Spring 2011

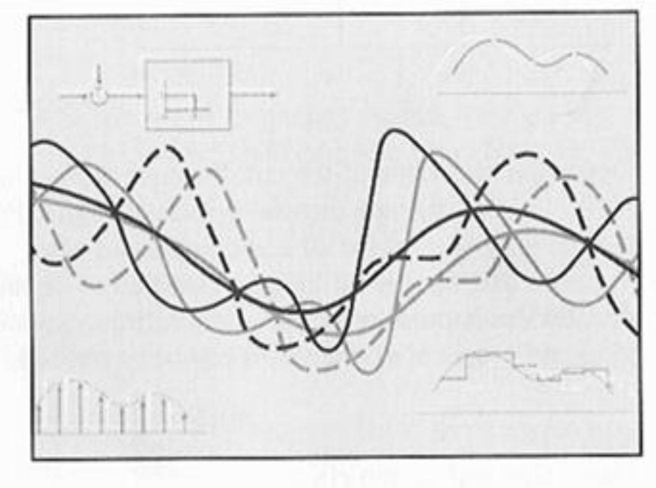
信號與系統
Signals and Systems

Chapter SS-7
Sampling

Feng-Li Lian

NTU-EE

Feb11 – Jun11

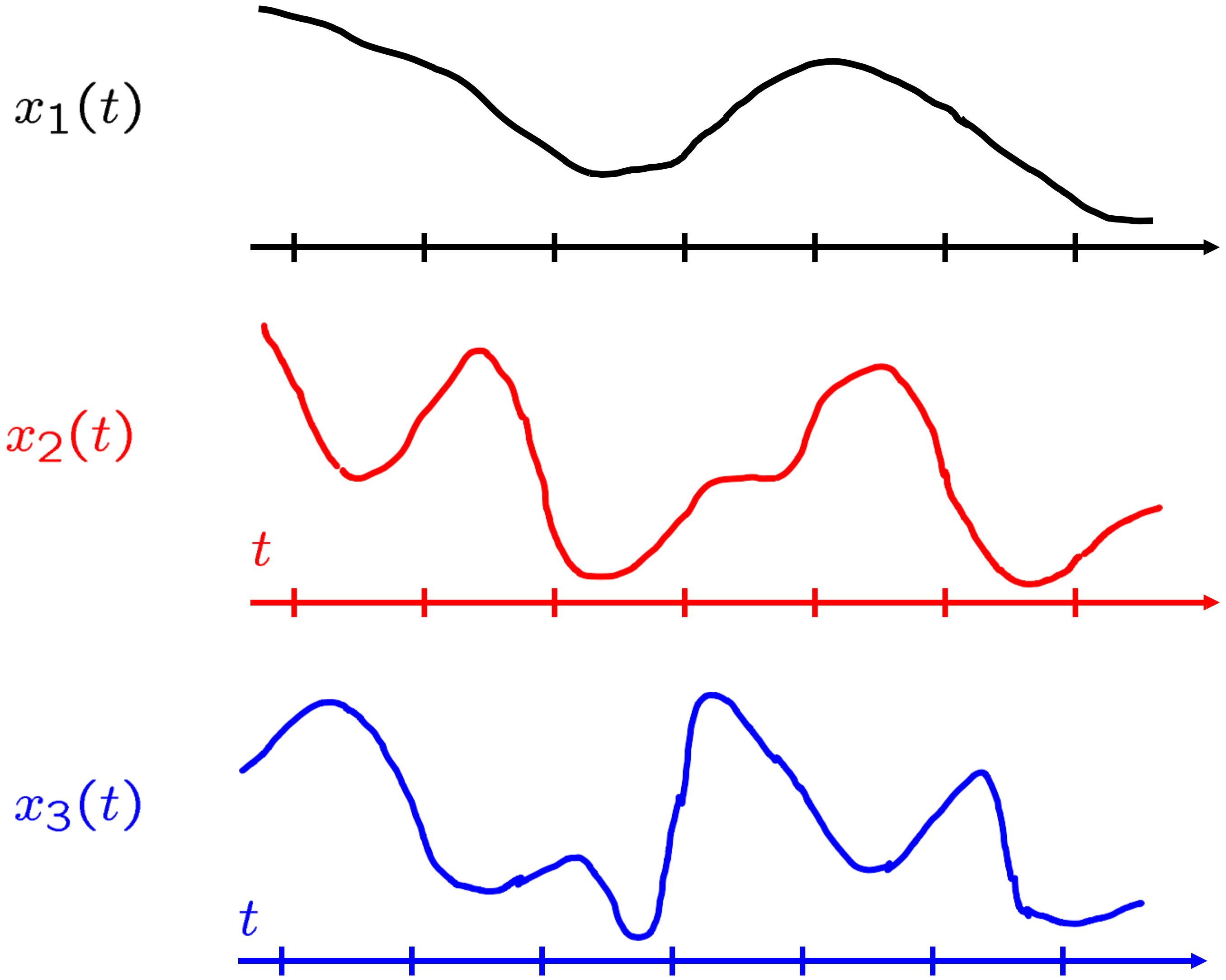


Figures and images used in these lecture notes are adopted from
“Signals & Systems” by Alan V. Oppenheim and Alan S. Willsky, 1997

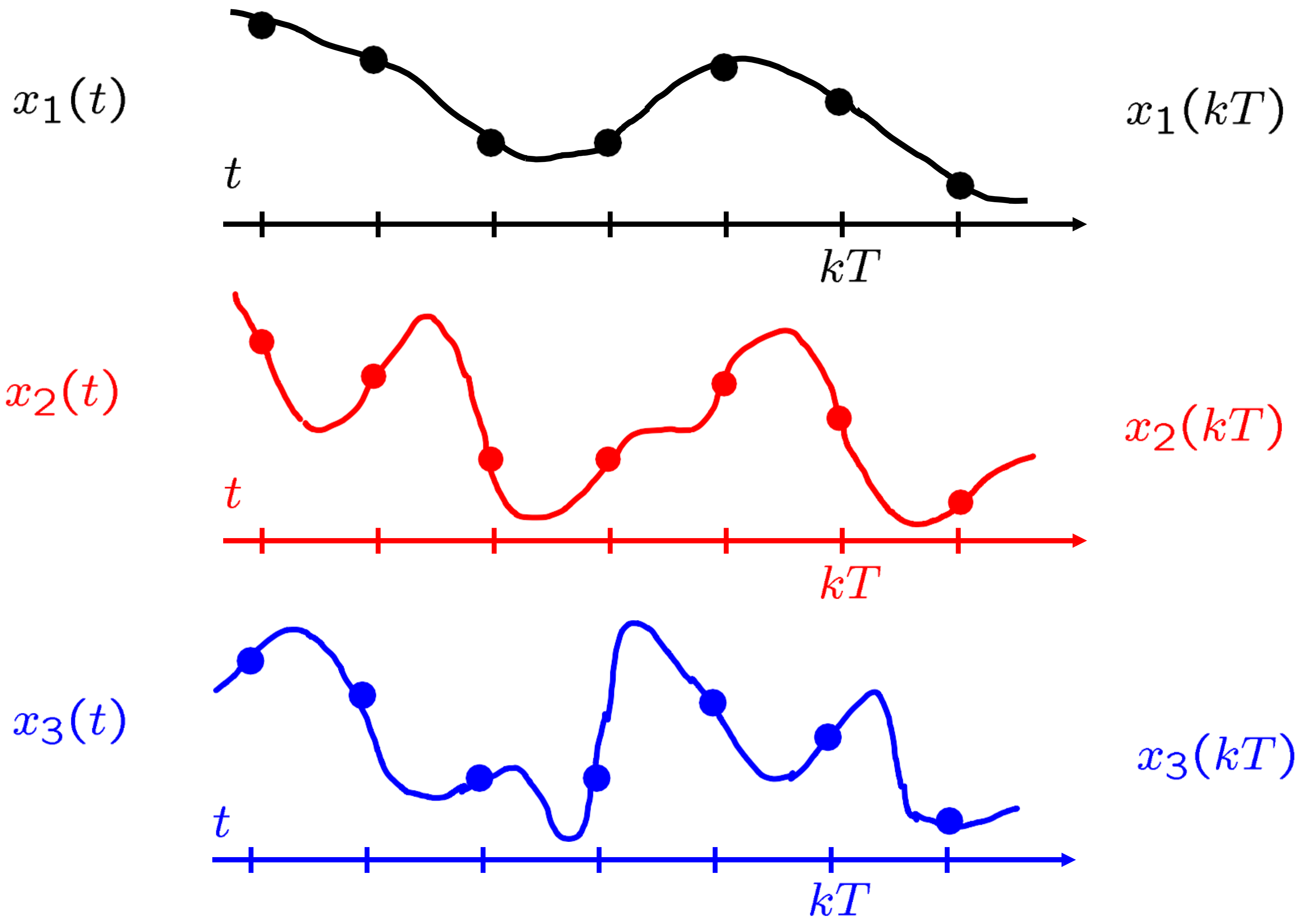
Outline

- Representation of of a Continuous-Time Signal by Its Samples: The Sampling Theorem
- Reconstruction of of a Signal from Its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

Representation of CT Signals by its Samples

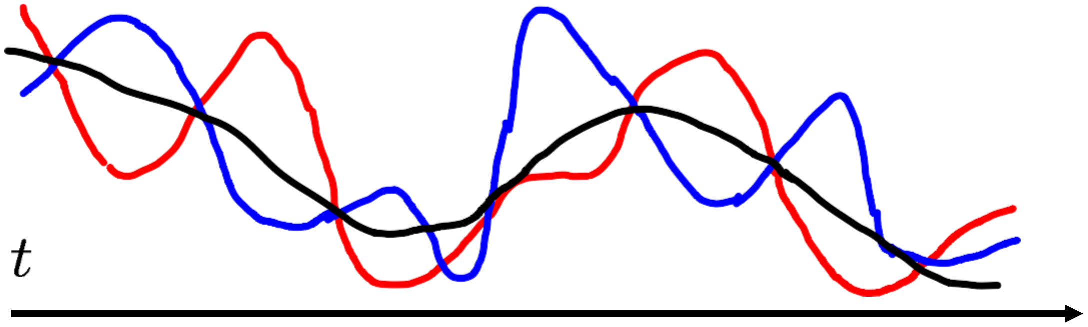


Representation of CT Signals by its Samples

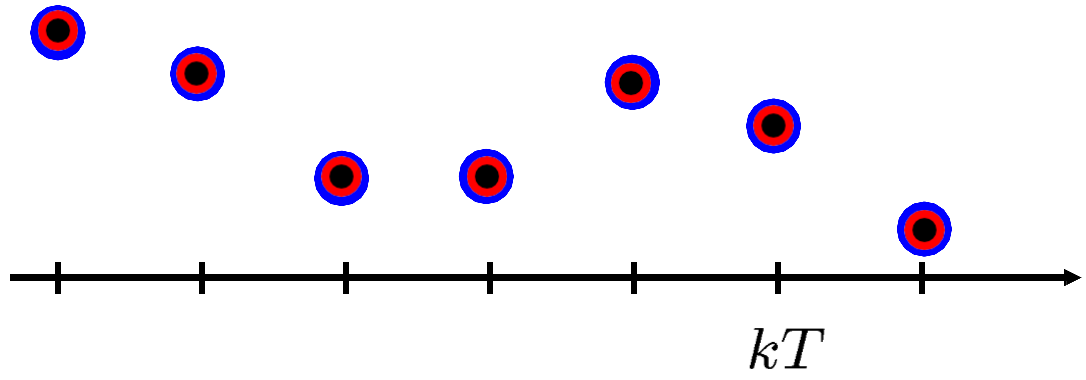


Representation of CT Signals by its Samples

$x_1(t) \neq x_2(t) \neq x_3(t)$



$x_1(kT) = x_2(kT) = x_3(kT)$

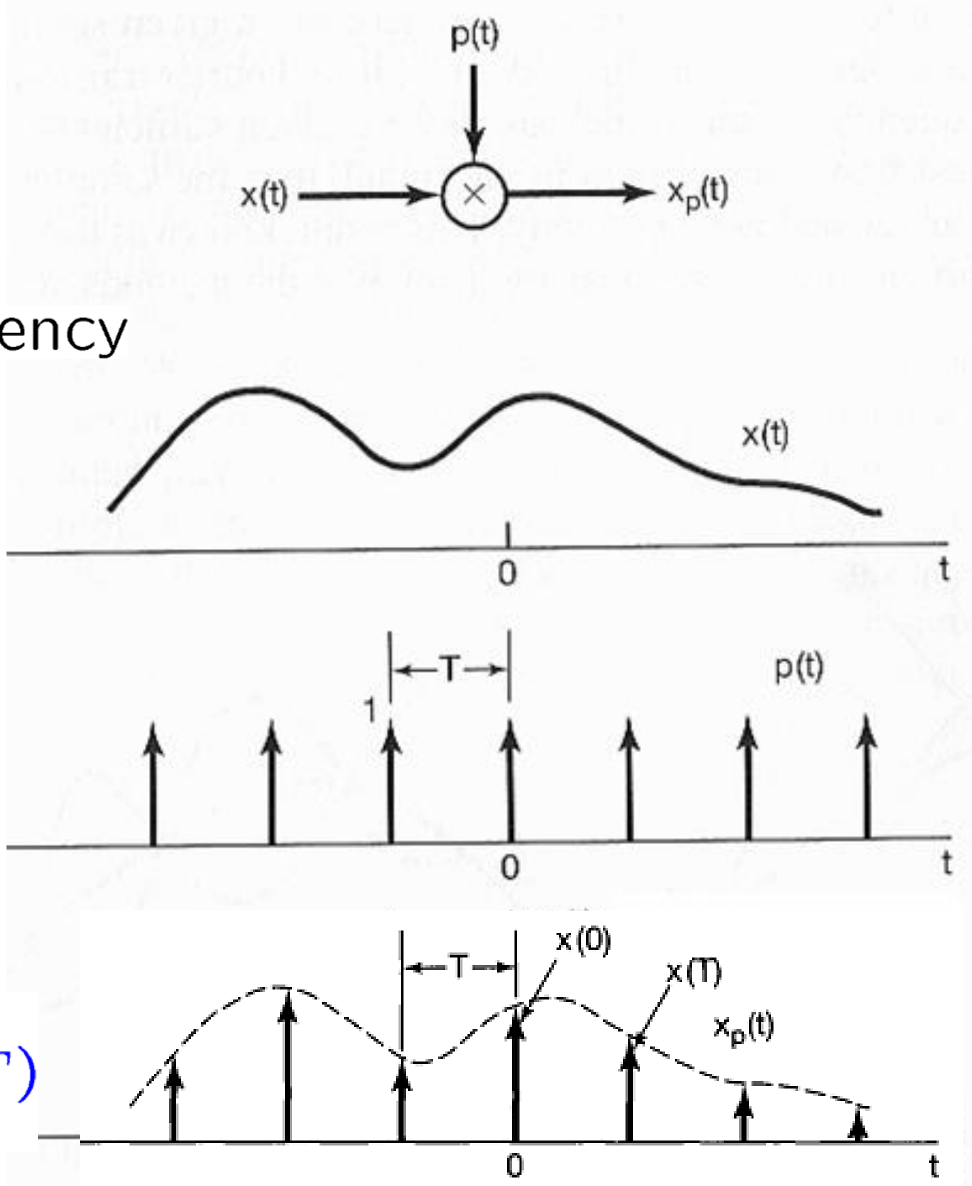


Impulse-Train Sampling:

