

Seiminar II : Financial Data Analysis Using Expert Bayesian Framework For Bankruptcy Prediction



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Abstract

in recent years, bankruptcy forecasting has gained lot of attention from researchers as well as practitioners in the field of financial risk management. For bankruptcy prediction, various approaches proposed in the past and currently in practice relies on accounting ratios and using statistical modeling or machine learning methods. These models have had varying degrees of successes. Models such as Linear Discriminant Analysis or Artificial Neural Network employ discriminative classification techniques. They lack explicit provision to include prior expert knowledge. In this paper, we propose another route of generative modeling using *Expert Bayesian Framework*.

The biggest advantage of the proposed framework is an explicit inclusion of expert judgment in the modeling process. Also the proposed methodology provides a way to quantify uncertainty in prediction. As a result the model built using Bayesian framework is highly flexible, interpretable and intuitive in nature. The proposed approach is well suited for highly regulated or safety critical applications such as in finance or in medical diagnosis. In such cases accuracy in the prediction is not the only concern for decision makers. Decision makers and other stakeholders are also interested in uncertainty in the prediction as well as interpretability of the model. We empirically demonstrate these benefits of proposed framework on real world dataset using Stan, a probabilistic programming language. We found that the proposed model is either comparable or superior to the other existing methods. Also resulting model has much greater specificity compared to many existing state of the art methods. The corresponding R code for the experiments is available on Github.

Keywords: Bayesian Data Analysis, Interpretable Machine Learning, Uncertainty Quantification, Financial Credit Risk, Bankruptcy prediction

Contents

1	Introduction	1
2	Literature Survey	4
2.1	Related Work	4
2.2	Dataset	5
3	Methodology	7
3.1	Introduction	7
3.2	Generalized Linear Model (GLM)	7
3.2.1	Variable Selection	11
3.2.2	Choice of Priors	12
3.2.3	Model Convergence	12
3.2.4	Model Selection	13
3.3	Bayesian Insight	14
4	Comparative Analysis	19
5	Conclusion	23
A	Exploratory Variables	27
B	Posterior Distribution	30

List of Figures

3.1	Methodology:In all the phases inputs from domain expert are vital . .	8
3.2	Posterior Density plot for Model#1	15
3.3	Equivalence Test Plot	18
4.1	Neural Network	20
4.2	Performance Comparison	22

Chapter 1

Introduction

The adverse effect of uncertainty across most of the global economies, aggravated by the series of shocks to the financial sector, will likely be felt increasingly across most sectors with ongoing pandemic as outlined by International Monetary Fund (IMF 2020). Worsening conditions in the capital markets, increasing economic instability and declining consumer confidence has meant that sound liquidity positions, prudent treasury management and resilient cash flows are important for credit quality, particularly for speculative grade corporates.

Bankruptcy is a state wherein a corporation is unable to meet or repay its statutory obligations from its operating activities. Financial ratio analysis helps in identifying such types of bankrupt corporations or corporations headed for bankruptcy in the near future.

Ratio Analysis is important for relevant stakeholders such as banks, financial institutions, investors, and others in order to analyze the financial position, liquidity, profitability, risk, solvency, efficiency, and ultimately the health of any corporation. These are quantitative metrics which help stakeholders in taking timely decisions as well as identifying corporations which will go bankrupt. They are early warning indicators of credit risk and financial distress. Broad set of financial ratios would include liquidity ratios, solvency ratios, profitability ratios, efficiency ratios, coverage ratios,

and earnings ratios. The numerator and denominator of the ratio to be calculated are taken from the financial statements. Financial ratios can be seen as a tool that can be used by every company to determine the financial liquidity, the debt burden, and the profitability of the company and whether it would be able to sustain and meet its obligations in the near future. In fact, bond rating agencies like S&P, Moody's and Fitch also use financial ratio analysis to rate corporate bonds. These ratios are critical component of any statistical or machine learning model built to analyze and forecast financial distress.

There are various statistical techniques such as Linear Discriminant Analysis (LDA), Multi-discriminant Analysis(MDA) as well as machine learning techniques such as Support Vector Machines (SVM), Artificial Neural Networks(ANN) used for bankruptcy prediction (Devi and Radhika 2018). However none of these approaches allow quantification of uncertainty in prediction or intuitive interpretability of model. These are specially important in the fields such as healthcare diagnosis or finance where the decision based on the predictions of models have critical implications.

On the other hand, Bayesian framework provides capability to include expert judgement, uncertainty quantification as well as interpretability of model right out of the box. The proposed model will help stakeholders to be more vigilant, alleviate the above risks and in predicting bankruptcies. The methodology presented in this article helps investors, banks, financial institutions and other relevant stakeholders to take corrective measures, timely decisions, strategic business planning in the funding and investment process. In our knowledge this is the first of it's kind study to apply Bayesian treatment to bankruptcy forecasting.

Reminder of the seminar report is organized as follows. The second chapter presents the literature survey covering some of the earlier work done in the area of bankruptcy prediction, third chapter covers the dataset used for conducting our experiments. Fourth chapter gives a brief introduction to Generalized Linear Modeling method with underlying mathematical underpinnings followed by chapter on

experimental results, comparative performance with other methods and conclusions.

Chapter 2

Literature Survey

2.1 Related Work

Altman's Z-score (E. I. Altman 1968) is a popular method to determine whether a company would go bankrupt or not based on the selected financial ratios. Vast literature on the study of distressed companies relies on this method as first tool of choice available in their toolbox. However it was not able to perform as expected as demonstrated later in the comparative study section.

