# ICPSR 2015 "Advanced Maximum Likelihood": Survival Analysis

## Day Six

August 10, 2015

# Stratification

- Allow different groups to have <u>different baseline hazards</u>

- Akin to different intercepts, but more flexible.

- *Assumes covariate effects are otherwise identical*

- Uses:
  - · Unit/group heterogeneity
  - · Nonproportional hazards
  - · Simple models for duration dependence

# Stratification, Simulated

```
> set.seed=7222009
> Z<-rnorm(200)
> X0<-rep(0,times=200)
> X1<-rep(1,times=200)
> T0<-rweibull(200,shape=1,scale=1/exp(2+0.5*Z))
> T1<-rweibull(200,shape=0.5,scale=1/exp(2+0.5*Z))
> C<-rep(1,times=400)
> X<-append(X0,X1)
> T<-append(T0,T1)
> data<-as.data.frame(cbind(T,C,X,rep(Z,times=2)))
> colnames(data)<-c("T","C","X","Z")
```

# Stratified Weibull Hazards
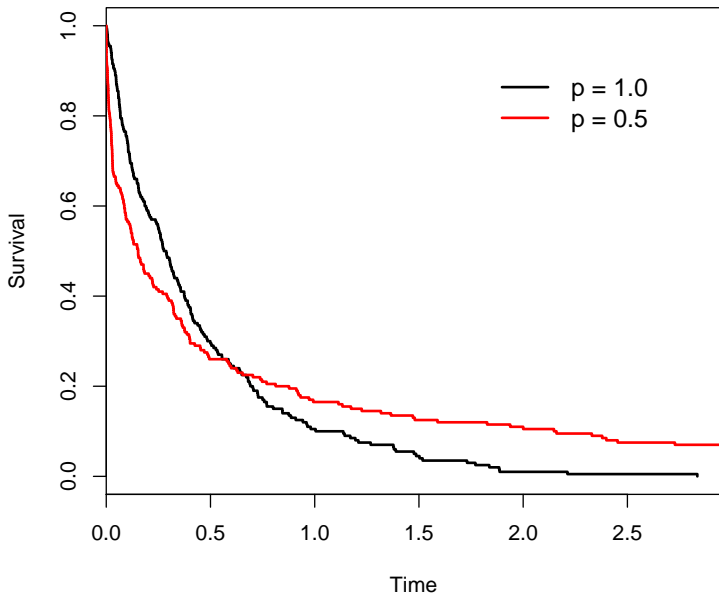
# Stratification, Simulated

```
> cox<-coxph(S~Z+X,data=data)
> summary(cox)
Call:
coxph(formula = S ~ Z + X, data = data)

  n= 400, number of events= 400

      coef exp(coef) se(coef)     z Pr(>|z|)
Z  0.28286   1.32692  0.05133  5.510 3.58e-08 ***
X -0.22866   0.79560  0.10639 -2.149   0.0316 *
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

  exp(coef) exp(-coef) lower .95 upper .95
Z    1.3269     0.7536    1.1999    1.4674
X    0.7956     1.2569    0.6459    0.9801

Concordance= 0.571  (se = 0.017 )
Rsquare= 0.08   (max possible= 1 )
Likelihood ratio test= 33.25  on 2 df,   p=6.022e-08
Wald test            = 33.02  on 2 df,   p=6.749e-08
Score (logrank) test = 33.07  on 2 df,   p=6.601e-08
```

