University of Tehran

COMPUTER SCIENCE DEPARTMENT

COMPUTATIONAL NEUROSCIENCE

REPORT 1

IZHIKEVICH NEURON ANALYSIS

Student Name	Student ID
AMIR NADERI	610398126

Lecturer in charge:

M.Ganjtabesh

Submission Date : 18/05/2023

1 Izhikevich Neuron Model

The Izhikevich neuron model reproduces spiking and bursting behavior of known types of cortical neurons. It takes the biological plausibility of Hodgkin and Huxley neurons while being as computationally efficient as integrate-and-fire neurons. It can reproduce the spiking and bursting of cortical neurons and we can simulate thousands of them on a regular computer.
Hodgkin and Huxley's neurons are very accurate, but we can only simulate a handful of these neurons since it's so computationally expensive. On the other hand, we have integrate-and-fire neurons. These are computationally effective, but it's unrealistic simple, thus not able to reproduce rich spiking and bursting that is in real neurons. So this is the most efficient and powerful model in computational neuroscience.
We can represent this model through a 2-D system of differential equations:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \tag{1}$$

$$\frac{du}{dt} = a(bv - u) \tag{2}$$

Now we got the resetting part:

$$if\ v > 30\,mv,\ then\ [v = c\ and\ u = u + d] \tag{3}$$

Here are the parameters used in equations:

- v: Represents the membrane potential of the neuron
- u: Represents a membrane recovery variable, which accounts for the activation of k^+ ionic currents and inactivation of Na^+ ionic currents, and it provides negative feedback to v. After the spike reaches to its threshold, the membrane voltage and the recovery variable are reset according to the (3).
- I: The input current or injected dc-currents
- a: It describes the time scale of the recovery variable u. Smaller values result in slower recovery. A typical value is $a = 0.02$.
- b: It describes the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential v. A typical value is $b = 0.2$. Greater values couple and more strongly resulting in possible subthreshold oscillations and low-threshold spiking dynamics, because u parameter will try to decrease the neuron's potential.
- c: after-spike reset value of v. A typical value of -65 mV. It will be reset because of the fast high-threshold k^+ conductances.
- d: after-spike reset value of membrane recovery variable. A typical value of 2. It will be reset because the slow high-threshold Na^+ and K^+ conductances. It will cause to lower the membrane potential value, so the spike frequency adaption will happen and after a while the neurons will spike with bigger and distinguished inter-spike values, in other word the inter-spike period of neurons will increase. This keeps homeostasis of neurons.

The voltage in this case is the membrane potential v. And the K^+ is the membrane recovery variable u.
Here you can see these parameters in diagrams:

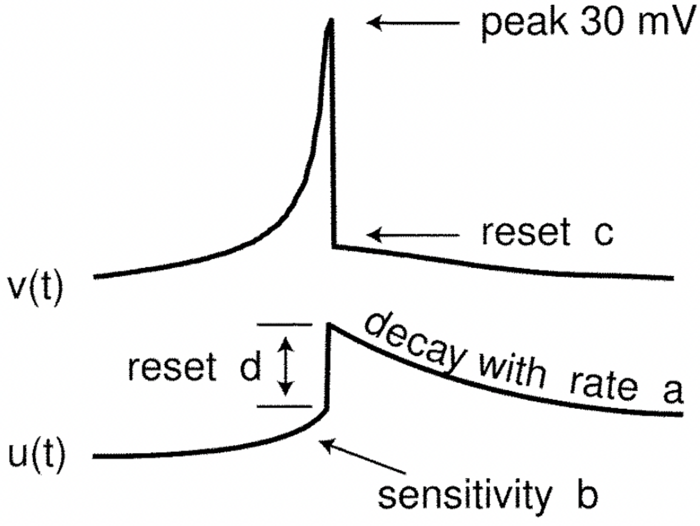


Figure 1: parameters of Izhikevich Neuron Model

2 Different Types of Neurons With Step-input Current

In this part, we use step-input current and analyze the behavior of the neurons. In the code the threshold voltage is +30 mV and we initialized the membrane recovery variable with value of -8 mV. Also we can use different value of parameters such as a,b,c,d and I. Also we made it possible to use neuron populations. For this section we used a single neuron and applied different parameters and got the results using the Izhikevich model.

2.1 simulate neurons in mammalian brain

There are lots of types of patterns spiking and bursting inside of cells in mammalian brain. Below are the descriptions of the different types and simulation of our model with diagrams. Just notice that our input current is I = 15 A and the time resolution is 0.5 ms:

1. Regular Spiking (RS) neurons: These are our normal neurons. the parameters are:

    - $a : 0.02$
    - $b : 0.2$
    - $c : -65$
    - $d : 8$

    It makes neurons to fire a few times in short periods, and the periods will be increased. It is called spike frequency adaptation. If we increase the

injected dc-current, we also increase the frequency, but it's never too fast because we have long after hyperpolarization(because of parameter d. It increses the u parameter, so the hyperpolarization will be long).
Below, you can see the diagram:

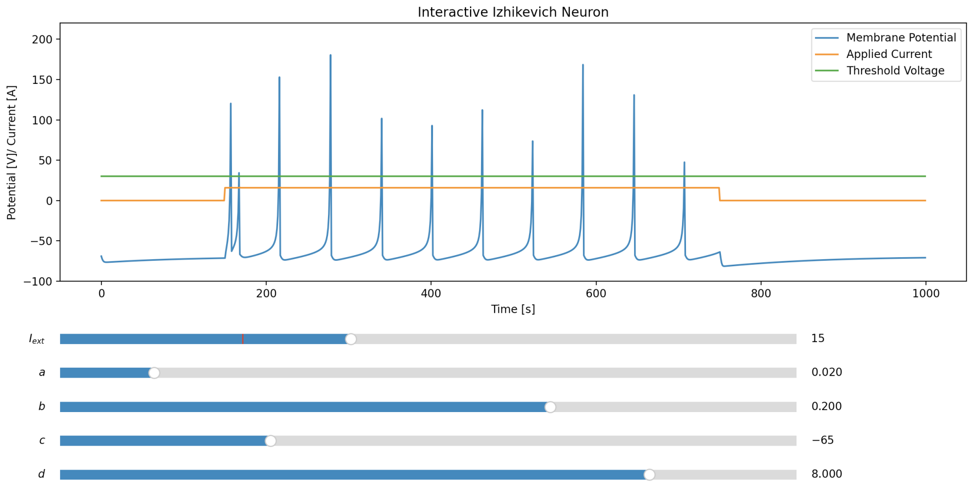


Figure 2: Regular Spiking with I=15 A

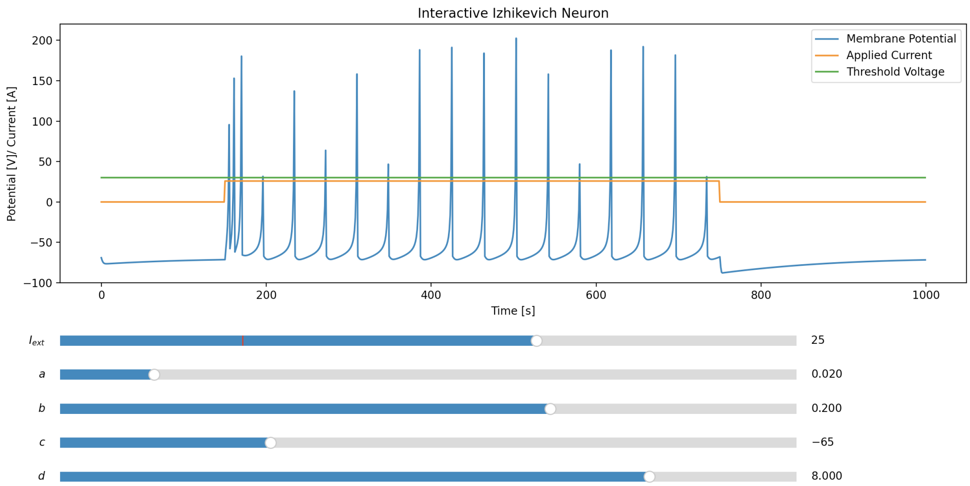


Figure 3: Regular Spiking with I=25 A

Now see the raster plot of 40 neurons with I = 15 A:

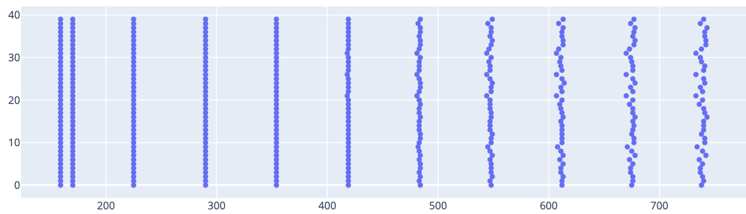


Figure 4: Regular Spiking raster plot with I=15 A

As you can see at first neurons spike with short inter-spike period. But then the adaption will happen because of parameter of membrane recovery variable u which plays the role of K^+ channels. As we increase the input current, the frequency of spikes will never be too fast. We can see this behavior in another way: At first, when the neuron fire a lot of spike, the u value will increase so after a while it takes a longer time for the neuron to spike.
An interesting part of this test is when we cut off the input current, the membrane potential shoots down the way below of the resting potential. That's because of the u value is still high and when we don't have input current, the voltage won't rise and it takes a while for that u to decay. This is called under shooting. You can see the diagram of u variable below:

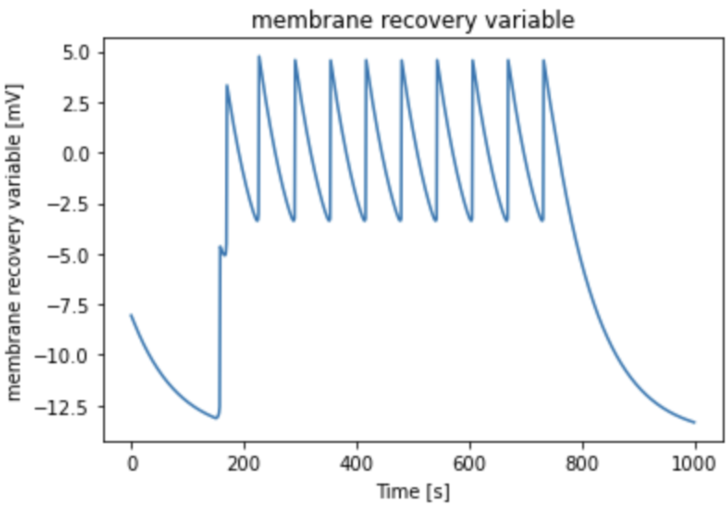


Figure 5: membrane recovery variable for regular spiking

Now it's the time to see the F-I curve of this type of neuron. The F-I curve is a diagram of frequencies related to the input current. We applied current between the values of 0.0 to 40.0 and computed the frequency per current. the result is:

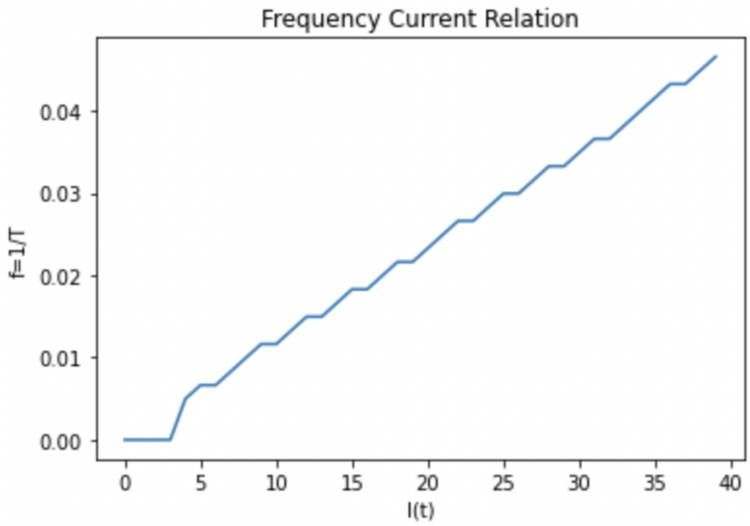


Figure 6: F-I curve of Regular Spiking neuron

As you see with applying more current to the model, our frequency increases. Until I = 3 A, there is no spikes and after that neuron starts firing.

2. Intrinsically Bursting (IB) neurons: First, we fire a lot of bursts, but after a while, we end up with normal spiking. This is because the u builds up over time. Here are the parameters:

   - $a : 0.02$
   - $b : 0.2$
   - $c : -55$
   - $d : 4$

   In this case the parameter d is lower than the previous part. so the increase of parameter u will be lower. this change makes the model to spike faster, so we have more spike in the beginning. But after a while adaption will happen and the neuron fires with less inter-spike period. check the diagram below for this type:

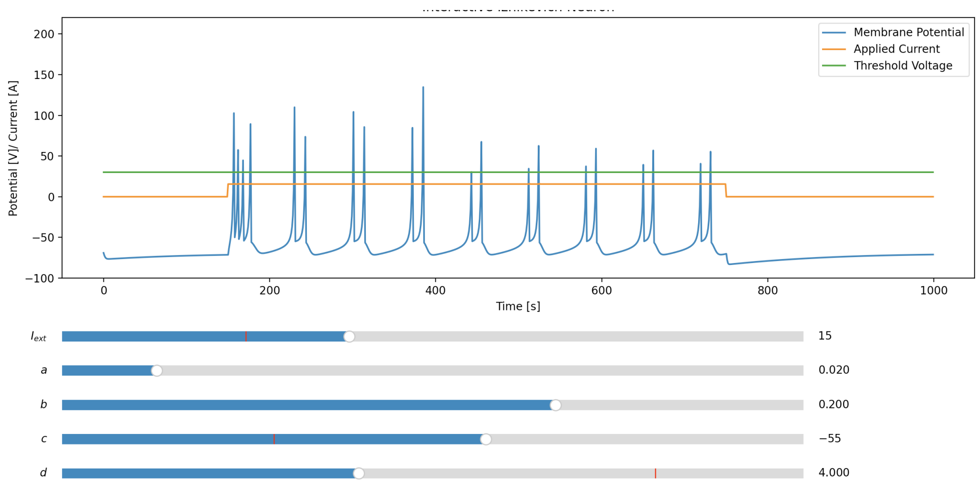


Figure 7: Intrinsically Bursting with I = 15 A

Now check the changes of membrane recovery variable u:

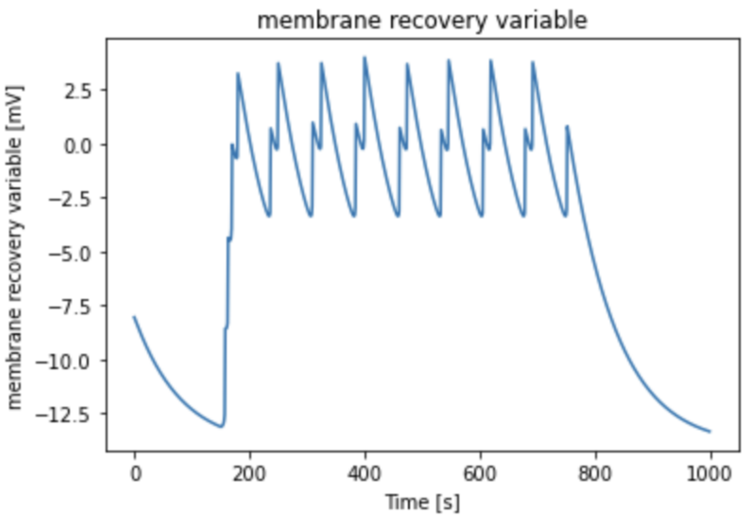


Figure 8: membrane recovery variable for Intrinsically Bursting

As you can see, the maximum value of u is lower than the maximum value in the previous type of neuron. Check the F-I curve of this type of neuron:

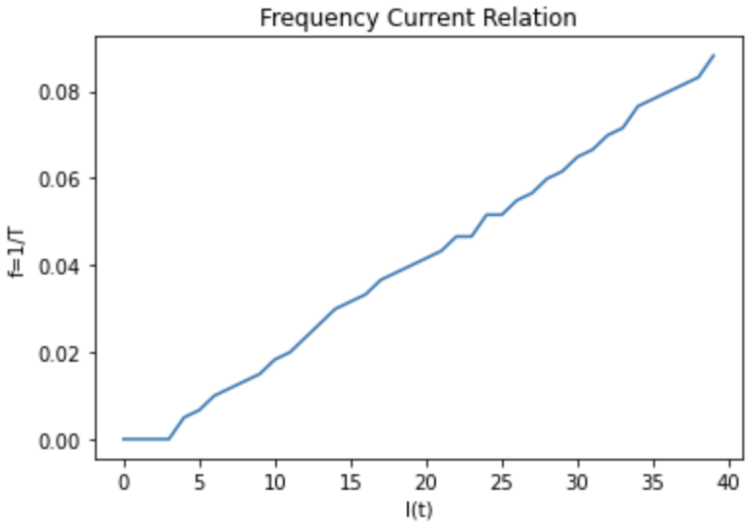


Figure 9: F-I curve of Intrinsically Bursting neuron

Well that's a reasonable figure. the parameter d is lower than the previous type, so the spikes are faster and by applying more current, the model spikes with more frequencies. As you see the frequencies are reaching to 0.09 comparing the last part which the value was 0.05