On the other side of spectrum, machine learning techniques such as logistic regression was used by Ohlson (Ohlson 1980). Qi Yu et al. (Yu et al. 2014) used Extreme Learning Machines for the prediction which relies on single layer Artificial Neural Network which is evolved with the knowledge of financial ratios to discover the most optimum neural network structure. Arindam Chaudhary et al. (Chaudhuri and Ghosh 2017) proposed the soft computing approach using fuzzy rough sets. Lahmiri, S et al. (Lahmiri and Bekiros 2019) proposed the Generalized Regression Topology as an efficient method to arrive at optimized neural architecture search. Barboza et al. (Barboza, Kimura, and E. Altman 2017) provides detailed survey on the statical techniques such as Linear Discriminant Analysis (LDA) as well as machine learning techniques such as Support Vector Machine and their comparative analysis.

While each of the above methods have their merits and demerits none of them provide uncertainty quantification of the prediction. On top of that many models including those based on Artificial Neural Networks (ANN) are black box models. They provide no opportunity to interpret them and facilitate the discussion between experts, business stakeholders even if they could perform well on test data. Secondly, many of these models need the tuning of the hyperparameters, for example, number of neurons or layers and learning rate which is a time consuming and considered as black art (Snoek, Larochelle, and Adams 2012).

Many of these challenges can be overcome with Bayesian modeling techniques. Bayesian modeling techniques allows us to provide uncertainty quantification in terms of posterior probability density (Gelman et al. 2013). While in the past there have been challenges in sampling from this distribution due to computational limitations however with the increased capacity and power of modern computation sampling has become much more practical. Also advances in sampling algorithms such as Markov Chain Monte Carlo (MCMC) have helped alleviate further the pain point in this regard when exact posterior computation is intractable specially for multivariate data.

2.2 Dataset

For this study, bankruptcy information of Polish companies (Kaggle 2018 (accessed October 29, 2020)) is used which is based on dataset by Zeiba M et al. (Zięba, S. K. Tomczak, and J. M. Tomczak 2016) for the period between year 2000 and 2012. This dataset consists of various accounting ratios such as different types of assets to liabilities in both long and short terms, and whether that company went bankrupt or not. Such 10,000 sample data points are available. Dataset was divided into training and test dataset with class label distribution as shown in Table 2.1

Table 2.1: Class distribution in training and test dataset

Dataset	#Non-bankrupt	#Bankrupt
Training	5487	113
Test	2105	47

All the 64 financial ratios available in the dataset are shown in Appendix in the Table A

Chapter 3

Methodology

3.1 Introduction

Generalized Linear Model was chosen which is well known for its flexibility. To implement Bayesian GLM regression model we have used R package *rstanarm* (Muth, Oravecz, and Gabry 2018) that provides an abstraction over the underlying probabilistic programming language compiler of Stan.

Data is preprocessed by using standard scaling method. Then the variables to be included in the model are selected along with suitable prior distribution for model's priors. After building the models their predictive performance was compared using K Fold cross validation. Once the model is selected their significance is assessed using Bayesian hypothesis testing methodology. in all the phases of these process domain expertise was utilized.

This process is illustrated in the flowchart in Figure 3.1

3.2 Generalized Linear Model (GLM)

Generalized Linear Models (GLM) are similar to linear regression models. Here instead of having a normally distributed output variable it could be made constrained,

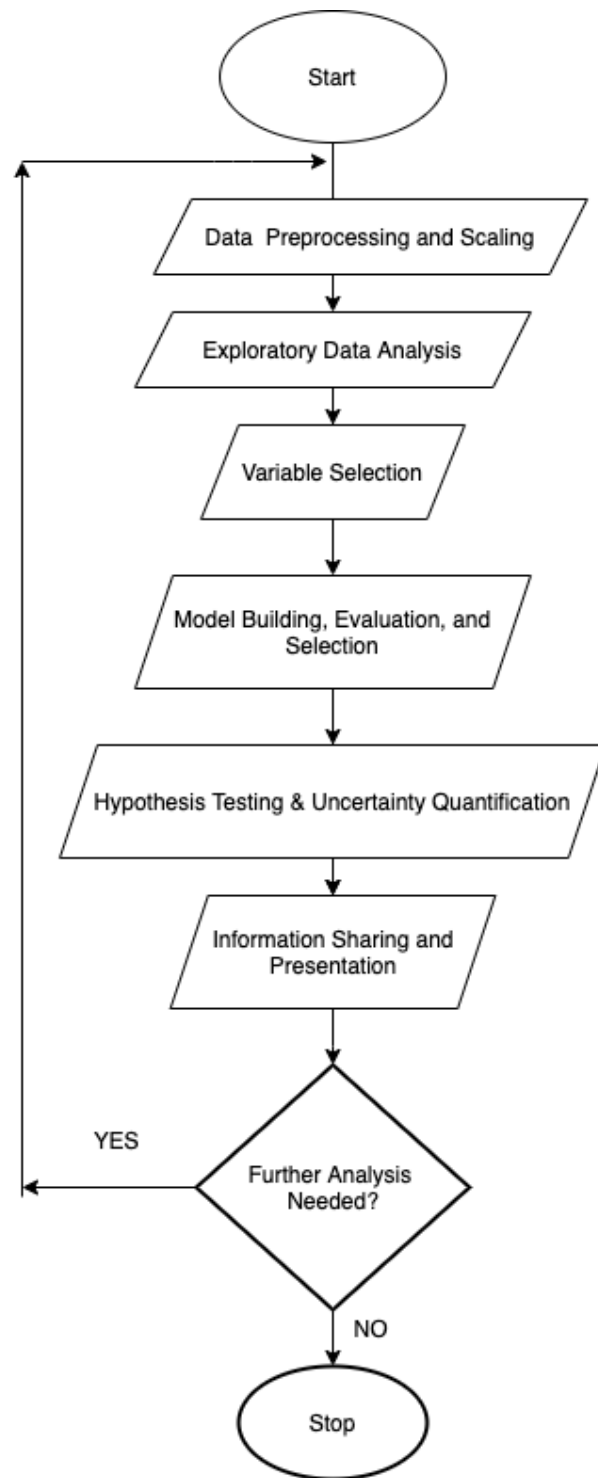


Figure 3.1: Methodology: In all the phases inputs from domain expert are vital

for example, for a binary logistic regression, the output or response variable can be restricted to be between 0 and 1 or positive values only.

