})x_j^{(k)} \frac{\partial a\left(net^{(k)}\right)}{\partial net^{(k)}}$$

Sigmoid

| Adaline | Unipolar | Bipolar |
|---|---|---|
| $a(net) = net$ | $a(net) = \frac{1}{1+e^{-\lambda net}}$ | $a(net) = \frac{2}{1+e^{-\lambda net}} - 1$ |
| $\frac{\partial a(net)}{\partial net} = 1$ | $\frac{\partial a(net)}{\partial net} = \lambda y^{(k)}(1 - y^{(k)})$ | **Exercise** |

$$\nabla_w E(\mathbf{w}) = \left( \frac{\partial E(\mathbf{w})}{\partial w_1}, \frac{\partial E(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial E(\mathbf{w})}{\partial w_m} \right)^T$$

# Learning Rule – Unipolar Sigmoid

$$\delta^{(k)} = d^{(k)} - y^{(k)}$$

Minimize $E(\mathbf{w}) = \frac{1}{2}\sum_{k=1}^{p}(d^{(k)} - y^{(k)})^2$

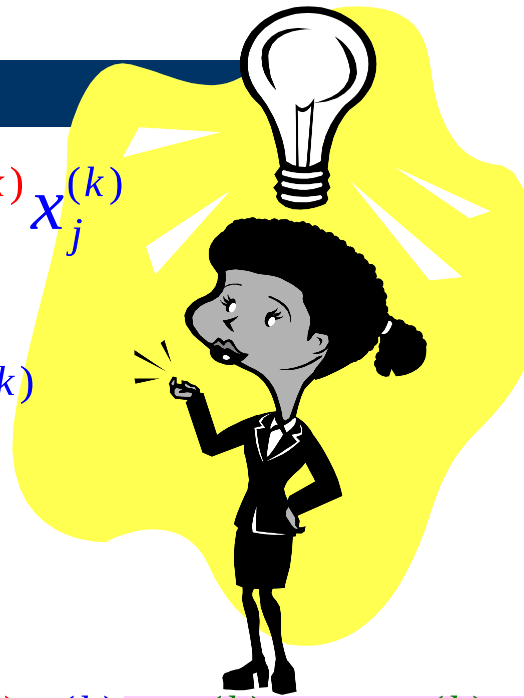
$$\frac{\partial E(\mathbf{w})}{\partial w_j} = -\sum_{k=1}^{p}(d^{(k)} - y^{(k)})x_j^{(k)}\lambda y^{(k)}(1 - y^{(k)})$$

$$= -\sum_{k=1}^{p}\delta^{(k)}x_j^{(k)}\lambda y^{(k)}(1 - y^{(k)})$$

$$\Delta w_j = \eta\sum_{k=1}^{p}\delta^{(k)}x_j^{(k)}\lambda y^{(k)}(1 - y^{(k)})$$ — Weight Modification Rule

# Comparisons

$$\lambda y^{(k)}(1-y^{(k)})$$



**Adaline**

Batch $$\Delta w_j = \eta \sum_{k=1}^{p} \delta^{(k)} x_j^{(k)}$$

Incremental $$\Delta w_j = \eta \delta^{(k)} x_j^{(k)}$$

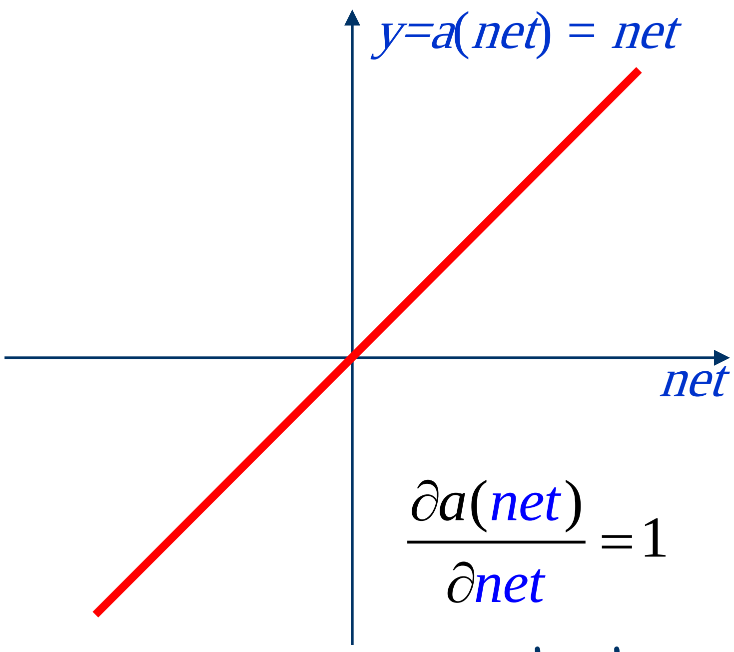
**Sigmoid**

Batch $$\Delta w_j = \eta \sum_{k=1}^{p} \delta^{(k)} x_j^{(k)} \lambda y^{(k)}(1-y^{(k)})$$

Incremental $$\Delta w_j = \eta \delta^{(k)} x_j^{(k)} \lambda y^{(k)}(1-y^{(k)})$$

# The Learning Efficacy

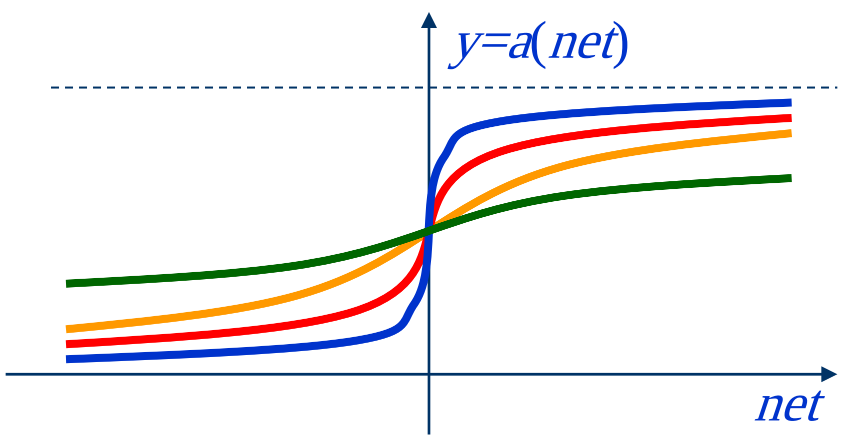
## Adaline




$$\frac{\partial a(net)}{\partial net} = 1$$

constant

## Sigmoid

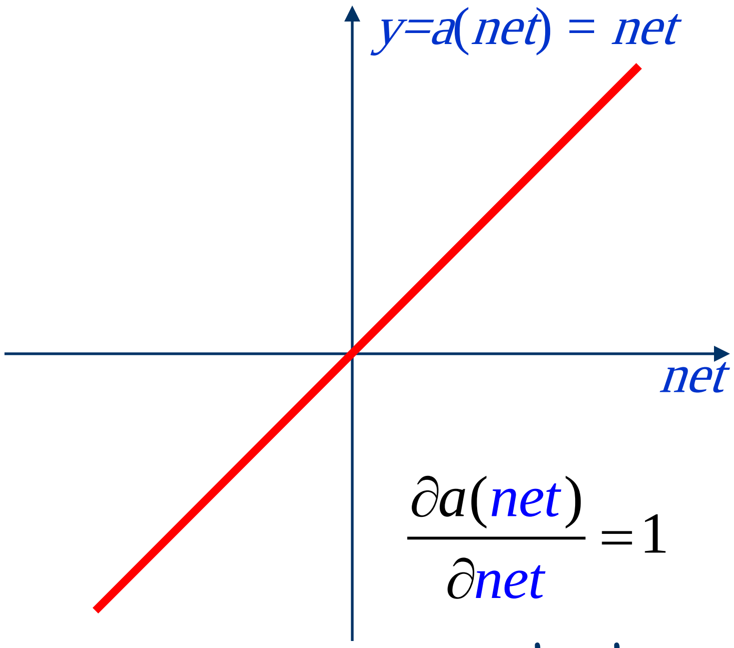



$$\frac{\partial a(net)}{\partial net} = \lambda y(1-y)$$

depends on output

# The Learning Efficacy

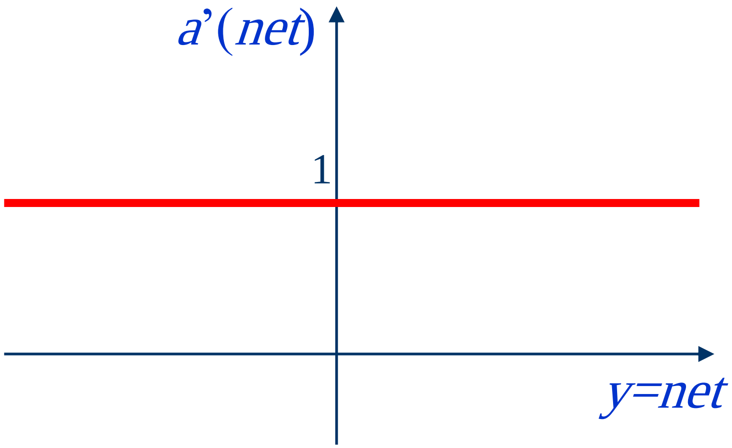
Adaline




$$\frac{\partial a(net)}{\partial net} = 1$$

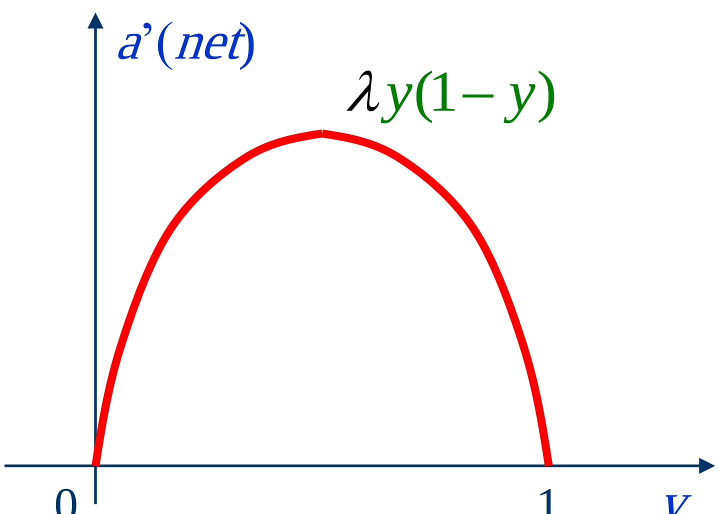
constant

The learning efficacy of Adaline is constant meaning that the Adline will never get saturated.

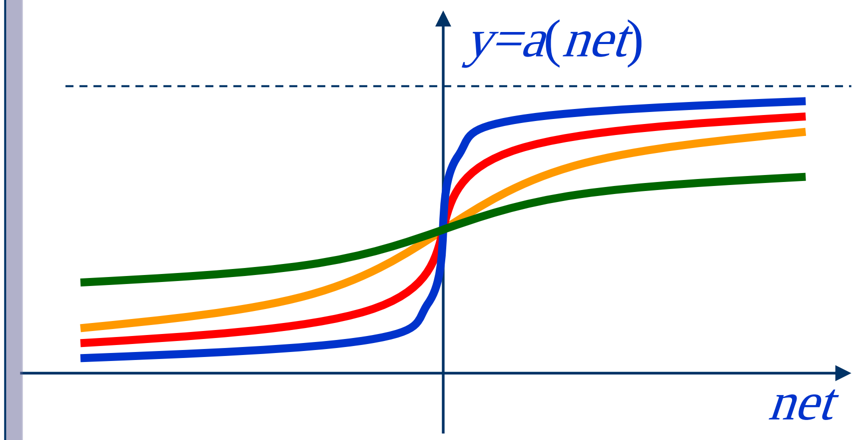


# The Learning Efficacy

The sigmoid will get saturated if its output value nears the two extremes.




## Sigmoid



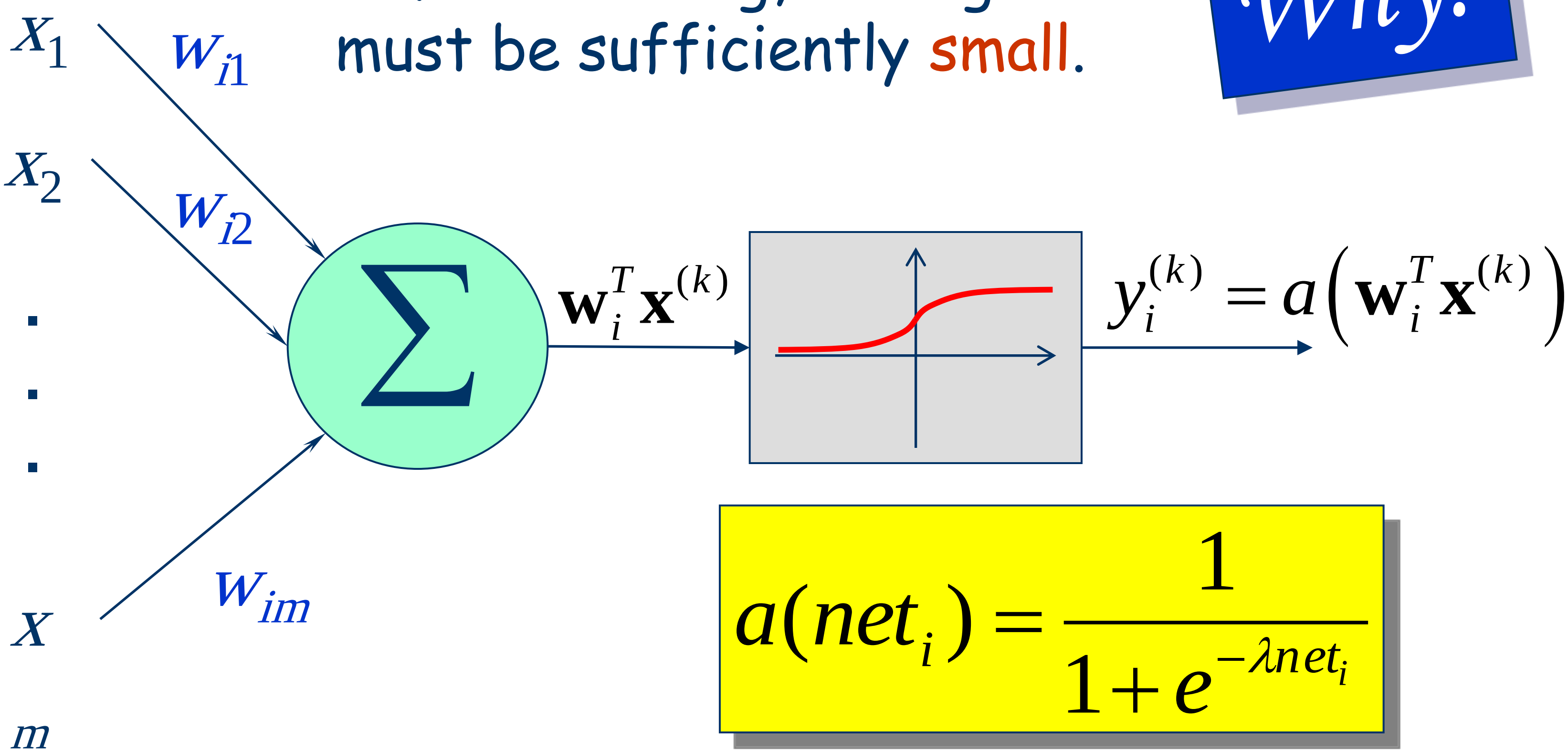

$$\frac{\partial a(net)}{\partial net} = \lambda y(1-y)$$

depends on output

# Initialization for Sigmoid Neurons

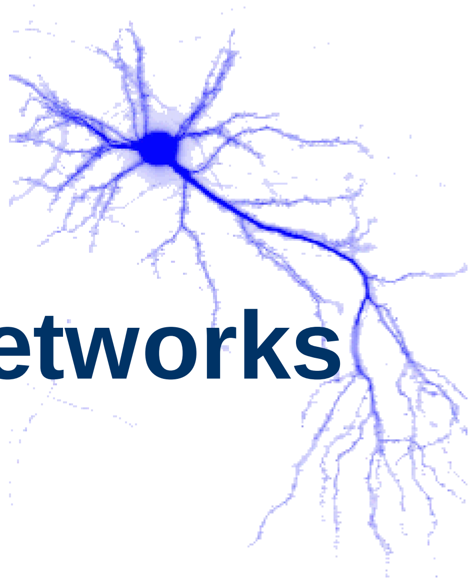
Before training, it weight must be sufficiently small.

Why?



