Midterm - Program Analysis

Seyedamirhosseinn Hesamian
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1.1)
Forward analysis:

$$E = init(S_*) = \{1\}$$

Backward analysis:

$$E = final(S_*) = \{7\}$$

1.2)

Very busy expressions

$$\iota = \emptyset$$

Reaching definitions

$$\iota = \{(x, ?) | x \in FV(S_*)\} = \{(a, ?), (b, ?), (x, ?), (y, ?)\}$$

Or in Scala:

```
val tuples: [(String, Long)] = Utils.vars(stmt).map((_, ?))
```

1.3)
Very busy expressions

$$\perp = AExp = \{\text{x} > 0, \text{a * a}, \text{y} + \text{b}, \text{x - 1}\}$$

Or in Scala:

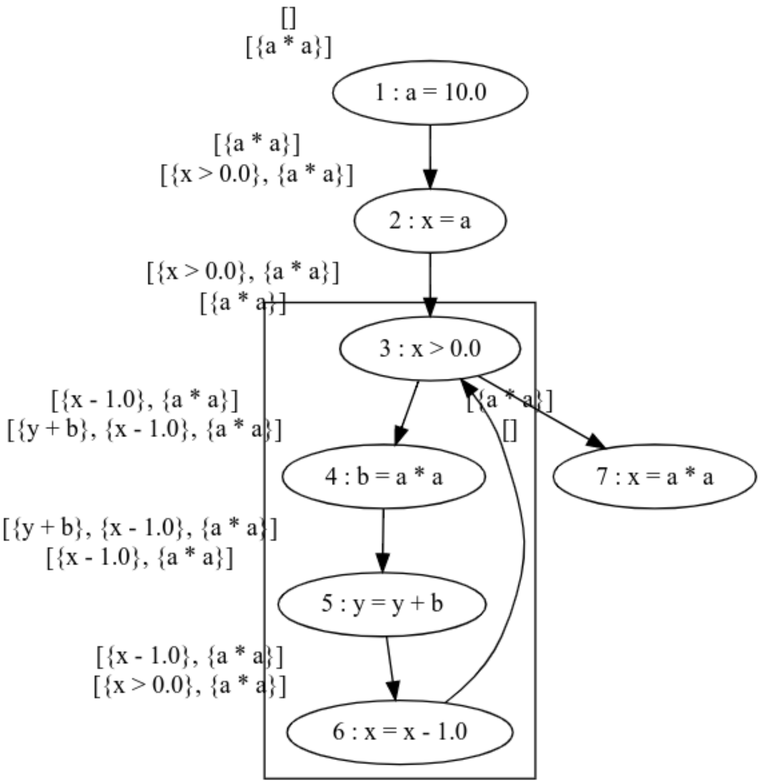
```
val exprs: Ser[Expression] = Utils.aexpr(strmt)
```

Reaching definitions

$$\perp = \emptyset$$

1.4)
Very busy expression is a **must** analysis and **backward**. So we need to use a backward CFG and **intersection** lattice.

- We need to start from the last
- Go backwards and if there are two paths going up, then we need to use intersection to join the paths
- We can change the direction of arrows to be backward
  - To get to label 3 we need to **intersect** the results 6 and 7



1.5)

1.6)

Definitions:

$$Kill(B) = \{ \text{ X op Y } | \text{ either X or Y defined before use of x op Y in B } \}$$

$$Gen(B) = \{ \text{ x op Y } | \text{ X op Y used in B before any definition of X or Y } \}$$

Transfer Equation

$$In(B) = (Out(B) - Kill(B)) \cup Gen(B)$$

Confluence Equation

$$Out(B) = \cap In(S) \text{ forall } S \in Succ(B)$$

Formula for GEN:

```
[x := a] then AExp(a)
[b] = AExp(b)


GEN(1) = { }
GEN(2) = { a*a }
GEN(3) = { x-1, x>0 }
GEN(4) = { a*a }
GEN(5) = { y+b }
GEN(6) = { x-1, x>0 }
GEN(7) = { a*a }
```