More formally, for binary outcome y_i , which is Bernoulli independent and identically distributed (iid) given the probability of parameters as θ_i ,

$$f(y_i|\theta_i) \sim \text{Bernoulli}(\theta_i), i = 1, 2, \dots, n \quad (3.1)$$

Bernoulli distribution is a special case of Binomial distribution where out of n inputs we model the probability of k outputs being 1 (success) or zero (failure).

The expected value for discrete random variable X with a finite number of outcomes x_1, x_2, \dots, x_k occurring with probabilities $\theta_1, \theta_2, \dots, \theta_k$ respectively, is given by:

$$E[X] = \sum_{i=1}^k x_i \theta_i = x_1 \theta_1 + x_2 \theta_2 + \dots + x_k \theta_k \quad (3.2)$$

For a linear regression model, the expectation of θ_i is given by:

$$E(\theta_i) = \beta_0 + \beta_1 x_i \quad (3.3)$$

However in order to constrain the value of θ_i to be between 0 and 1 we use *log-odds ratio* as,

$$\log\left(\frac{\theta}{1-\theta}\right) = \beta_0 + \beta_1 x_i \quad (3.4)$$

This function, log of odds, is also called as logit-link function. It called as *Link* since it helps to link given linear expression to valid probability between 0 and 1 of successful outcome.

Therefore the expected value for the linear expression with probability of success as θ_i and probability of failure as $(1 - \theta_i)$ is given as :

$$E[\theta_i] = 1 * \theta_i + 0 * (1 - \theta_i) = \frac{e^{(\beta_0 + \beta_1 x)}}{1 + e^{(\beta_0 + \beta_1 x)}} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \quad (3.5)$$

This model can be extended to multidimensional as well as multinomial response variables by adding number of coefficients accordingly.

In order to build Bayesian linear regression model, we can select priors on intercept and coefficient, for example, to be normally distributed as,

$$Intercept : \beta_0 \sim Normal(a, b) \quad (3.6)$$

$$\beta_1 \sim Normal(a, b) \quad (3.7)$$

In case of *Hierarchical Bayesian models* we can include the priors for a and b as well. However in this study due to lack obvious population subgroups in data such as sector or region hierarchical model were not utilized.

In order to compute the posterior distribution of parameter vector β using Bayesian inference,

$$posterior = \frac{likelihood * prior}{evidence} \quad (3.8)$$

$$p(\beta|X) = \frac{f(X|\beta) * p(\beta)}{\int f(X|\beta) * p(\beta)d\beta} \quad (3.9)$$

In above equation, denominator i.e. the normalizing constant is most of the times a multidimensional integral and is analytically intractable to compute. Therefore we approximate the posterior distribution using Markov Chain Monte Carlo (MCMC) sampling algorithm. Probabilistic programming language such as Stan (Carpenter et al. 2017) helps in generating dependent samples from such distribution as an approximation to the true posterior distribution in an efficient manner.

The posterior predictive distribution for a new example is given by,

$$p(y_{new}|X, \beta) = \int p(y_{new}|\beta) * p(\beta|X)d\beta \quad (3.10)$$

3.2.1 Variable Selection

In the dataset there are total of 64 financial ratios that are explanatory variables and a class variable indicating whether the company went bankrupt or not indicated by 1 or 0 respectively. Expertise in credit risk management available to us played a critical role in this phase. We started with including variables in the model as shown in Table 3.1 In Model#1 with focus on ratios involving total liabilities.

Table 3.1: Ratios in Model#1

Ratios
[(cash + short-term securities + receivables - short-term liabilities) / (operating expenses - depreciation)] * 365
gross profit (in 3 years) / total assets
(equity - share capital) / total assets
(net profit + depreciation) / total liabilities
operating expenses / total liabilities

For the second model Model#2 we included mostly short term liability ratios as shown in Table 3.2

Table 3.2: Ratios in Model#2

Ratios
book value of equity / total liabilities
equity / total assets
gross profit / short-term liabilities
(inventory * 365) / sales
operating expenses / short-term liabilities
(current assets - inventory - receivables) / short-term liabilities
profit on operating activities / sales
(current assets - inventory) / short-term liabilities
EBITDA (profit on operating activities - depreciation) / sales
long-term liabilities / equity
sales / short-term liabilities
sales / fixed assets

3.2.2 Choice of Priors

Since we believe that intercept as well coefficient should be close to zero however there is chance that they could be large. Therefore for both the models Student's t-distribution was used as it provides fatter tails than a Normal distribution as prior for all parameters with following hyperparameters:

$$priors \sim Student_t(df = 7, location = 0, scale = 2.5) \quad (3.11)$$

3.2.3 Model Convergence

Once the model is setup up, it is fitted on training data. In this process we draw samples from posterior distribution using Markov Chain Monte Carlo(MCMC) sampling, specifically using No U Turn Sampler (NUTS) (Hoffman and Gelman 2014).

Both the models converged with 4000 total iterations with 2000 warmup iterations with 4 chains. For all the coefficients we got an \hat{R} of less than 1.1. \hat{R} helps in analyzing the mixing of chains, value of less than 1.1 is an indication of proper mixing of chains (Vehtari, Gelman, Simpson, et al. 2020) .

Analysis of posterior samples is shown in Table B.1 for Model#1 and in Table B.2 for Model#2.