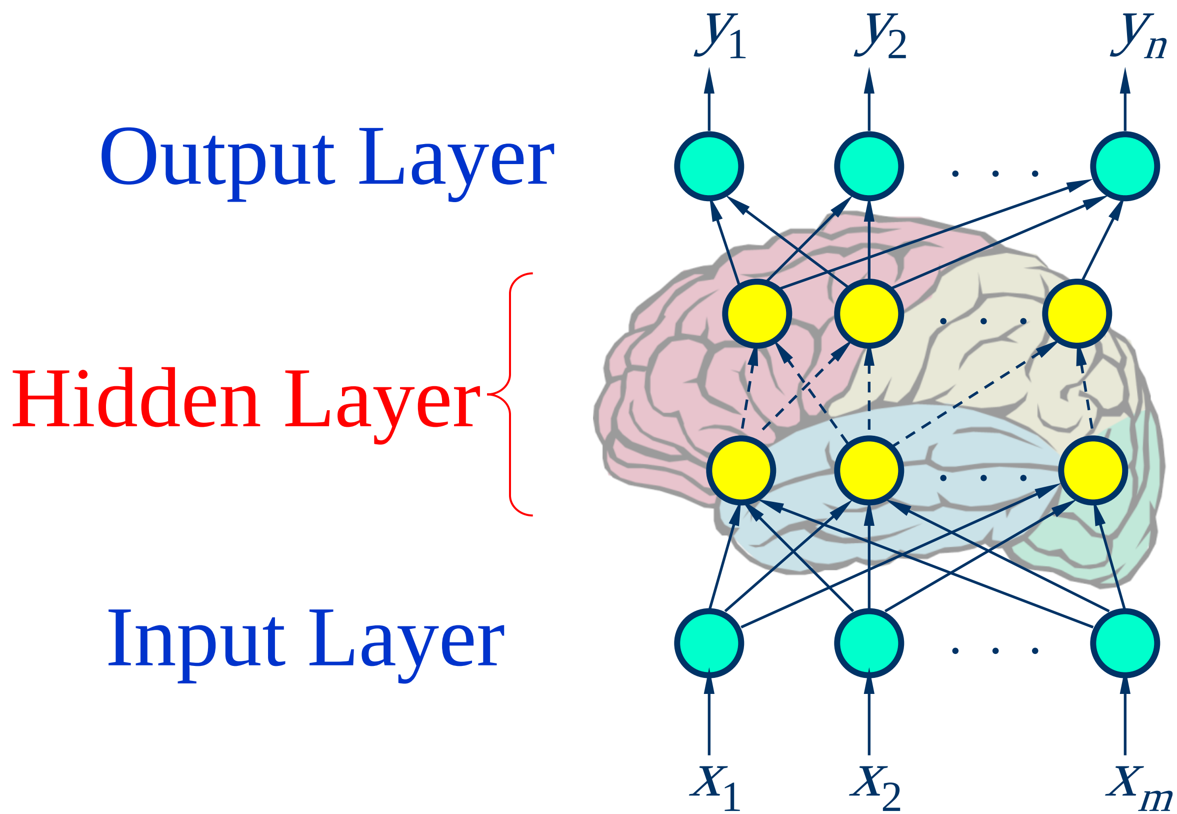

$$a(net_i) = \frac{1}{1+e^{-\lambda net_i}}$$

# Feed-Forward Neural Networks



## Multilayer Perceptron

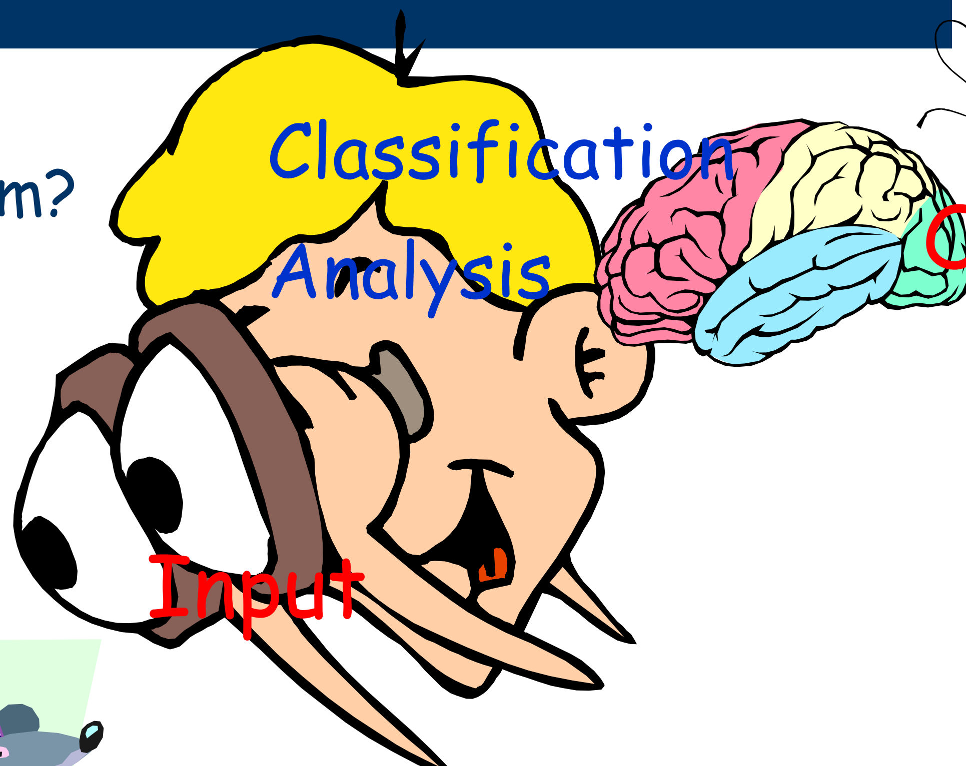
# Multilayer Perceptron



# Multilayer Perceptron

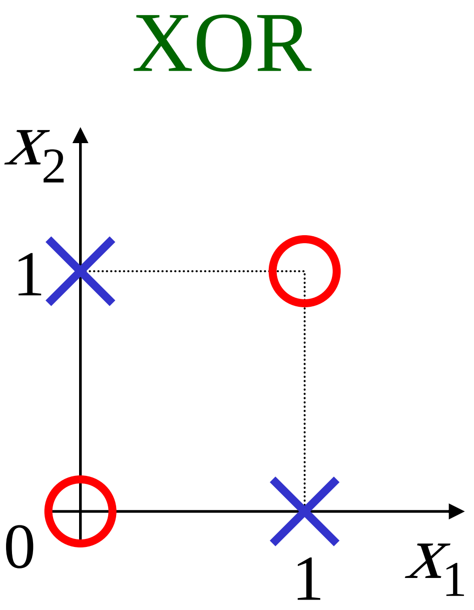
Where the knowledge from?

Learning




# How an MLP Works?

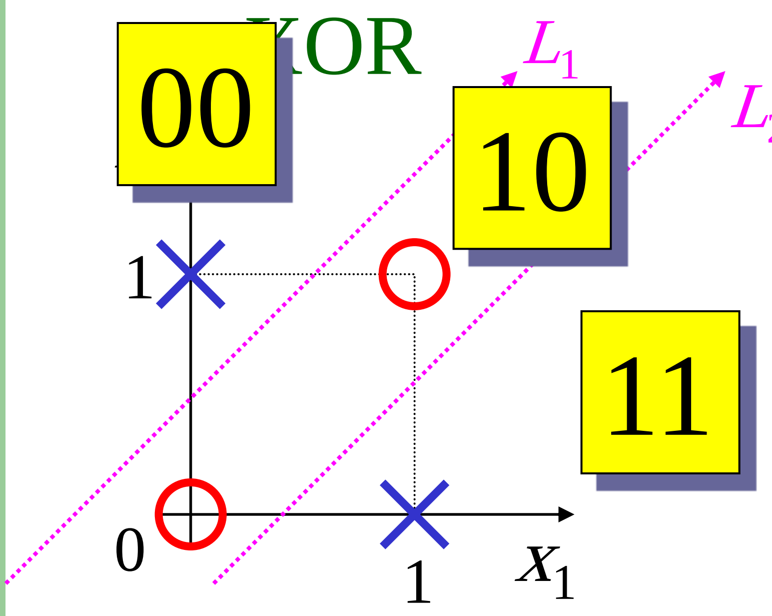
Example:

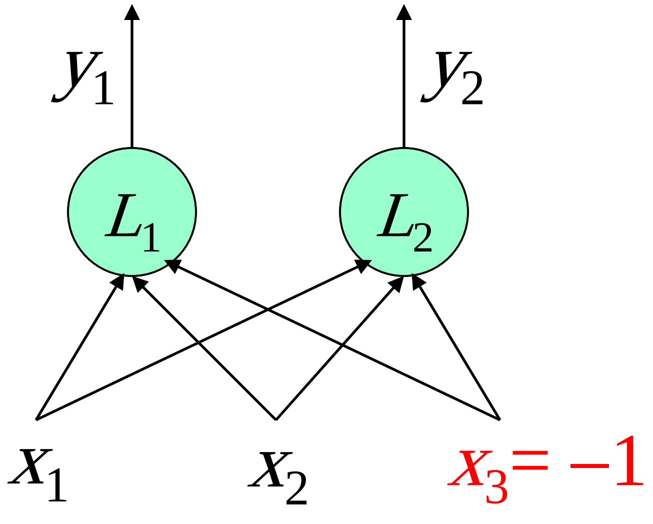



- Not linearly separable.
- Is a single layer perceptron workable?

# How an MLP Works?

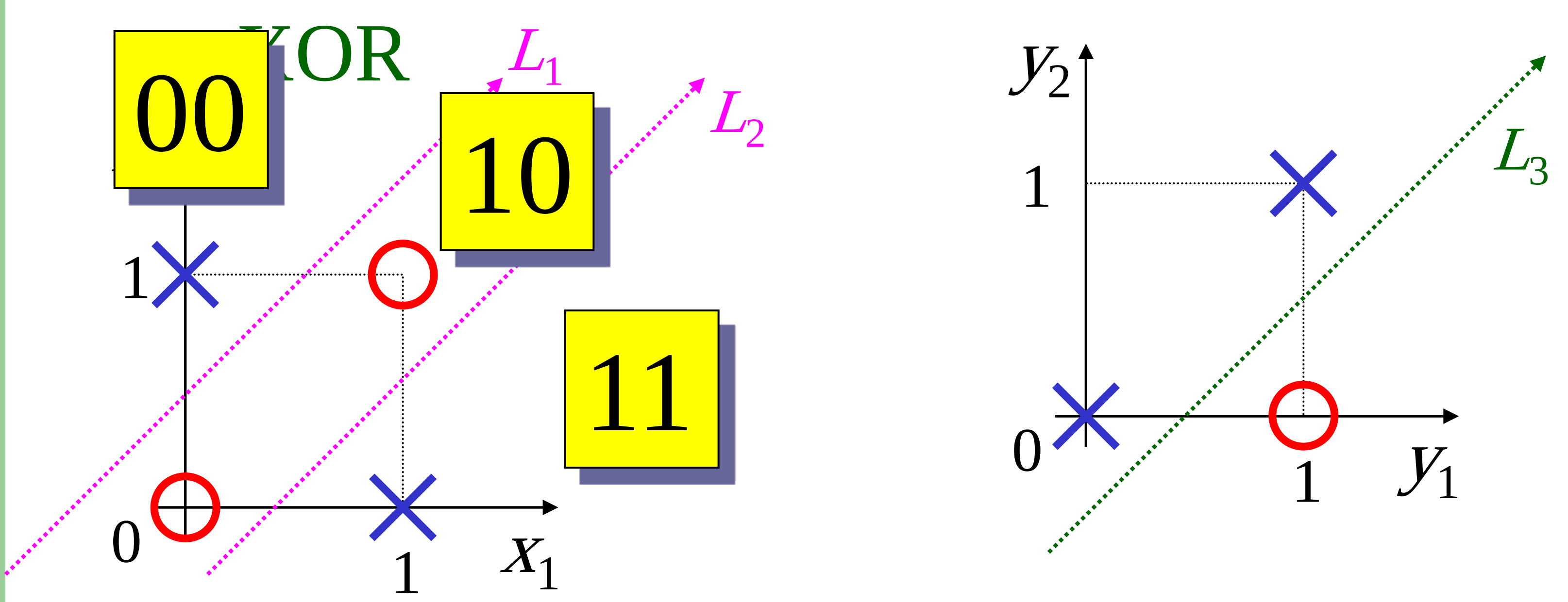
Example:



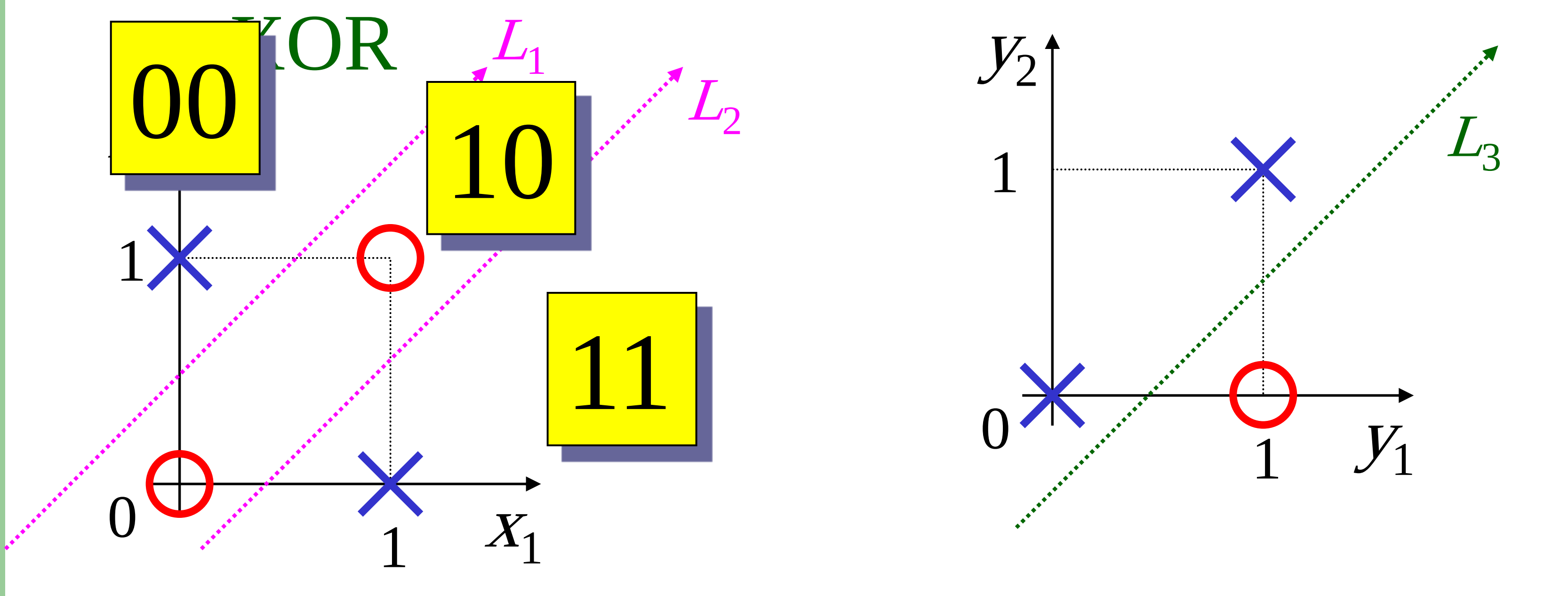
# How an MLP Works?

Example:



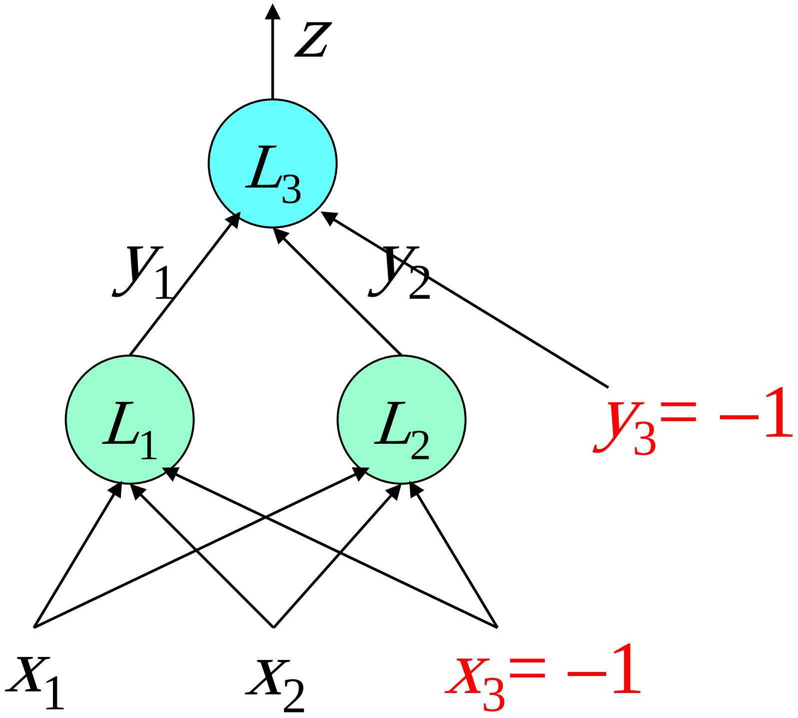
# How an MLP Works?