3. Chattering (CH) neurons: neurons fire spaced apart bursts. The frequency inside of bursts can be as high as 40 Hz.It has bursts of closely spaced spikes, means the inter-burst of every period is very low. The parameters are:

    - $a : 0.02$
    - $b : 0.2$
    - $c : -50$
    - $d : 2$

    check the behaviour if the neuron:

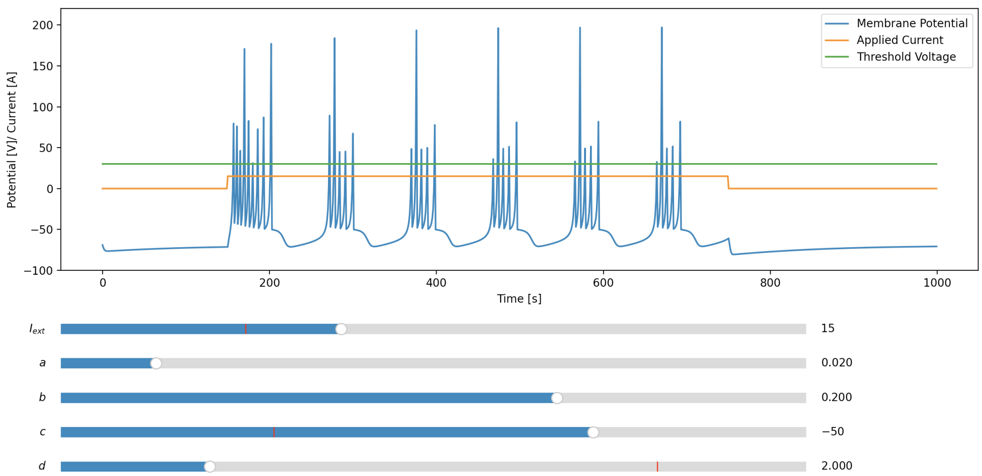


Figure 10: chattering neuron with I = 15 A

So here the parameter d is very low. What does it mean? we can say when d is low, the membrane recovery variable won't be increased enough to hold the membrane voltage to reach to the threshold. so the spikes are fast. Now see the changes of u for this type of neuron:

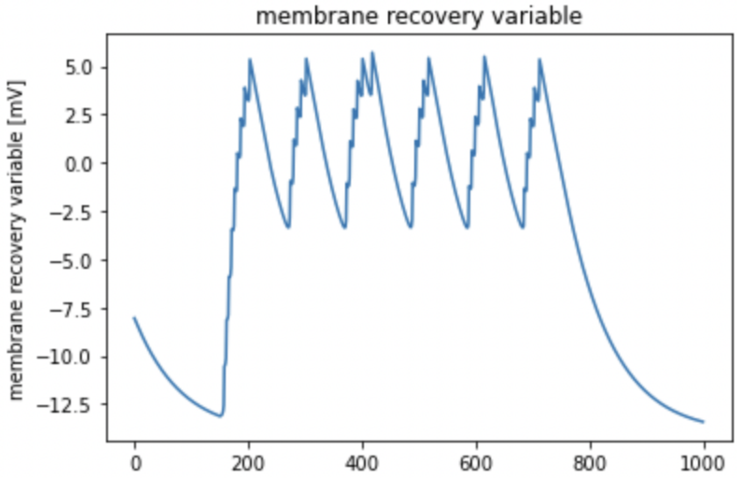


Figure 11: membrane recovery variable for chattering neuron

Now we increase the input current(25 A) and see the results:

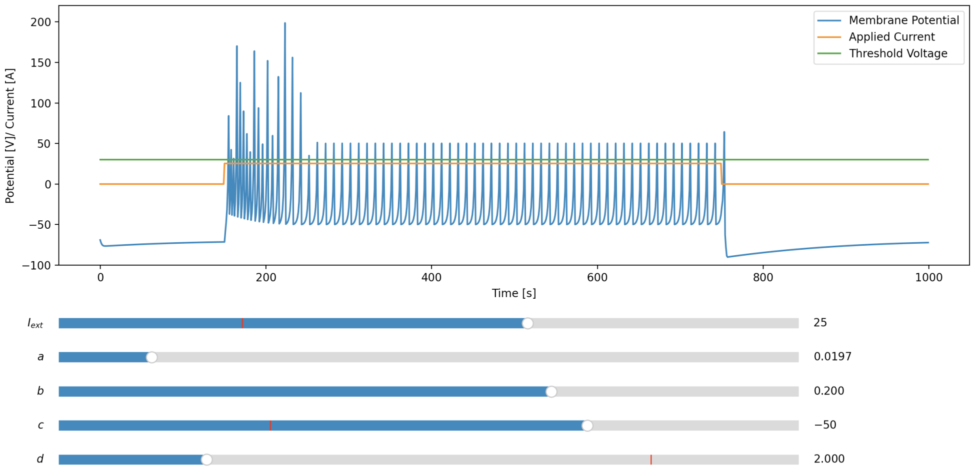


Figure 12: chattering neuron with I = 25 A

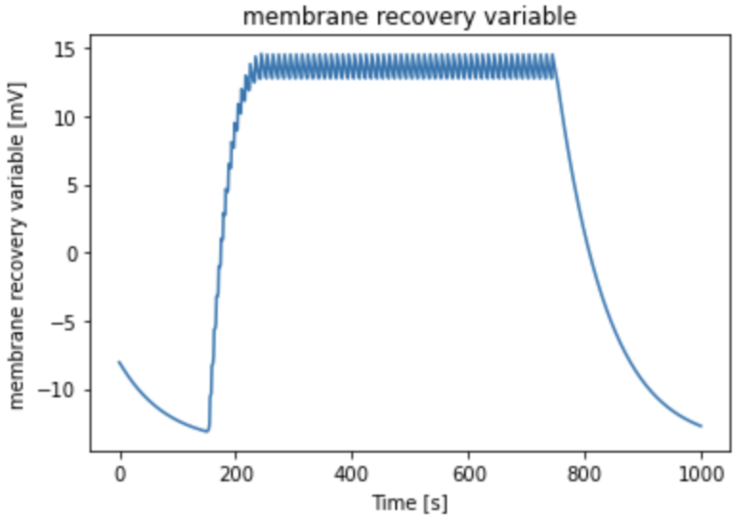


Figure 13: membrane recovery variable for chattering neuron with I = 25 A

According to the Figure 13, the value of us are high. Why does it happen? As you can see when the input current is very high, the neuron will reach to the threshold very soon and it spikes very fast with short inter-spike period. so with every spike the value of u will increase by d. on the other hand parameter I is high and it compensates the increase of u. So here we got u which it wants to lower the membrane potential and got I which is very high and it doesn't let the u parameter to do its job. Finally it spikes very fast and the value of u will be high. In biology we can interpret this behaviour as LDP, which the Na^+ channels will increase and because parameter d is low, membrane recovery variable can't do its job perfectly(less k^+ channels), so in short period the neuron spikes. Now see the F-I curve:

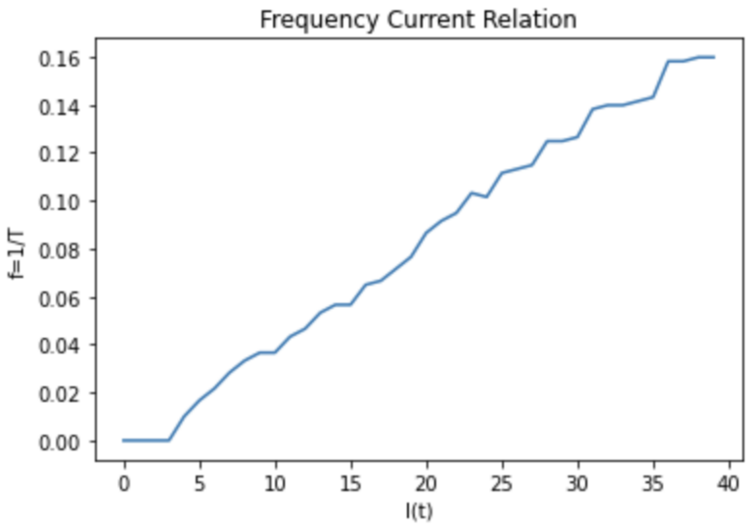


Figure 14: F-I curve of chattering neuron

Here because of parameter d which is set to 2, frequencies are higher than the two above types of neurons.