$p(t)$: sampling function

T : sampling period

$w_s = \frac{2\pi}{T}$: sampling frequency



$$\Rightarrow \; x_p(t) = x(t)\, p(t)$$

$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

Impulse-Train Sampling:

From multiplication property,

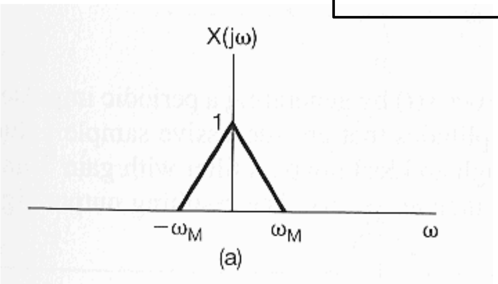
$$x_p(t) = x(t)\, p(t) \quad \xleftrightarrow{\mathcal{F}} \quad X_p(jw) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)P(j(w-\theta))d\theta$$

Eq 4.70, p. 322

$$= \frac{1}{T}\sum_{k=-\infty}^{+\infty} X(j(w-kw_s))$$

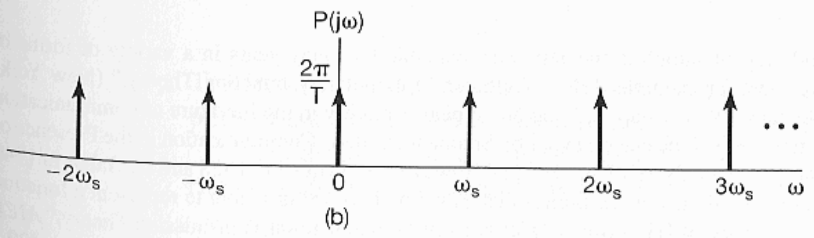
Ex 4.21, p. 323

$$x(t) \quad \xleftrightarrow{\mathcal{F}} \quad X(jw)$$



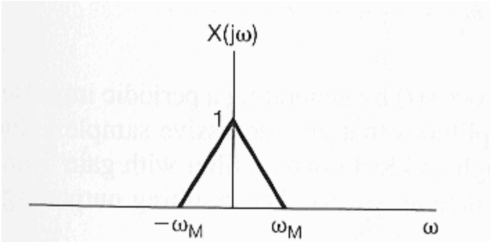
$$p(t) \quad \xleftrightarrow{\mathcal{F}} \quad P(jw) = \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty} \delta(w-kw_s)$$

Ex 4.8, pp. 299-300

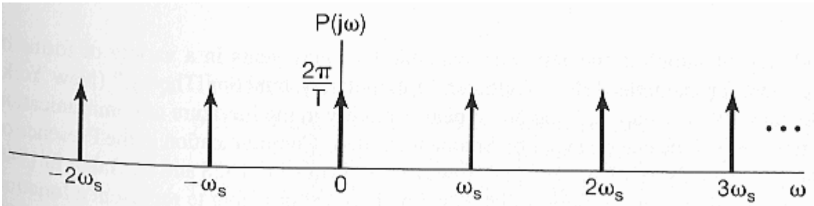


■ Impulse-Train Sampling:

Ex 4.21, 4.22, pp. 323-4

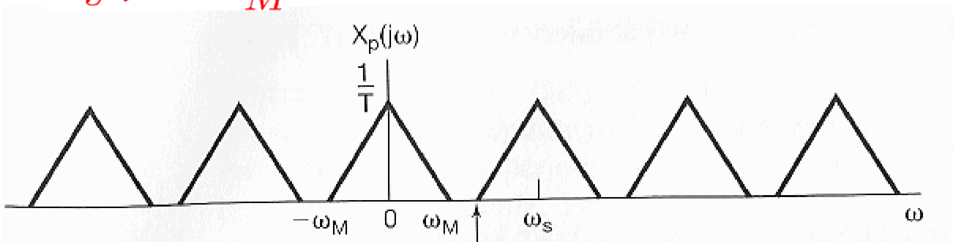


(a)



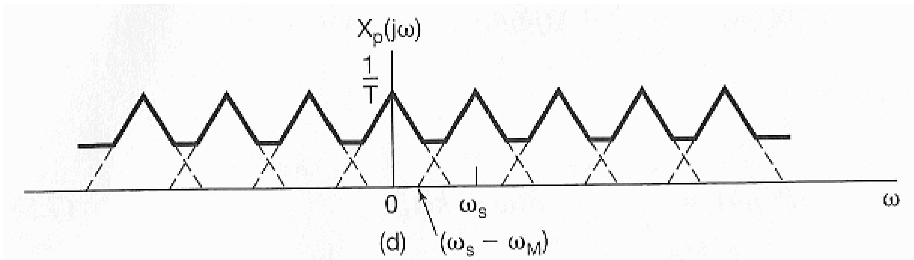
(b)

$w_s > 2w_M$



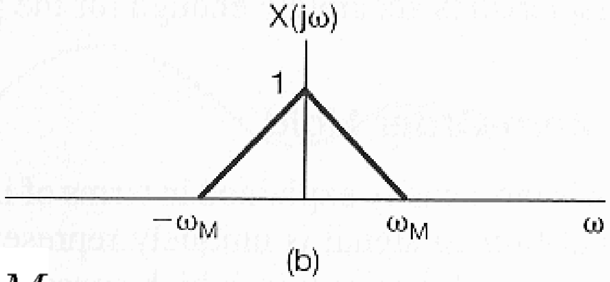
(c)

$w_s < 2w_M$



(d)

- The Sampling Theorem:



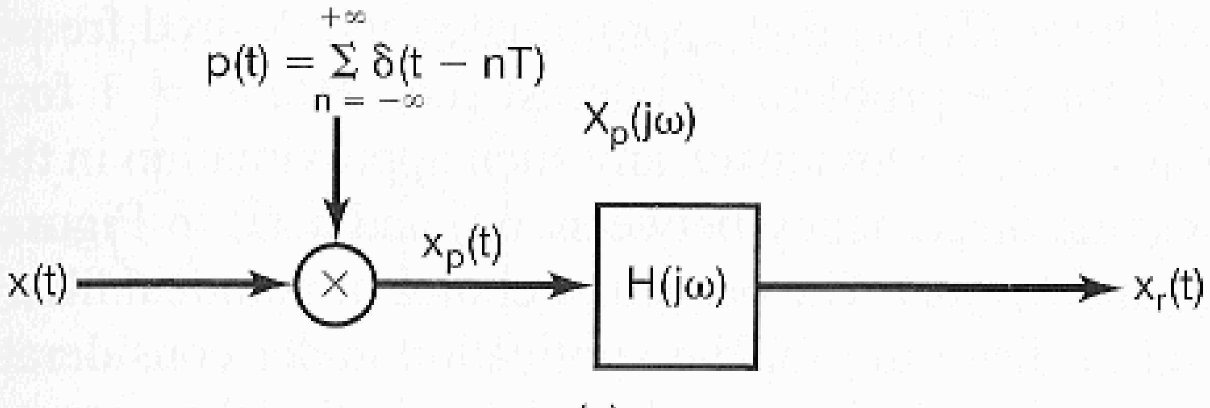
(b)

$x(t)$: a band-limited signal

with $X(jw) = 0$ for $|w| > w_M$

if $w_s > 2w_M$ where $w_s = \dfrac{2\pi}{T}$

$\Rightarrow$ $x(t)$ is uniquely determined by $x(nT), n = 0, \pm 1, \pm 2, ...,$

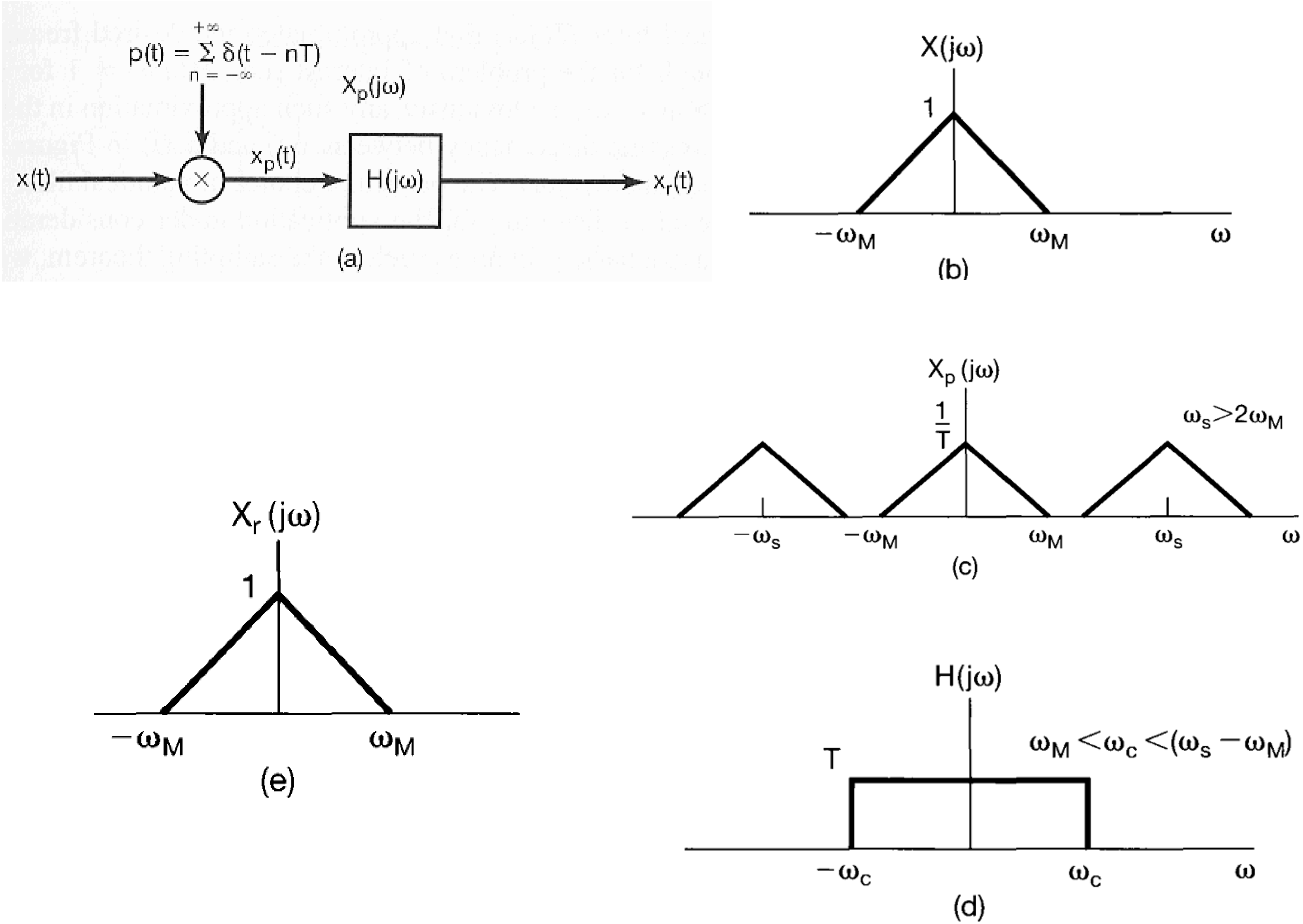


(a)

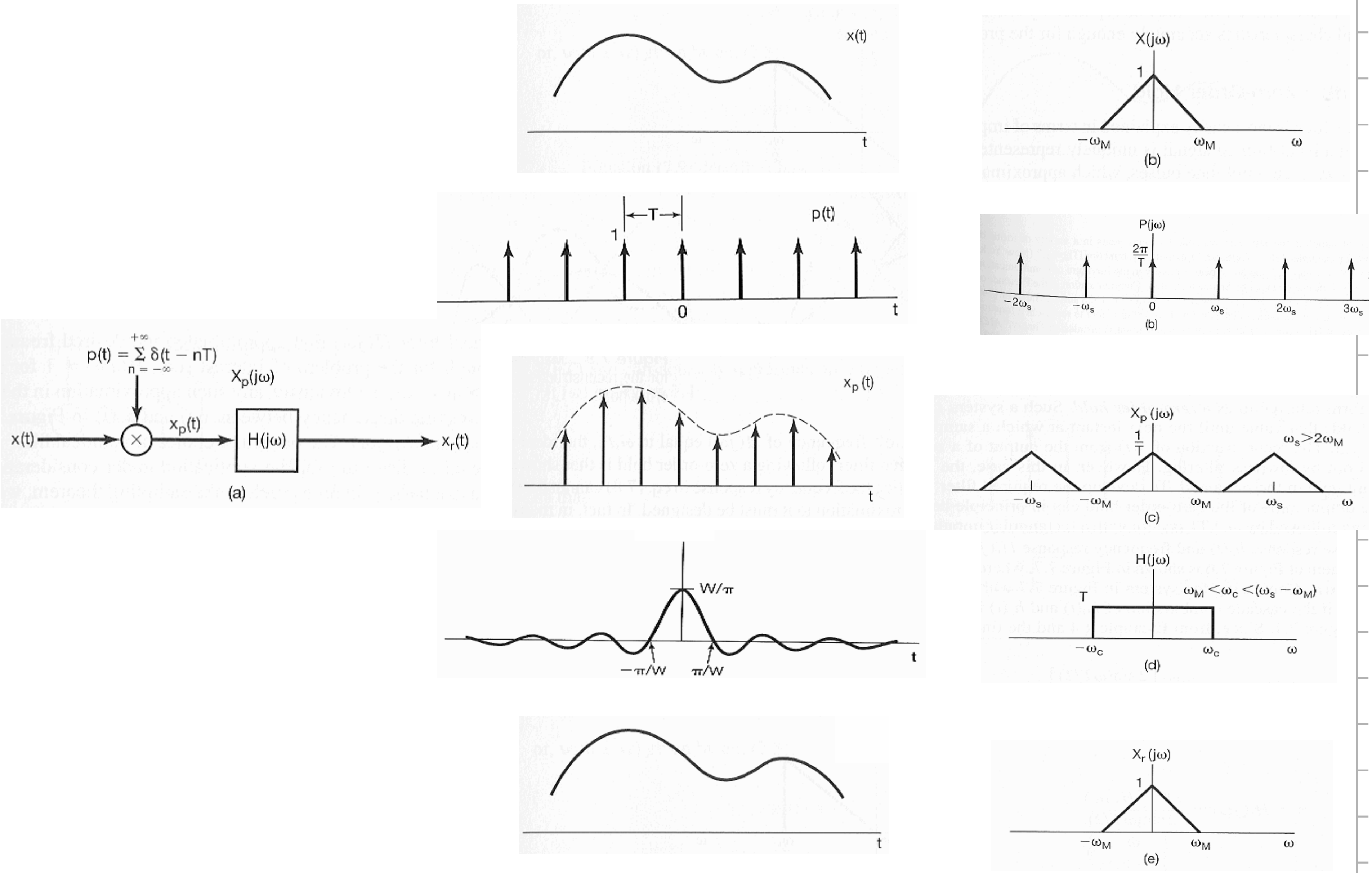
$\Rightarrow$ $2w_M$: Nyquist rate

w_M : Nyquist frequency

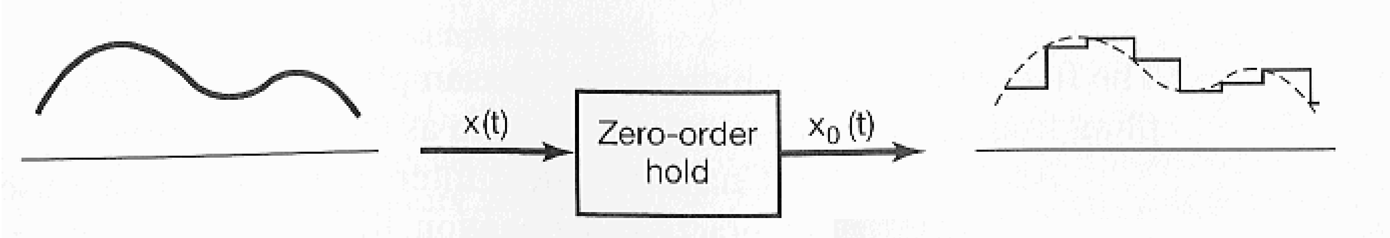
Exact Recovery by an Ideal Lowpass Filter:

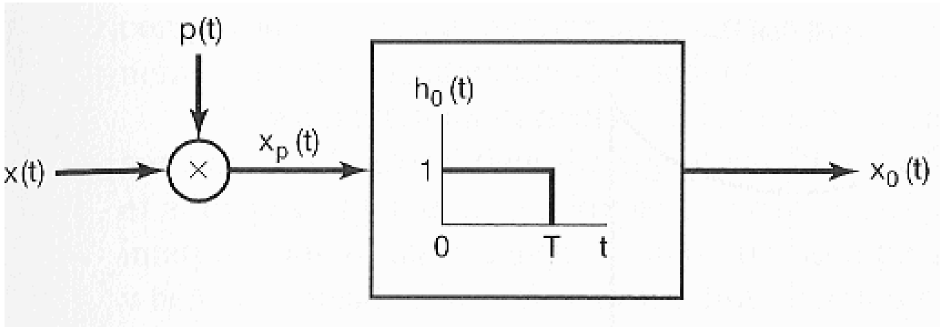


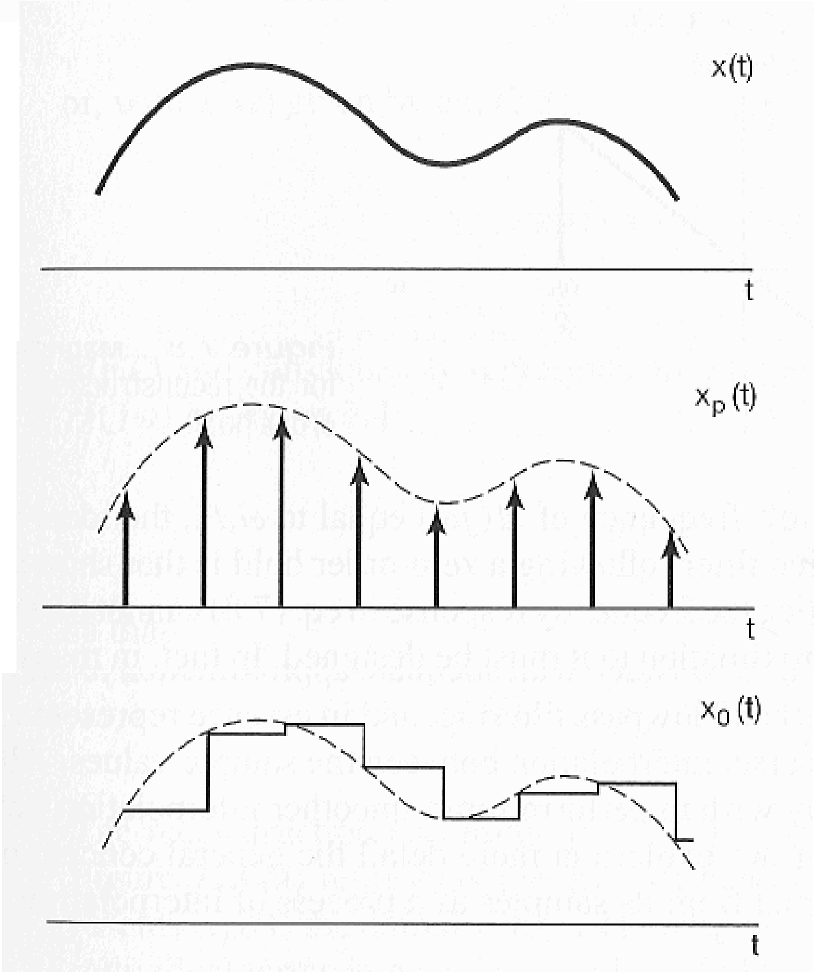
- <u>Exact Recovery by an Ideal Lowpass Filter:</u>



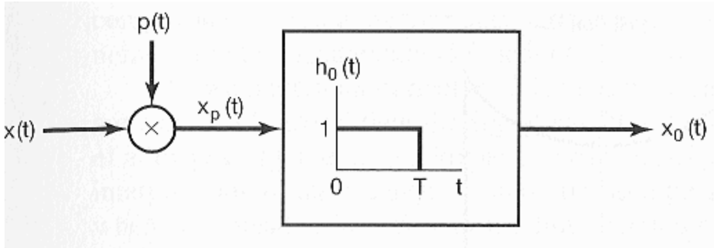
- Sampling with Zero-Order Hold:





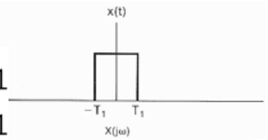


Sampling with Zero-Order Hold:

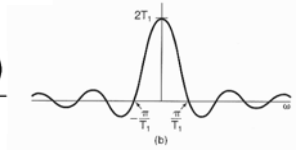


Ex 4.4, p. 293

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}$$

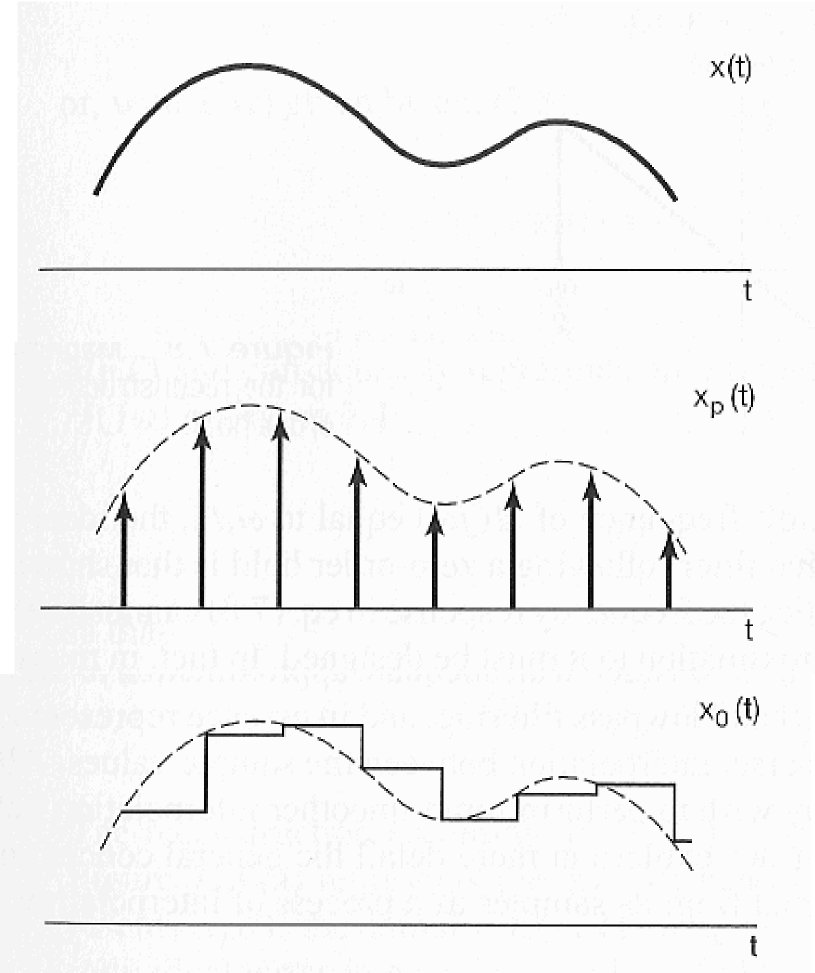


$$X(jw) = 2\frac{\sin(wT_1)}{w}$$

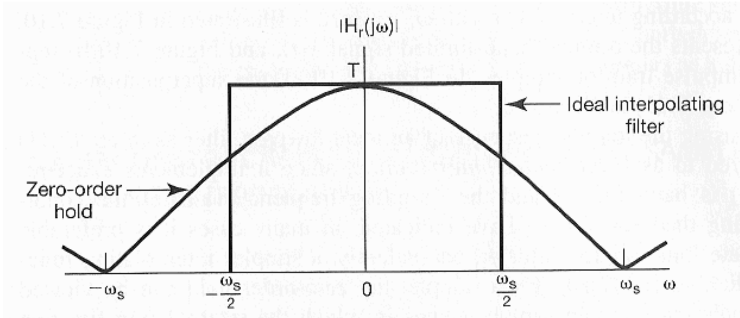


Eq 4.27, p. 301

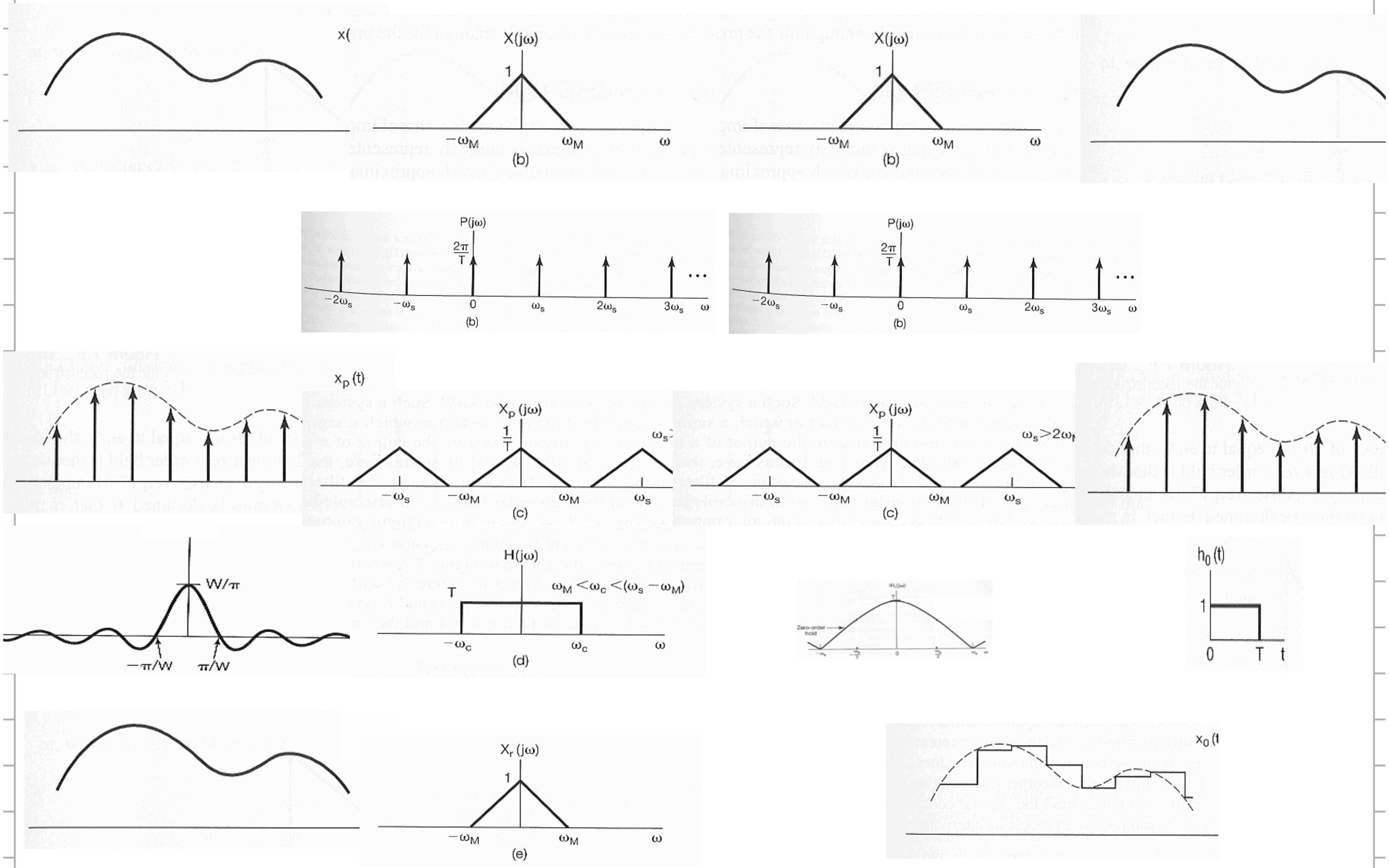
$$x(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-jwt_0}X(jw)$$



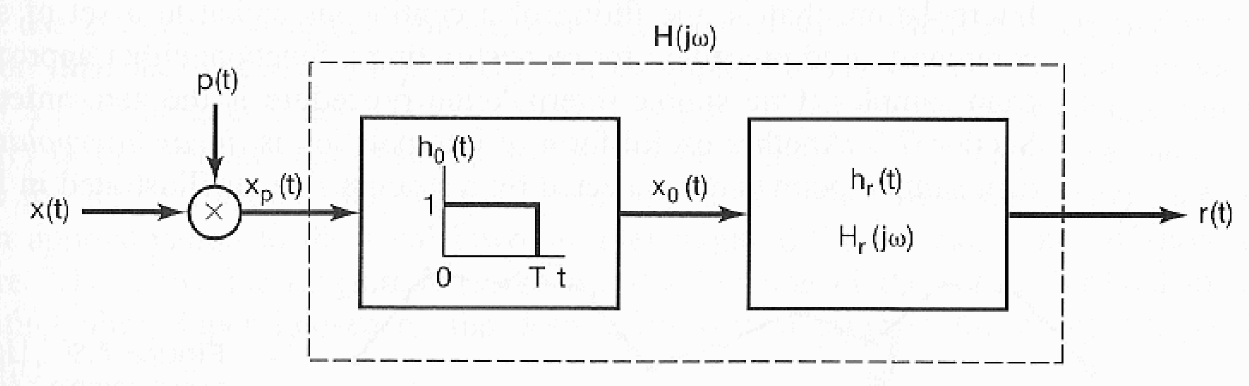
$$H_0(jw) = e^{-jwT/2}\left[\frac{2\sin(wT/2)}{w}\right]$$



With Ideal Lowpass Filter & with Zero-Order Hold:



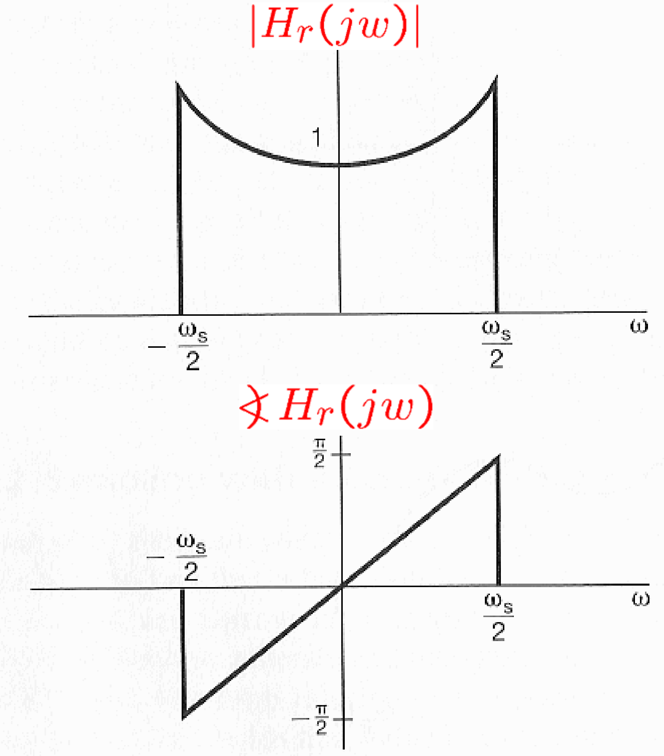
▪ Sampling with Zero-Order Hold:



$$H_0(jw) = e^{-jwT/2}\left[\frac{2\sin(wT/2)}{w}\right]$$

$$H(jw) = H_0(jw)H_r(jw)$$

$$\Rightarrow \quad H_r(jw) = \frac{e^{jwT/2}H(jw)}{\frac{2\sin(wT/2)}{w}}$$



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- Reconstruction of of a Signal from Its Samples Using Interpolation
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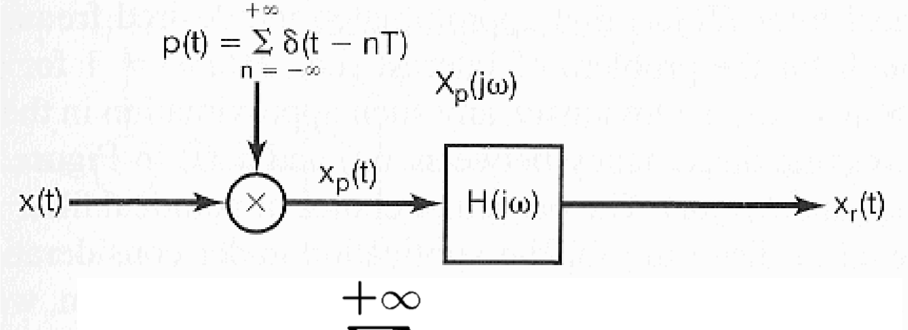
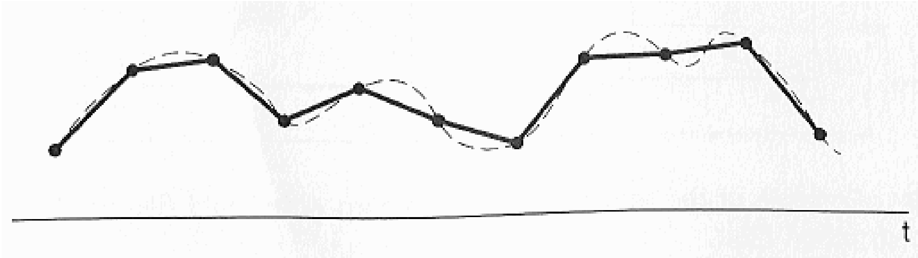
- **Exact Interpolation:**





$$x_p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

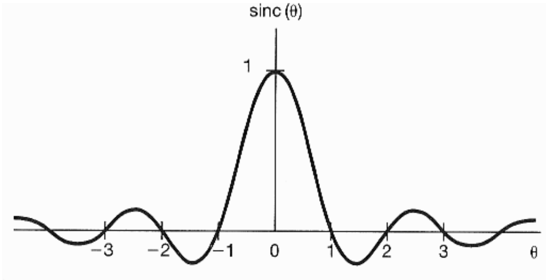
ideal lowpass filter with a magnitude of T

$$x_r(t) = x_p(t) * h(t)$$

$$h(t) = T\,\frac{w_c}{\pi}\,\frac{\sin(w_c t)}{w_c t}$$

Ex 2.11, p. 110

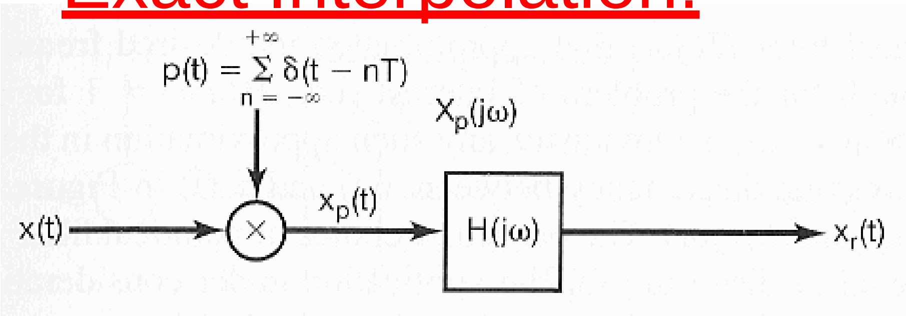
$$x(t - t_0) = x(t) * \delta(t - t_0)$$



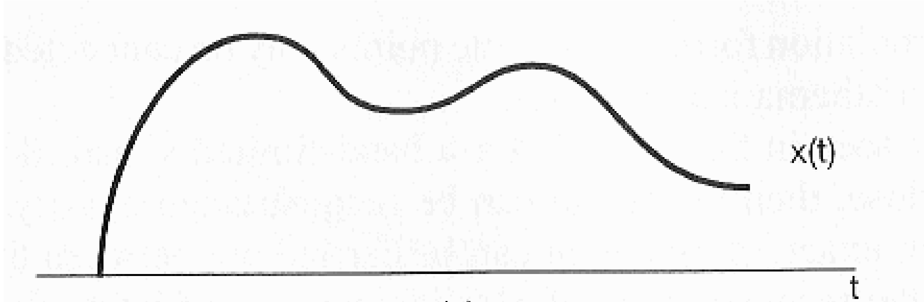
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)h(t - nT)$$

$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\frac{w_c T}{\pi}\,\frac{\sin(w_c(t - nT))}{w_c(t - nT)}$$

Exact Interpolation:



(a)

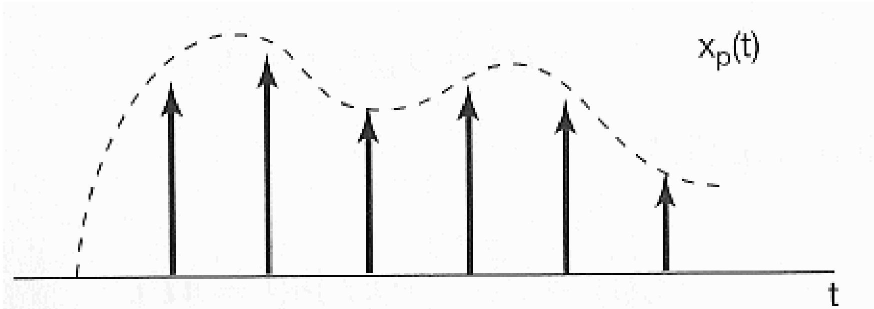


(a)

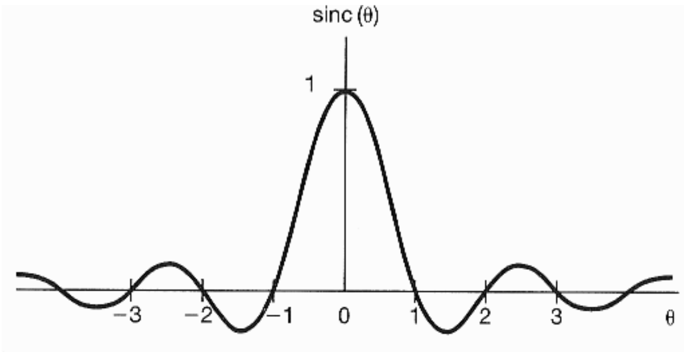
$$x_r(t) = \sum_{n=-\infty}^{+\infty} x(nT)\, \frac{w_c T}{\pi} \frac{\sin(w_c(t-nT))}{w_c(t-nT)}$$

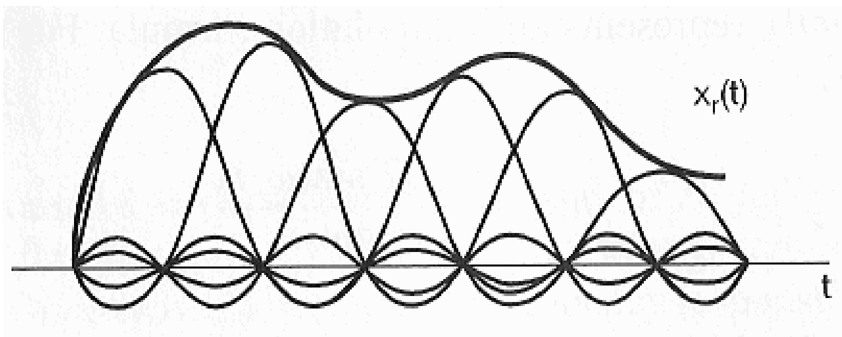
$$\frac{w_c}{\pi} \frac{2\pi}{w_s} \frac{\sin \pi(w_c(t-nT)/\pi)}{\pi w_c(t-nT)/\pi}$$

$$\frac{2w_c}{w_s} \mathrm{sinc}(w_c(t-nT)/\pi)$$



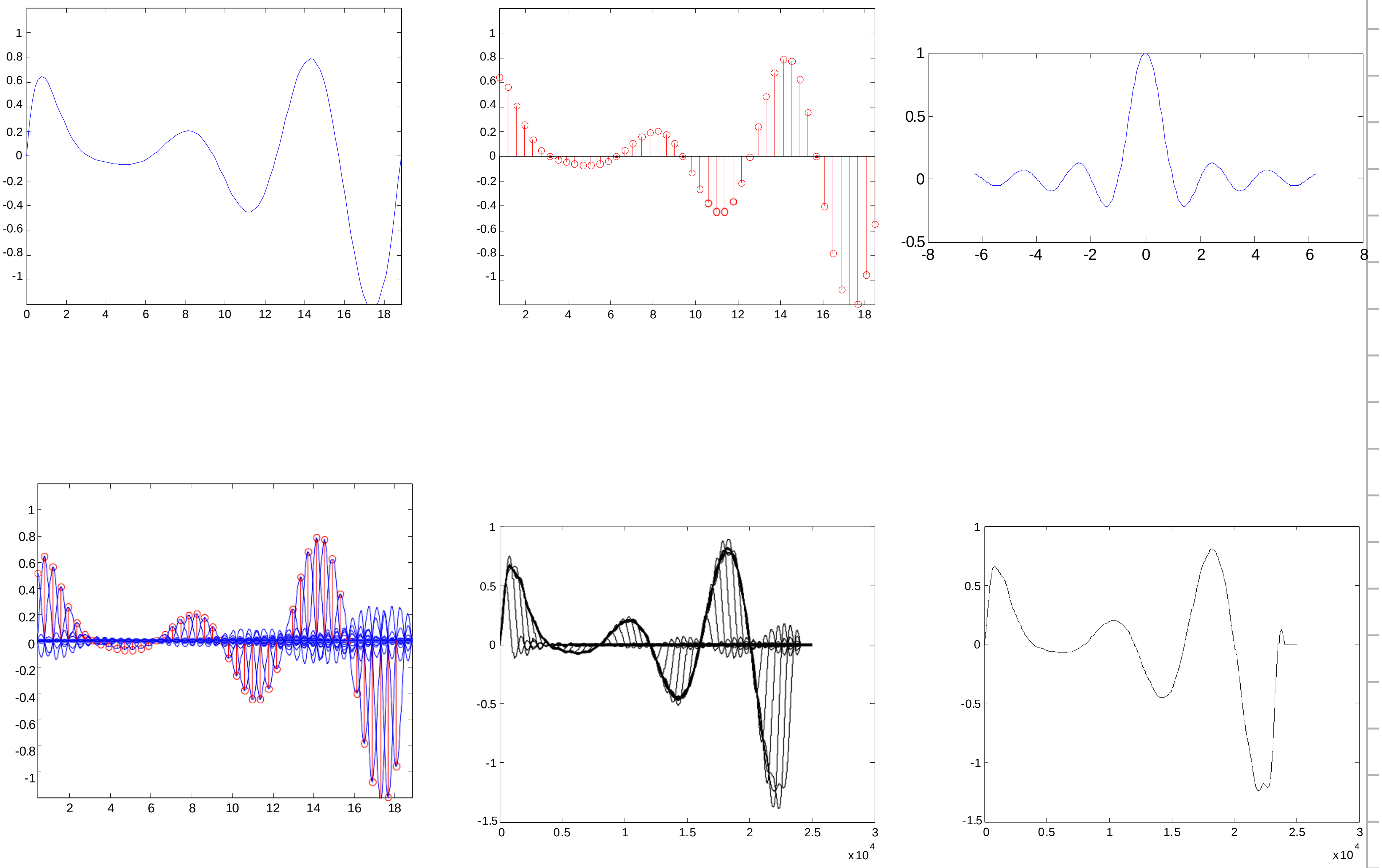
(b)



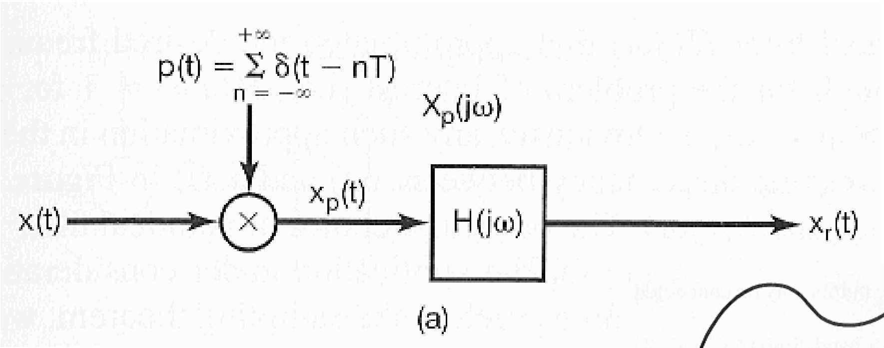


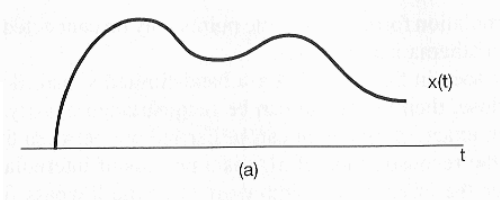
(c)

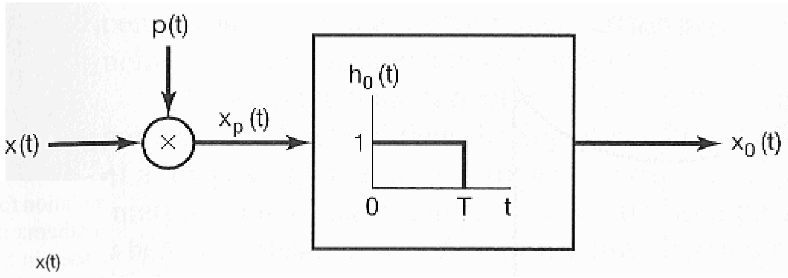
Reconstruction of a Signal from its Samples Using Interpolation