3.2.4 Model Selection

Bayesian K Fold cross validation (Vehtari, Gelman, and Gabry 2017) is used to compare both the models with 10 folds. The Expected Log Point wise Density (elpd) difference between the two models is shown in Table 3.3. The elpd helps in comparing the model and hence in model selection. From the difference in the elpd values we see that Model#2, even though it has more number of variables there is difference with high standard error. Therefore we choose *Model#2* as better amongst the two models.

Table 3.3: ELPD Difference

	ELPD difference	Std Error
Model#2	0.0	0.0
Model#1	-0.1	13.6

Also Model#1 as well Model#2 was tested on unseen test dataset. Confusion matrix from the predictions of Model#1 and Model#2 is shown in Table 3.4 and Table 3.5 respectively. Model#2 performs better than Model#1 for the test data specifically in detecting with positive cases. Here threshold of 0.5 for column means of the posterior predictive samples was used to map the real values between 0 and 1 to discrete 0s and 1s.

From the test results clearly Model#2 performs better than Model#1 on all measures.

Table 3.4: Model#1 Confusion Matrix

	Predicted NO	Predicted YES
True NO	2101	4
True YES	47	0

Table 3.5: Model#2 Confusion Matrix

	Predicted NO	Predicted YES
True NO	2087	18
True YES	39	8

3.3 Bayesian Insight

To determine if an attribute is significant or not we used ROPE (Region of Practical Equivalence) (Kruschke 2014). ROPE value provides the percentage of Credible Interval(CI) that is in the null region. With sufficiently high value of ROPE the null hypothesis is accepted otherwise rejected. The significant attributes are highlighted in Table B.1 and B.2. Figure 3.2 shows posterior density plot using the drawn samples with uncertainty intervals. We can see that the attributes with significant values are away from zero.

From this analysis we can see that ratios involving short-term liabilities are the best predictors for bankruptcy based on our dataset.

We further analyze these ratios to get more insight using *report* (Makowski and Lüdtke 2019) R package. After generating report we can examine significance of an attribute. Following is the list of the analysis of the attributes in Model#2. Significant attributes in the below summary are underlined.

- The effect of (Intercept) has a probability of 100% of being negative and can be considered as large and significant (median = -5.09, 89% CI [-5.44, -4.72], 0% in ROPE, std. median =). The algorithm successfully converged (Rhat = 1.004) and the estimates can be considered as stable (ESS = 1261).

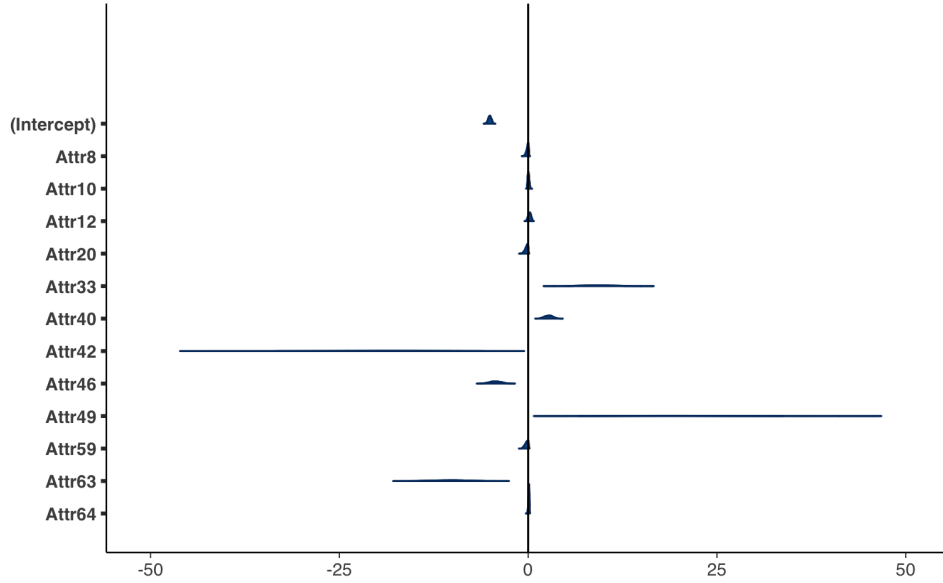


Figure 3.2: Posterior Density plot for Model#1

- The effect of Attr8 (book value of equity / total liabilities) has a probability of 67.90% of being negative and can be considered as very small and not significant (median = -0.06, 89% CI [-0.32, 0.12], 76.15% in ROPE, std. median = -0.06). The algorithm successfully converged (Rhat = 1.001) and the estimates can be considered as stable (ESS = 2197).
- The effect of Attr10 (equity / total assets) has a probability of 63.68% of being positive and can be considered as very small and not significant (median = 0.04, 89% CI [-0.12, 0.20], 90.33% in ROPE, std. median = 0.04). The algorithm successfully converged (Rhat = 1.000) and the estimates can be considered as stable (ESS = 3208).
- The effect of Attr12 (gross profit / short-term liabilities) has a probability of 83.23% of being positive and can be considered as very small and not significant (median = 0.20, 89% CI [-0.11, 0.50], 43.60% in ROPE, std. median = 0.20). The algorithm successfully converged (Rhat = 1.000) and the estimates can be considered as stable (ESS = 3673).