4. Fast Spiking (FS) neurons: Neurons can fire spikes super quickly at high frequencies with no slowing down(No adaption). Here are the parameters:

    - $a : 0.1$
    - $b : 0.2$
    - $c : -65$
    - $d : 8$

    The only parameters that has changed from our first type of neurons(regularly spiking neurons) is a. The value of a indicates the decay rate of membrane recovery variable. When this rate is high, the parameter u will recover fast and the neuron's membrane potential reaches to the threshold faster. Now check the diagrams below:

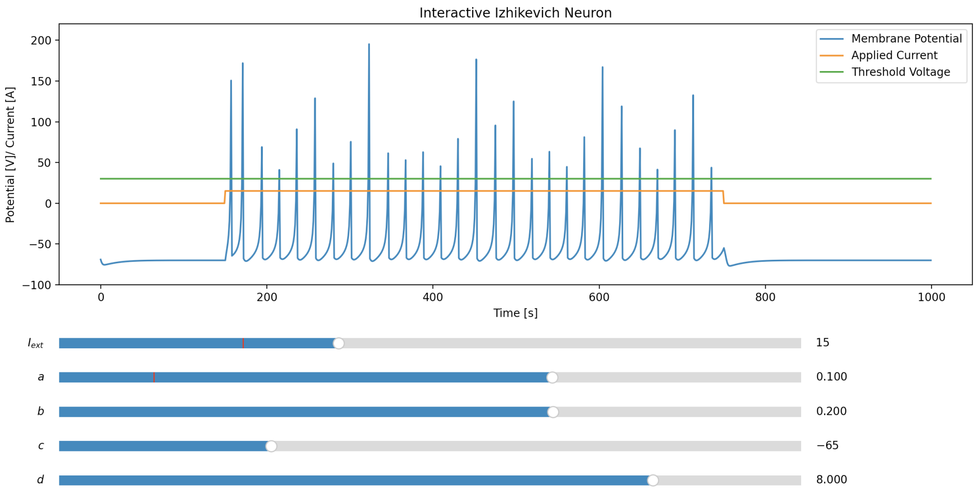


Figure 15: fast spiking neuron with I = 15 A

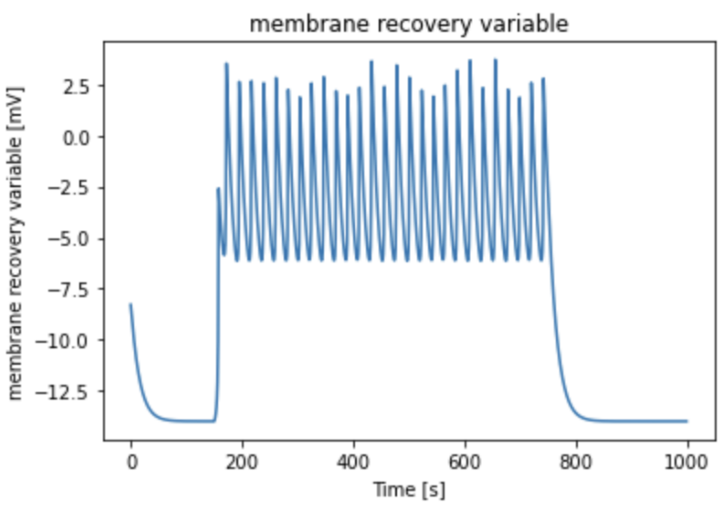


Figure 16: membrane recovery variable for fast spiking neuron

Check the F-I curve. what is your guess about this diagram? Let's see:

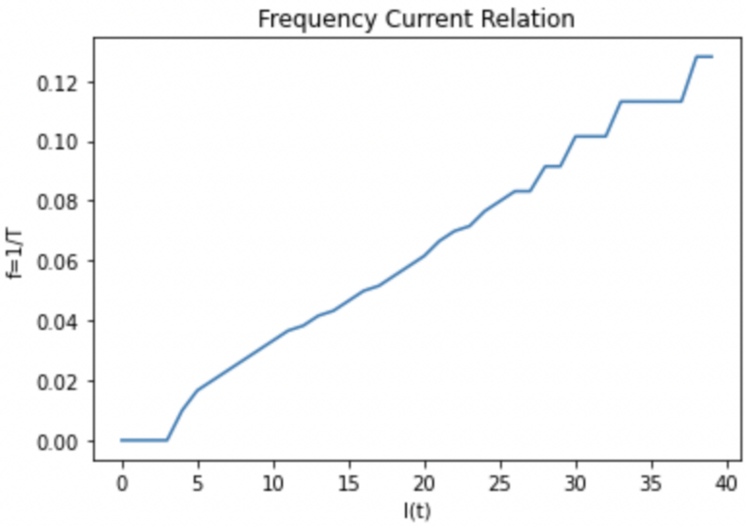


Figure 17: F-I curve of fast spiking neuron

There is a point in this figure! If you compare this type of neuron with the previous neuron(chattering), you see that the frequencies of chattering neurons are higher than the fast spiking ones. It means the effect of parameter d is more than the effect of parameter a with these settings. In other word if the neuron's resetting parameter decreases(after spikes, the parameter u increase slowly), the neuron spikes faster comparing to the condition when neuron's membrane recovery variable recovers faster(a is high).

5. Low-Threshold Spiking neurons(LTS): Low-threshold spikes (LTS) refer to membrane depolarizations by the T-type calcium channel. LTS occur at low, negative, membrane depolarizations. They often follow a membrane hyperpolarization, which can be the result of decreased excitability or increased inhibition. This type of neurons spike at low thresholds. The intervals increase with respect to time. These neurons can also fire high-frequency trains of action potentials but with a noticeable spike frequency adaptation. Here are the parameters:

    - $a : 0.02$
    - $b : 0.25$
    - $c : -65$
    - $d : 8$

    In this type, the parameter b is set to the value 0.25. b is the parameter in Izhikevich model that indicates the sensitivity of the membrane recovery variable u to the subthreshold fluctuations of the membrane potential v. Also you can say that b indicates how fast membrane recovery variable can reach to the point which u will be reset. When b is higher than the default value, the neuron will have more fluctuation in subthreshold part and will have low-threshold spiking. That's because when the membrane potential goes up, we have more changes in membrane recovery($\frac{du}{dt}$). So it tries to keep the model's voltage under the threshold. So it behaves like a inhibitory neuron. Now see the diagrams:

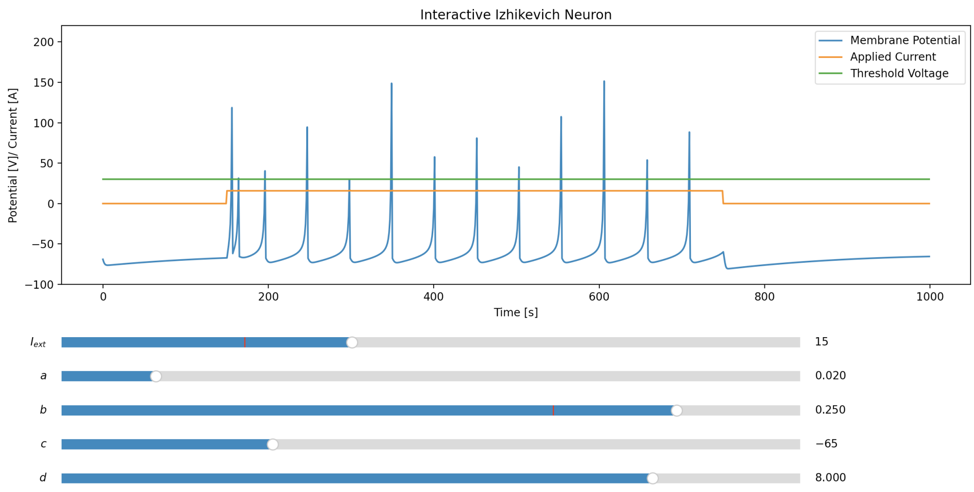


Figure 18: low threshold spiking neuron with I = 15 A

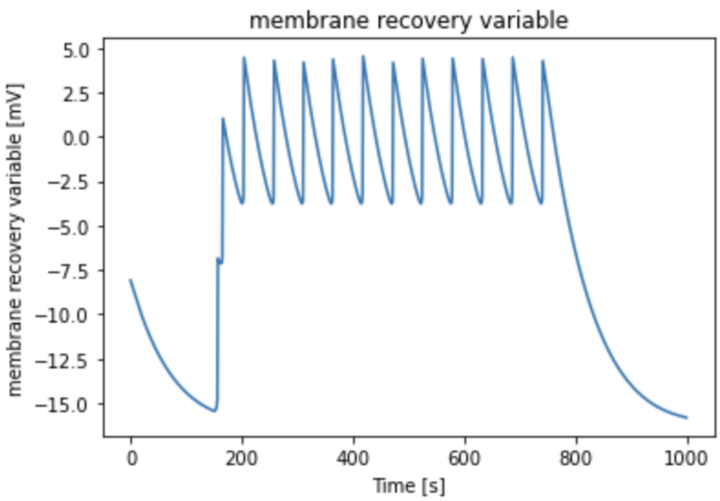


Figure 19: membrane recovery variable for low threshold spiking neuron

Check the F-I curve:

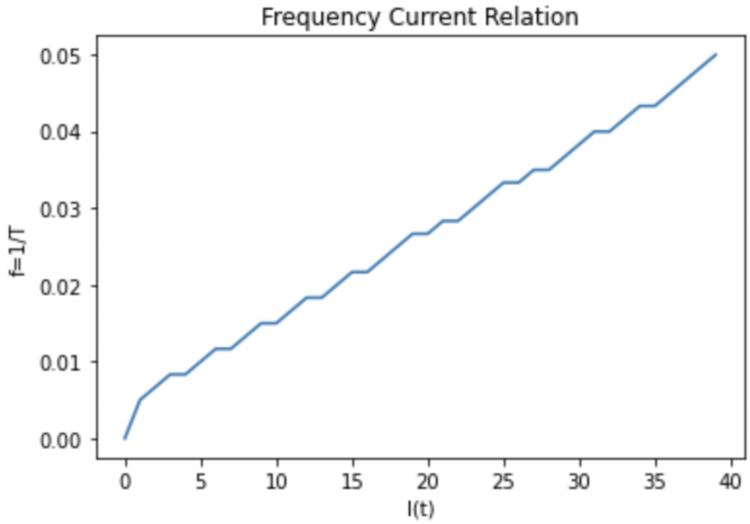


Figure 20: F-I curve of low threshold spiking neuron

6. Resonator (RZ) neurons: These neurons have sustained subthreshold oscillations, and if we increase the input current for a little time, it will spike. In the beginning of this test, we applied a low input current(I = 0.2 A) to the neuron. Then after a while for a brief time we increased the current(I = 2 A) and after that we set the current as its default. In the diagram below you can see the input current based on time:

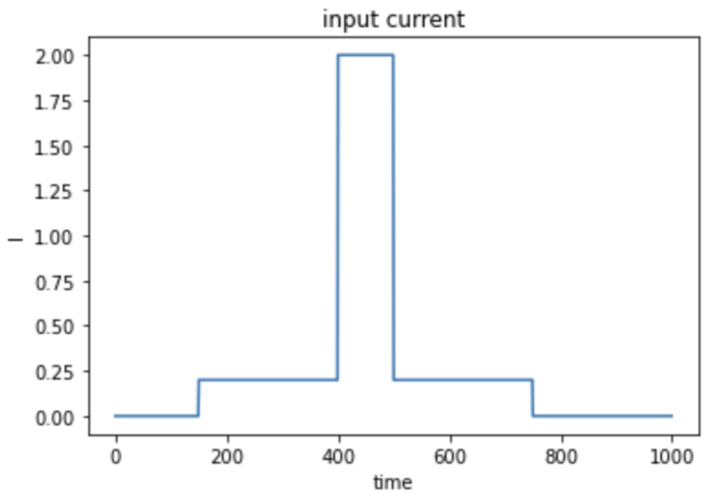
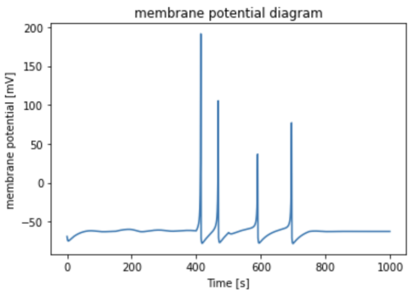


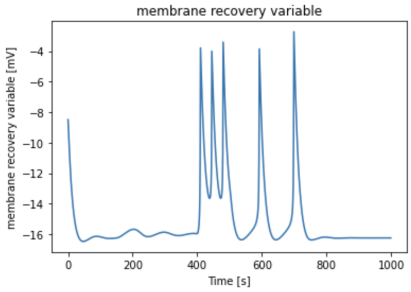
Figure 21: input current for resonator neuron

Now set the parameters of this type and see the results:

- a : 0.1
- b : 0.26
- c : -65
- d : 8



(a) membrane potential



(b) membrane recovery variable

Figure 22: behaviour of resonator neuron

In this test, the parameter a is set to the value of 0.1. So the u variable will recover very fast and the parameter b is set to 0.26 which it means the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential v is high. In result we have fluctuation in the subthreshold section.
In figure 22 part (a), up to time 400, we see that the membrane potential is changing with an oscillations and when we increase the input current, we see that the neuron spikes fast. It is fast because of high value of current and the parameter a which makes the u parameter recover fast. Now when we set the input current to the previous value, we see normal spikes.
Now in part (b) we see when the first spike happens, value of u increases and because of parameter a it will recover fast, After a while when the input current decreases, we see that u shoots down way below(about -16 mV). That happens because when less input current is injected to the model, it gets more time for membrane potential to get to the threshold and fire. If we observe the behavior of this type of neurons more accurately, we see that they are behaving like inhibitory neurons. because of the high parameter of b, the membrane recovery variable effect membrane potential more than before and it holds the membrane potential not to get to the threshold or it gets to the threshold slowly.

7. When $b < a$: In the mathematical area of bifurcation theory a saddle-node bifurcation is a local bifurcation in which two fixed points of a dynamical system collide. Now due to the equations (1) and (2), the saddle-node point is when $b < a$. So we set the parameters and see the result:

   - $a : 0.1$
   - $b : 0.05$
   - $c : -65$
   - $d : 8$

   Now check the diagram below:

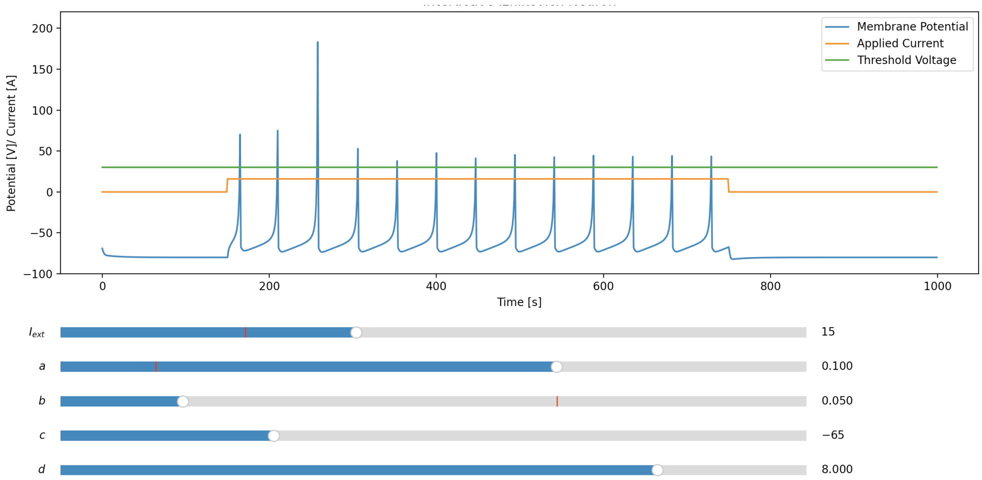


Figure 23: membrane potential changing when $b < a$

And this is the membrane recovery variable changing plot:

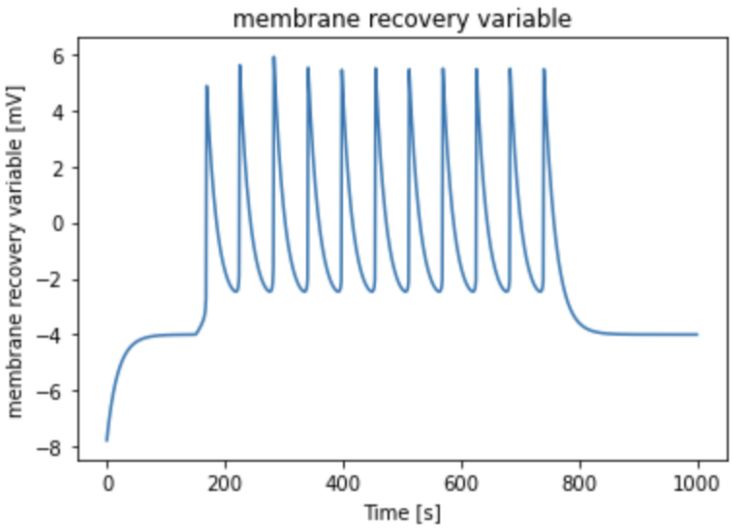


Figure 24: membrane recovery variable when $b < a$

Just remember that this is the saddle-node bifurcation of the resting state. So in this test, the parameter a is 0.1, means the membrane recovery variable recovers fast and the parameter b is 0.05 which is low. in this case the sensitivity of the recovery variable u to the subthreshold fluctuations of the membrane potential v is low. When b is low, the parameter u change slower than before. On the other hand parameter a is high, so when the parameter u is recovering very fast, we expect than v can spike fast too. But the parameter b won't let that happen, because there will be less change in membrane recovery variable. So we can say there is a balance in this type of the neuron and neuron fires its spikes with a relatively much inter-spike periods.
Now check the F-I curve for this type of neuron:

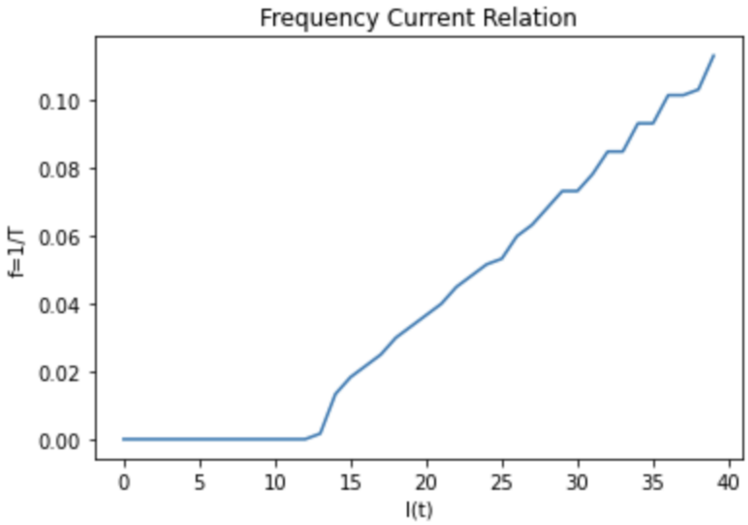


Figure 25: F-I curve when $b < a$

As you see despite of the previous types of neuron, we should apply higher amount of input current to the model for the neuron to start spiking. The reason of this behaviour is that when is low the change of voltage is slow and in this case when the parameter a is high the membrane recovery parameter decays fast and if you see the figure 24, the membrane recovery variable is increasing in the beginning. It means that it tries to hold the membrane potential not to reach to the threshold voltage, but in the previous types of neurons, the parameter u is decreasing in the beginning.

3 Different Types of Neurons With Noisy-input Current

In this section, we applied different kinds of noisy-input current and analyze the results. These are some selective types of neurons that we test in the last part with the step-input and new conditions. Now we test with different noisy-input currents:

1. Regular spiking: the parameters were:

   - $a : 0.02$
   - $b : 0.2$
   - $c : -65$
   - $d : 8$

   First, we applied a normal distribution input current with mean of 20 and standard deviation of 3. the result are in diagrams below:

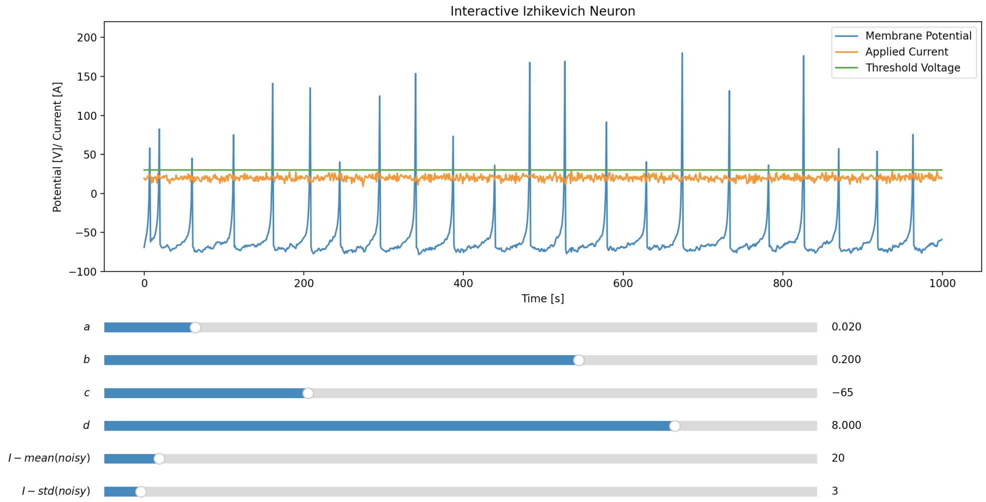


Figure 26: regular spiking with noisy input current

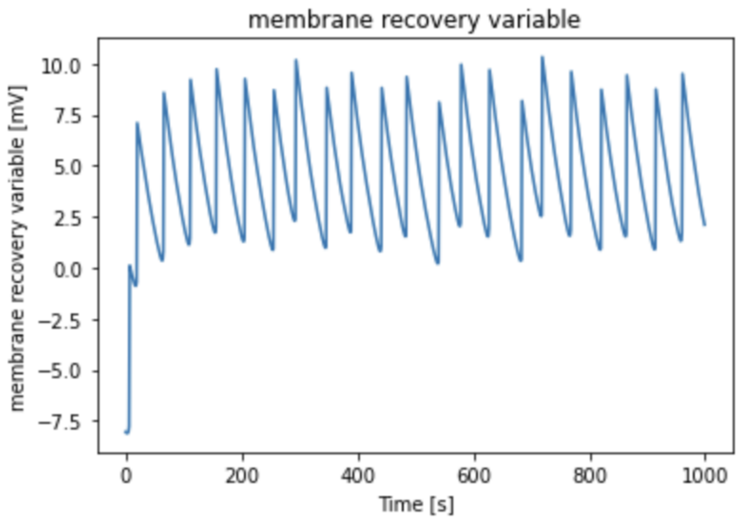


Figure 27: membrane recovery variable changing for regular spiking

The difference between figure 26 and figure 2 is in the subthreshold part. the Input current is noisy, so there is more fluctuation in the subthreshold part, but neuron fires with a distinguished inter-spike periods. So at first neuron fires with short inter-spike period but then using parameter u, adaption happens and neuron spikes with a larger inter-spike periods. Just notice there is more fluctuation in the subthreshold.
If you look at the membrane recovery variable diagram, you'll notice there is no pattern in changing. That's because of the noise of input current which effect on the membrane potential.
Now if we increase the value of standard deviation, means add more noisy term, the fluctuation will increase in the subthreshold. See the diagram here:

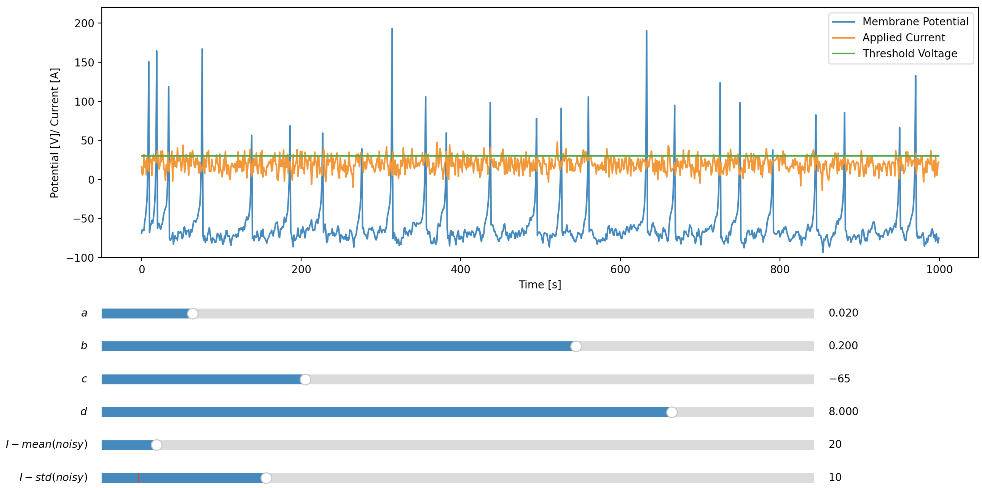


Figure 28: regular spiking

There is an interesting behaviour in the above diagram which is that unlike the pattern pf spikes in the step-input current, there is no adaption after some spikes here. that's because the change in the input current in high(standard deviation is high), so there will be disorder in the behavior of spikes. In result we realized that if the input current is more noisy, adaption will not happen. Now check the membrane recovery diagram:

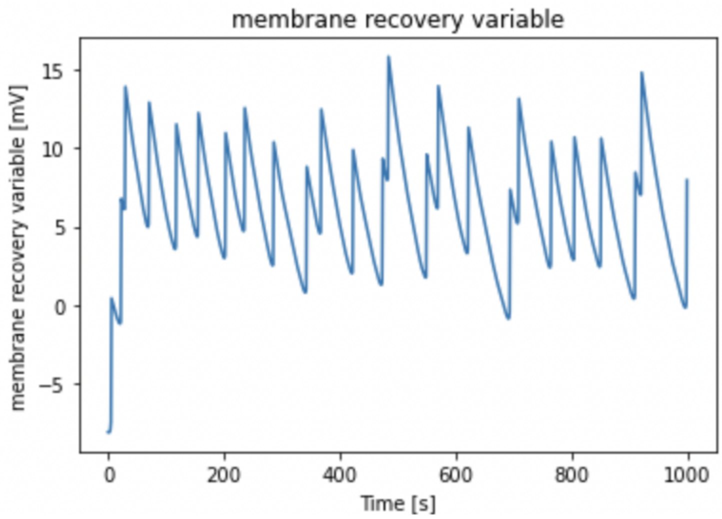
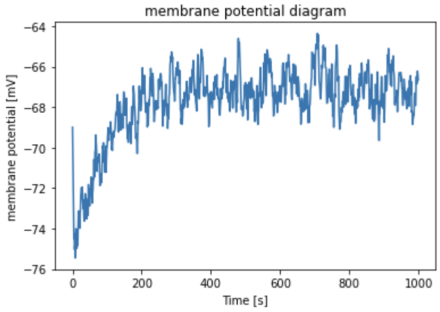
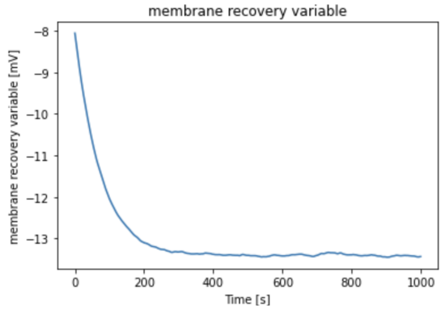


Figure 29: membrane recovery variable changing for regular spiking with more noisy term

Now Let's see a condition where there is no spike with these parameters. The mean of the current is 2 and the std is 1. Check the two diagram below:



(a) membrane potential



(b) membrane recovery variable

Figure 30: behaviour of RS neuron

As you see the membrane potential of this neuron doesn't reach the threshold and it never spikes. So it just has fluctuation under the threshold line and if we see figure 30 part (b), there is no change in membrane recovery variable neither.

2. Chattering (CH) neurons: the parameters are:
   - a : 0.02
   - b : 0.2
   - c : -50
   - d : 2

   Check the diagram for changing membrane potential:

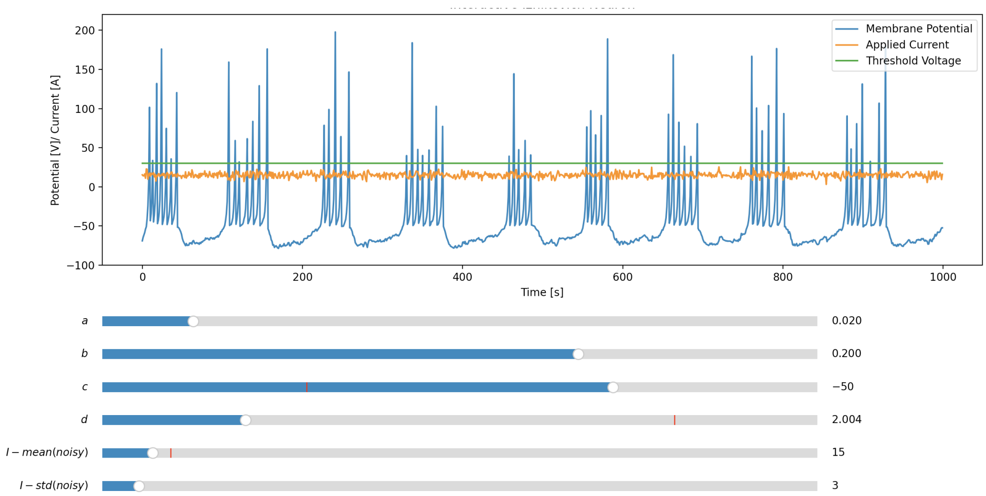


Figure 31: membrane recovery variable changing for regular spiking with more noisy term

The mean of the current is 15 and the standard deviation is 3.
We can say when d is low, the membrane recovery variable won't be increased enough to hold the membrane voltage to reach the threshold. so the spikes are fast. Now see the changes of u for this type of neuron:

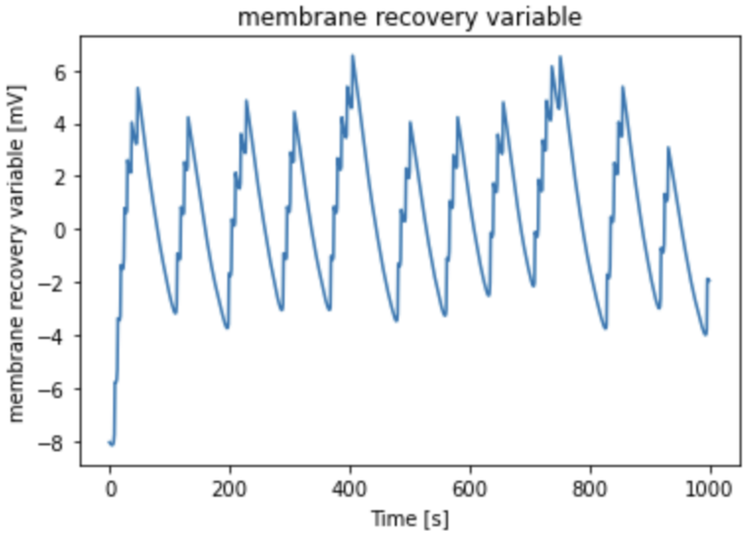


Figure 32: membrane recovery variable changing for chattering

3. Fast spiking neuron: the parameters are:

   - $a : 0.1$
   - $b : 0.2$
   - $c : -65$
   - $d : 8$

   The value of a indicates the decay rate of membrane recovery variable. When this rate is high, the parameter u will recover fast and the neuron's membrane potential reaches the threshold faster. See the diagram below for changing the voltage of this type of neuron:

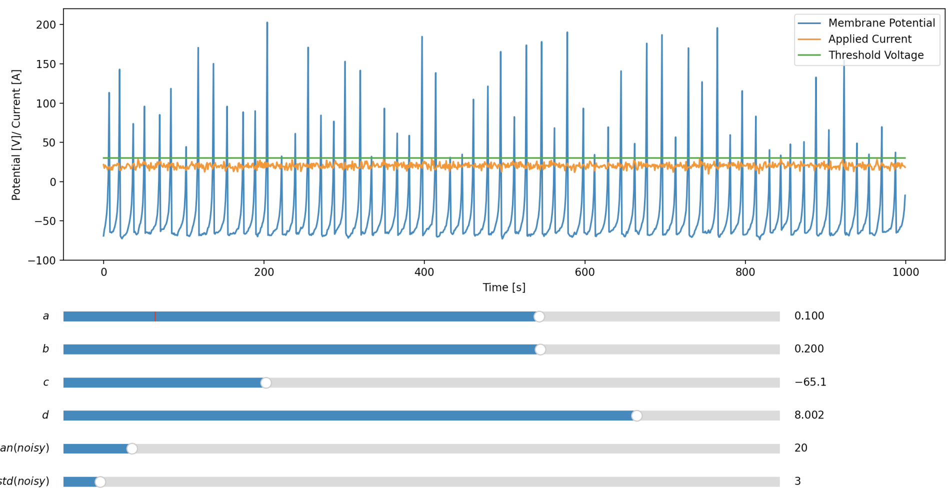


Figure 33: membrane potential changing for fast spiking neuron

Now we set the parameter d to be 2. In this case the membrane recovery variable decays faster than before, so neuron spikes faster. Let's see:

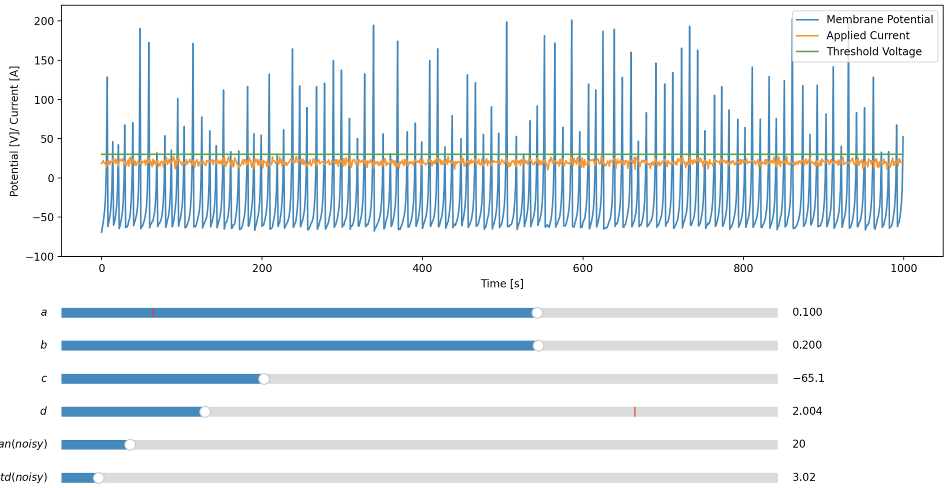


Figure 34: membrane potential changing for fast spiking neuron when d is 2

Now we see the result of the behavior of the neuron with these parameters:

- $a : 0.1$
- $b : 0.3$
- $c : -65$
- $d : 1.5$

So, here the parameter a is set to 0.1 which it means it recovers faster and the neuron spikes fast. Also the parameter d is set to 1.5. It means after every spike, u increases by the value of 1.5 which is not a large number. So the membrane recovery variable won't hold the membrane potential for reaching to the threshold and it reaches faster. On the other hand parameter b is set to 3. b indicates how fast the u variable reaches to the time when u should increase to $u + d$. So in total we set parameters to values which neuron spike very fast. You can see the result in the diagrams below:

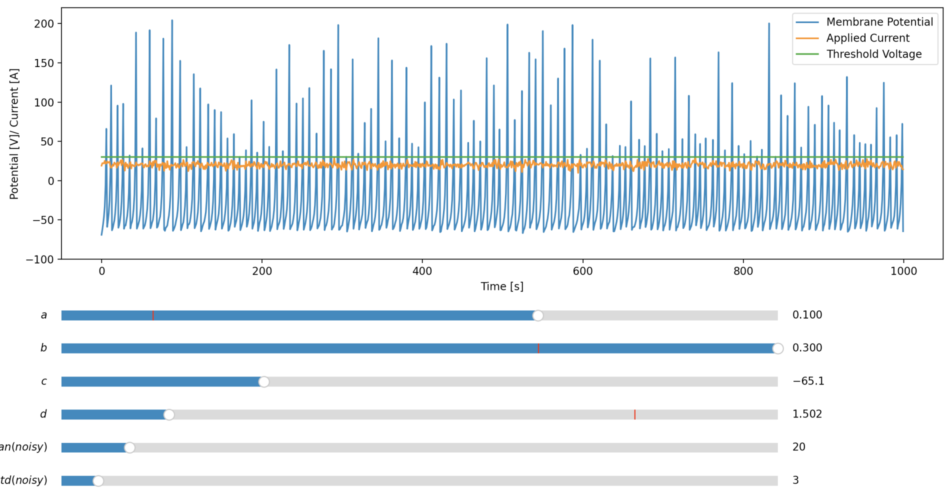


Figure 35: membrane potential changing

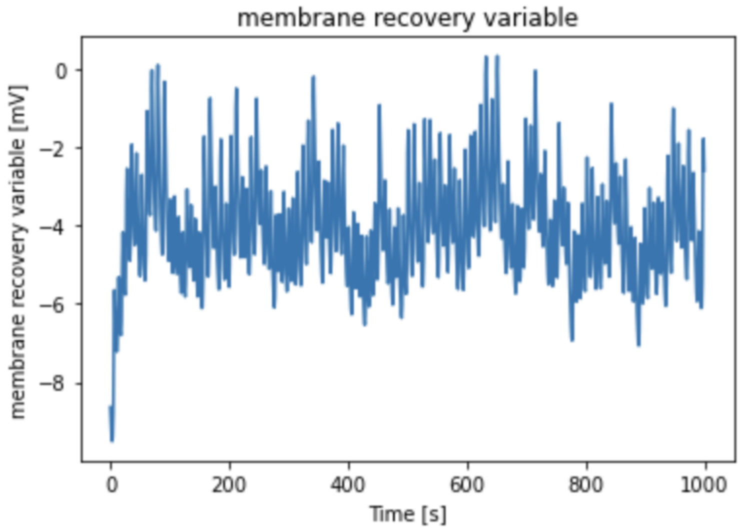


Figure 36: membrane recovery variable changing

As you see, the membrane recovery variable reaches to the $u+d$ and recovers very fast.

4 Interactive Izhikevich Model

I implemented an interactive izhikevich model using matplotlib library. You can choose two mode for this part. One is step which is applying the step-input current to the model and the other is noise which is applying of noisy input current to the model. In both mode you can change the value of parameter and see the membrane potential changing in the diagram. You can see your current and threshold line in the diagram too. Also you can change the value of current and in the step mode, there are six button for different types of the neurons. Let's see an image of this graphical program:

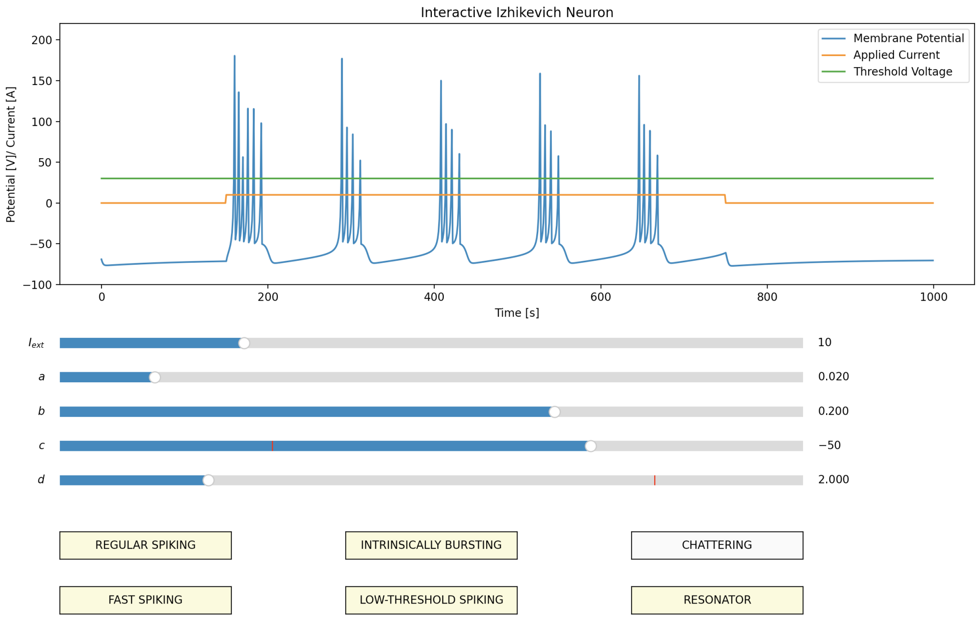


Figure 37: Interactive Izhikevich Model

You can play with the parameter and see the results in the diagram. Here are the steps to run interactive Izhikevich model interface:

```
# run git clone https://github.com/cnrl/PymoNNtorch.git
in the terminal
# pip install -e .
# copy interactive_Izhikevich_model.py to
the PymoNNtorch folder
# run python3 interactive_Izhikevich_mode.py
```

HAVE FUN !

5 References

[1] Izhikevich, E.M., 2003. Simple model of spiking neurons. IEEE Transactions on neural networks, 14(6), pp.1569-1572.

[2] Simple Model of Spiking Neurons https://www.izhikevich.org/publications/spikes.html.