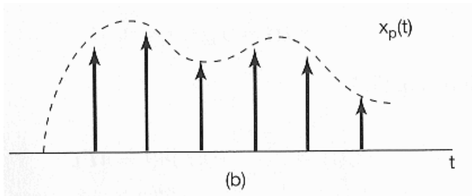


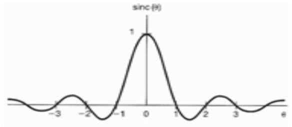
Ideal Interpolating Filter & The Zero-Order Hold:

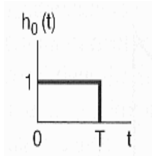


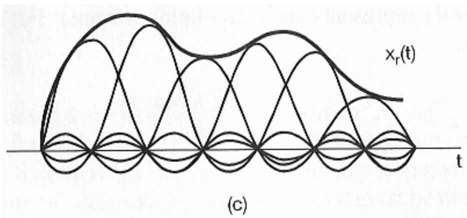


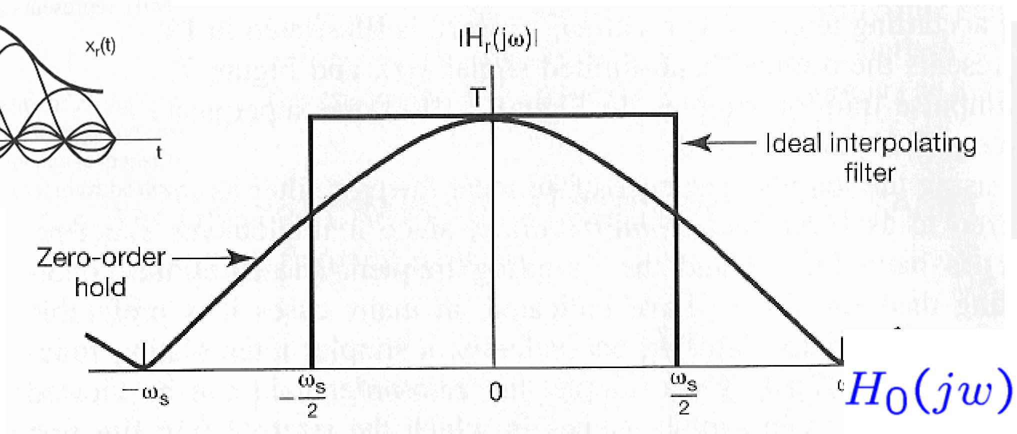


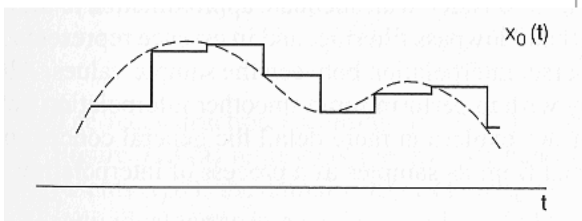












$$H_0(jw) = e^{-jwT/2} \left[\frac{2\sin(wT/2)}{w} \right]$$

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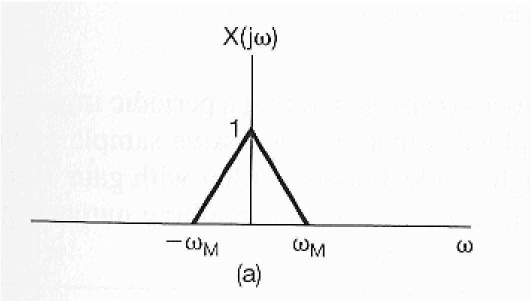
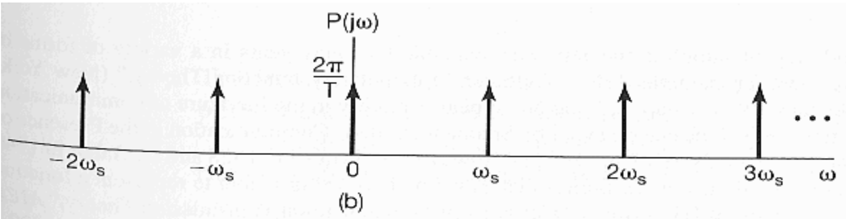
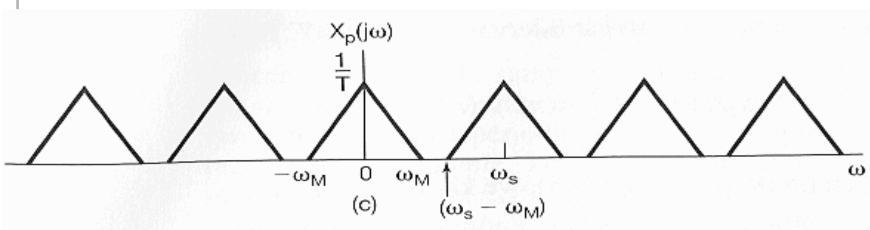
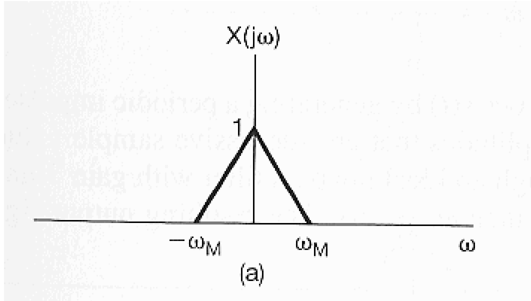
- <u>Overlapping in Frequency-Domain: Aliasing</u>



$$w_s > 2w_M$$







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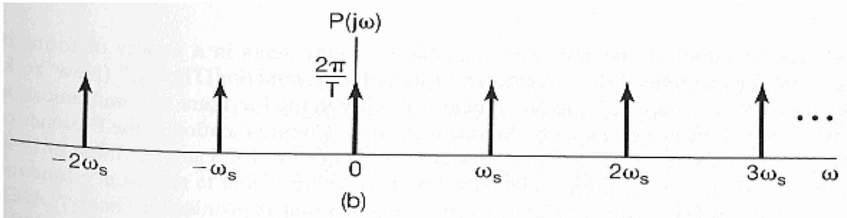
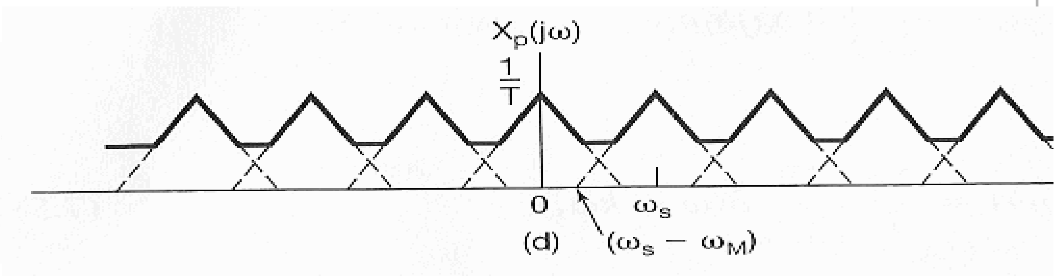
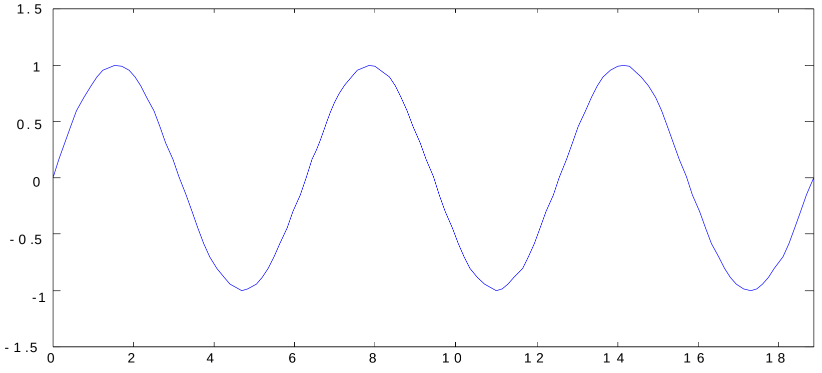
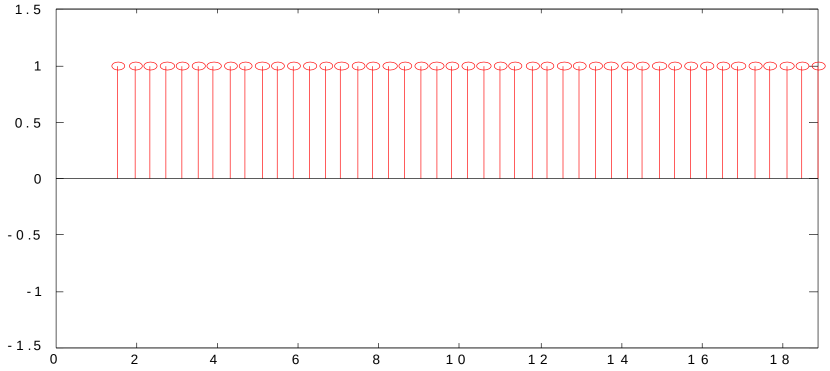
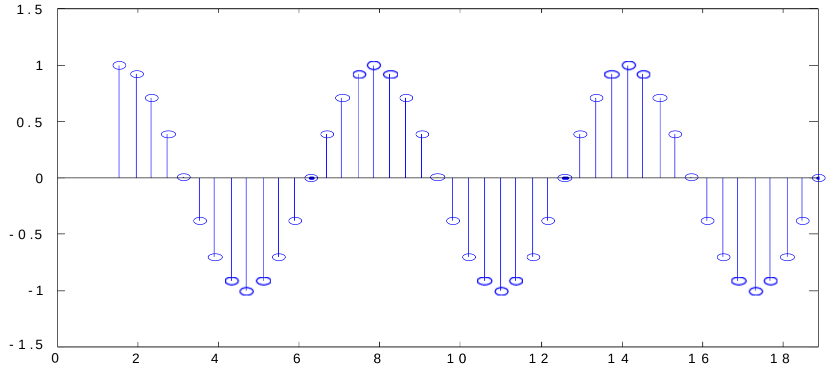


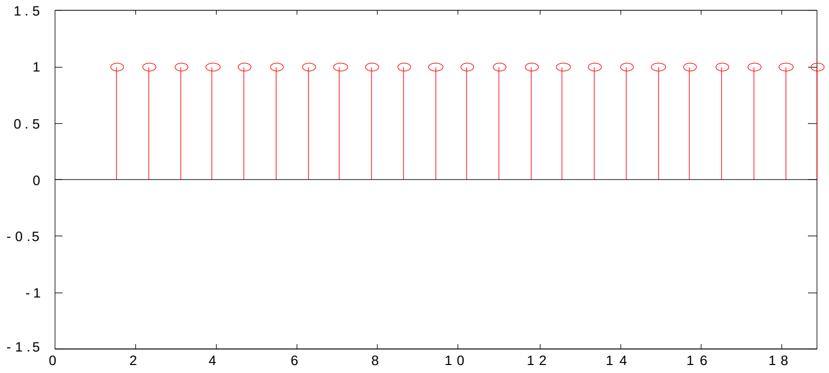


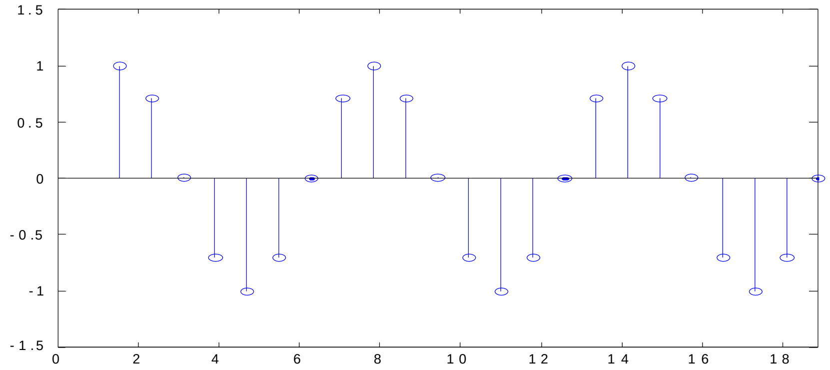
- <u>Overlapping in Frequency-Domain: Aliasing</u>











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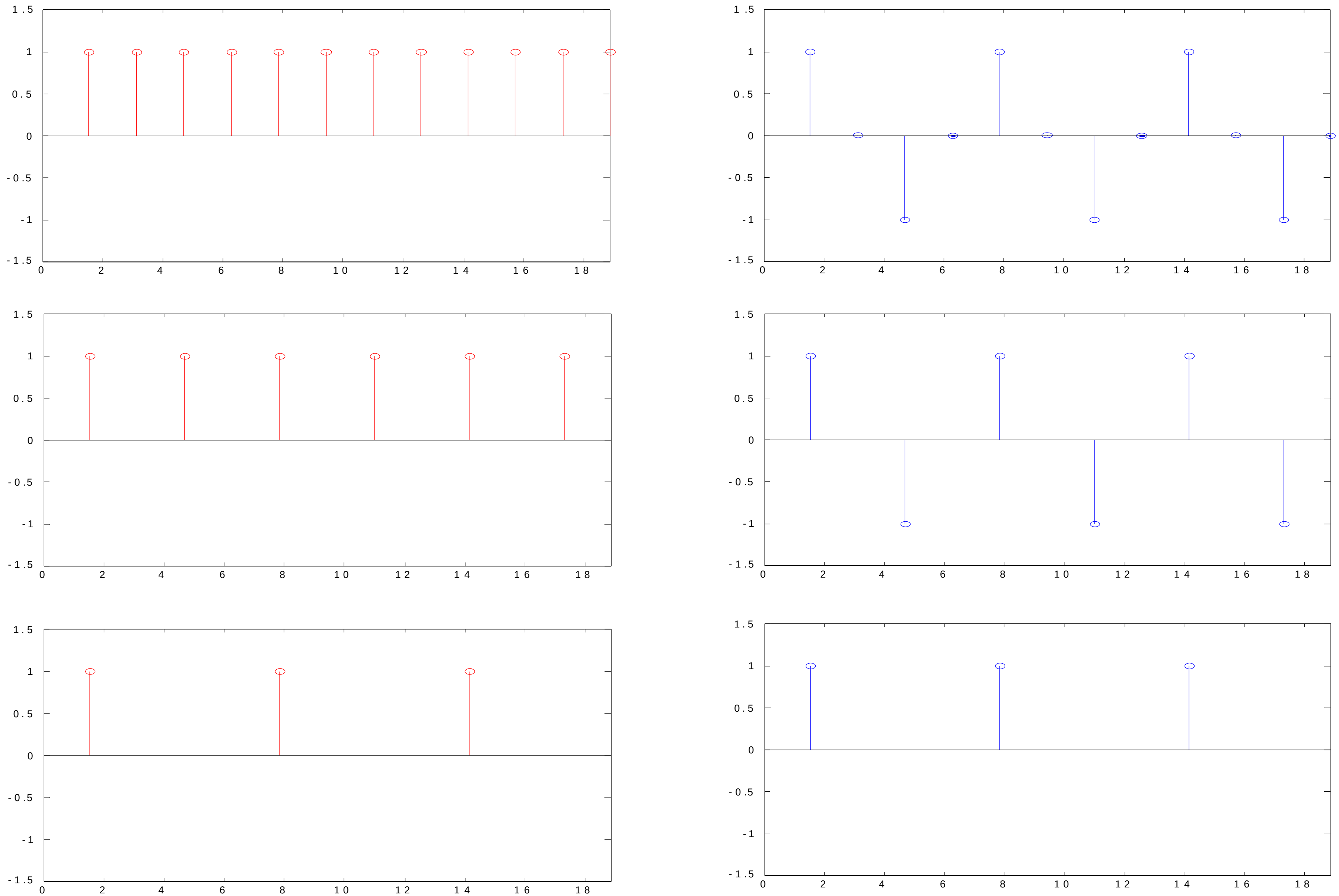