Formula for KILL:

```
[x := a] then { a' \in AExp | x \in FV(a')}
[b] = \emptyset
```

```
KILL(1) = { a*a }
KILL(2) = { x-1, x>0 }
KILL(3) = { }
KILL(4) = { y+b }
KILL(5) = { y+b }
KILL(6) = { x-1, x>0 }
KILL(7) = { x-1, x>0 }


OUT(1) = { a*a }
OUT(2) = { a*a, x>0 }
OUT(3) = { a*a }
OUT(4) = { a*a, x-1, y+b }
OUT(5) = { a*a, x-1 }
OUT(6) = { a*a, x>0 }
OUT(7) = { }


IN(1) = OUT(1) - KILL(1) + GEN(1) = { }
IN(2) = OUT(2) - KILL(2) + GEN(2) = { a*a }
IN(3) = OUT(3) - KILL(3) + GEN(3) = { a*a, x>0}
IN(4) = OUT(4) - KILL(4) + GEN(4) = { a*a, x-1 }
IN(5) = OUT(5) - KILL(5) + GEN(5) = { a*a, x-1, y+b }
IN(6) = OUT(6) - KILL(6) + GEN(6) = { a*a, x-1 }
IN(7) = OUT(7) - KILL(7) + GEN(7) = { a*a }
```

Basically `a*a` is always available and it could be replaced everywhere it is used.

Implementation

```scala
case class AExp(exps: Set[Expression]) extends Lattice[AExp] {
  override def lub(that: AExp): AExp = AExp(exps intersect (that.exps))
}

case class VB(stmt: Statement) extends Analysis[AExp] {
  override val cfg: CFG = BackwardCFG(stmt)
  override val extremalValue: AExp = AExp(Set())
  override val bottom: AExp = AExp(Set() ++ Util.aexp(stmt))
  override val entry: mutable.Map[Node, AExp] = real_exit
  override val exit: mutable.Map[Node, AExp] = real_entry

  override def transfer(stmt: Statement, l: AExp): AExp = {
    def kill_gen(y: String, e: Expression) = {
      AExp((l.exps).filter(!Util.fv(_).contains(y)) ++ Util.aexp(e))
    }

    def gen(e: Expression) = AExp(l.exps ++ Util.aexp(e))

    stmt match {
      case ExprStmt(AssignExpr(_, LVarRef(name), e)) => kill_gen(name, e)
      case ExprStmt(expr) => gen(expr)
      case VarDeclStmt(IntroduceVar(y), expr) => expr match {
        case EmptyExpr() => l
        case _=> kill_gen(y, expr)
      }
      case IfStmt(cond, _, _) => gen(cond)
      case WhileStmt(cond, _) => gen(cond)
      case _ => l // no change!
    }
  }
}
```

2.1)

Constant Propagation is a **forward analysis**.

Forward analysis:

$$E = init(S_*) = \{1\}$$

Backward analysis:

$$E = final(S_*) = \{10\}$$

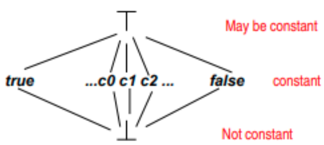
2.2)

For constant propagation extremal value

$$\iota_{CP} = \lambda x.\top$$

The least element (opposite of extremal value)

$$\bot$$



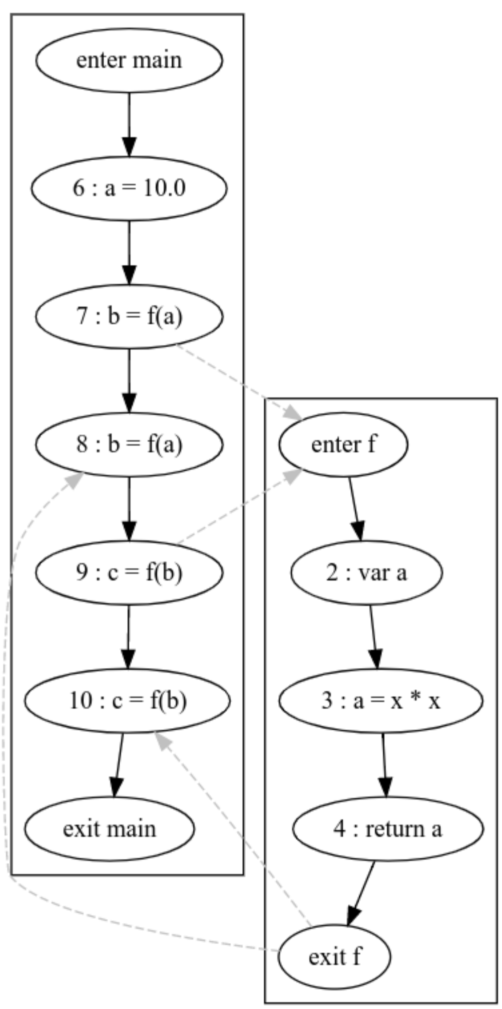
Basically the idea is the worst conclusion of running constant propagation analysis could be *everything may be a constant*.