- The effect of Attr20 ((inventory * 365) / sales) has a probability of 91.50% of being negative and can be considered as very small and not significant (median = -0.20, 89% CI [-0.54, 0.05], 45.88% in ROPE, std. median = -0.20). The algorithm successfully converged (Rhat = 1.001) and the estimates can be considered as stable (ESS = 3112).
- The effect of Attr33 (operating expenses / short-term liabilities) has a probability of 100% of being positive and can be considered as large and significant (median = 9.22, 89% CI [5.93, 12.31], 0% in ROPE, std. median = 9.22). The algorithm successfully converged (Rhat = 1.001) and the estimates can be considered as stable (ESS = 2182).
- The effect of Attr40 ((current assets - inventory - receivables) / short-term liabilities) has a probability of 100% of being positive and can be considered as large and significant (median = 2.71, 89% CI [1.81, 3.54], 0% in ROPE, std. median = 2.71). The algorithm successfully converged (Rhat = 1.001) and the estimates can be considered as stable (ESS = 2086).
- The effect of Attr42 (profit on operating activities / sales) has a probability of 100% of being negative and can be considered as large and significant (median = -19.73, 89% CI [-31.69, -8.29], 0.03% in ROPE, std. median = -19.73). The algorithm successfully converged (Rhat = 1.000) and the estimates can be considered as stable (ESS = 2313).
- The effect of Attr46 ((current assets - inventory) / short-term liabilities) has a probability of 100% of being negative and can be considered as large and significant (median = -4.33, 89% CI [-5.56, -3.11], 0% in ROPE, std. median = -4.33). The algorithm successfully converged (Rhat = 1.003) and the estimates can be considered as stable (ESS = 1610).
- The effect of Attr49 (EBITDA (profit on operating activities - depreciation) /

sales) has a probability of 100% of being positive and can be considered as large and significant (median = 20.25, 89% CI [9.48, 32.78], 0.03% in ROPE, std. median = 20.25). The algorithm successfully converged (Rhat = 1.000) and the estimates can be considered as stable (ESS = 2360).

- The effect of Attr59 (long-term liabilities / equity) has a probability of 83.62% of being negative and can be considered as very small and not significant (median = -0.19, 89% CI [-0.52, 0.10], 49.45% in ROPE, std. median = -0.19). The algorithm successfully converged (Rhat = 1.005) and the estimates can be considered as stable (ESS = 1727).
- The effect of Attr63 (sales / short-term liabilities) has a probability of 100% of being negative and can be considered as large and significant (median = -10.33, 89% CI [-14.01, -7.09], 0% in ROPE, std. median = -10.33). The algorithm successfully converged (Rhat = 1.001) and the estimates can be considered as stable (ESS = 2019).
- The effect of Attr64 (sales / fixed assets) has a probability of 88.62% of being positive and can be considered as very small and not significant (median = 0.10, 89% CI [-8.84e-03, 0.21], 93.80% in ROPE, std. median = 0.10). The algorithm successfully converged (Rhat = 1.002) and the estimates can be considered as stable (ESS = 1703).

Figure 3.3 shows the the plot of equivalence test that helps in deciding weather the parameter should be accepted or rejected based on 89% Credible Interval (CI).

From above analysis, we conclude that declining sales and disproportionate short-term liabilities are the main predictors of credit default in near term future.

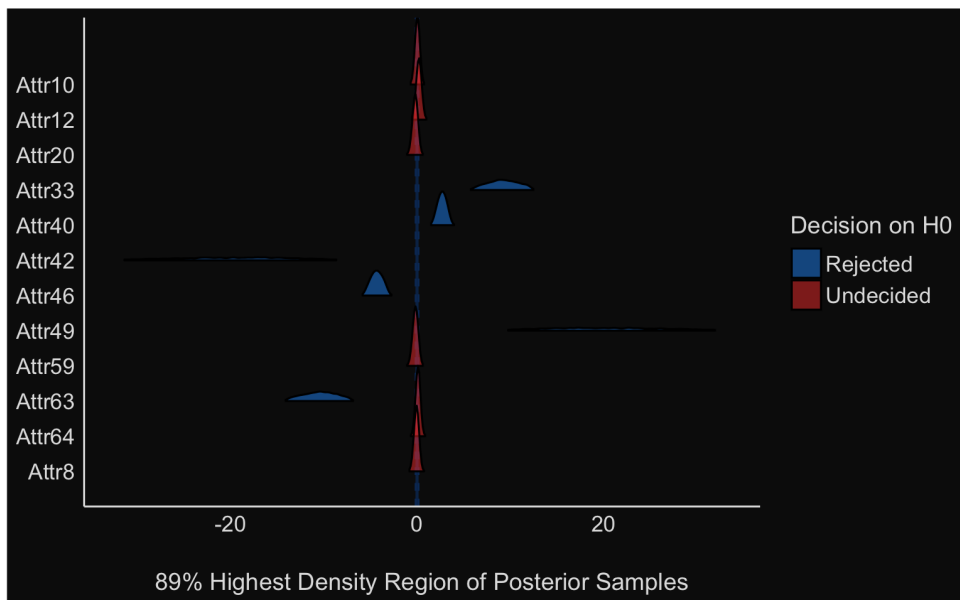


Figure 3.3: Equivalence Test Plot

Chapter 4

Comparative Analysis

Using Altman’s Z-Score method (E. I. Altman 1968) we found that it was inadequate and gave a very low accuracy of 21% on training data and 20% on test data with high False Positive Rate (FPR).

We also conducted experiments using other machine learning methods such as Support Vector Machines, Random Forest, Neural Network, and Logistic Regression.

SVM classifier with linear and radial basis kernels could not classify a single true positive case from test data correctly. Random Forest was able to classify one as true positive out of total 47 true positive cases from test data.

The experiments with neural network with 3 hidden layers were also carried out. ANN is shown in Figure 4.1 . As shown in Table 4.1 it was able to classify 8 out of 47 true bankrupt cases.