# Stratification, Simulated

```
> cox.strata<-coxph(S~Z+strata(X),data=data)
> summary(cox.strata)
Call:
coxph(formula = S ~ Z + strata(X), data = data)

  n= 400, number of events= 400

     coef exp(coef) se(coef)    z Pr(>|z|)
Z 0.32140   1.37906  0.05176 6.21  5.3e-10 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

  exp(coef) exp(-coef) lower .95 upper .95
Z     1.379     0.7251     1.246     1.526

Concordance= 0.597  (se = 0.024 )
Rsquare= 0.092    (max possible= 1 )
Likelihood ratio test= 38.69  on 1 df,   p=4.955e-10
Wald test            = 38.56  on 1 df,   p=5.303e-10
Score (logrank) test = 38.62  on 1 df,   p=5.151e-10
```
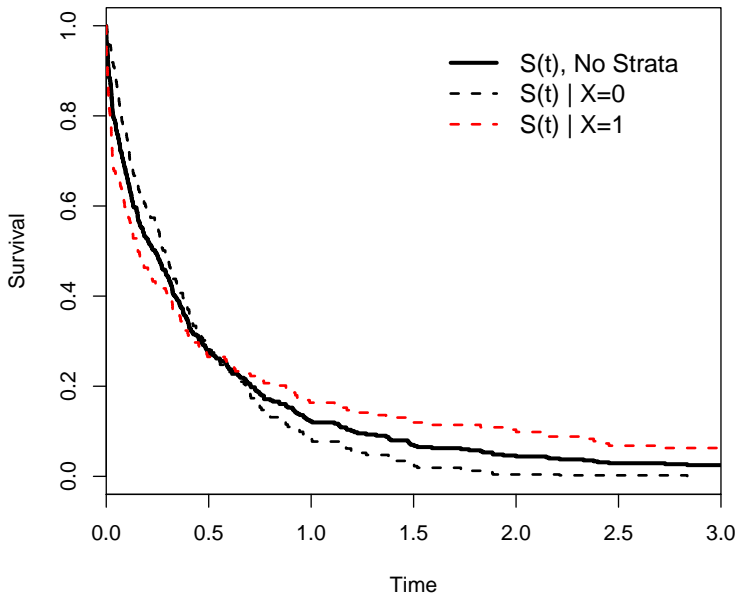
# Stratified Weibull Model

```
> summary(survreg(S~Z+strata(X),data=data,dist="weibull"))

Call:
survreg(formula = S ~ Z + strata(X), data = data, dist = "weibull")
             Value Std. Error       z         p
(Intercept) -0.9976     0.0675 -14.781 1.93e-49
Z           -0.4140     0.0577  -7.178 7.06e-13
X=0          0.0152     0.0555   0.274 7.84e-01
X=1          0.6864     0.0543  12.650 1.11e-36

Scale:
 X=0  X=1   # Recall: scale = 1 / p
1.02 1.99

Weibull distribution
Loglik(model)= -7.8   Loglik(intercept only)= -31.4
Chisq= 47.36 on 1 degrees of freedom, p= 5.9e-12
Number of Newton-Raphson Iterations: 6
n= 400
```

# Competing Risks

$R$ multiple kinds of events:

$$T_i \in T_{i1}, ..., T_{iR}$$

Observed duration:

$$T_i = \min(T_{i1}, ... T_{iR})$$

Event indicator:

$$D_i = r \text{ iff } T_i = T_{ri}$$

$R$ censoring indicators:

$$C_{ir} = \begin{cases} 1 \text{ if observation } i \text{ experienced event } r \\ 0 \text{ otherwise} \end{cases}$$

$$L_i = f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r)$$

$$
\begin{aligned}
L &= \prod_{i=1}^{N} \left\{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \prod_{r \neq D_i} S_r(T_i|\mathbf{X}_{ir}, \beta_r) \right\} \\
&= \prod_{r=1}^{R} \prod_{i=1}^{N_r} \left\{ f_r(T_i|\mathbf{X}_{ir}, \beta_r) S_r(T_i|\mathbf{X}_{ir}, \beta_r) \right\} \\
&= \prod_{r=1}^{R} \prod_{i=1}^{N} \left[ f_r(T_i|\mathbf{X}_{ir}, \beta_r) \right]^{C_{ir}} \left[ S_r(T_i|\mathbf{X}_{ri}, \beta_r) \right]^{1-C_{ir}}
\end{aligned}
$$

# Practical Estimation

- Independent risks $=$ separate models

- Otherwise identical estimation, interpretation, etc.

- No identification problem

- Discrete-Time $\rightarrow$ MNL

- See (e.g.) Diermeier and Stevenson 1999; Zorn and Van Winkle 2000; Goemans 2008

# Independent Risks

- Key: <u>Conditional</u> independence

- $\rightsquigarrow$ Model specification

- Dependent risks:
  - Using frailties (Gordon 2002)
  - Discrete-time: strategic (Fukumoto 2009)
  - Discrete-time: bivariate probit (Quiros Flores 2012)
  - SUR?