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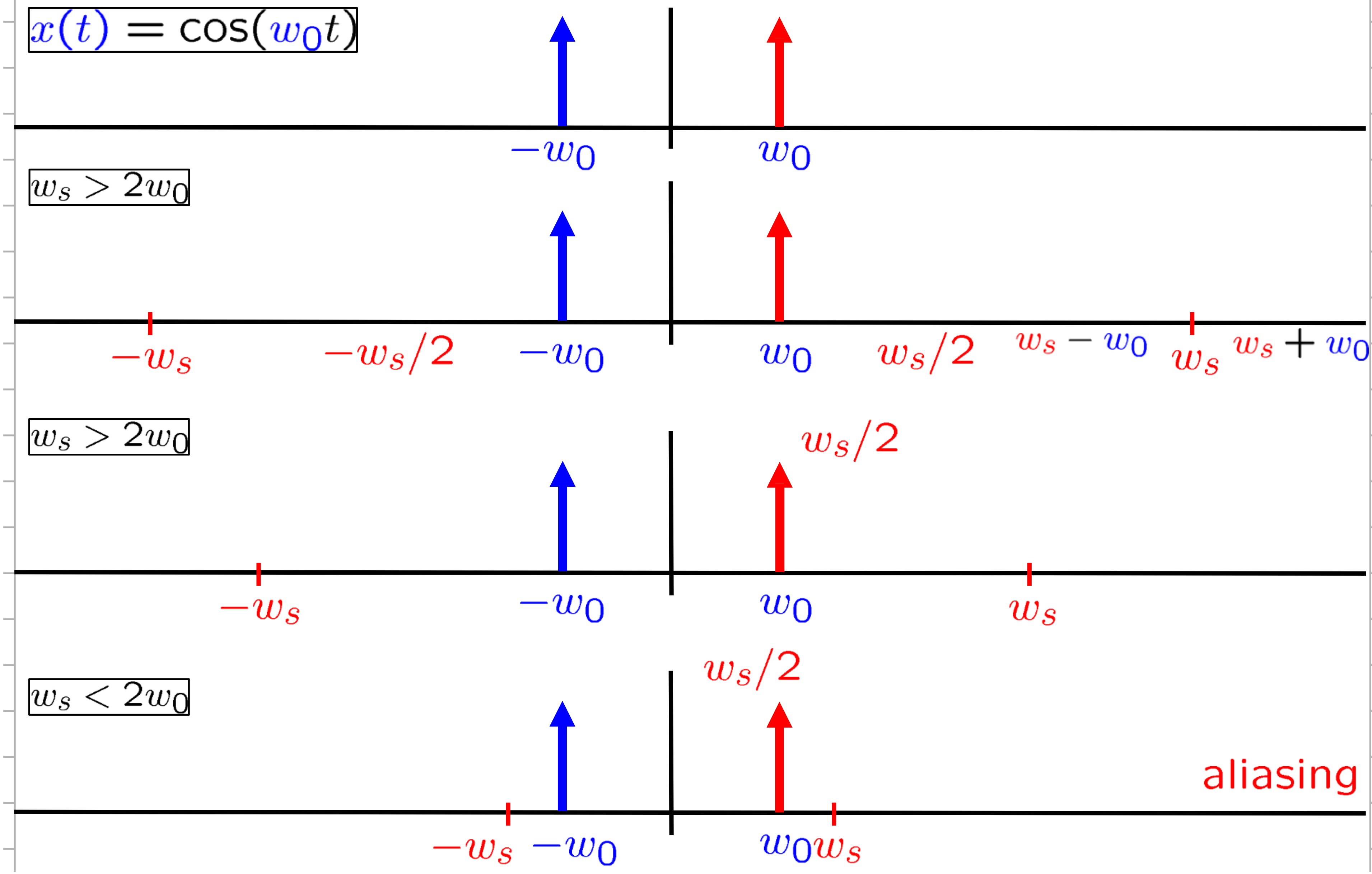

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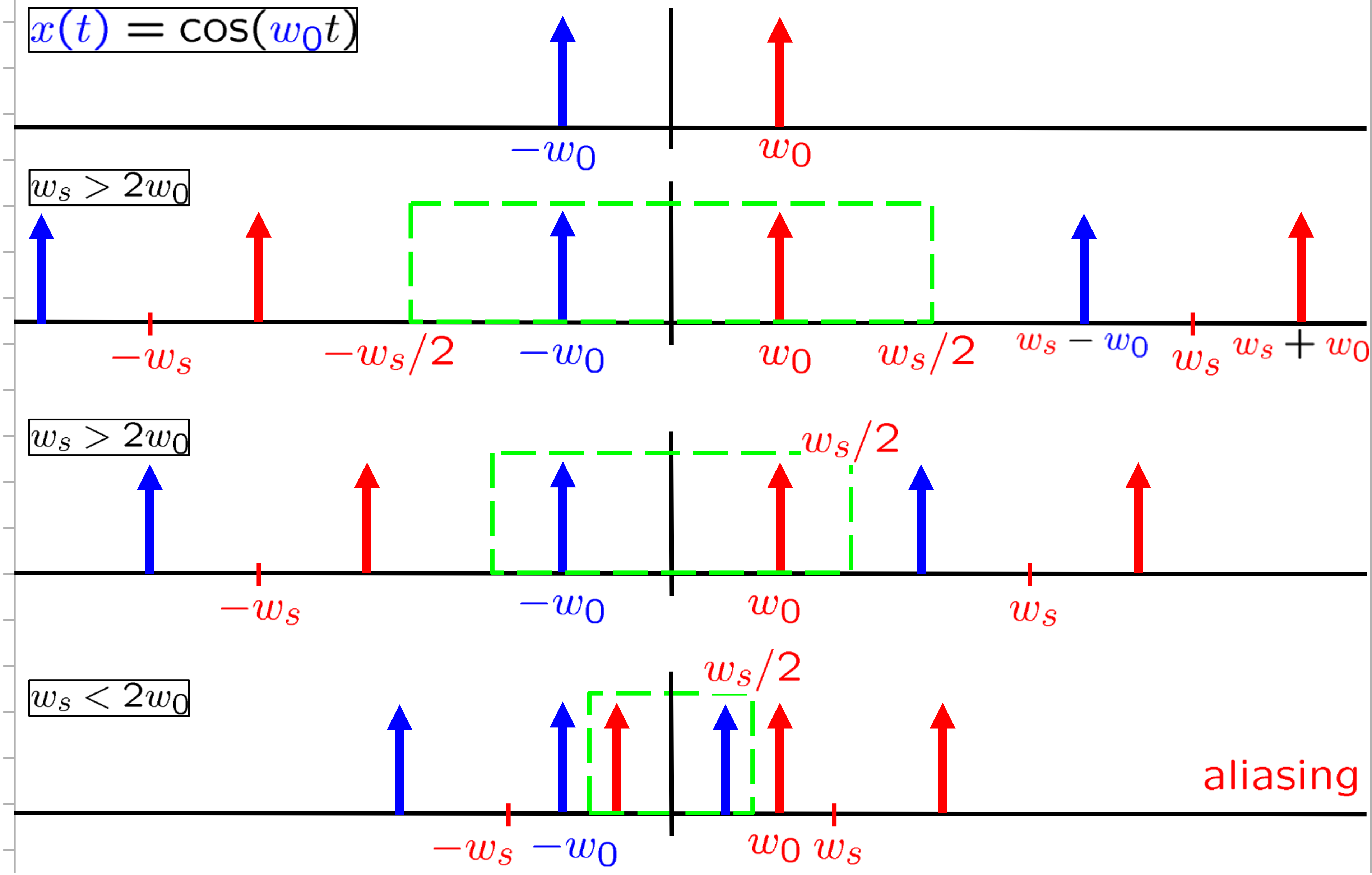

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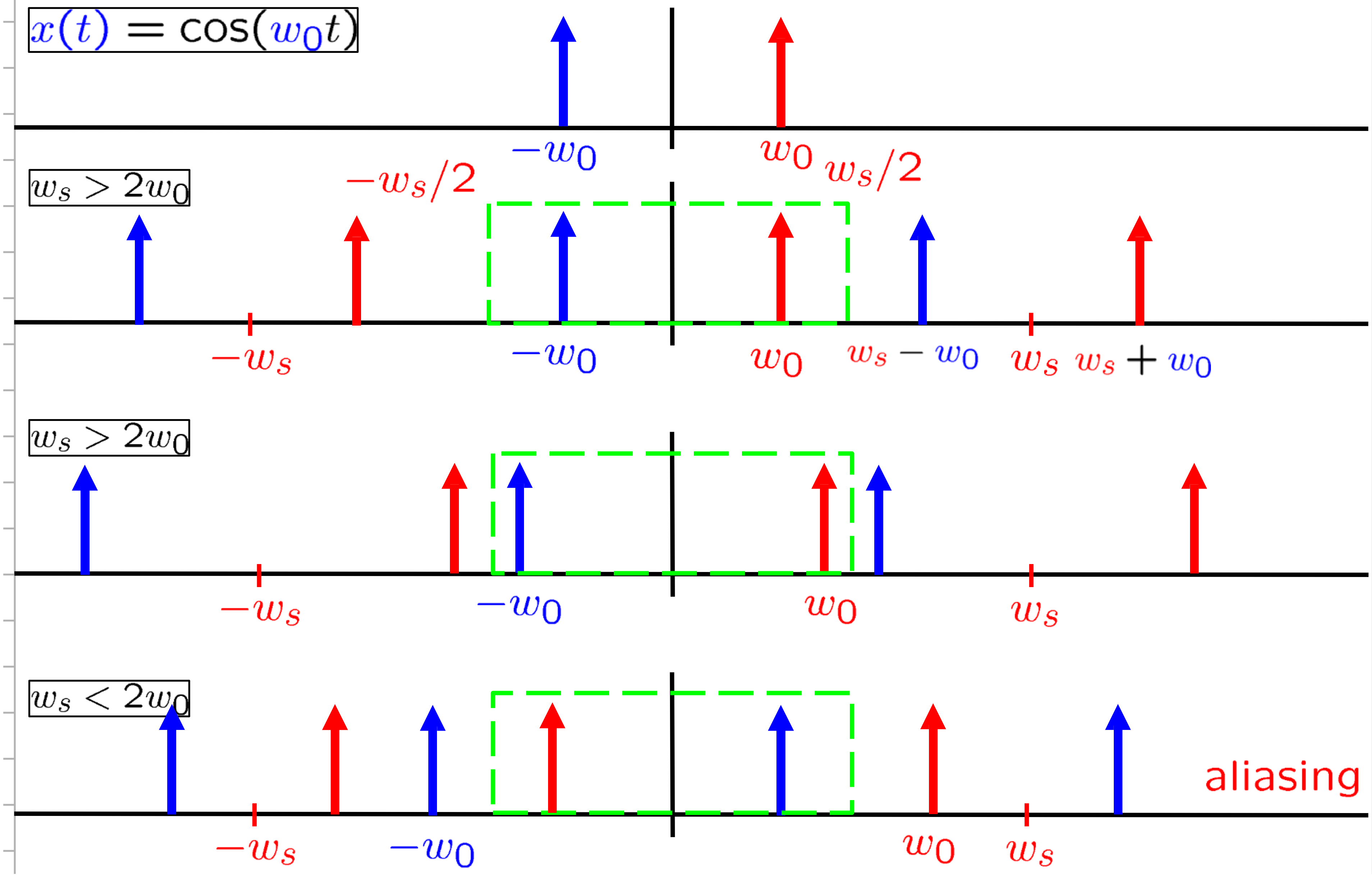
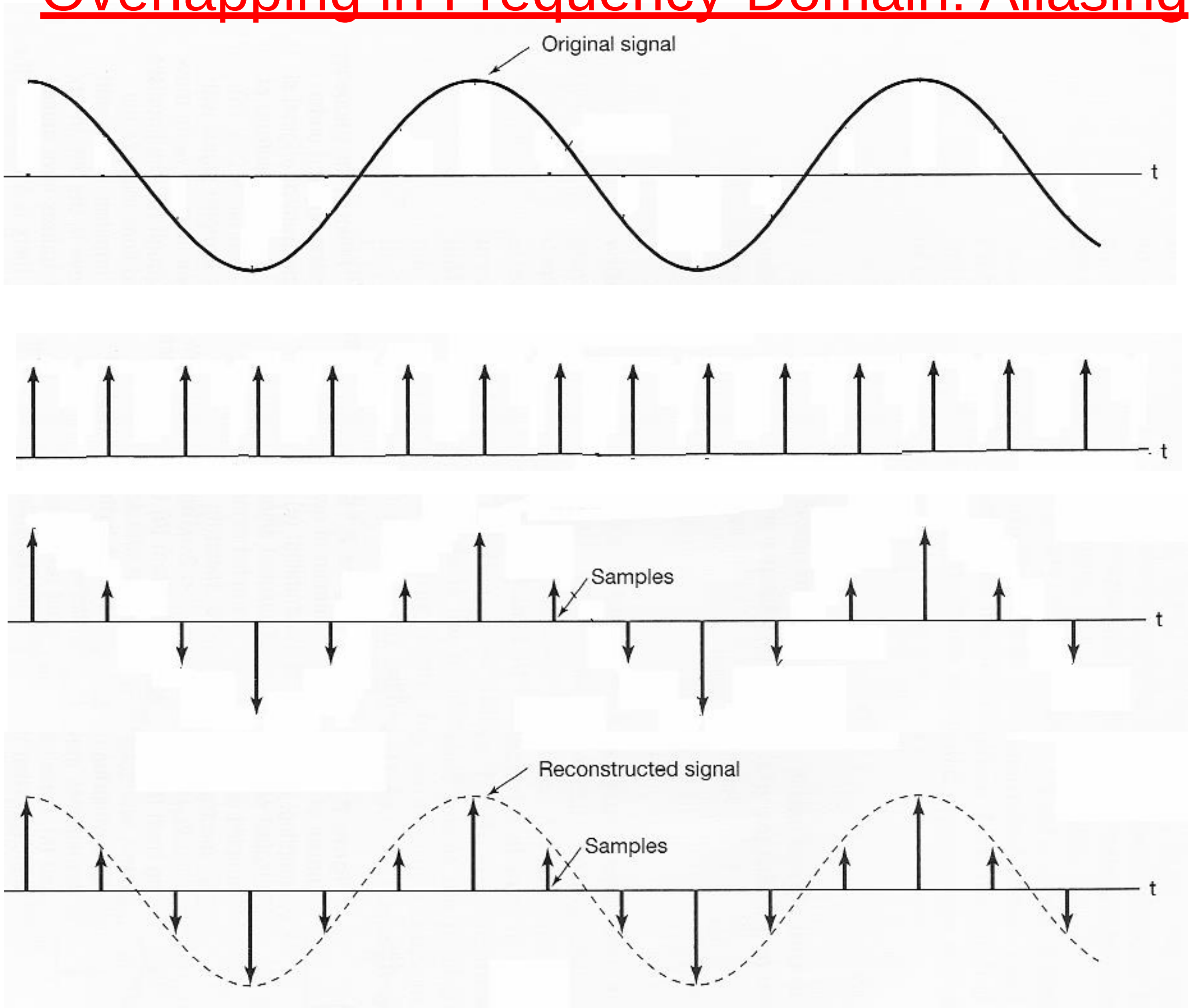
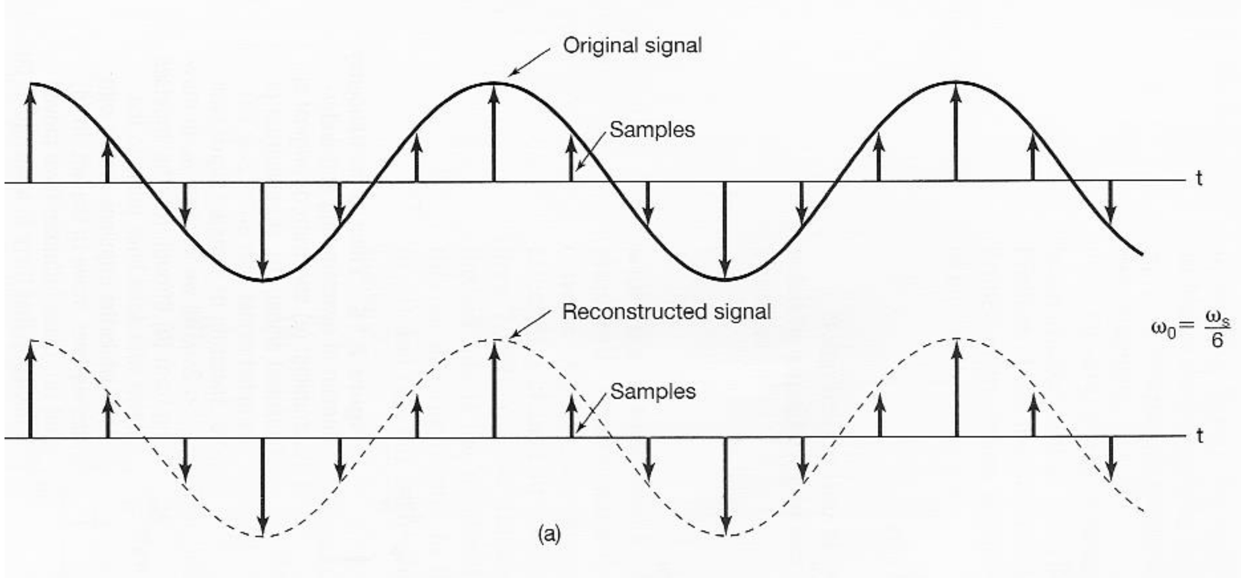


