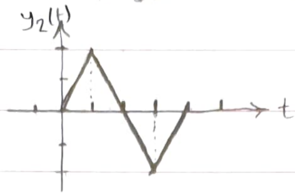
۹۷۳۱۰۹۶

امیرحسین علی‌بخشی

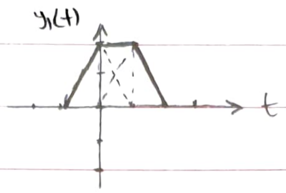
« بخش تئوری تمرین ۲ سیگنال »

۱) الف)

$x_2(t) = x_1(t) - x(t-2) \Rightarrow y_2(t) = y_1(t) - y_1(t-2)$



$x_3(t) = x_1(t) + x_1(t+1) \Rightarrow y_3(t) = y_1(t) + y_1(t+1)$



ب)

$y_1(t) = e^{-t}u(t) + u(-1-t) = e^{-t}x_1(t) + x_1(-1-t)$

$x_2(t) = u(t-1) - u(t-2) = \underbrace{x_1(t-1)}_{*} - \underbrace{x_1(t-2)}_{**}$

$$\left.\begin{aligned} *)\ & y_1(t-1) = e^{-t+1}u(t-1) + u(-t) \\ **)\ & y_1(t-2) = e^{2-t}u(t-2) + u(-t+1) \end{aligned}\right\} \Rightarrow \boxed{y_2(t) = e^{1-t}u(t-1) + u(-t) - e^{2-t}u(t-2) - u(-t+1)}$$

* تمام عملیات انجام شده به دلیل LTI بودن سیستم ها قابل قبول میباشد.

2-

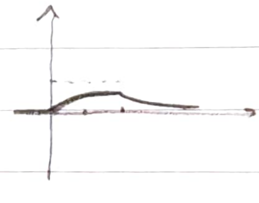
a) $x(t) = u(t) - u(t-2)$ / $h(t) = e^{-2t}u(t)$

$t<0$ no overlap $\rightarrow y(t) = 0$

$0 \le t \le 2 \quad y(t) = \int_0^t e^{-2(t-\tau)}d\tau = e^{-2t} \times \frac{1}{2}e^{2\tau}\Big|_0^t = \frac{e^{-2t}}{2}(e^{2t}-1)$

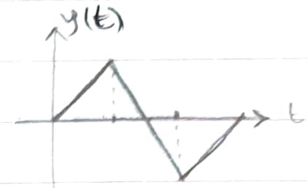
$t > 2 \quad y(t) = \int_0^2 e^{-2(t-\tau)}d\tau = \frac{e^{-2t}}{2}(e^4 - 1)$

$$\Rightarrow y(t) = \begin{cases} 0 & , t<0 \\ \frac{e^{-2t}}{2}(e^{2t}-1) & , 0 \le t < 2 \\ \frac{e^{-2t}}{2}(e^4-1) & , t > 2 \end{cases}$$



b) $x(t) = u(t) - u(t-1) - u(t-1) + u(t-2)$ / $h(t) = u(t) - u(t-1)$

$t<0$, no overlap $\rightarrow y(t) = 0$



$0 \leq t < 1$ $\rightarrow y(t) = \int_0^t 1\, d\tau = t$

$1 < t \leq 2$ $\rightarrow y(t) = \int_{t-1}^{1} 1\, d\tau - \int_1^t d\tau = 1 - t + 1 - t + 1 = 3 - 2t$

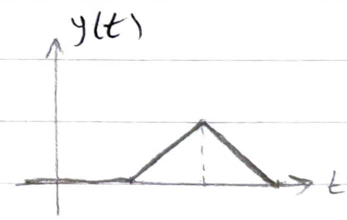
$2 < t \leq 3$ $\rightarrow y(t) = \int_{t-1}^{2} -1 = -2 + t - 1 = -3 + t$