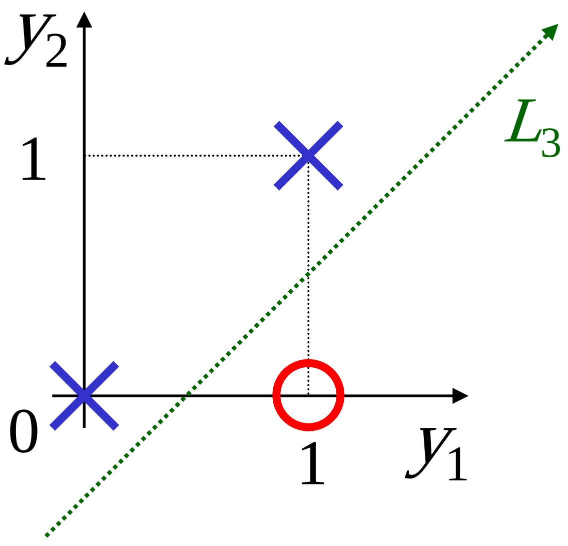
Example:



# How an MLP Works?

Example:

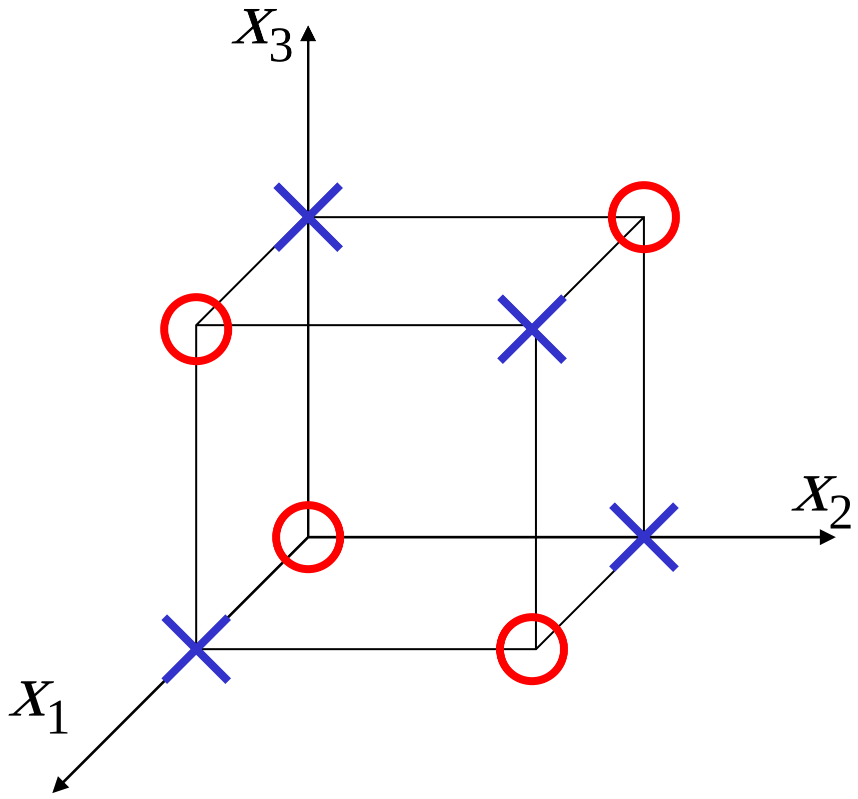


*Is the problem linearly separable?*

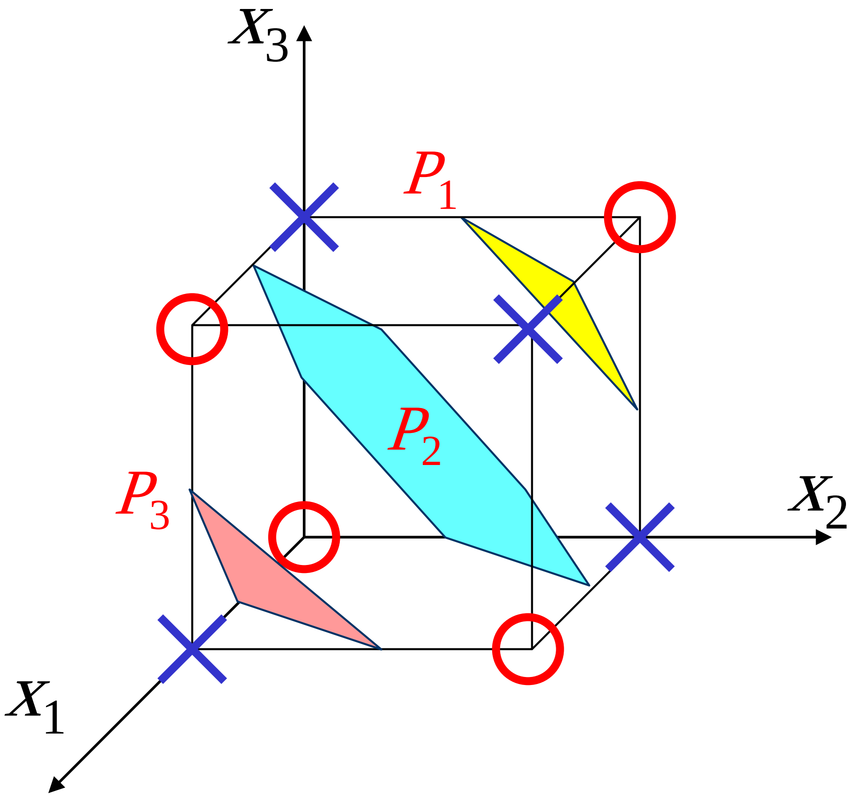
# Parity Problem

| $x_1$ $x_2$ $x_3$ | |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 0 |
| 111 | 1 |



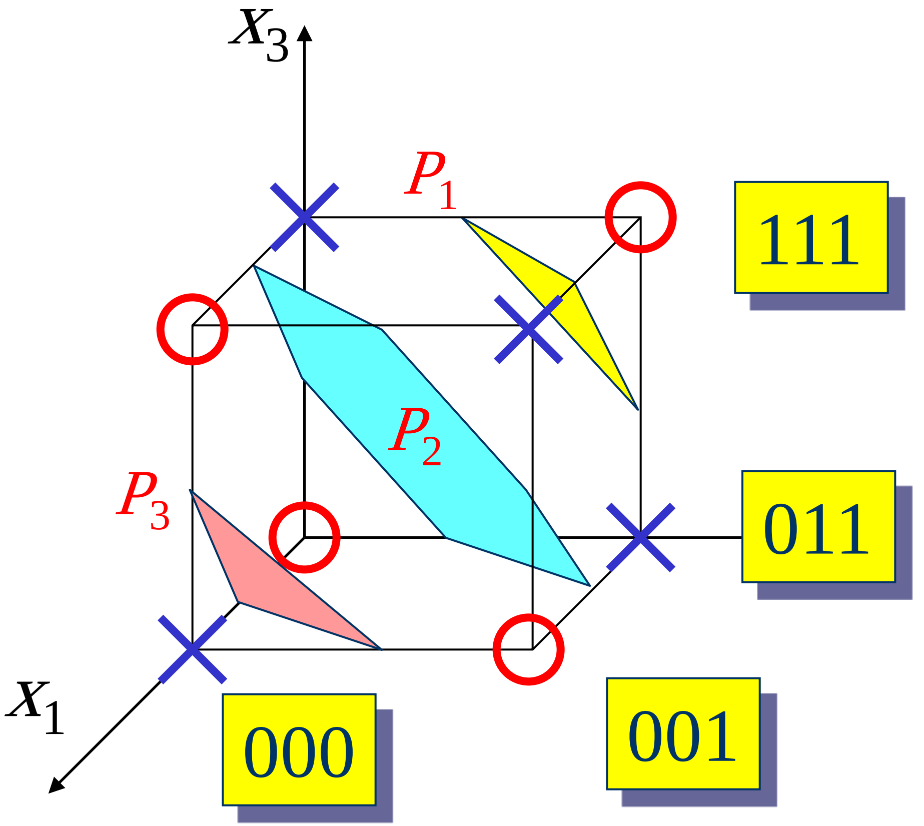
# Parity Problem

| $x_1\ x_2\ x_3$ | |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 0 |
| 111 | 1 |

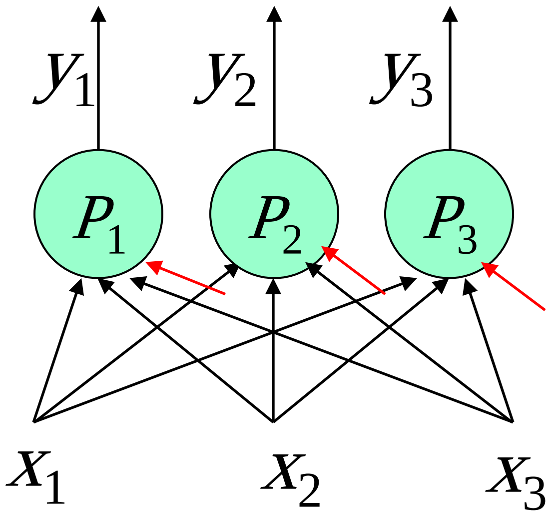


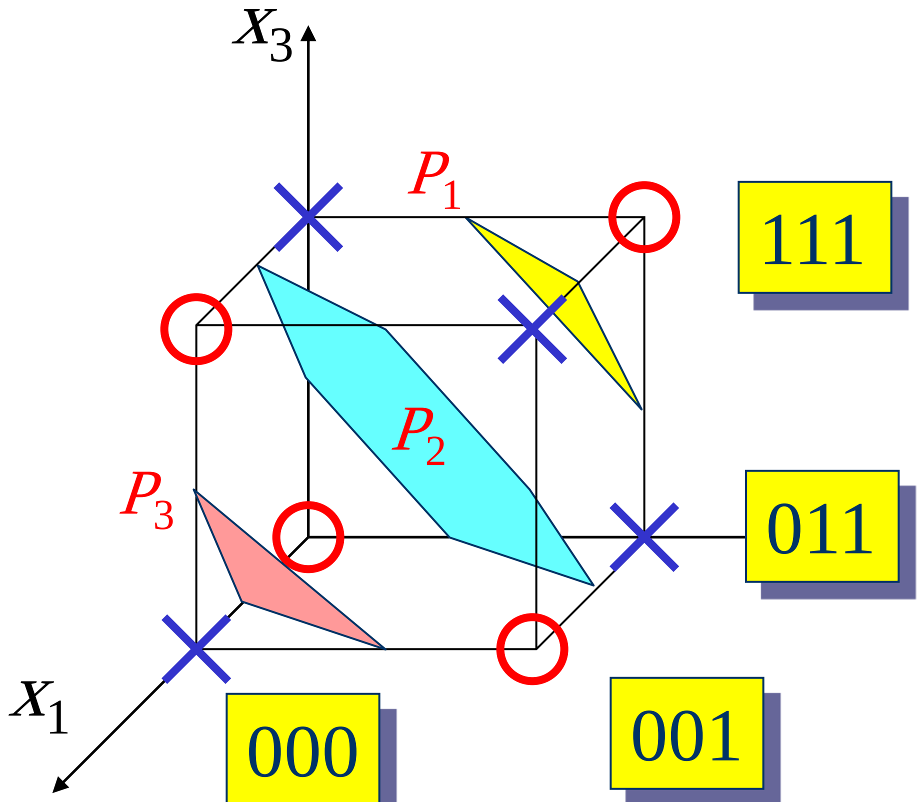
# Parity Problem

| $x_1\ x_2\ x_3$ | |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 0 |
| 111 | 1 |