# Example: SCOTUS Vacancies

- Supreme Court Vacancies, 1789-1992 ($NT = 1783$)

- Departures $\in$ {Retirement, Mortality}

- Independent competing risks models: Cox + MNL

# SCOTUS Data

```
> summary(scotus)
    justice        svcstart        service        retire
 Min.   :  1   Min.   : 0    Min.   : 1    Min.   :0.00
 1st Qu.: 26   1st Qu.: 4    1st Qu.: 5    1st Qu.:0.00
 Median : 51   Median : 9    Median :10    Median :0.00
 Mean   : 53   Mean   :11    Mean   :12    Mean   :0.03
 3rd Qu.: 79   3rd Qu.:16    3rd Qu.:17    3rd Qu.:0.00
 Max.   :109   Max.   :36    Max.   :37    Max.   :1.00
    death          chief           south            age
 Min.   :0.00   Min.   :0.00   Min.   :0.00    Min.   :32
 1st Qu.:0.00   1st Qu.:0.00   1st Qu.:0.00    1st Qu.:56
 Median :0.00   Median :0.00   Median :0.00    Median :62
 Mean   :0.03   Mean   :0.12   Mean   :0.31    Mean   :62
 3rd Qu.:0.00   3rd Qu.:0.00   3rd Qu.:1.00    3rd Qu.:69
 Max.   :1.00   Max.   :1.00   Max.   :1.00    Max.   :91
    pension         pagree         threecat
 Min.   :0.0    Min.   :0.00   Min.   :0.00
 1st Qu.:0.0    1st Qu.:0.00   1st Qu.:0.00
 Median :0.0    Median :1.00   Median :0.00
 Mean   :0.2    Mean   :0.61   Mean   :0.08
 3rd Qu.:0.0    3rd Qu.:1.00   3rd Qu.:0.00
 Max.   :1.0    Max.   :1.00   Max.   :2.00
```

# SCOTUS: Death and Retirement
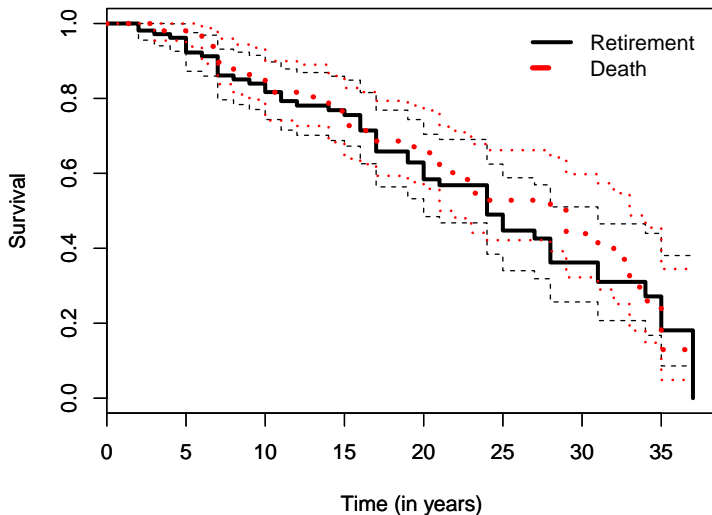
# Independent Risks (Cox) Models

|                     | Combined | Retirement | Death  |
|---------------------|----------|------------|--------|
| Age                 | 0.06     | 0.07       | 0.04   |
|                     | (0.02)   | (0.03)     | (0.02) |
| Chief               | −0.03    | −0.23      | 0.09   |
|                     | (0.30)   | (0.44)     | (0.40) |
| South               | 0.29     | 0.06       | 0.45   |
|                     | (0.23)   | (0.34)     | (0.33) |
| Pension Eligibility | 0.59     | 2.04       | −0.48  |
|                     | (0.28)   | (0.55)     | (0.41) |
| Party Agreement     | −0.01    | 0.10       | −0.10  |
|                     | (0.21)   | (0.29)     | (0.31) |
| AIC                 | 713.26   | 356.70     | 348.83 |
| Num. events         | 99       | 52         | 47     |

