

# Graph Neural Network

## Lecture 3



- 1 Introduction to GCNs
- 2 Mathematical Formulation

# What is a Graph Convolutional Network (GCN)?

- A type of Graph Neural Network (GNN) designed for graph-structured data.
- Extends the concept of convolutional neural networks (CNNs) to graphs.
- Captures local graph structure and node features.

- **Graph-Structured Data:** Many real-world problems involve graphs (e.g., social networks, molecular structures).
- **Node-Level Tasks:** Node classification, link prediction.
- **Graph-Level Tasks:** Graph classification, clustering.

The GCN layer is defined as:

$$H^{(l+1)} = \sigma \left( \tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(l)} W^{(l)} \right)$$

- $\tilde{A} = A + I$ : Adjacency matrix with self-loops.
- $\tilde{D}$ : Degree matrix of  $\tilde{A}$ .
- $H^{(l)}$ : Node features at layer  $l$ .
- $W^{(l)}$ : Learnable weight matrix.
- $\sigma$ : Activation function (e.g., ReLU).

- **Normalization:**  $\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}$  ensures normalized aggregation.
- **Feature Propagation:** Each node aggregates features from its neighbors.
- **Nonlinearity:** Activation function introduces nonlinearity.