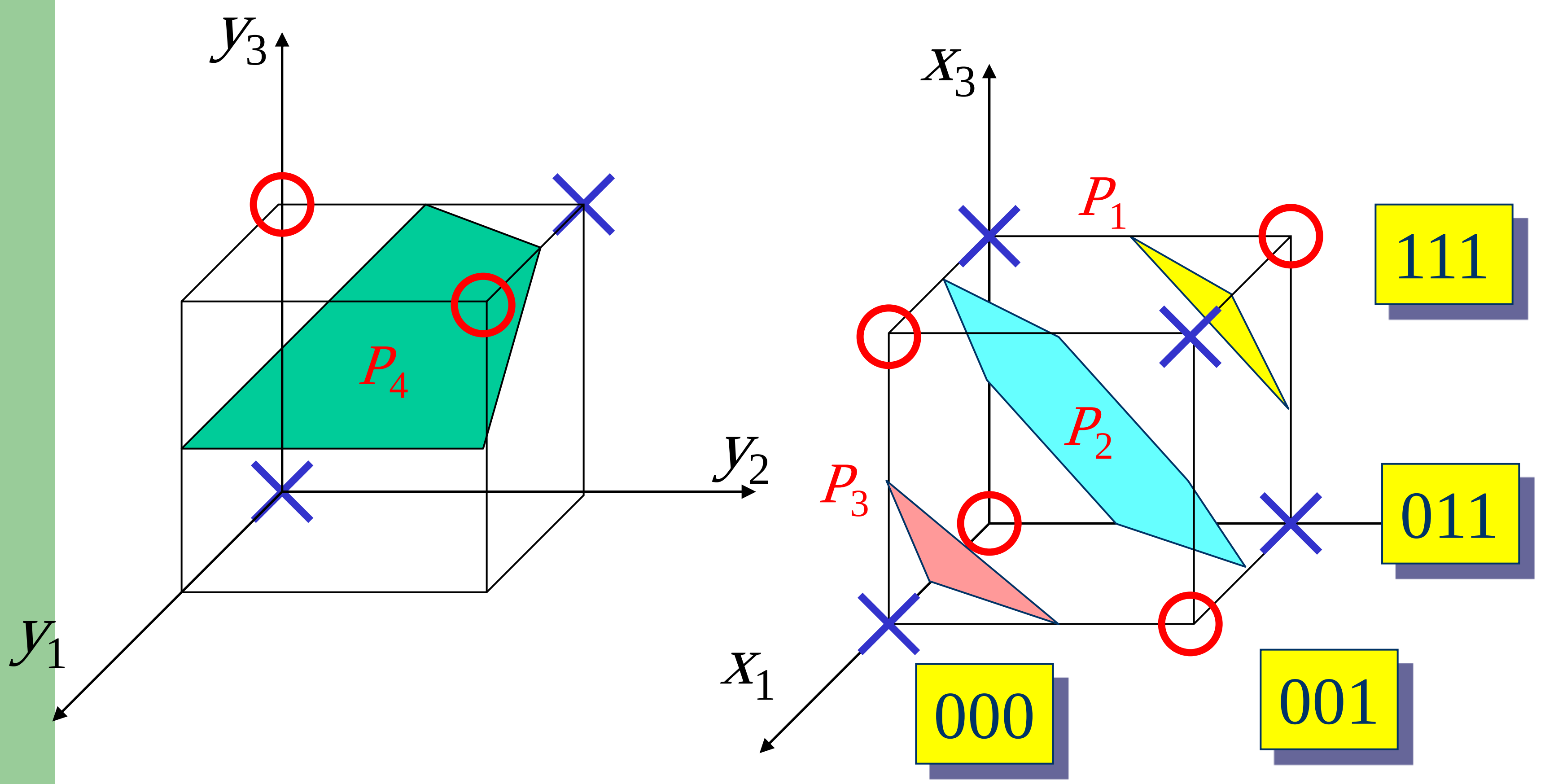


# Parity Problem

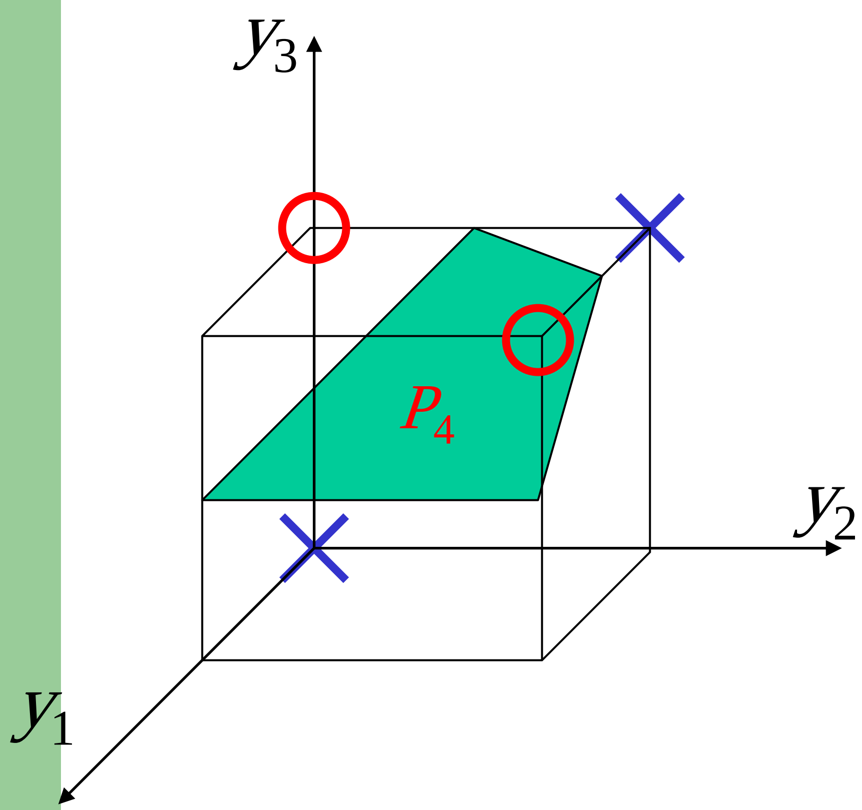


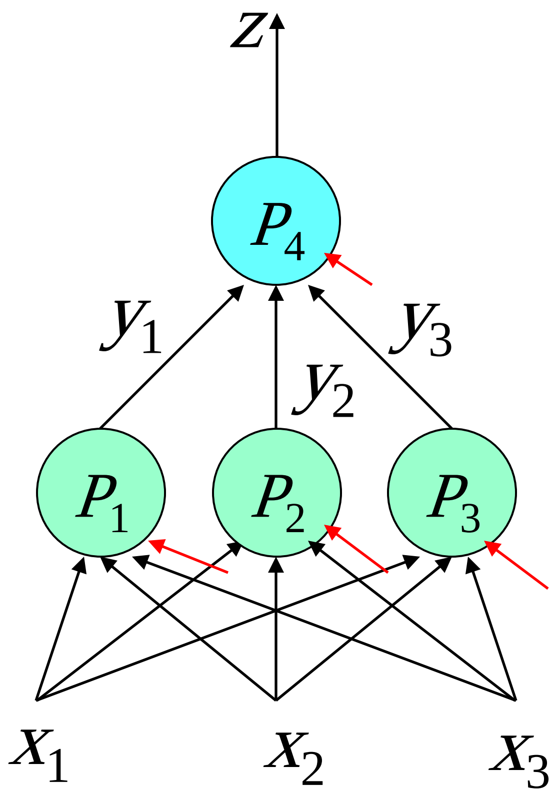



# Parity Problem

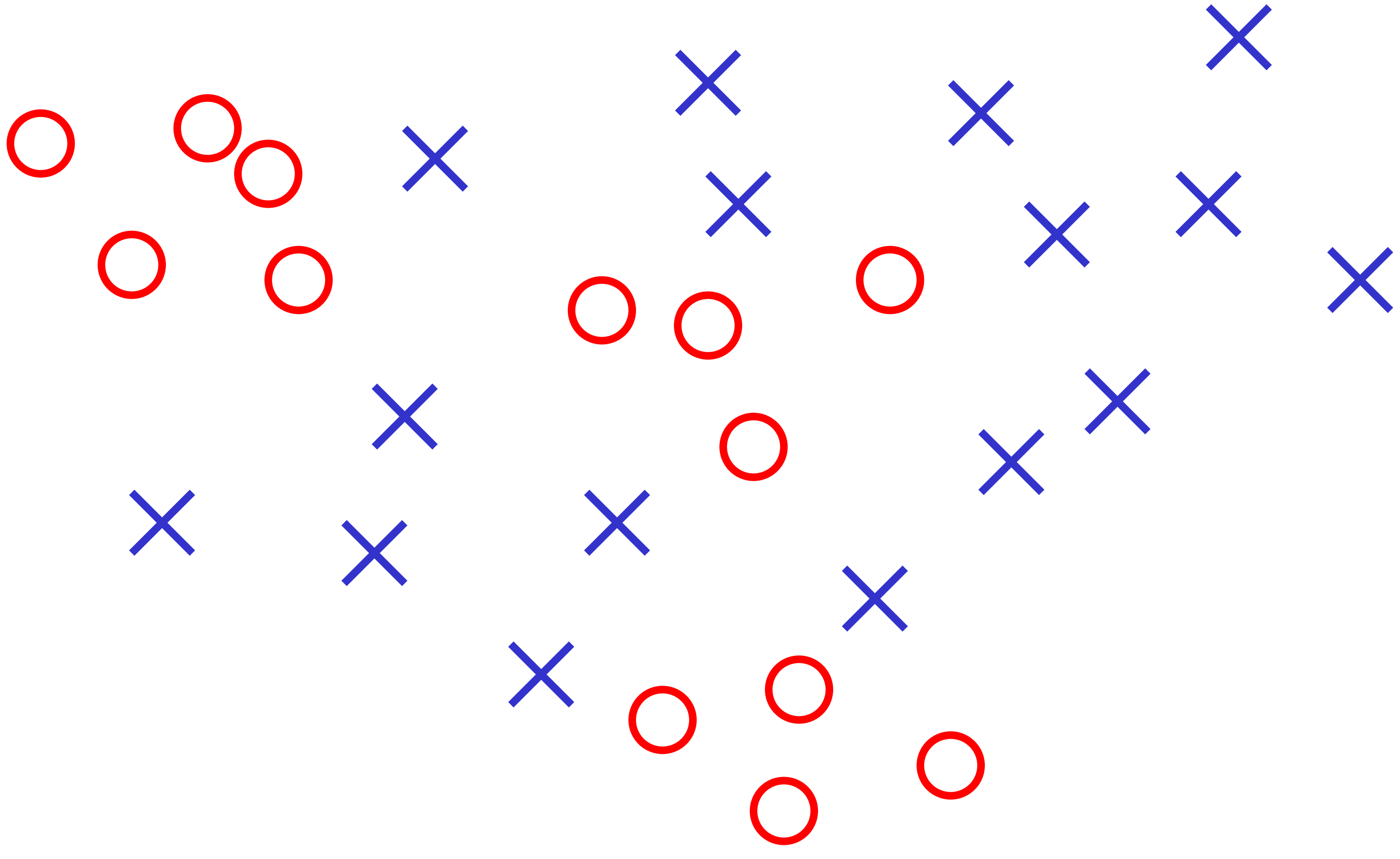


# Parity Problem

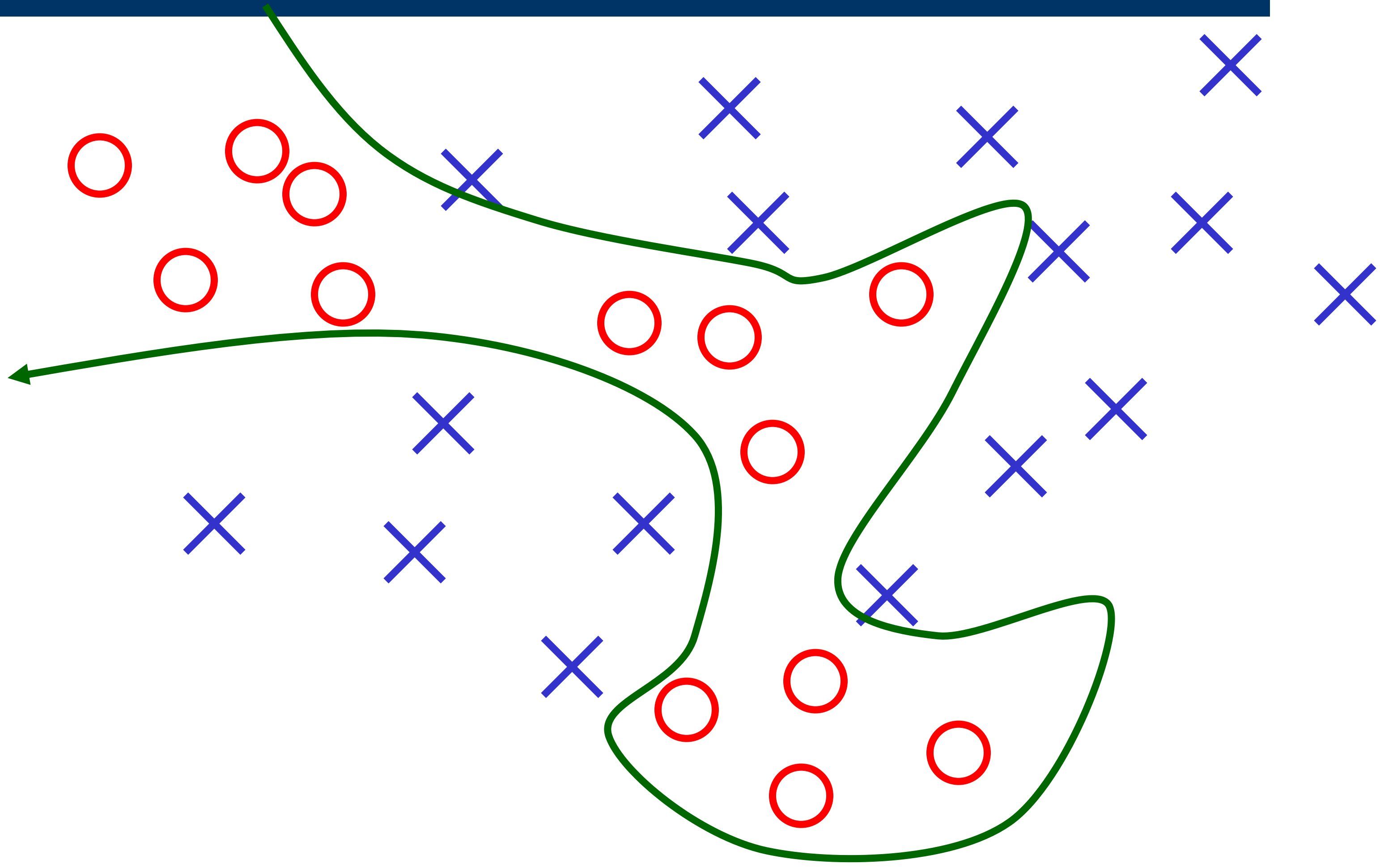


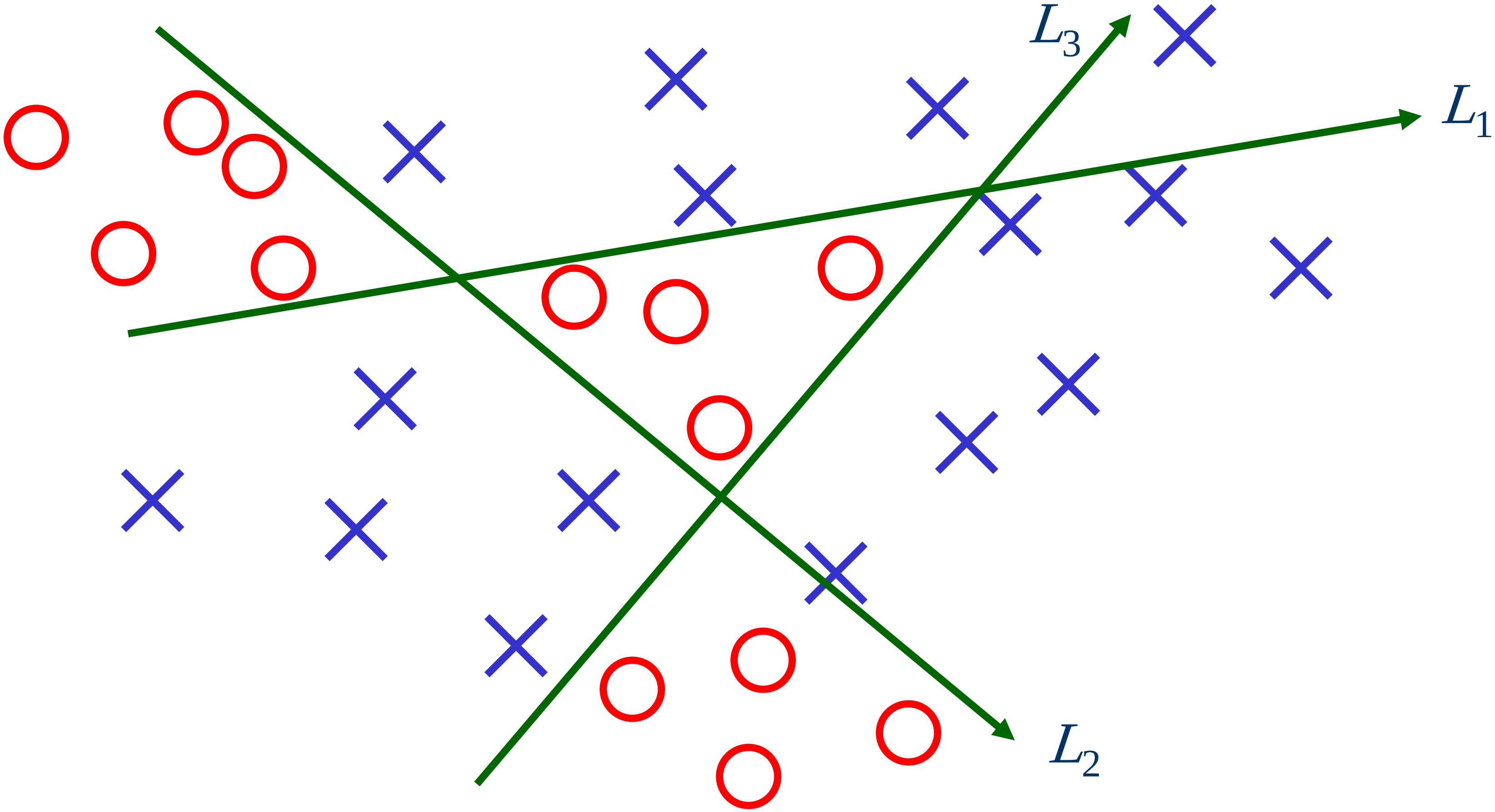
# General Problem



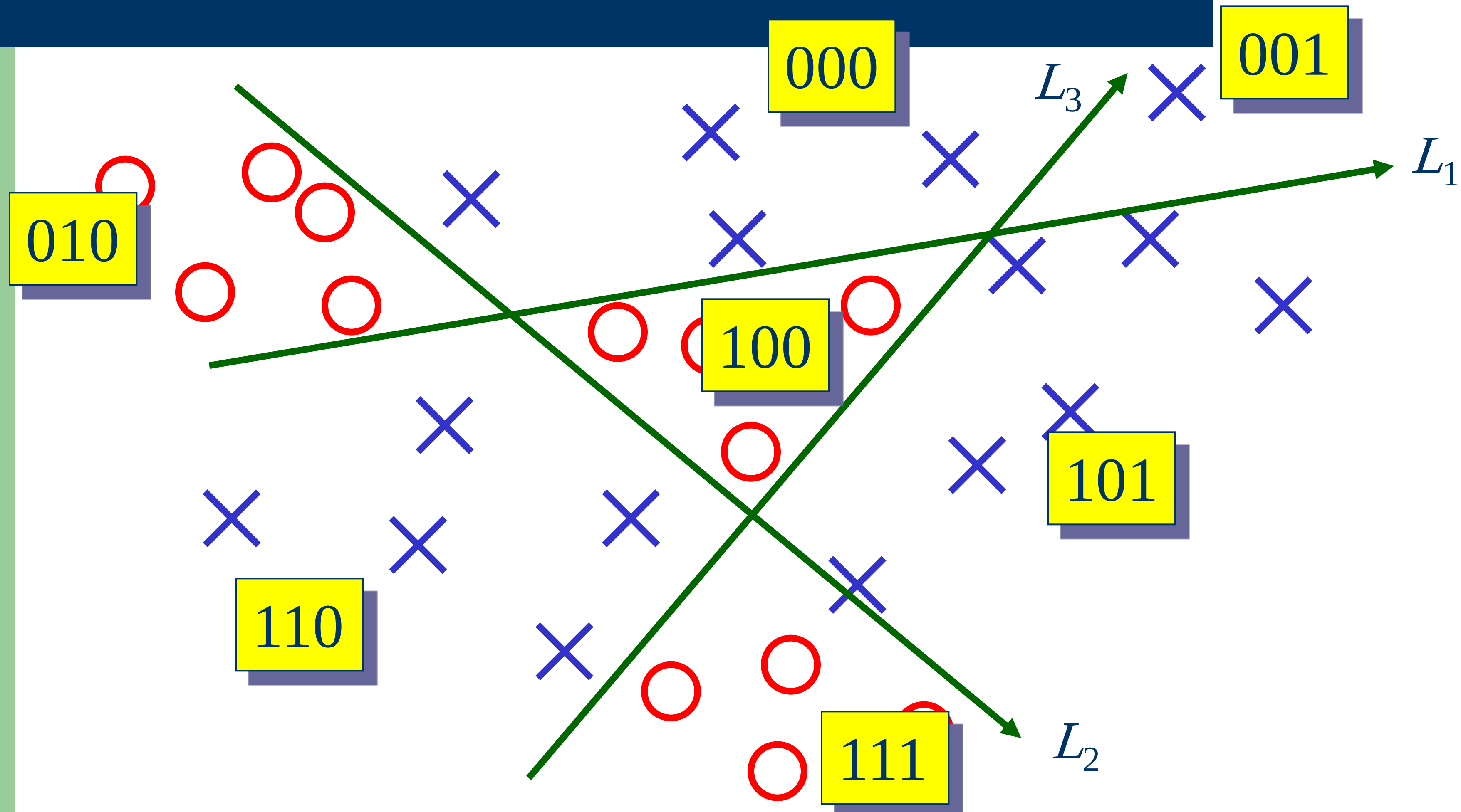
# General Problem



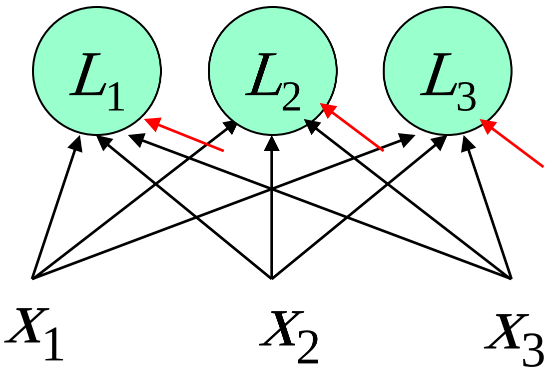
# Hyperspace Partition

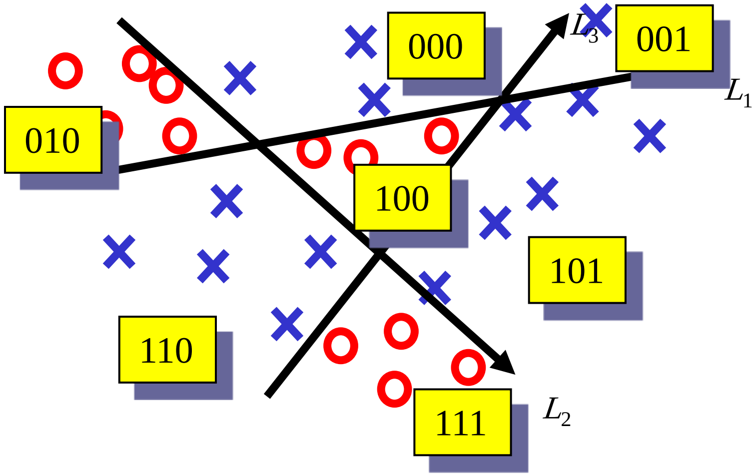


# Region Encoding

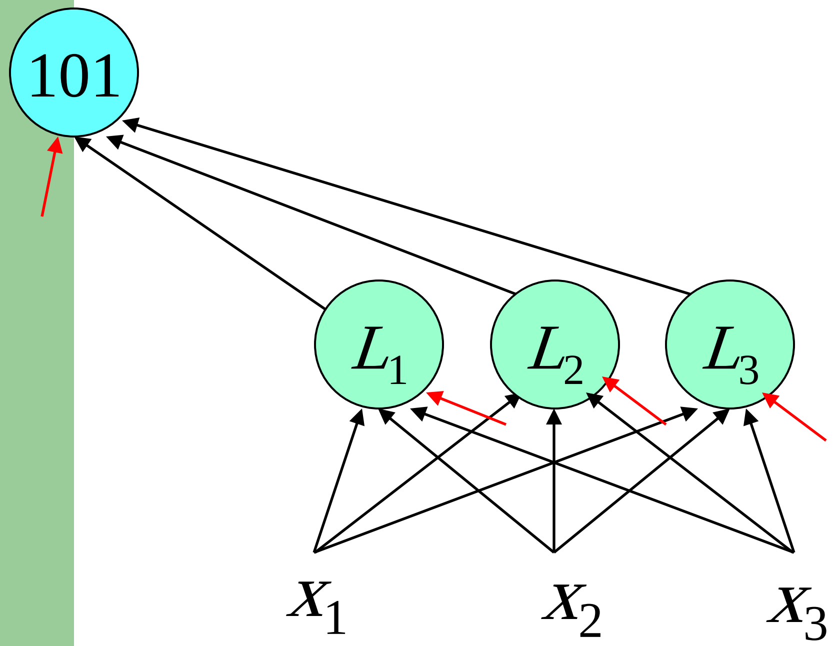


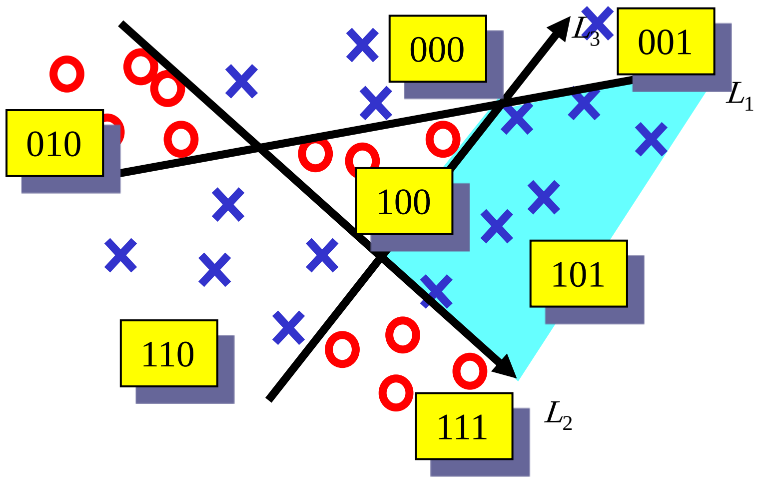
# Hyperspace Partition & Region Encoding Layer