- <u>Overlapping in Frequency-Domain: Aliasing</u>

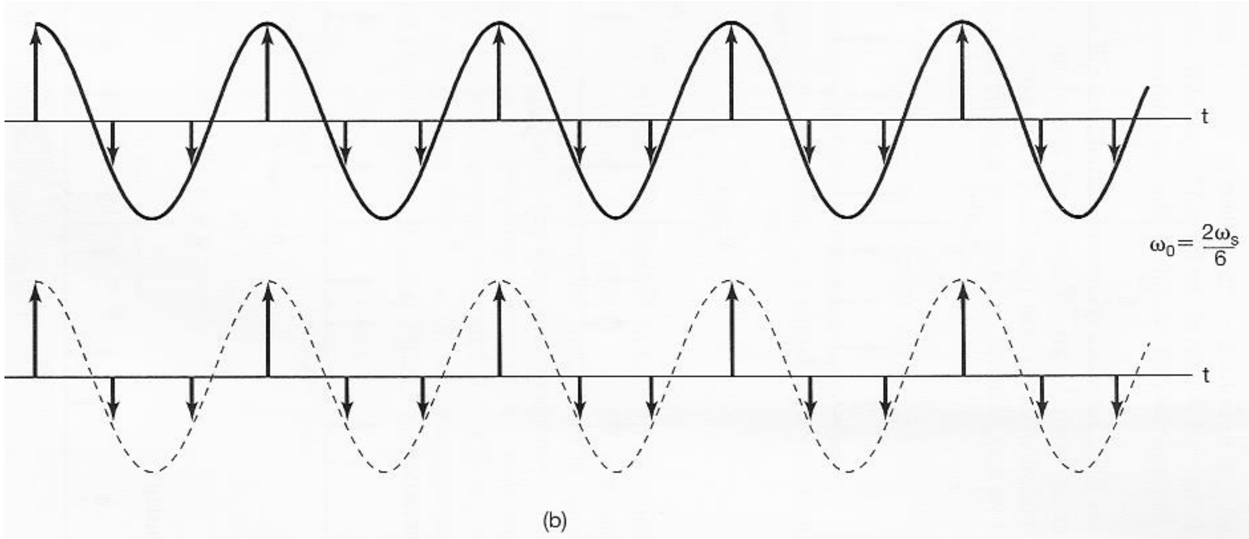


$$w_0 = \frac{w_s}{6}$$

- Overlapping in Frequency-Domain: Aliasing

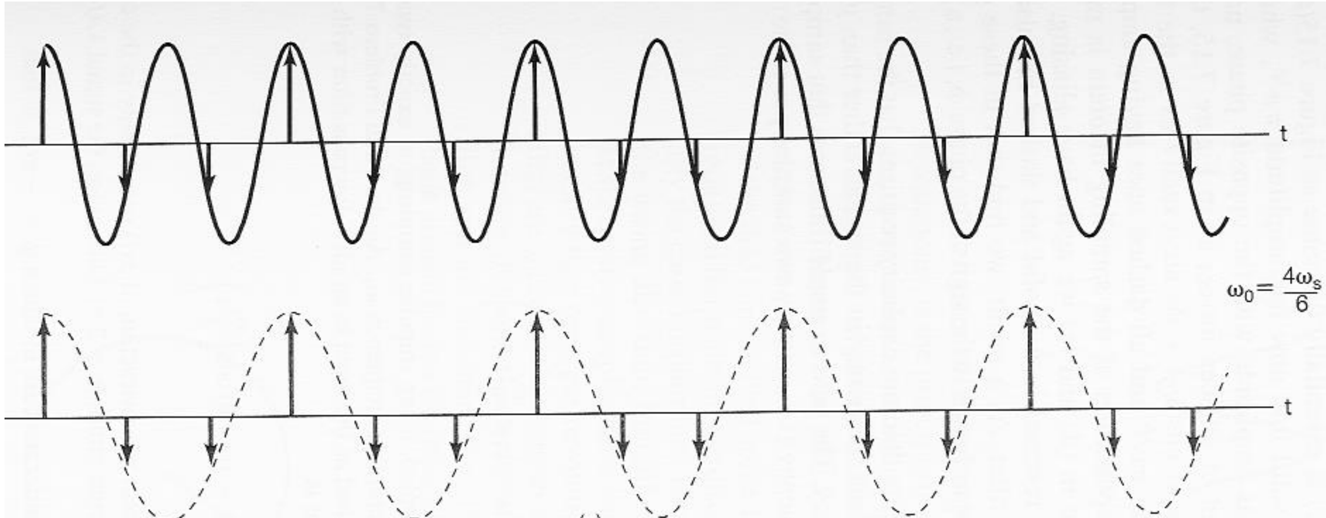


$$w_0 = \frac{w_s}{6}$$



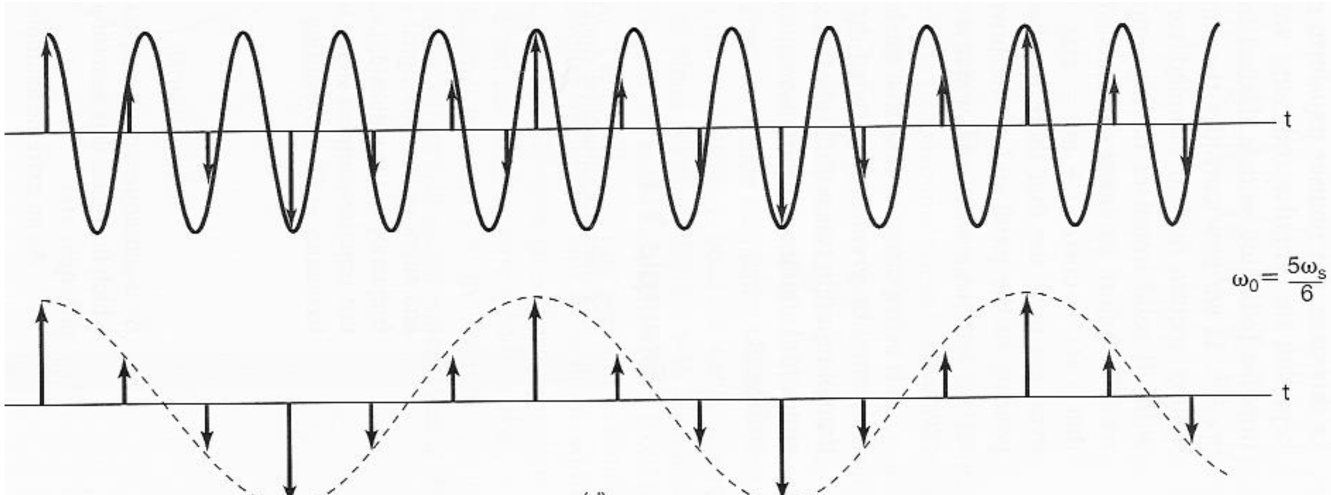
$$w_0 = \frac{2w_s}{6}$$

- Overlapping in Frequency-Domain: Aliasing



(c)

$$w_0 = \frac{4w_s}{6}$$



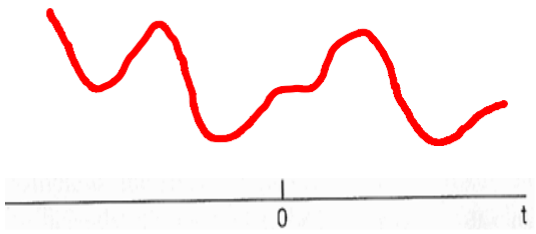
(d)

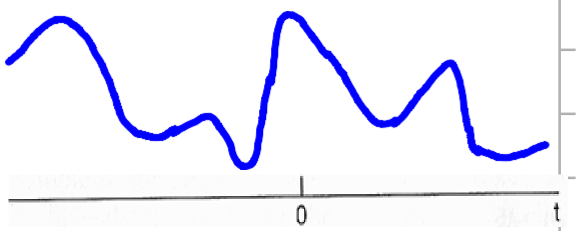
$$w_0 = \frac{5w_s}{6}$$

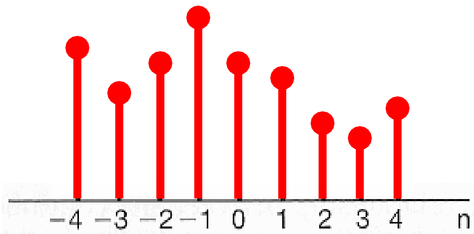
Outline

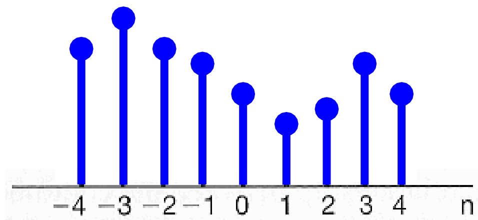
- Representation of of a Continuous-Time Signal by Its Samples: The Sampling Theorem
- Reconstruction of of a Signal from Its Samples Using Interpolation
- The Effect of Under-sampling: Aliasing
- Discrete-Time Processing of Continuous-Time Signals
- Sampling of Discrete-Time Signals

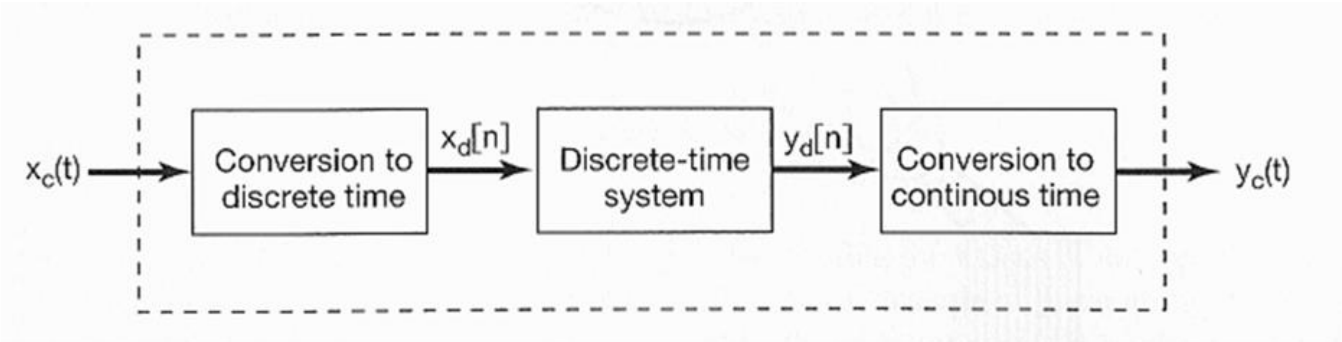
- Discrete-Time Processing of CT Signals:



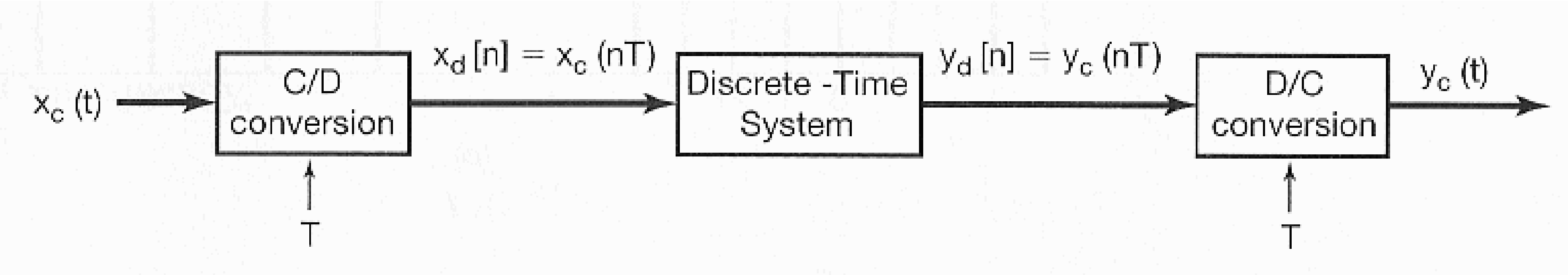


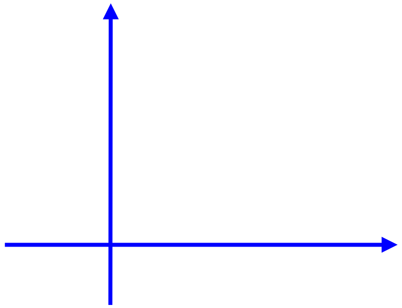






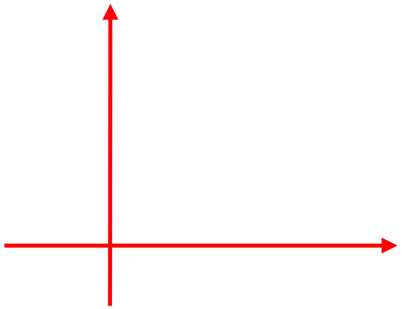
- C/D or A-to-D (ADC) and D/C or D-to-A (DAC):