# Multinomial Logit

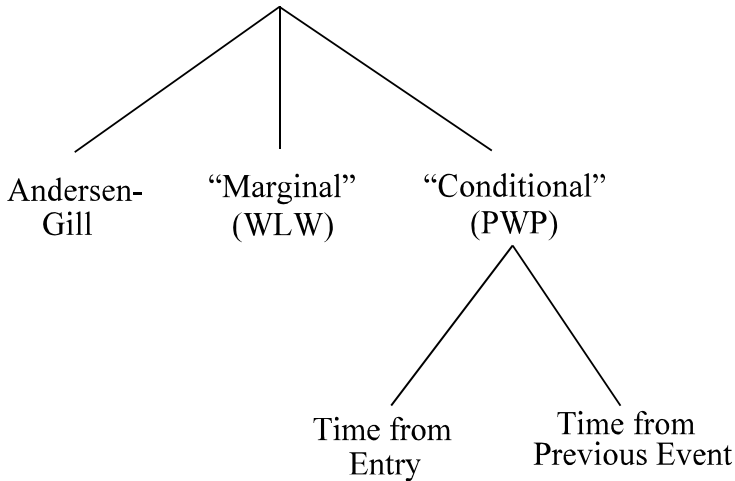|  | Retirement | Death |
|---|---|---|
| Intercept | −7.77 | −8.28 |
|  | (1.45) | (1.28) |
| Age | −0.29 | 0.00 |
|  | (0.45) | (0.42) |
| Chief | 0.06 | 0.48 |
|  | (0.34) | (0.32) |
| South | 0.07 | 0.06 |
|  | (0.03) | (0.02) |
| Pension Eligibility | 1.40 | −0.56 |
|  | (0.42) | (0.41) |
| Party Agreement | 0.03 | −0.26 |
|  | (0.30) | (0.31) |
| log(Time) | −0.30 | 0.51 |
|  | (0.27) | (0.29) |
| AIC | 847.51 | 847.51 |
| BIC | 924.31 | 924.31 |
| Log Likelihood | -409.75 | -409.75 |

# Multiple / Repeated Events

Events are not "absorbing" $\rightarrow$ capable of repetition

Raises (at least) two issues:

- Dependence across events

- Parameter variability

**Variance-Correction Models**



Andersen-Gill

"Marginal" (WLW)

"Conditional" (PWP)

Time from Entry

Time from Previous Event

# Variance Correction Model Properties

| Model Property | Andersen-Gill (AG) | Marginal (WLW) | Conditional (PWP), Elapsed Time | Conditional (PWP), Gap Time |
|---|---|---|---|---|
| Risk Set for Event $k$ at Time $t$ | Independent Events | All Subjects that Haven't Experienced Event $k$ at Time $t$ | All Subjects that Have Experienced Event $k$ - 1, and Haven't Experienced Event $k$, at Time $t$ | |
| Time Scale | Duration Since Starting Observation | Duration Since Starting Observation | Duration Since Starting Observation | Duration Since Previous Event |
| Robust standard errors? | Yes | Yes | Yes | |
| Stratification by Event? | No | Yes | Yes | |

# Data Organization

```
> OR<-OR[order(OR$dyadid,OR$year),]
> OR$one<-rep(1,times=nrow(OR))
> OR<-ddply(OR,"dyadid",mutate,eventno=cumsum(dispute)+1,
          altstart=cumsum(one)-1,altstop=cumsum(one))
```

|     | dyadid | year | start | stop | altstart | altstop | dispute | eventno |
|-----|--------|------|-------|------|----------|---------|---------|---------|
| 461 | 2130   | 1951 | 0     | 1    | 0        | 1       | 0       | 1       |
| 462 | 2130   | 1952 | 1     | 2    | 1        | 2       | 1       | 1       |
| 463 | 2130   | 1953 | 0     | 1    | 2        | 3       | 0       | 2       |
| 464 | 2130   | 1954 | 1     | 2    | 3        | 4       | 1       | 2       |
| 465 | 2130   | 1956 | 0     | 1    | 4        | 5       | 0       | 3       |
| 466 | 2130   | 1957 | 1     | 2    | 5        | 6       | 0       | 3       |
| 467 | 2130   | 1958 | 2     | 3    | 6        | 7       | 0       | 3       |
| 468 | 2130   | 1959 | 3     | 4    | 7        | 8       | 0       | 3       |
| 469 | 2130   | 1960 | 4     | 5    | 8        | 9       | 0       | 3       |
| 470 | 2130   | 1961 | 5     | 6    | 9        | 10      | 0       | 3       |
| 471 | 2130   | 1962 | 6     | 7    | 10       | 11      | 0       | 3       |
| 472 | 2130   | 1963 | 7     | 8    | 11       | 12      | 1       | 3       |
| 473 | 2130   | 1964 | 0     | 1    | 12       | 13      | 0       | 4       |
| 474 | 2130   | 1965 | 1     | 2    | 13       | 14      | 0       | 4       |

.
.
.