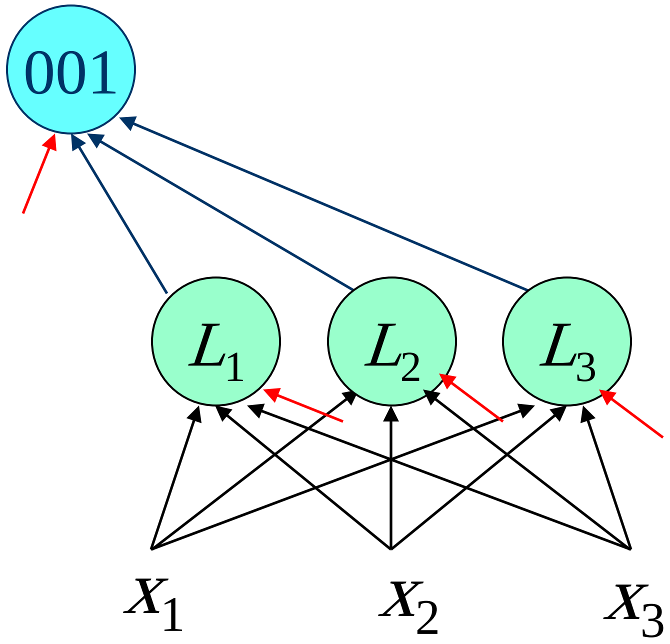



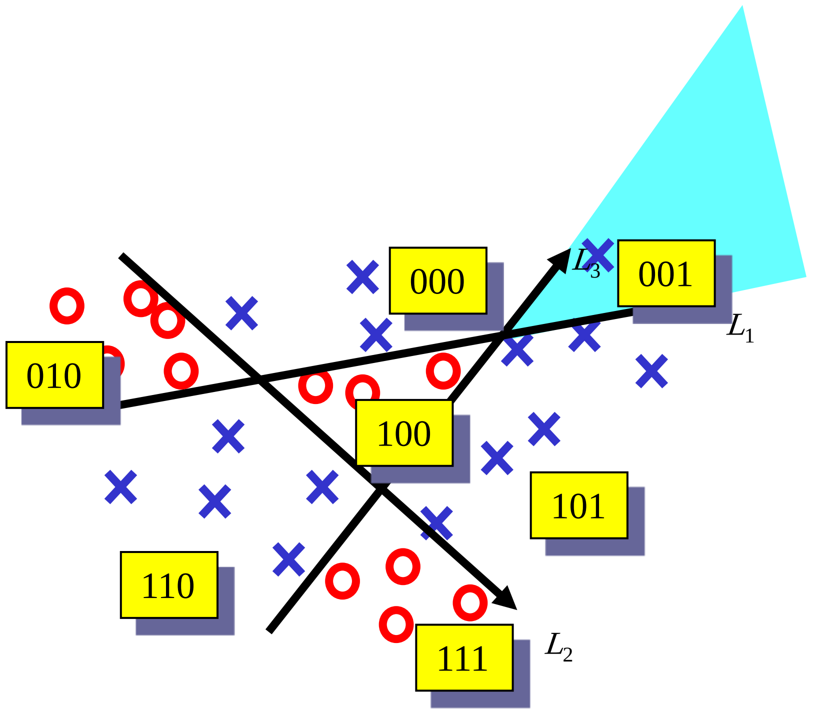
# Region Identification Layer



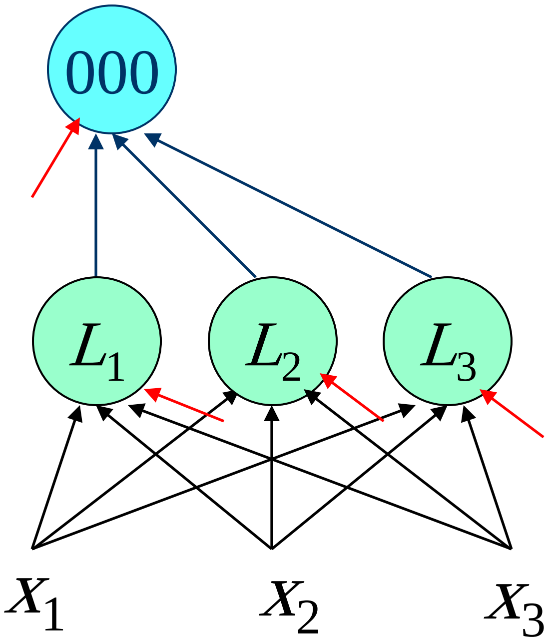



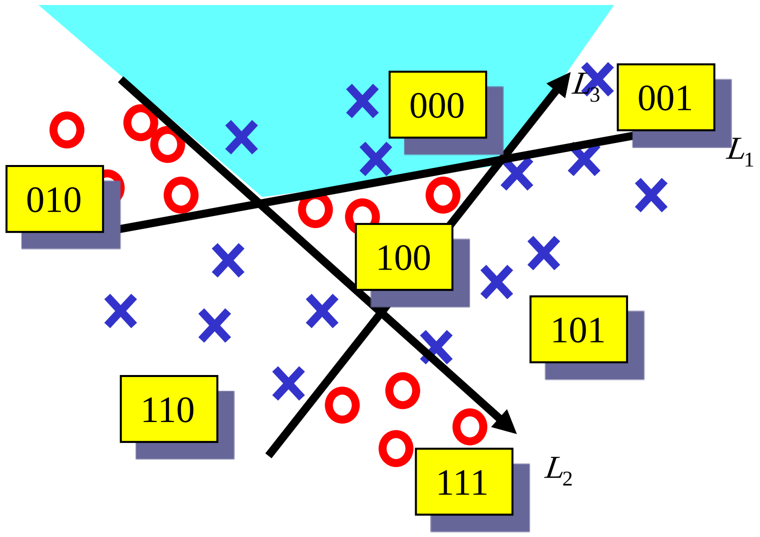
# Region Identification Layer



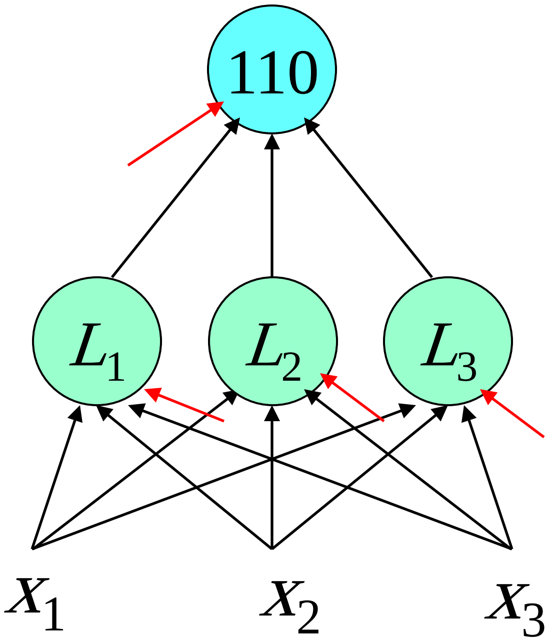



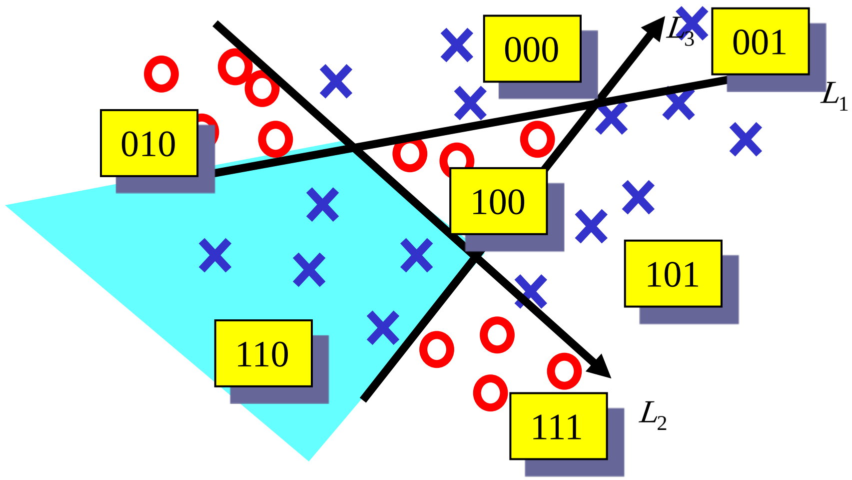
# Region Identification Layer



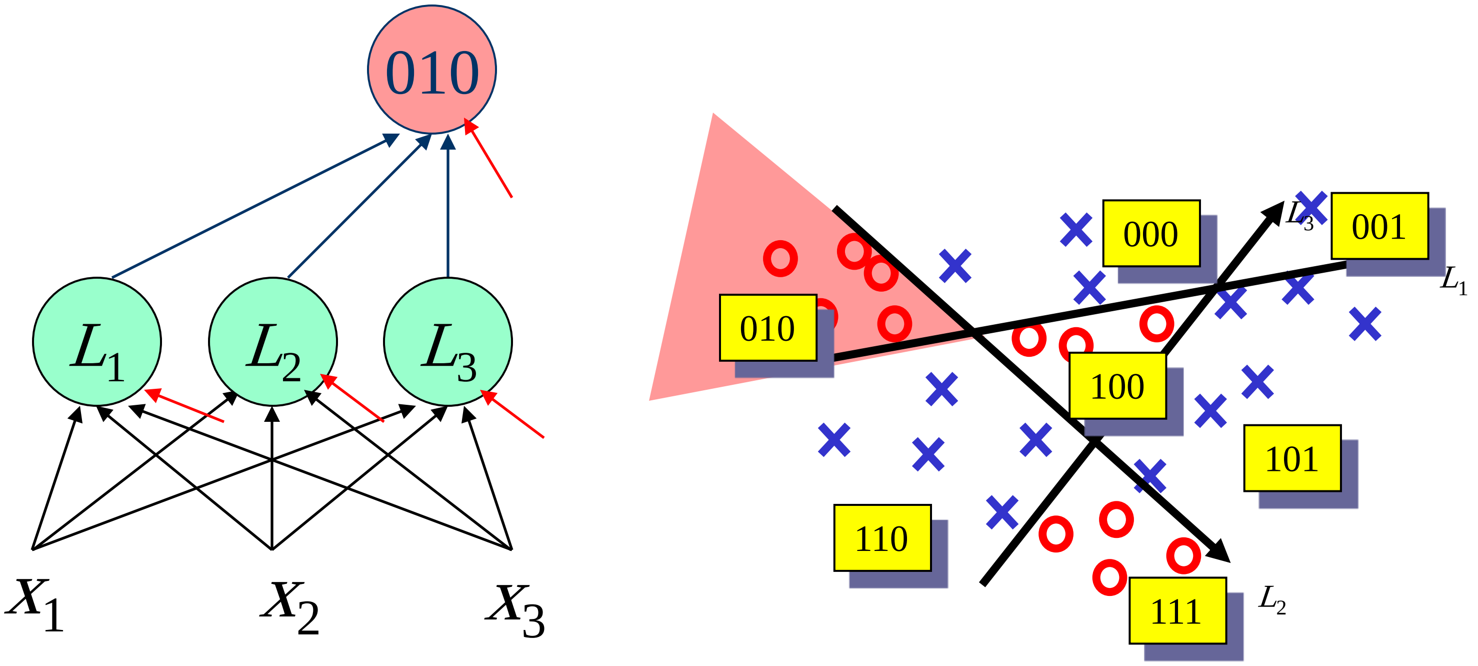



# Region Identification Layer

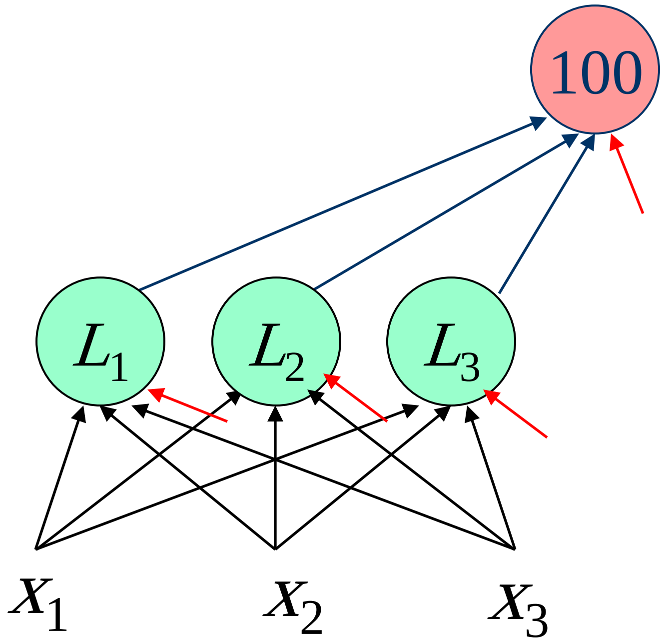


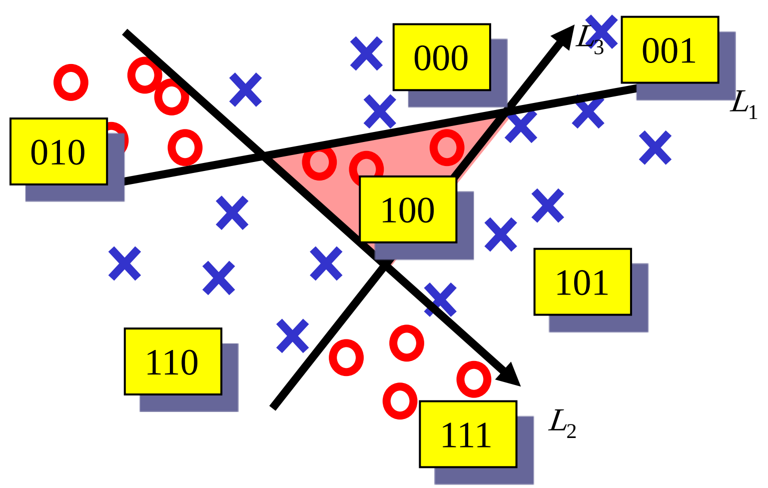



# Region Identification Layer

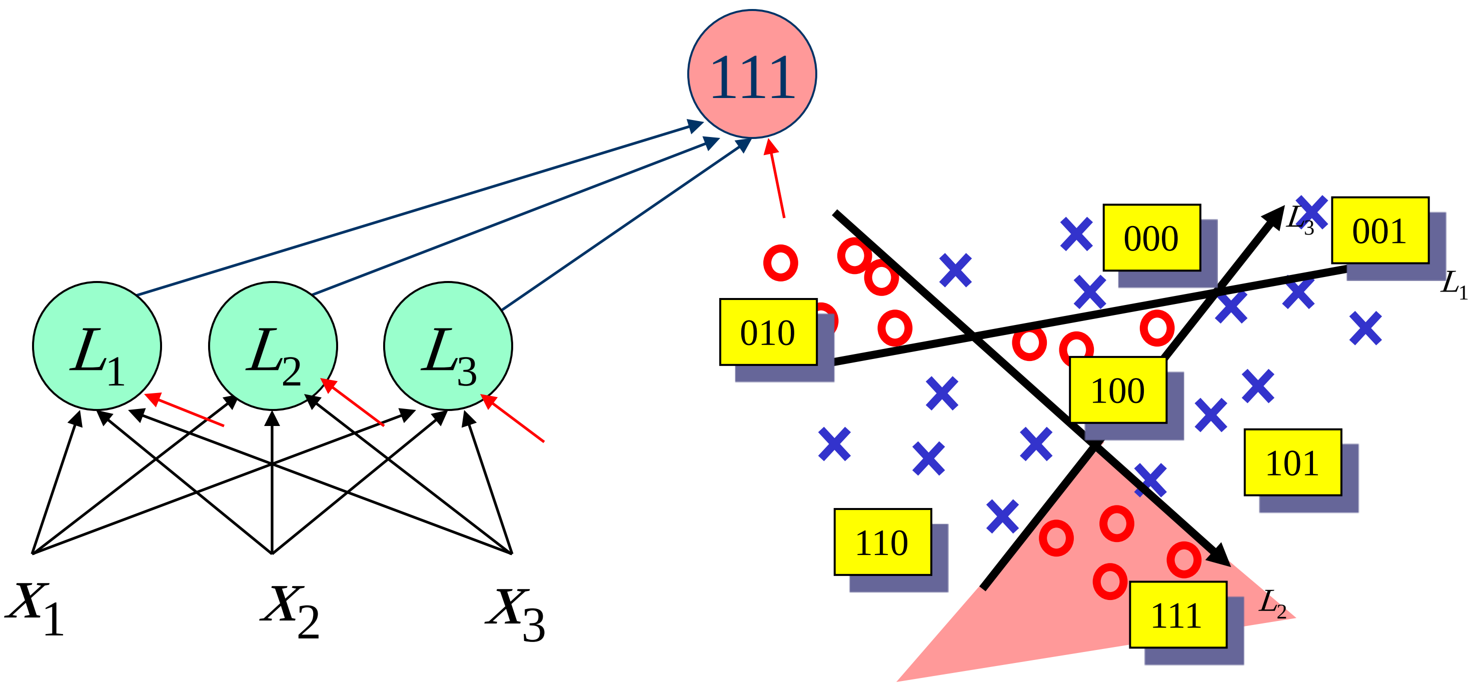


# Region Identification Layer

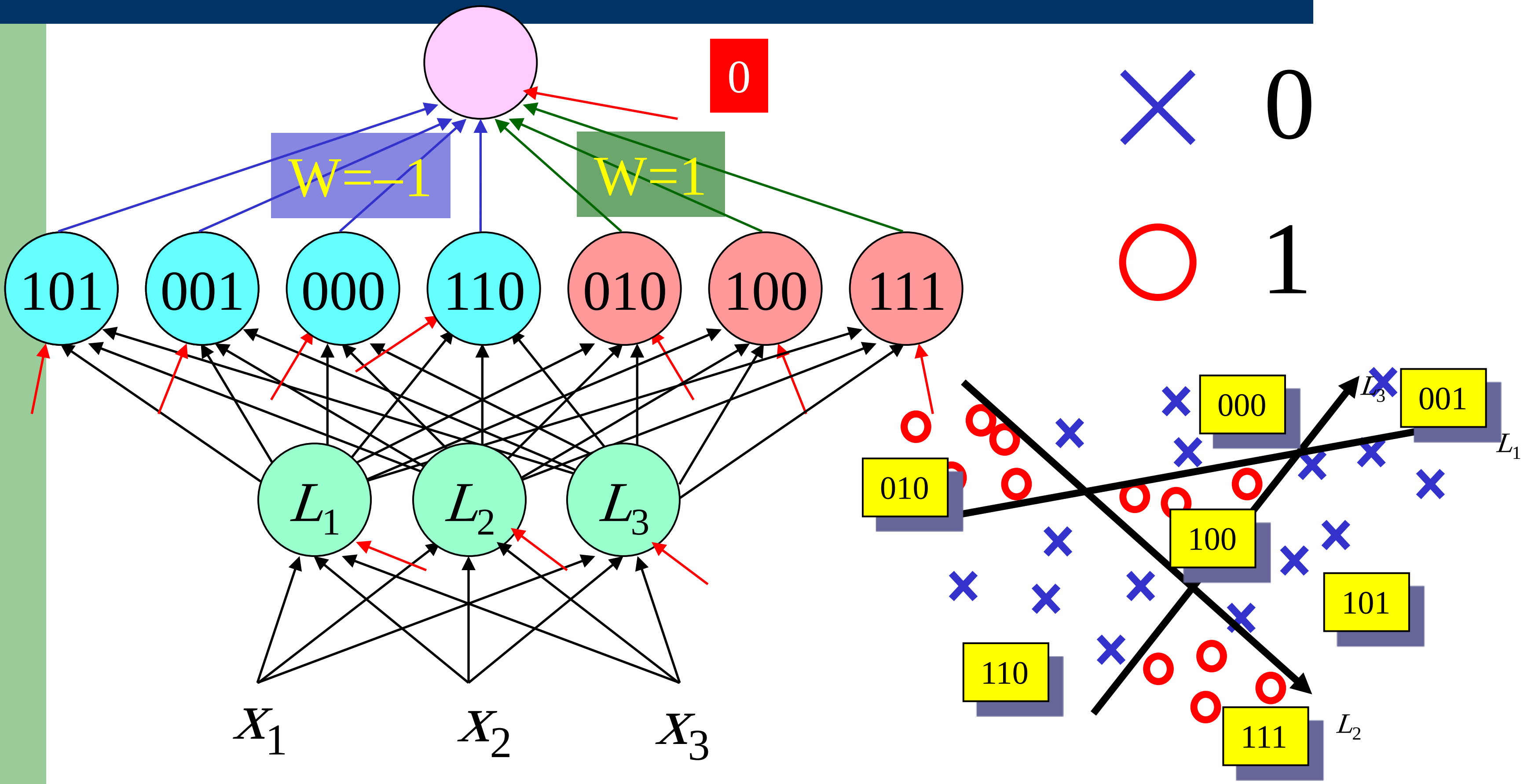


# Region Identification Layer



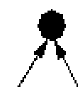
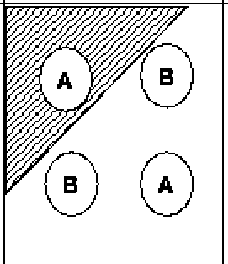
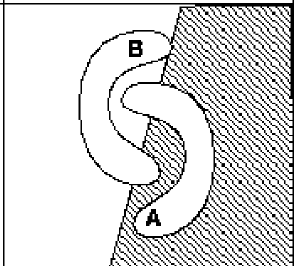
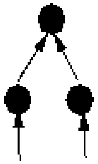
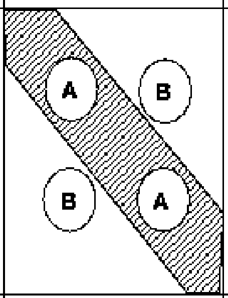
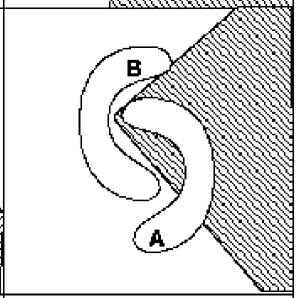
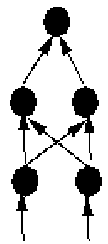
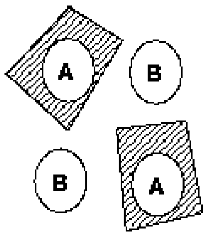
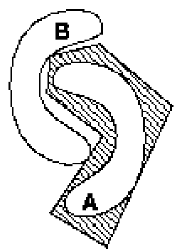
# Classification



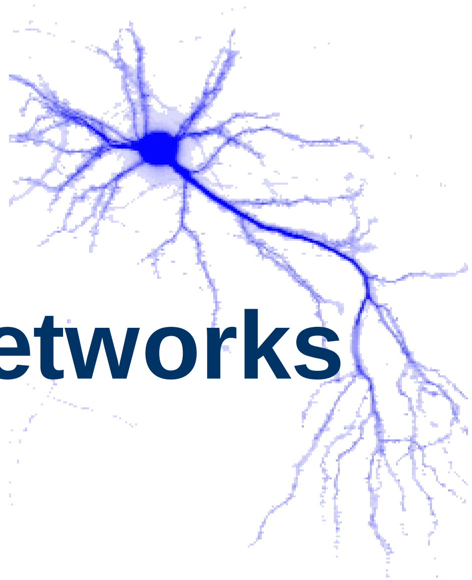
# Multi-Layer Perceptron

The shape of regions in pattern space that can be separated by a Multi-Layer Perceptron is shown in the table.

We can see that a three layer MLP can learn arbitrary areas while a two layer MLP can learn convex regions. (if you can draw a line from any point in the region to any other in the region and the line passes out of the region then that region is not convex).