C/D: continous-to-discrete-time conversion

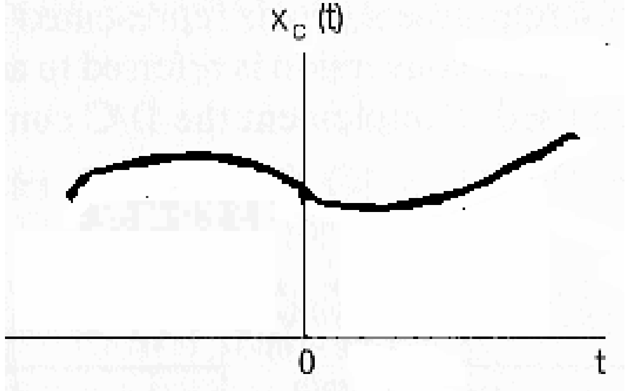
A-to-D: analog-to-digital converter

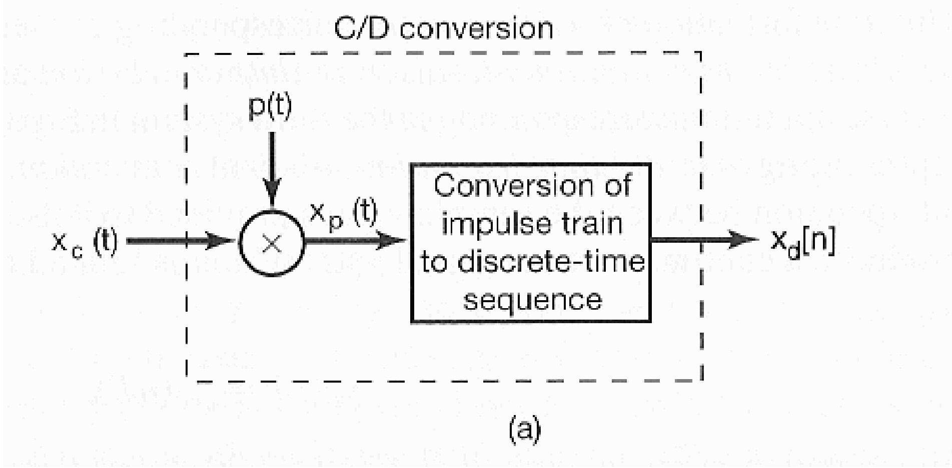


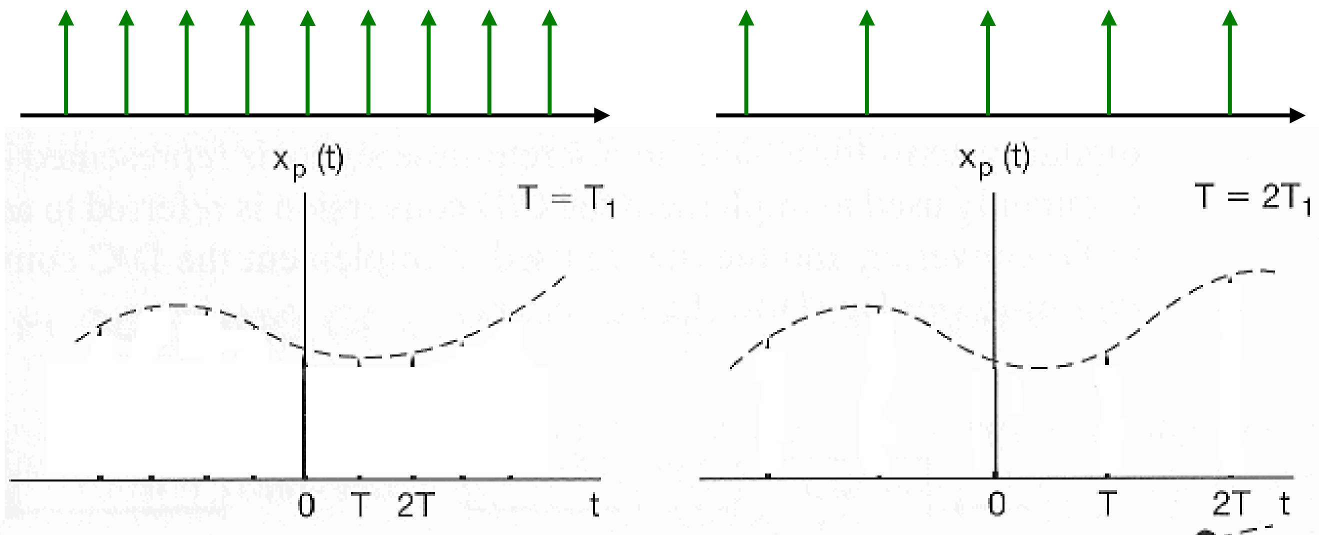
D/C: discrete-to-continous-time conversion

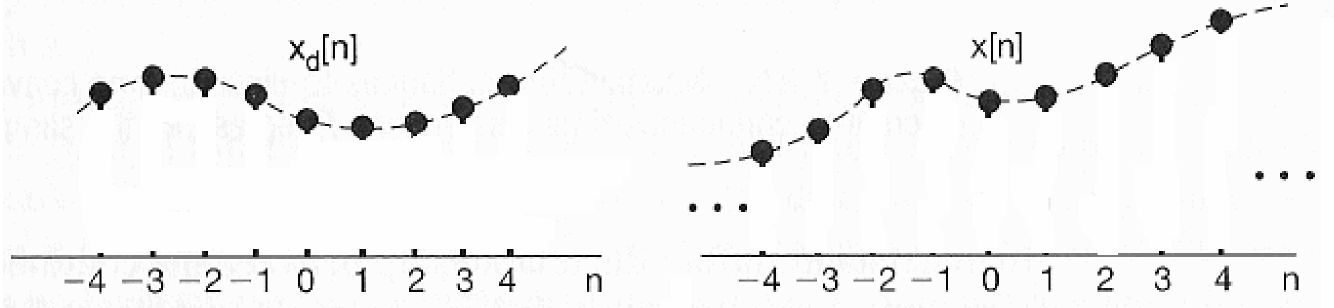
D-to-A: digital-to-analog converter

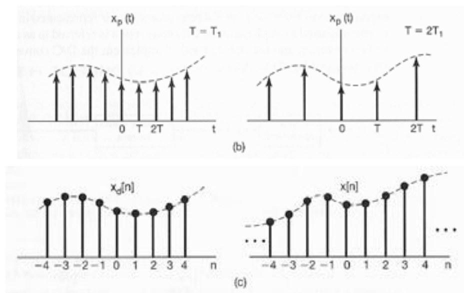
- C/D Conversion:



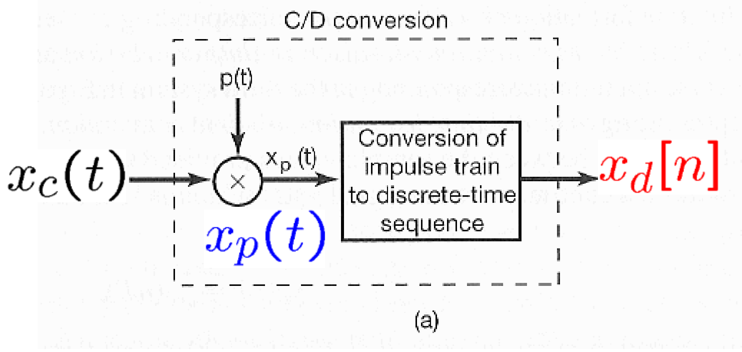


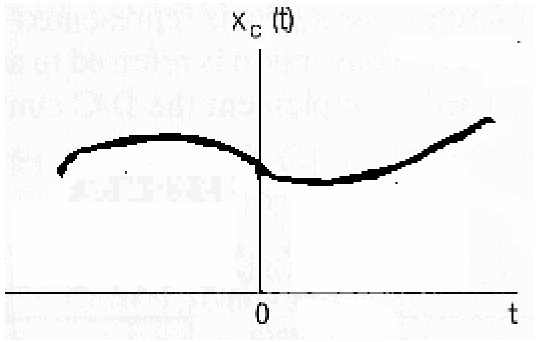




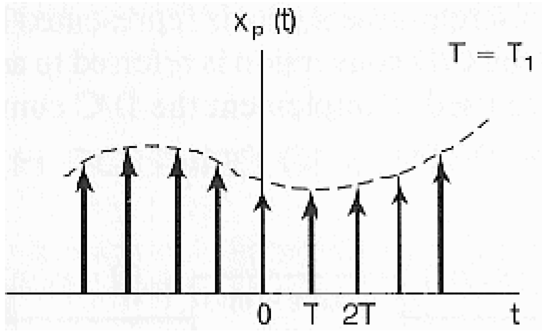


C/D Conversion:

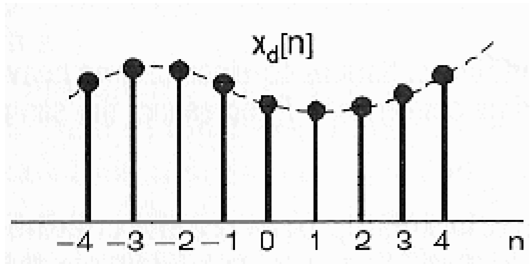




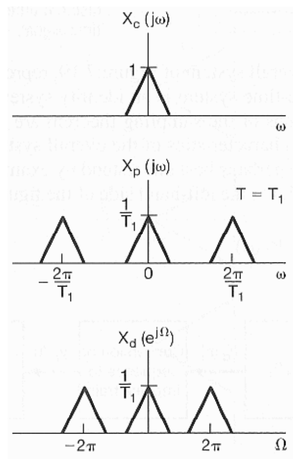
$$X_c(jw)$$



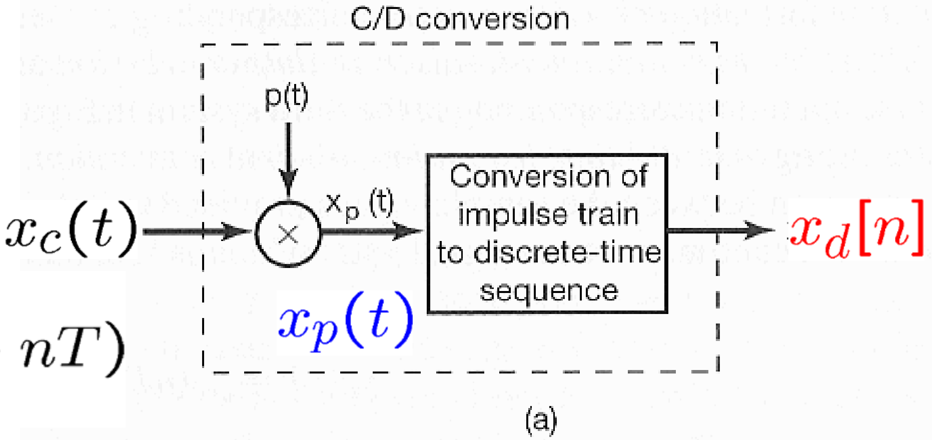
$$X_p(jw)$$



$$X_d(e^{j\Omega})$$



- C/D Conversion:



$$x_p(t) = \sum_{n=-\infty}^{+\infty} x_c(nT)\delta(t - nT)$$

Table 4.2, p. 329

$$\delta(t-t_0) \stackrel{\mathcal{F}}{\longleftrightarrow} e^{-jwt_0}$$

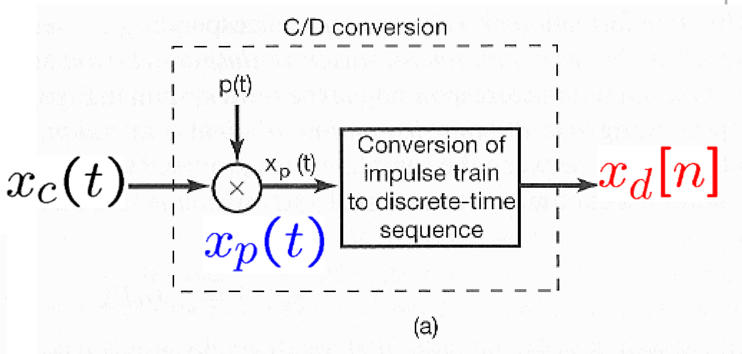
Eq 7.3, 7.6, p. 517

$$X_p(jw) = \sum_{n=-\infty}^{+\infty} x_c(nT)e^{-jwnT} = \frac{1}{T}\sum_{K=-\infty}^{+\infty} X_c(j(w - kw_s))$$

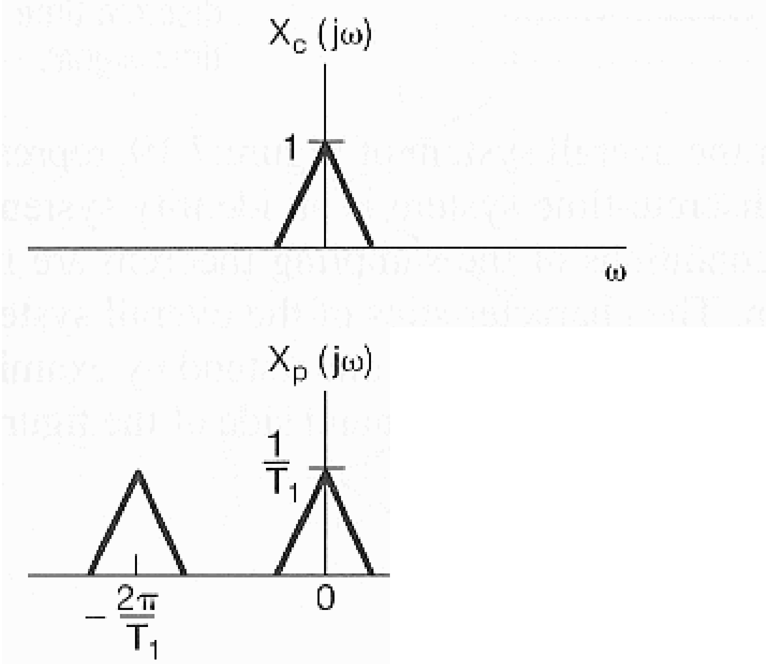
$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x_d[n]e^{-j\Omega n} = \sum_{n=-\infty}^{+\infty} x_c(nT)e^{-j\Omega n}$$

$$\Rightarrow X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right) = \frac{1}{T}\sum_{K=-\infty}^{+\infty} X_c\left(j\left(\frac{\Omega}{T} - k\frac{2\pi}{T}\right)\right)$$

C/D Conversion:



$X_c(jw)$



$X_p(jw)$

$X_d(e^{j\Omega})$