2.3)

Each call site's `stmt.id` can be a context: { 7, 9 }

2.4)

P.S. I copied this from my solution of Homework 5 (context sensitive uninitialized variable). The IDs may not completely match the Ids in the question.



2.5)

$$CP_{\circ}(1) = \{[7] \mapsto \{a \mapsto 10, x \mapsto 10\}, [9] \mapsto \{a \mapsto 10, x \mapsto 100\}\}$$

$$CP_\bullet(1) = \{[7] \mapsto \{a \mapsto 10, x \mapsto 10\}, [9] \mapsto \{a \mapsto 10, x \mapsto 100\}\}$$

$$CP_\circ(2) = CP_\bullet(1)$$

$$CP_\bullet(2) = \hat{f}_2(CP_\circ(2)) = \{[7] \mapsto \{a \mapsto \top, x \mapsto 10\}, [9] \mapsto \{a \mapsto \top, x \mapsto 100\}\}$$

$$CP_\circ(3) = CP_\bullet(2)$$

$$CP_\bullet(3) = \hat{f}_3(CP_\circ(3)) = \{[7] \mapsto \{a \mapsto 10 \hat{*} 10, x \mapsto 10\}, [9] \mapsto \{a \mapsto 100 \hat{*} 100, x \mapsto 100\}\} = \{[7] \mapsto \{a \mapsto 100, x \mapsto 10\}, [9] \mapsto \{a \mapsto 100$$

$$CP_\circ(4) = CP_\bullet(3)$$

$$CP_\bullet(4) = \hat{f}_4(CP_\circ(4)) = \{[7] \mapsto \{a \mapsto 100, x \mapsto 10, \_\mathrm{r} \mapsto 100\}, [9] \mapsto \{a \mapsto 10000, x \mapsto 100, \_\mathrm{r} \mapsto 10000\}\}$$

$$CP_\circ(5) = CP_\bullet(4)$$

$$CP_\bullet(5) = \hat{f}_5(CP_\bullet(5)) = CP_\circ(5)$$

$$CP_\circ(6) = CP_\bullet(5)$$

$$CP_\bullet(6) = \hat{f}_6(CP_\bullet(6)) = \{[7] \mapsto \{a \mapsto 10, x \mapsto 10, \_\mathrm{r} \mapsto 100\}, [9] \mapsto \{a \mapsto 10, x \mapsto 100, \_\mathrm{r} \mapsto 10000\}\}$$

$$CP_\circ(7) = CP_\bullet(6)$$

$$CP_\bullet(7) = \hat{f}_7(CP_\bullet(7)) = \{[7] \mapsto \{a \mapsto 10, x \mapsto 10, \_\mathrm{r} \mapsto 100\}\}$$

$$CP_\circ(8) = CP_\bullet(7)$$

$$CP_\bullet(8) = \hat{f}_8(CP_\bullet(4), CP_\circ(8)) = \{[7] \mapsto \{a \mapsto 100, b \mapsto 100, x \mapsto 10, \_\mathrm{r} \mapsto 100\}\}$$

$$CP_\circ(9) = CP_\bullet(8)$$

$$CP_\bullet(9) = \hat{f}_9(CP_\circ(9)) = \{[7] \mapsto \{a \mapsto 100, b \mapsto 100, x \mapsto 100, \_\mathrm{r} \mapsto 100\}\}$$

$$CP_\circ(10) = CP_\bullet(9)$$

$$CP_\bullet(10) = \hat{f}_{10}(CP_\bullet(4), CP_\circ(9)) = \{[9] \mapsto \{a \mapsto 10000, c \mapsto 100, x \mapsto 100, \_\mathrm{r} \mapsto 10000\}\}$$

The final result or `_r` is 10000 = 10 * 10 * 10 * 10 = 100 * 100 which makes sense. Therefore our constant propagation run successfully.