| Structure | Regions | XOR | Meshed regions |
|---|---|---|---|
| single layer  | Half plane bounded by hyper-plane |  |  |
| two layer  | Convex open or closed regions |  |  |
| three layer  | Arbitrary (limited by # of nodes) |  |  |

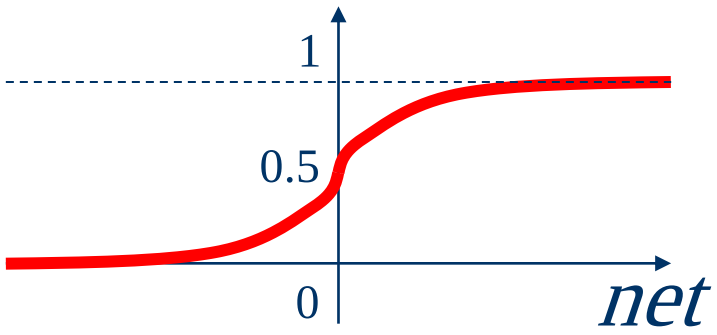
# Feed-Forward Neural Networks



Back Propagation
Learning algorithm

# Activation Function — Sigmoid

$$y = a(net) = \frac{1}{1+e^{-\lambda net}}$$




$$a'(net) = -\left(\frac{1}{1+e^{-\lambda net}}\right)^2 \cdot (-\lambda) e^{-\lambda net}$$

$$e^{-\lambda net} = \frac{1-y}{y}$$

$$a'(net) = \lambda y(1-y)$$

Remember this

Training Set

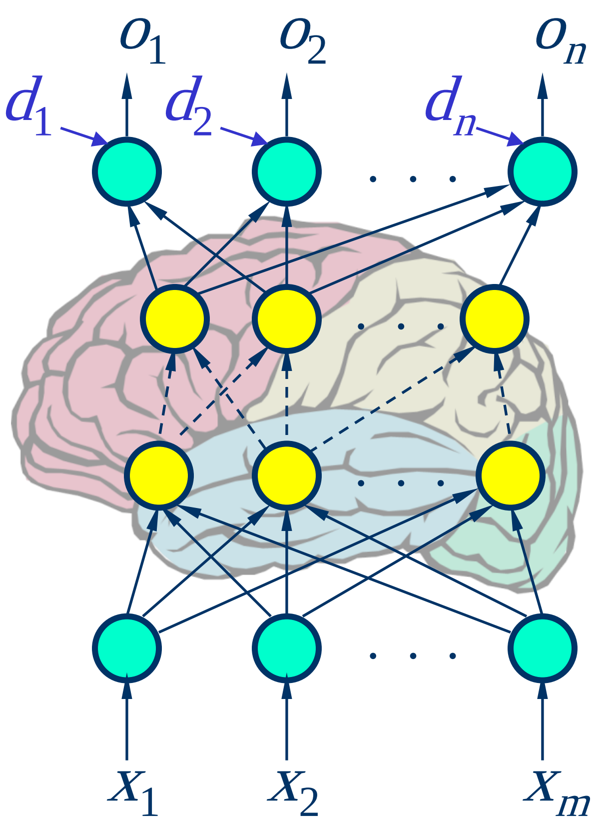
$$\mathbf{T} = \left\{ (\mathbf{x}^{(1)}, \mathbf{d}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{d}^{(2)}), \ldots, (\mathbf{x}^{(p)}, \mathbf{d}^{(p)}) \right\}$$

# Supervised Learning

Output Layer

Hidden Layer

Input Layer



Training Set

$$\mathbf{T} = \left\{ (\mathbf{x}^{(1)}, \mathbf{d}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{d}^{(2)}), \ldots, (\mathbf{x}^{(p)}, \mathbf{d}^{(p)}) \right\}$$

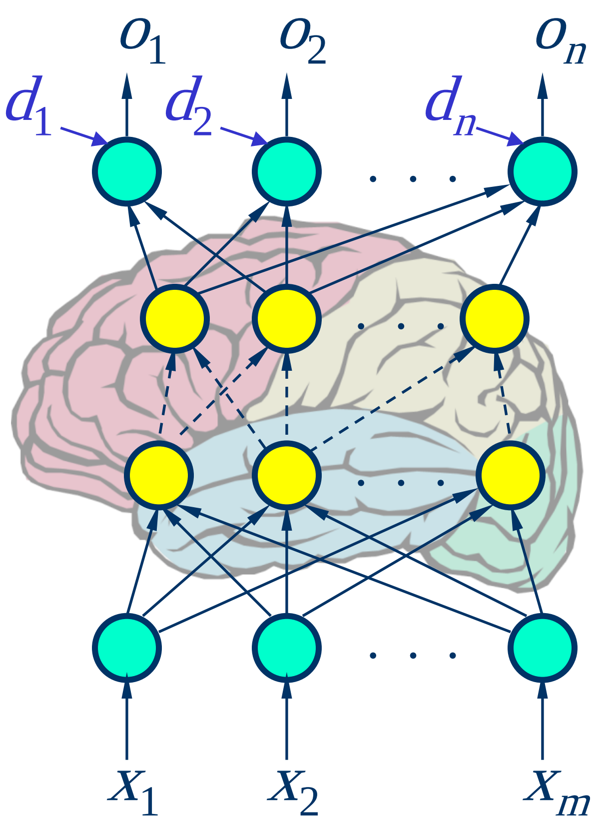
# Supervised Learning

Sum of Squared Errors

$$E^{(l)} = \frac{1}{2} \sum_{j=1}^{n} \left[ d_j^{(l)} - o_j^{(l)} \right]^2$$

Goal:

Minimize $E = \sum_{l=1}^{p} E^{(l)}$

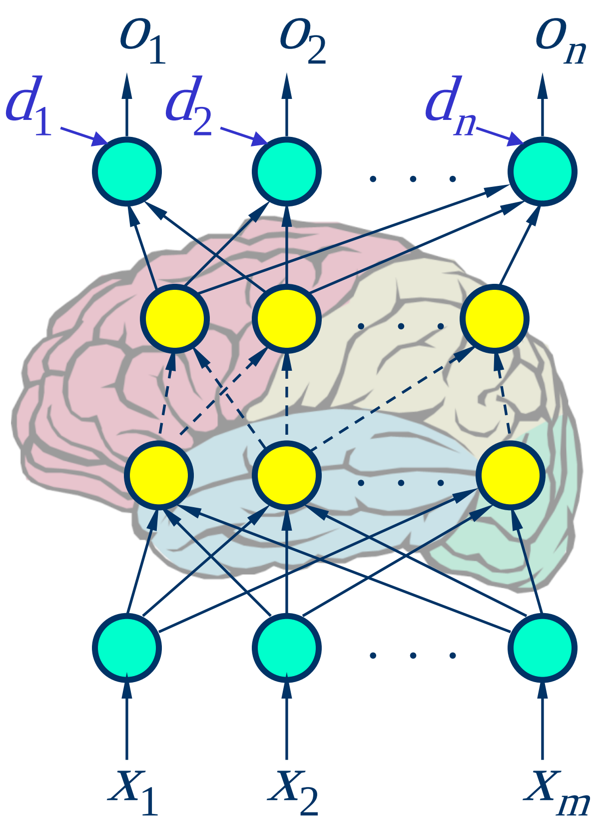


$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Back Propagation Learning Algorithm

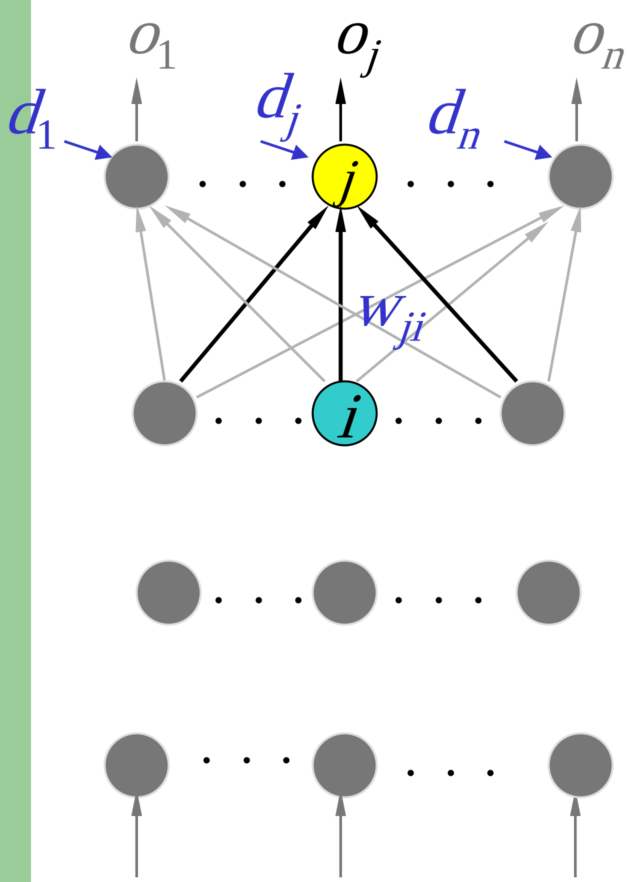
- Learning on Output Neurons
- Learning on Hidden Neurons



$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons




$$o_j^{(l)} = a(net_j^{(l)})$$

$$net_j^{(l)} = \sum w_{ji} o_i^{(l)}$$

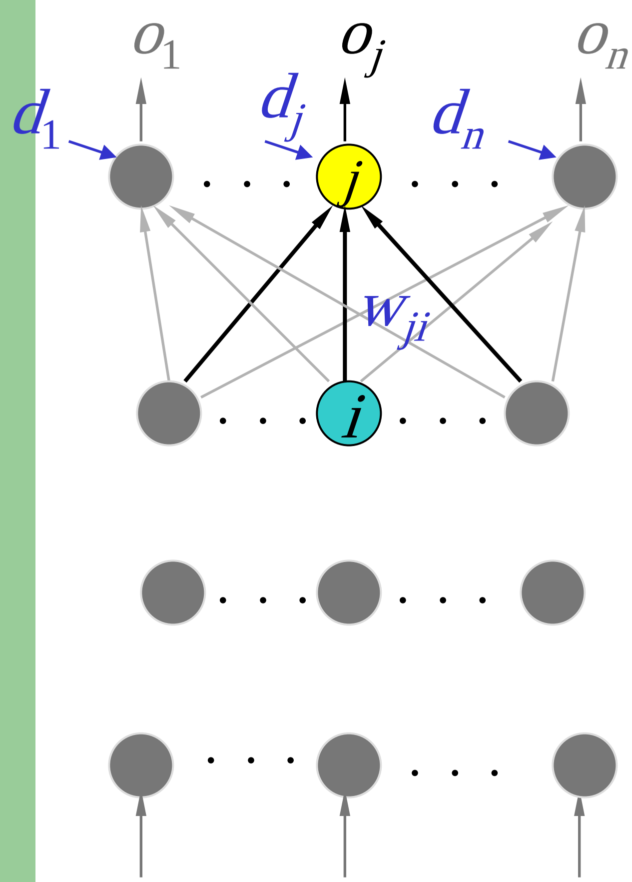
$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p} \frac{\partial E^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \underbrace{\frac{\partial E^{(l)}}{\partial net_j^{(l)}}}_{?} \underbrace{\frac{\partial net_j^{(l)}}{\partial w_{ji}}}_{?}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons




$$o_j^{(l)} = a(net_j^{(l)})$$

$$net_j^{(l)} = \sum w_{ji} o_i^{(l)}$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p} \frac{\partial E^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \boxed{\frac{\partial E^{(l)}}{\partial net_j^{(l)}}} \frac{\partial net_j^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial net_j^{(l)}} = \frac{\partial E^{(l)}}{\partial o_j^{(l)}} \frac{\partial o_j^{(l)}}{\partial net_j^{(l)}}$$

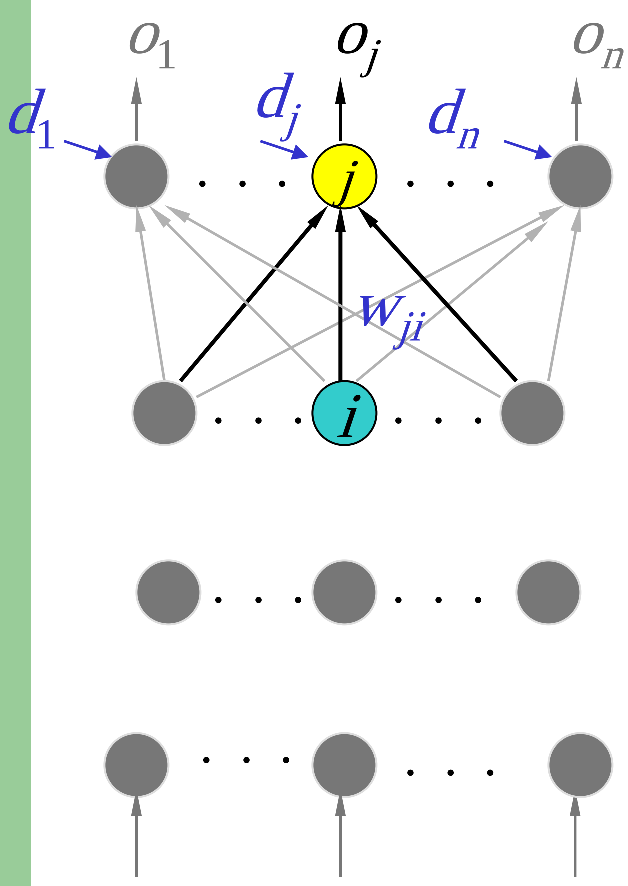
$$-(d_j^{(l)} - o_j^{(l)})$$

depends on the activation function

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons




$$o_j^{(l)} = a(net_j^{(l)})$$

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$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p} \frac{\partial E^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \boxed{\frac{\partial E^{(l)}}{\partial net_j^{(l)}}} \frac{\partial net_j^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial net_j^{(l)}} = \frac{\partial E^{(l)}}{\partial o_j^{(l)}} \frac{\partial o_j^{(l)}}{\partial net_j^{(l)}}$$

$-(d_j^{(l)} - o_j^{(l)})$

Using sigmoid,

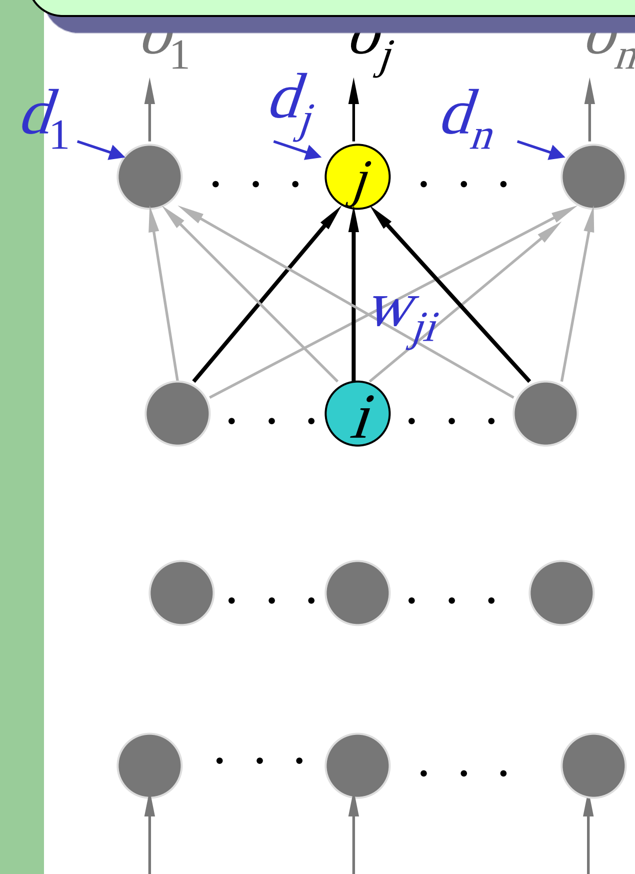
$\lambda o_j^{(l)}(1 - o_j^{(l)})$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons

$$\delta_j^{(l)} = \frac{\partial E^{(l)}}{\partial net_j^{(l)}} = -(d_j^{(l)} - o_j^{(l)})\lambda o_j^{(l)}(1-o_j^{(l)})$$




$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ji}}$$

$\delta_j^{(l)}$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \boxed{\frac{\partial E^{(l)}}{\partial net_j^{(l)}}}\frac{\partial net_j^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial net_j^{(l)}} = \frac{\partial E^{(l)}}{\partial o_j^{(l)}}\frac{\partial o_j^{(l)}}{\partial net_j^{(l)}}$$

$-(d_j^{(l)} - o_j^{(l)})$

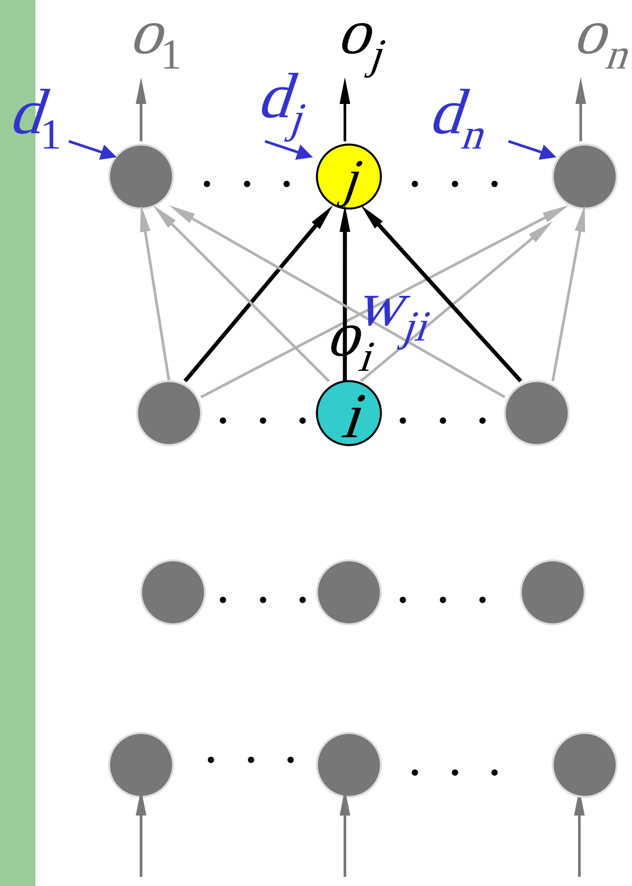
Using sigmoid,

$\lambda o_j^{(l)}(1-o_j^{(l)})$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons




$$o_j^{(l)} = a(net_j^{(l)})$$

$$net_j^{(l)} = \sum w_{ji} o_i^{(l)}$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p} \frac{\partial E^{(l)}}{\partial w_{ji}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \frac{\partial E^{(l)}}{\partial net_j^{(l)}} \boxed{\frac{\partial net_j^{(l)}}{\partial w_{ji}}} \rightarrow o_i^{(l)}$$

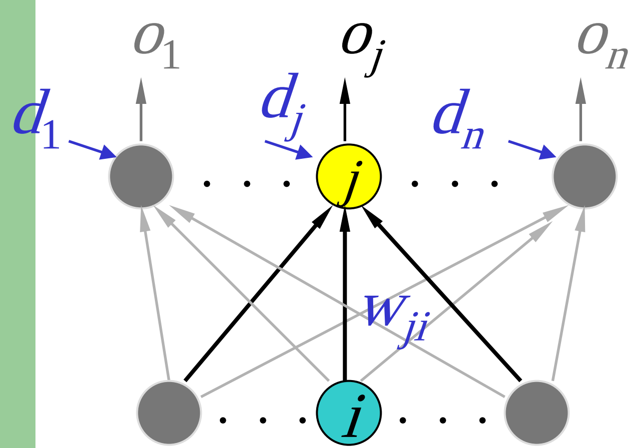
$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \delta_j^{(l)} o_i^{(l)}$$

$$= -(d_j^{(l)} - o_j^{(l)})\lambda o_j^{(l)}(1 - o_j^{(l)}) o_i^{(l)}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Output Neurons




$$o_j^{(l)} = a(net_j^{(l)})$$

$$net_j^{(l)} = \sum w_{ji} o_i^{(l)}$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial}{\partial w_{ji}} \sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p} \frac{\partial E^{(l)}}{\partial w_{ji}}$$

**How to train the weights connecting to output neurons?**

$$\partial w_{ji} \qquad \partial net_j \qquad \boxed{\partial w_{ji}} \rightarrow o_i^{(l)}$$

$$\frac{\partial E^{(l)}}{\partial w_{ji}} = \delta_j^{(l)} o_i^{(l)}$$

$$= -(d_j^{(l)} - o_j^{(l)})\lambda o_j^{(l)}(1 - o_j^{(l)}) o_i^{(l)}$$

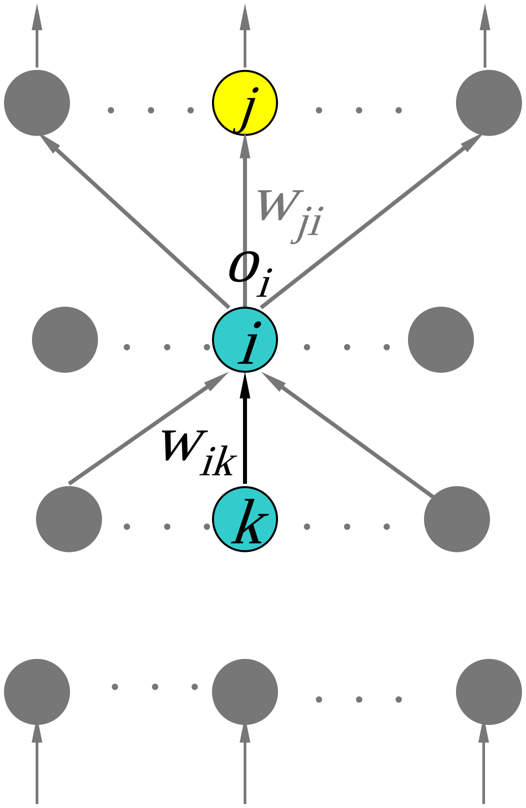
$$\frac{\partial E}{\partial w_{ji}} = \sum_{l=1}^{p} \delta_j^{(l)} o_i^{(l)}$$

$$\Delta w_{ji} = -\eta \sum_{l=1}^{p} \delta_j^{(l)} o_i^{(l)}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Hidden Neurons