On the other hand, non-Bayesian Generalized Linear Model with logit link function, we found the results with the test data as shown in Table 4.2. Given the flexibility of the GLMs it was able to classify many more true positive cases correctly. How-

Table 4.1: Neural Network Confusion Matrix

	Predicted NO	Predicted YES
True NO	2087	18
True YES	39	8

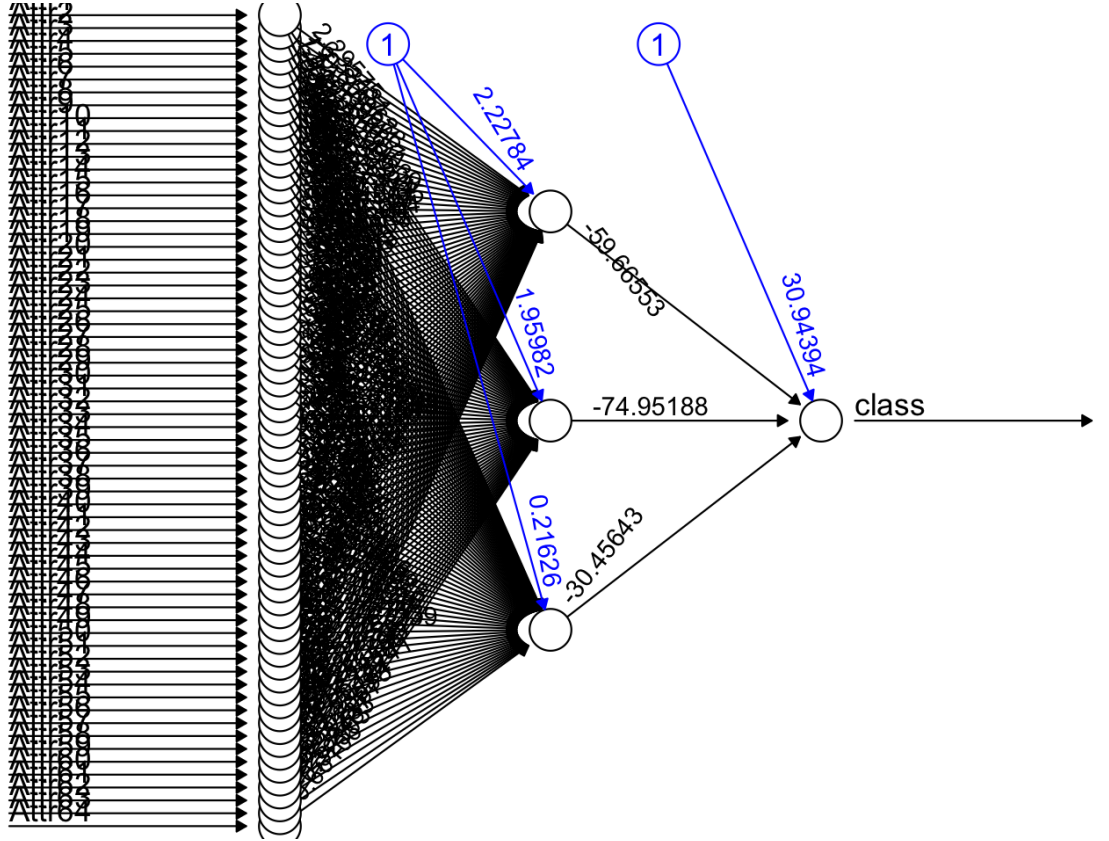


Figure 4.1: Neural Network

Table 4.2: Non-Bayesian GLM Confusion Matrix

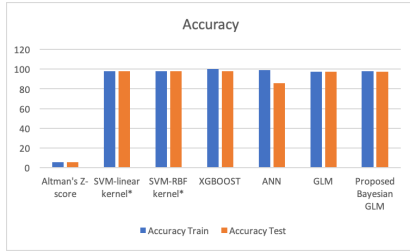
	Predicted NO	Predicted YES
True NO	2088	17
True YES	41	6

Table 4.3: Performance Comparison

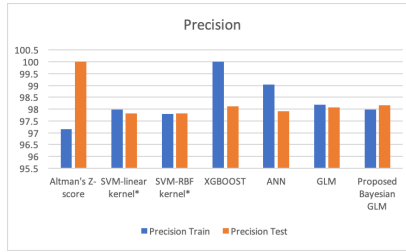
<i>Methodology</i>	<i>Accuracy</i>		<i>Precision</i>		<i>Recall</i>		<i>F1 Score</i>	
	Train	Test	Train	Test	Train	Test	Train	Test
Altman's Z-score	5.6	6.0	97.16	100	3.75	4.0	7.22	7.7
SVM-linear kernel	97.98	97.81	97.98	97.81	100	100	98.98	98.89
SVM-RBF kernel	97.79	97.81	97.79	97.81	100	100	98.98	98.89
XGBOOST	100	97.86	100	98.12	100	99.71	100	98.91
ANN	99.05	86.01	99.04	97.92	100	87.55	99.51	92.45
GLM	97.44	97.3	98.19	98.07	99.21	99.19	98.7	98.63
Proposed Bayesian GLM	97.98	97.25	97.98	98.16	100	99.0	98.98	98.60

ever it also ended up classifying more False positives in the process. XGBOOST (Extreme Gradient Boosting Machine) was closest to the proposed Bayesian GLM in detecting the positive cases. Performance metrics of various methodologies that were tried against the proposed approach are listed in Table 4.3. Graphical visualization of the same is shown in Figure 4.2. Also it was found that oversampling the minority class i.e. records with bankruptcy class did not improve their performance any further for any of the methods.

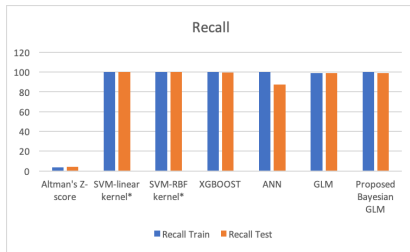
Critics of Bayesian methodology have objected to inclusion of prior knowledge as being subjective however other method viz. frequentist do also use expert inputs without explicitly stating it (Brownstein et al. 2019). Frequentist models have no provision to explicitly include the same while building a model. Bayesian framework allows the inclusion of prior judgment explicitly and updating it based on the evidence as we proceed in our experiments. This helps in directly driving decision making process using data as well as taking benefit of an expert judgement.