$t > 3$ no overlap $\rightarrow y(t) = 0$

$$\Rightarrow y(t) = \begin{cases} 0 & , t<0 \\ t & , 0 \leq t \leq 1 \\ 3-2t & , 1 < t \leq 2 \\ t-3 & , 2 < t \leq 3 \\ 0 & , t > 3 \end{cases}$$

c) $x(t) = u(t) - u(t-1)$ / $h(t) = u(t-1) - u(t-2)$

$t<1$ no overlap $\rightarrow y(t) = 0$



$1 \leq t \leq 2$ $y(t) = \int_0^{t-1} 1\, d\tau = t-1$

$2 < t \leq 3$ $y(t) = \int_{t-2}^{1} 1\, d\tau = 1 - t + 2 = 3 - t$

$t > 3$ no overlap $\rightarrow y(t) = 0$

$$\Rightarrow y(t) = \begin{cases} 0 & , t<1 \\ t-1 & , 1 \leq t < 2 \\ 3-t & , 2 \leq t \leq 3 \\ 0 & , t > 3 \end{cases}$$

d) $x[n] = u[n] - u[n-4]$ / $h[n] = 2u[n] - 2u[n-4]$

$n<0$ or $n>6$ no overlapping $\rightarrow y[n] = 0$

$0 \leq n \leq 3$ $y[n] = \sum_{k=0}^{n} 2 = 2n + 2$

$3 < n \leq 6$ $\rightarrow y[n] = \sum_{k=n-3}^{3} 2 = \sum_{k=0}^{6-n} 2 = 14 - 2n$

$$\Rightarrow y[n] = \begin{cases} 2n+2 & , 0 \leq n \leq 3 \\ 14-2n & , 3 < n \leq 6 \\ 0 & , \text{otherwise} \end{cases}$$

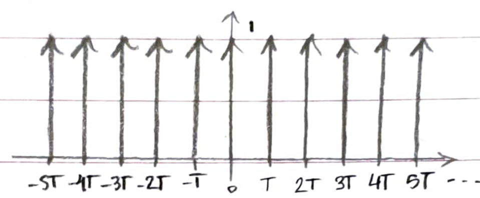
$$x(t) * \Big(\big(\big(\cancel{h_1(t) * h_2(t)} + h_2(t) * h_2(t) - \cancel{h_2(t) * h_1(t)}\big) * h_1(t)\big) + h_1^{-1}(t)\Big) * h_2^{-1}(t)$$ -3

$$= x(t) * \big(\big(h_1(t) * h_2(t) * h_2(t)\big) + h_1^{-1}(t)\big) * h_2^{-1}(t)$$

$$= x(t) * \Big(\big(h_1(t) * h_2(t)\big) + \big(h_1^{-1}(t) * h_2^{-1}(t)\big)\Big)$$

$$x(t) = \sum_{K=-\infty}^{+\infty} \delta(t - KT) = \begin{cases} 1 & , \ t = TK \\ 0 & , \ \text{otherwise} \end{cases}$$ -4

الف)



ب)

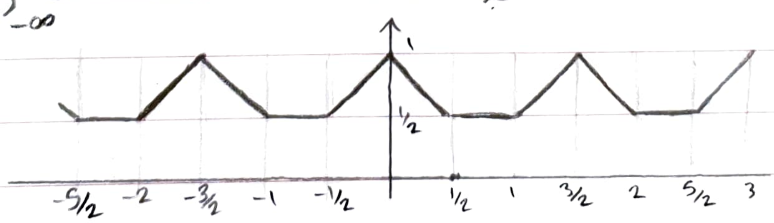
$$T = \frac{3}{2} \Rightarrow x(t) = \sum_{K=-\infty}^{+\infty} \delta\left(t - \frac{3K}{2}\right)$$

یک بازه (مثلاً $[0, \frac{3}{2}]$) را بررسی می‌کنیم و نتیجه را برای سایر بازه‌ها تعمیم می‌دهیم:

$$0 \leq t < \tfrac{1}{2} : y(t) = \int_{-\infty}^{+\infty} \delta(\tau) h(t-\tau)\, d\tau = 1 - t$$

$$\tfrac{1}{2} < t < 1 : y(t) = \int_{-\infty}^{+\infty} \delta(\tau) h(t-\tau)\, d\tau = 1 - \tfrac{1}{2} = \tfrac{1}{2}$$

$$1 \leq t \leq \tfrac{3}{2} : y(t) = \int_{-\infty}^{+\infty} \delta(\tau) h(t-\tau)\, d\tau = t - \tfrac{1}{2}$$



$$a)\ \boxed{I} : x[n] * (h[n] \cdot g[n]) = (x[n] * h[n]) \cdot (x[n] * g[n])$$ -5

$$= \Big(\sum_{K=-\infty}^{+\infty} x[K] h[n-K]\Big)\Big(\sum_{K=-\infty}^{+\infty} x[K] g[n-K]\Big)$$

$$\boxed{II}\ (x[n] * h[n]) \cdot g[n] = \Big(\sum_{K=-\infty}^{+\infty} x[K] h[n-K]\Big) g[n]$$

$$\boxed{I} \neq \boxed{II} \Rightarrow$$ نادرست

b) $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \Rightarrow y(2t)=\int_{-\infty}^{+\infty} x(\tau)h(2t-\tau)d\tau \xrightarrow{\tau\to 2\tau}$