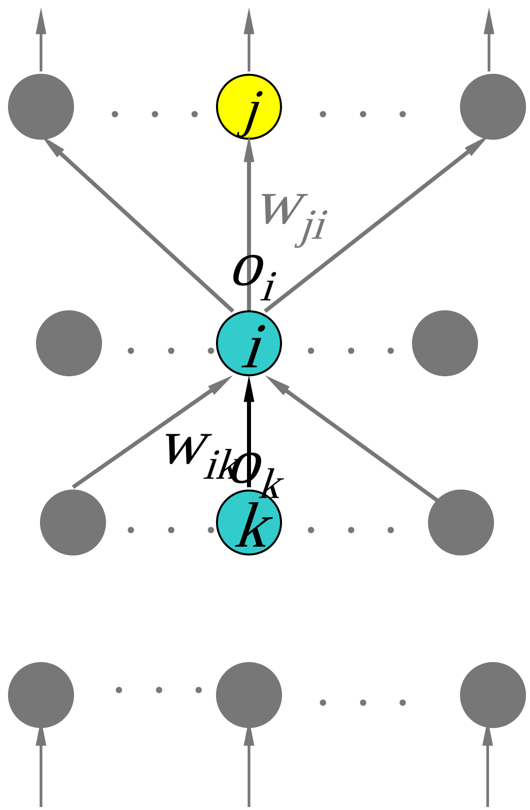

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial}{\partial w_{ik}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ik}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ik}} = \underbrace{\frac{\partial E^{(l)}}{\partial net_i^{(l)}}}_{?}\ \underbrace{\frac{\partial net_i^{(l)}}{\partial w_{ik}}}_{?}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Hidden Neurons




$\delta_i^{(l)}$

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial}{\partial w_{ik}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ik}}$$

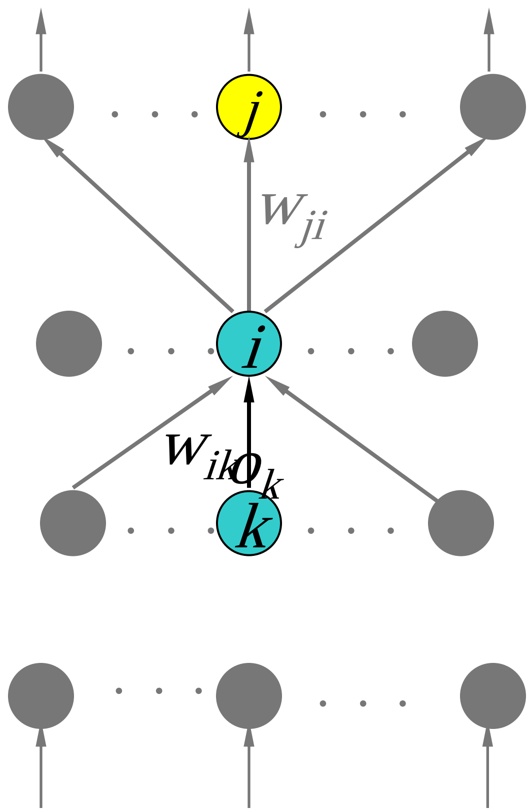
$$\frac{\partial E^{(l)}}{\partial w_{ik}} = \frac{\partial E^{(l)}}{\partial net_i^{(l)}}\boxed{\frac{\partial net_i^{(l)}}{\partial w_{ik}}} \rightarrow o_k^{(l)}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Hidden Neurons

$$\delta_i^{(l)}$$




$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial}{\partial w_{ik}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ik}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ik}} = \boxed{\frac{\partial E^{(l)}}{\partial net_i^{(l)}}}\frac{\partial net_i^{(l)}}{\partial w_{ik}} \rightarrow o_k^{(l)}$$

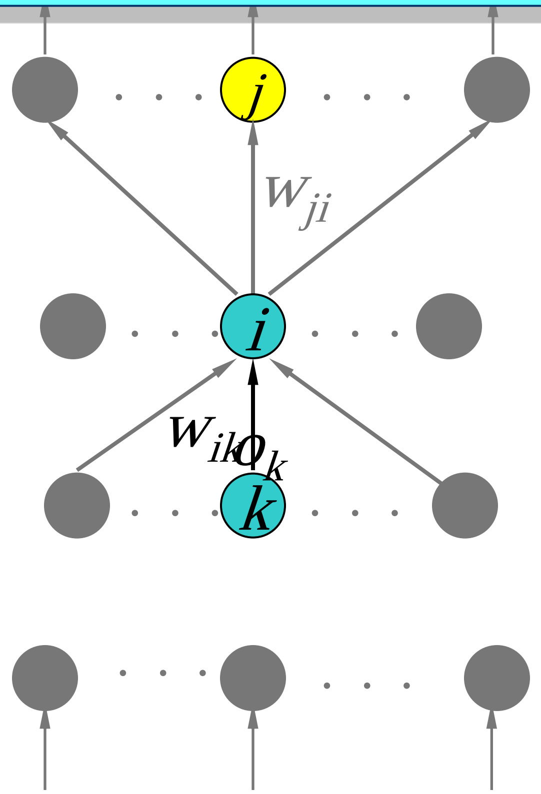
$$\frac{\partial E^{(l)}}{\partial net_i^{(l)}} = \underbrace{\frac{\partial E^{(l)}}{\partial o_i^{(l)}}}_{?}\frac{\partial o_i^{(l)}}{\partial net_i^{(l)}} \rightarrow \lambda o_i^{(l)}(1 - o_i^{(l)})$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Hidden Neurons

$$\delta_i^{(l)} = \frac{\partial E^{(l)}}{\partial net_i^{(l)}} = \lambda o_i^{(l)}(1-o_i^{(l)})\sum_j w_{ji}\delta_j^{(l)}$$




$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial}{\partial w_{ik}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ik}}$$

$\delta_i^{(l)}$

$$\frac{\partial E^{(l)}}{\partial w_{ik}} = \boxed{\frac{\partial E^{(l)}}{\partial net_i^{(l)}}}\frac{\partial net_i^{(l)}}{\partial w_{ik}}$$

$o_k^{(l)}$

$$\frac{\partial E^{(l)}}{\partial net_i^{(l)}} = \boxed{\frac{\partial E^{(l)}}{\partial o_i^{(l)}}}\frac{\partial o_i^{(l)}}{\partial net_i^{(l)}}$$

$\lambda o_i^{(l)}(1-o_i^{(l)})$

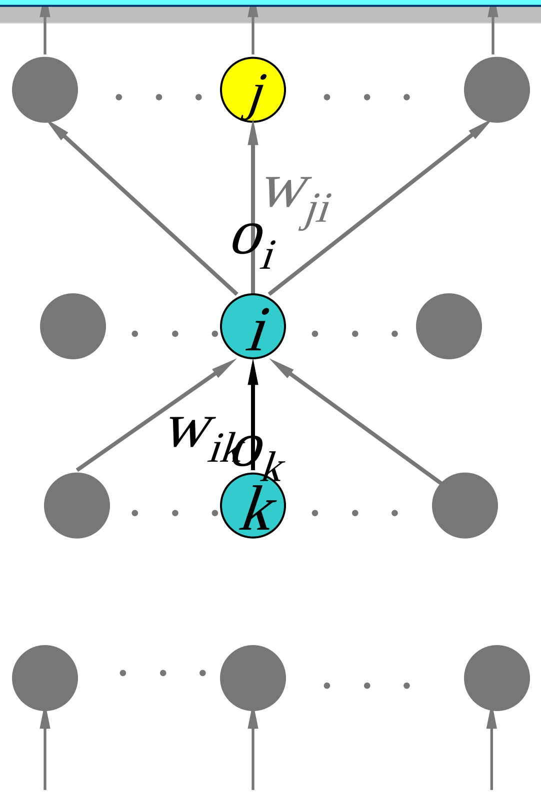
$$\frac{\partial E^{(l)}}{\partial o_i^{(l)}} = \sum_j \underbrace{\frac{\partial E^{(l)}}{\partial net_j^{(l)}}}_{\delta_j^{(l)}}\underbrace{\frac{\partial net_j^{(l)}}{\partial o_i^{(l)}}}_{w_{ji}}$$

$$E^{(l)} = \frac{1}{2}\sum_{j=1}^{n}\left[d_j^{(l)} - o_j^{(l)}\right]^2$$

$$E = \sum_{l=1}^{p} E^{(l)}$$

# Learning on Hidden Neurons

$$\delta_i^{(l)} = \frac{\partial E^{(l)}}{\partial net_i^{(l)}} = \lambda o_i^{(l)}(1-o_i^{(l)})\sum_j w_{ji}\delta_j^{(l)}$$



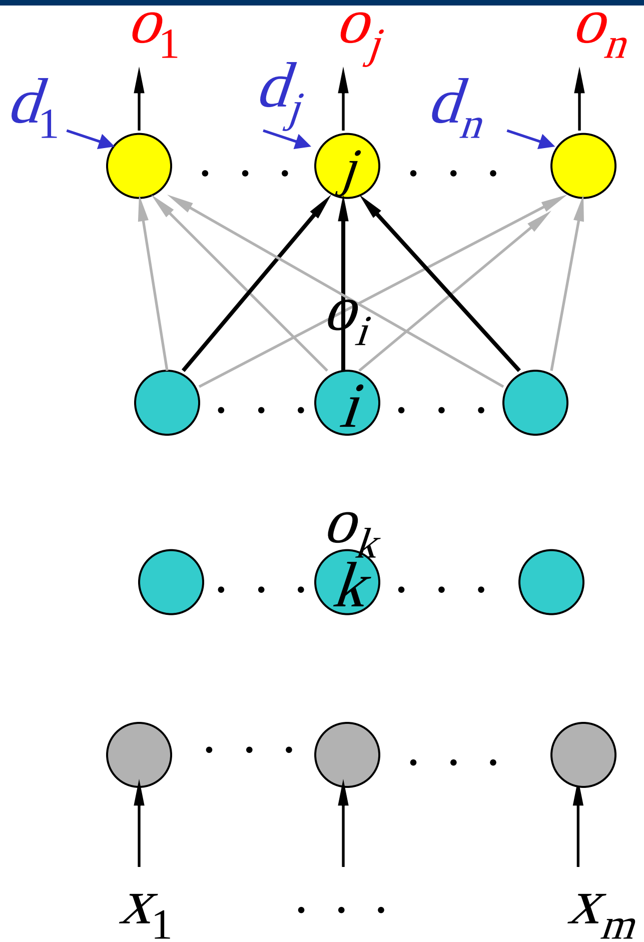

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial}{\partial w_{ik}}\sum_{l=1}^{p} E^{(l)} = \sum_{l=1}^{p}\frac{\partial E^{(l)}}{\partial w_{ik}}$$

$$\frac{\partial E^{(l)}}{\partial w_{ik}} = \boxed{\frac{\partial E^{(l)}}{\partial net_i^{(l)}}}\frac{\partial net_i^{(l)}}{\partial w_{ik}} \quad o_k^{(l)}$$

$$\frac{\partial E}{\partial w_{ik}} = \sum_{l=1}^{p}\delta_i^{(l)} o_k^{(l)}$$

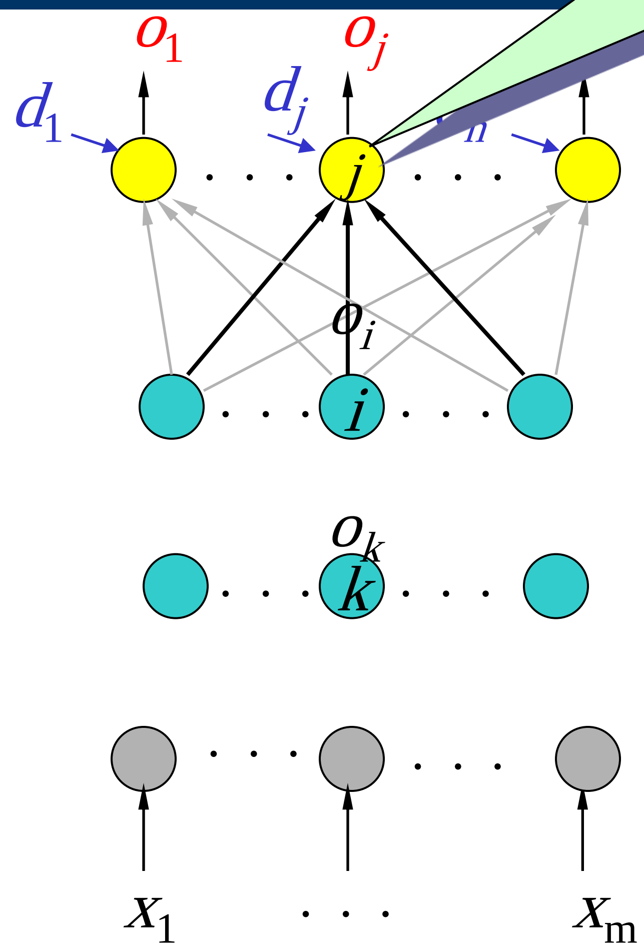
$$\Delta w_{ik} = -\eta\sum_{l=1}^{p}\delta_i^{(l)} o_k^{(l)}$$

# Back Propagation



# Back Pr

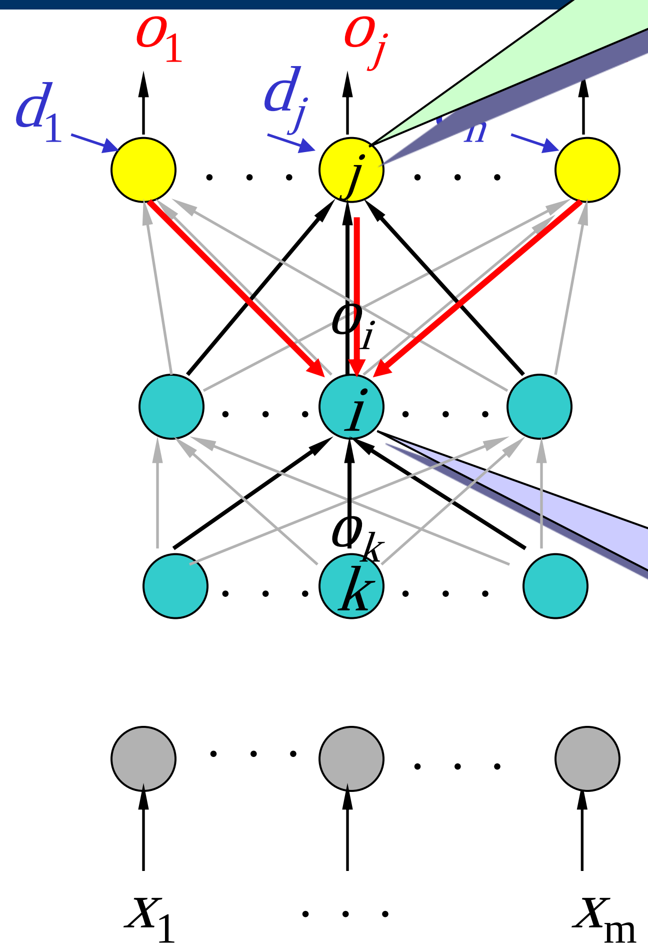
$$\delta_j^{(l)} = \frac{\partial E^{(l)}}{\partial net_j^{(l)}} = -\lambda(d_j^{(l)} - o_j^{(l)})o_j^{(l)}(1 - o_j^{(l)})$$




$$\Delta w_{ji} = -\eta \sum_{l=1}^{p} \delta_j^{(l)} o_i^{(l)}$$

# Back Pr

$$\delta_j^{(l)} = \frac{\partial E^{(l)}}{\partial net_j^{(l)}} = -\lambda(d_j^{(l)} - o_j^{(l)})o_j^{(l)}(1 - o_j^{(l)})$$




$$\Delta w_{ji} = -\eta \sum_{l=1}^{p} \delta_j^{(l)} o_i^{(l)}$$

$$\Delta w_{ik} = -\eta \sum_{l=1}^{p} \delta_i^{(l)} o_k^{(l)}$$

$$\delta_i^{(l)} = \frac{\partial E^{(l)}}{\partial net_i^{(l)}} = \lambda o_i^{(l)}(1 - o_i^{(l)}) \sum_j w_{ji} \delta_j^{(l)}$$

# Learning Factors

- Initial Weights
- Learning Constant ($\eta$)
- Cost Functions
- Momentum
- Update Rules
- Training Data and Generalization
- Number of Layers
- Number of Hidden Nodes

# Reading Assignments

- Shi Zhong and Vladimir Cherkassky, "Factors Controlling Generalization Ability of MLP Networks." In Proc. IEEE Int. Joint Conf. on Neural Networks, vol. 1, pp. 625-630, Washington DC. July 1999. (http://www.cse.fau.edu/~zhong/pubs.htm)

- Rumelhart, D. E., Hinton, G. E., and Williams, R. J. (1986b). "Learning Internal Representations by Error Propagation," in Parallel Distributed Processing: Explorations in the Microstructure of Cognition, vol. I, D. E. Rumelhart, J. L. McClelland, and the PDP Research Group. MIT Press, Cambridge (1986).

  (http://www.cnbc.cmu.edu/~plaut/85-419/papers/RumelhartETAL86.backprop.pdf).