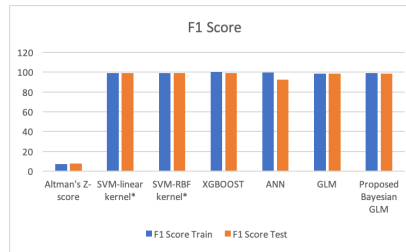
(a) Accuracy



(b) Precision



(c) Recall



(d) F1 Score

Figure 4.2: Performance Comparison

Chapter 5

Conclusion

With turbulent economic times ahead assessing and forecasting the financial well being of commercial entities will gain more and more traction. Most of the conventional approaches including Artificial Neural Network are black box in nature and provide no help in interpretation of model which is crucial in decision making process. The proposed Bayesian GLM model and methodology meets the critical requirements of interpretability as well as inclusion of expert judgement in all the phases of model building process. Secondly, the proposed method do not need any hyperparameter tuning or neural architecture search.

In this research, we built and tested model with this approach on the bankruptcy data of Polish companies. It was found that specifically looping in expert reduced the time for the model building and model selection considerably compared to other combinatorial and computationally intensive methods. Expert in the loop approach also helped in interpreting the model and in communication of results of analysis to the relevant stakeholders. At the end of the analysis we found that the ratios involving short term liabilities were strong predictors of companies going bankrupt. We believe this study will help open new avenues for further exploration of novel applications of Bayesian methodology in the areas of credit risk management and in investment decision making along with inclusion of domain expertise in the process.

References

- Altman, Edward I (1968). “Financial ratios, discriminant analysis and the prediction of corporate bankruptcy”. In: *The journal of finance* 23.4, pp. 589–609.
- Barboza, Flavio, Herbert Kimura, and Edward Altman (2017). “Machine learning models and bankruptcy prediction”. In: *Expert Systems with Applications* 83, pp. 405–417.
- Brownstein, Naomi C et al. (2019). “The role of expert judgment in statistical inference and evidence-based decision-making”. In: *The American Statistician* 73.sup1, pp. 56–68.
- Carpenter, Bob et al. (2017). “Stan: A probabilistic programming language”. In: *Journal of statistical software* 76.1.
- Chaudhuri, Arindam and Soumya K Ghosh (2017). *Bankruptcy prediction through soft computing based deep learning technique*. Springer.
- Devi, S Sarojini and Y Radhika (2018). “A survey on machine learning and statistical techniques in bankruptcy prediction”. In: *International Journal of Machine Learning and Computing* 8.2, pp. 133–139.
- Gelman, Andrew et al. (2013). *Bayesian data analysis*. CRC press.
- Hoffman, Matthew D and Andrew Gelman (2014). “The No-U-Turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo.” In: *J. Mach. Learn. Res.* 15.1, pp. 1593–1623.
- IMF (2020). “World Economic Outlook:A Long and Difficult Ascent”. In: *World Economic Outlook (WEO)* Washington, DC, October.

- Kaggle (2018 (accessed October 29, 2020)). *Companies bankruptcy forecast*. <https://www.kaggle.com/c/companies-bankruptcy-forecast/data>. URL: <https://www.kaggle.com/c/companies-bankruptcy-forecast/overview>.
- Kruschke, John (2014). *Doing Bayesian data analysis: A tutorial with R, JAGS, and Stan*. Academic Press.
- Lahmiri, Salim and Stelios Bekiros (2019). “Can machine learning approaches predict corporate bankruptcy? Evidence from a qualitative experimental design”. In: *Quantitative Finance* 19.9, pp. 1569–1577.
- Makowski, D and D Lüdtke (2019). “The report package for R: Ensuring the use of best practices for results reporting”. In: *CRAN*.
- Muth, Chelsea, Zita Oravecz, and Jonah Gabry (2018). “User-friendly Bayesian regression modeling: A tutorial with rstanarm and shinystan”. In: *Quantitative Methods for Psychology* 14.2, pp. 99–119.
- Ohlson, James A (1980). “Financial ratios and the probabilistic prediction of bankruptcy”. In: *Journal of accounting research*, pp. 109–131.
- Snoek, Jasper, Hugo Larochelle, and Ryan P Adams (2012). “Practical bayesian optimization of machine learning algorithms”. In: *Advances in neural information processing systems*, pp. 2951–2959.
- Vehtari, Aki, Andrew Gelman, and Jonah Gabry (2017). “Practical Bayesian model evaluation using leave-one-out cross-validation and WAIC”. In: *Statistics and computing* 27.5, pp. 1413–1432.
- Vehtari, Aki, Andrew Gelman, Daniel Simpson, et al. (2020). “Rank-normalization, folding, and localization: An improved Rhat for assessing convergence of MCMC”. In: *Bayesian Analysis*.
- Yu, Qi et al. (2014). “Bankruptcy prediction using extreme learning machine and financial expertise”. In: *Neurocomputing* 128, pp. 296–302.

Zięba, Maciej, Sebastian K Tomczak, and Jakub M Tomczak (2016). “Ensemble boosted trees with synthetic features generation in application to bankruptcy prediction”. In: *Expert systems with applications* 58, pp. 93–101.