$=\int_{-\infty}^{+\infty} x(2\tau)h(2t-2\tau)2d\tau = 2\int_{-\infty}^{+\infty} x(2\tau)h(2t-2\tau)d\tau = 2x(2t)*h(2t)$ ✓

صحیح

c) if $x(-t)=-x(t)$ and $h(-t)=-h(t)$ then $y(-t)=y(t)$

$y(-t)=-x(-t)*h(-t)=-[\cancel{-}x(t)*\cancel{-}h(t)]=-x(t)*h(t)=-y(t)$ × نادرست

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a) $h(t)=te^{-t}u(t)$ — 6

- causality : $\forall t<0,\ h(t)\overset{?}{=}0 \rightarrow$ ✓ $\Rightarrow$ $\boxed{\text{causal}}$

- stability : $\int_{-\infty}^{+\infty}|h(\tau)|d\tau = \int_{-\infty}^{+\infty}|\tau e^{-\tau}u(\tau)| = \int_{0}^{+\infty}\tau e^{-\tau}d\tau<\infty \Rightarrow$ $\boxed{\text{stable}}$

- memory : $\forall t\neq 0,\ h(t)\overset{?}{=}0 \rightarrow$ × $\Rightarrow$ $\boxed{\text{with memory}}$

b) $h[n]=(0.8)^n u[n+2]$

- causality : $\forall n<0,\ h[n]\overset{?}{=}0 \rightarrow$ × $\Rightarrow$ $\boxed{\text{not causal}}$

- stability : $\sum_{k=-\infty}^{+\infty}|0.8^k u[k+2]| = \sum_{k=-2}^{\infty}(0.8)^k < \infty \Rightarrow$ $\boxed{\text{stable}}$

- memory : $\forall n\neq 0,\ h[n]\overset{?}{=}0$ × $\Rightarrow$ $\boxed{\text{with memory}}$

c) $h(t)=e^{-6t}u(t+2)$

- causality : $\forall t<0,\ h(t)\overset{?}{=}0 \rightarrow$ × $\Rightarrow$ $\boxed{\text{not causal}}$

- stability : $\int_{-\infty}^{+\infty}|e^{-6\tau}u(\tau+2)|d\tau = \int_{-2}^{+\infty}e^{-6\tau}d\tau = -\frac{1}{6}e^{-6\tau}\Big]_{-2}^{\infty} = 0+\frac{1}{6}e^{12}<\infty \Rightarrow$ $\boxed{\text{stable}}$

- memory : $\forall t\neq 0,\ h(t)\overset{?}{=}0$ × $\Rightarrow$ $\boxed{\text{with memory}}$

d) $h[n] = 5^n u[3-n]$

- causality: $\forall n<0,\ h[n] \stackrel{?}{=} 0 \quad \times \Longrightarrow \boxed{\text{not causal}}$

- stability: $\sum_{k=-\infty}^{+\infty} |5^k u[3-k]| = \sum_{k=-\infty}^{3} 5^k = \sum_{k=3}^{+\infty} 5^{-k} = \frac{5^3}{1-1/5} = \frac{625}{4} < \infty \Rightarrow \boxed{\text{stable}}$

- memory: $\forall n \neq 0,\ h[n] \stackrel{?}{=} 0 \quad \times \Longrightarrow \boxed{\text{with memory}}$

$y[n] + 2y[n-1] = x[n] \quad , \quad x[n] = \delta[n] \xrightarrow{n<0} \delta[n] = 0 \ , \forall n<0 \ , y[n] = 0$ ✓

$y[n] + 2y[n-1] = \delta[n] \Big|_{n=0} = y[0] + \cancel{2y[-1]} = \delta[0] \Rightarrow y[0] = 1$

$n>0 : y[n] + 2y[n-1] = 0 \Longrightarrow y[n] = -2y[n-1]$ recursive function

$$\begin{cases} y[0] = 1 \\ y[1] = -2y[0] = -2 \\ y[2] = -2y[1] = -2(-2) = 4 \\ y[3] = -2y[2] = -2(4) = -8 \\ \vdots \\ y[n] = -2y[k-1] = -2^n \end{cases}$$

$\Rightarrow h[n] = (-2)^n u[n]$