Appendix A

Exploratory Variables

Variable	Variable
attr1 - net profit / total assets	attr33 - operating expenses / short-term liabilities
attr2 - total liabilities / total assets	attr34 - operating expenses / total liabilities
attr3 - working capital / total assets	attr35 - profit on sales / total assets
attr4 - current assets / short-term liabilities	attr36 - total sales / total assets
attr5 - [(cash + short-term securities + receivables - short-term liabilities) / (operating expenses - depreciation)] * 365	attr37 - (current assets - inventories) / long-term liabilities
attr6 - retained earnings / total assets	attr38 - constant capital / total assets
attr7 - EBIT / total assets	attr39 - profit on sales / sales
attr8 - book value of equity / total liabilities	attr40 - (current assets - inventory - receivables) / short-term liabilities

Variable	Variable
attr9 - sales / total assets	attr41 - total liabilities / ((profit on operating activities + depreciation) * (12/365))
attr10 - equity / total assets	attr42 - profit on operating activities / sales
attr11 - (gross profit + extraordinary items + financial expenses) / total assets	attr43 - rotation receivables + inventory turnover in days
attr12 - gross profit / short-term liabilities	attr44 - (receivables * 365) / sales
attr13 - (gross profit + depreciation) / sales	attr45 - net profit / inventory
attr14 - (gross profit + interest) / total assets	attr46 - (current assets - inventory) / short-term liabilities
attr15 - (total liabilities * 365) / (gross profit + depreciation)	attr47 - (inventory * 365) / cost of products sold
attr16 - (gross profit + depreciation) / total liabilities	attr48 - EBITDA (profit on operating activities - depreciation) / total assets
attr17 - total assets / total liabilities	attr49 - EBITDA (profit on operating activities - depreciation) / sales
attr18 - gross profit / total assets	attr50 - current assets / total liabilities
attr19 - gross profit / sales	attr51 - short-term liabilities / total assets
attr20 - (inventory * 365) / sales	attr52 - (short-term liabilities * 365) / cost of products sold)
attr21 - sales (n) / sales (n-1)	attr53 - equity / fixed assets
attr22 - profit on operating activities / total assets	attr54 - constant capital / fixed assets
attr23 - net profit / sales	attr55 - working capital

Variable	Variable
attr24 - gross profit (in 3 years) / total assets	attr56 - (sales - cost of products sold) / sales
attr25 - (equity - share capital) / total assets	attr57 - (current assets - inventory - short-term liabilities) / (sales - gross profit - depreciation)
attr26 - (net profit + depreciation) / total liabilities	attr58 - total costs / total sales
attr27 - profit on operating activities / financial expenses	attr59 - long-term liabilities / equity
attr28 - working capital / fixed assets	attr60 - sales / inventory
attr29 - logarithm of total assets	attr61 - sales / receivables
attr30 - (total liabilities - cash) / sales	attr62 - (short-term liabilities * 365) / sales
attr31 - (gross profit + interest) / sales	attr63 - sales / short-term liabilities
attr32 - (current liabilities * 365) / cost of products sold	attr64 - sales / fixed assets

Appendix B

Posterior Distribution

Table B.1: Posterior for Model#1

Parameter	Median	89% CI	pd	89% ROPE	% in ROPE	Rhat	ESS
(Intercept)	-4.670	[-4.951, -4.390]	1.000	[-0.181, 0.181]	0.000	1.001	1150.992
gross profit (in 3 years) / total assets	0.150	[-0.033, 0.311]	0.915	[-0.181, 0.181]	64.645	1.002	1779.815
(equity - share capital) / total assets	-0.160	[-0.263, -0.043]	0.989	[-0.181, 0.181]	65.319	0.999	6421.138
(net profit + depreciation) / total liabilities	-0.961	[-1.262, -0.688]	1.000	[-0.181, 0.181]	0.000	1.000	2235.707
operating expenses / total liabilities	0.661		1.000	[-0.181, 0.181]	0.000	1.000	1847.540
[(cash + short-term securities + receivables - short-term liabilities) / (operating expenses - depreciation)] * 365	0.380	[0.511, 0.802]	0.950	[-0.181, 0.181]	29.739	1.002	1353.509
(current assets - inventory) / short-term liabilities	-2.163	[-0.038, 0.897]	1.000	[-0.181, 0.181]	0.000	1.001	1239.331

Table B.2: Posterior for Model#2

Parameter	Median	89% CI	pd	89% ROPE	% in ROPE	Rhat	ESS
(Intercept)	-5.078	[-5.408, -4.727]	1.000	[-0.181, 0.181]	0.000	1.002	1273.642
book value of equity / total liabilities	-0.064	[-0.327, 0.122]	0.692	[-0.181, 0.181]	81.606	1.001	2215.902
equity / total assets	0.036	[-0.121, 0.202]	0.692	[-0.181, 0.181]	97.136	1.000	3249.143
gross profit / short-term liabilities	0.197	[-0.109, 0.501]	0.835	[-0.181, 0.181]	45.970	1.000	3994.138
(inventory * 365) / sales	-0.197	[-0.549, 0.046]	0.918	[-0.181, 0.181]	50.885	1.001	2686.980
operating expenses / short-term liabilities	9.176	[6.122, 12.508]	1.000	[-0.181, 0.181]	0.000	1.000	2287.606
(current assets - inventory - receivables) / short-term liabilities	2.720	[1.818, 3.576]	1.000	[-0.181, 0.181]	0.000	1.000	2260.580
profit on operating activities / sales	- 18.648	[-29.352, -7.436]	1.000	[-0.181, 0.181]	0.000	1.000	2439.790
(current assets - inventory) / short-term liabilities	-4.337	[-5.581, -3.128]	1.000	[-0.181, 0.181]	0.000	1.000	1772.371
EBITDA (profit on operating activities - depreciation) / sales	19.113	[7.767, 29.850]	1.000	[-0.181, 0.181]	0.000	1.000	2455.714
long-term liabilities / equity	-0.189	[-0.562, 0.094]	0.840	[-0.181, 0.181]	52.457	1.003	1579.627
sales / short-term liabilities	- 10.258	[-13.981, -6.939]	1.000	[-0.181, 0.181]	0.000	1.000	2111.063
sales / fixed assets	0.102	[-0.011, 0.193]	0.898	[-0.181, 0.181]	96.855	1.002	1982.562