# First Events

```
> OR1st<-OR[OR$eventno==1,]
> OR.1st<-Surv(OR1st$altstart,OR1st$altstop,OR1st$dispute)
> OR.Cox.1st<-coxph(OR.1st~allies+contig+capratio+growth+democracy+
                    trade+cluster(dyadid),data=OR1st,method="efron")

> OR.Cox.1st
Call:
coxph(formula = OR.1st ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR1st, method = "efron")


            coef exp(coef) se(coef) robust se      z            p
allies    -0.448    0.6389   0.1585    0.1640 -2.732 0.0063000000
contig     1.070    2.9167   0.1681    0.1767  6.059 0.0000000014
capratio  -0.196    0.8223   0.0603    0.0779 -2.510 0.0120000000
growth    -2.198    0.1110   1.7195    1.9005 -1.157 0.2500000000
democracy -0.424    0.6547   0.1298    0.1259 -3.365 0.0007600000
trade     -6.728    0.0012  12.3255   13.9025 -0.484 0.6300000000

Likelihood ratio test=121  on 6 df, p=0  n= 17158, number of events= 205
```

# Andersen-Gill

```
> OR.AGS<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.AG<-coxph(OR.AGS~allies+contig+capratio+growth+democracy+
                   trade+cluster(dyadid),data=OR,method="efron")

> OR.Cox.AG
Call:
coxph(formula = OR.AGS ~ allies + contig + capratio + growth +
    democracy + trade + cluster(dyadid), data = OR, method = "efron")


             coef exp(coef) se(coef) robust se      z       p
allies     -0.414 0.66090755   0.1107    0.1703 -2.431 1.5e-02
contig      1.213 3.36515975   0.1209    0.1782  6.811 9.7e-12
capratio   -0.214 0.80717357   0.0514    0.0817 -2.620 8.8e-03
growth     -3.227 0.03967003   1.2279    1.3169 -2.451 1.4e-02
democracy  -0.439 0.64437744   0.0998    0.1231 -3.571 3.6e-04
trade     -13.162 0.00000192  10.3266   13.8188 -0.953 3.4e-01

Likelihood ratio test=272  on 6 df, p=0  n= 20448, number of events= 405
```

# Prentice et al.: Elapsed Time

```
> OR.PWPES<-Surv(OR$altstart,OR$altstop,OR$dispute)
> OR.Cox.PWPE<-coxph(OR.PWPES~allies+contig+capratio+growth+democracy+
                trade+strata(eventno)+cluster(dyadid),data=OR,
                method="efron")

> OR.Cox.PWPE
Call:
coxph(formula = OR.PWPES ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


            coef exp(coef) se(coef) robust se      z       p
allies    -0.240    0.7865   0.1122    0.1283 -1.872 6.1e-02
contig     0.868    2.3811   0.1223    0.1329  6.526 6.8e-11
capratio  -0.162    0.8506   0.0472    0.0618 -2.618 8.8e-03
growth    -3.625    0.0266   1.2371    1.2032 -3.013 2.6e-03
democracy -0.273    0.7612   0.1036    0.1074 -2.541 1.1e-02
trade     -2.514    0.0810   9.2934    9.9432 -0.253 8.0e-01

Likelihood ratio test=133  on 6 df, p=0  n= 20448, number of events= 405
```

# Prentice et al.: Gap Time

```
> OR.PWPGS<-Surv(OR$start,OR$stop,OR$dispute)
> OR.Cox.PWPG<-coxph(OR.PWPGS~allies+contig+capratio+growth+democracy+
                     trade+strata(eventno)+cluster(dyadid),data=OR,
                     method="efron")

> OR.Cox.PWPG
Call:
coxph(formula = OR.PWPGS ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR,
    method = "efron")


            coef exp(coef) se(coef) robust se     z       p
allies    -0.329    0.7193   0.1119    0.1229 -2.68 7.3e-03
contig     0.885    2.4232   0.1222    0.1285  6.89 5.6e-12
capratio  -0.171    0.8431   0.0481    0.0636 -2.68 7.3e-03
growth    -3.459    0.0315   1.2189    1.2102 -2.86 4.3e-03
democracy -0.284    0.7530   0.1028    0.1016 -2.79 5.2e-03
trade     -4.287    0.0137   9.9352   10.4592 -0.41 6.8e-01

Likelihood ratio test=139  on 6 df, p=0  n= 20448, number of events= 405
```

```
> OR.expand<-OR[rep(1:nrow(OR),each=max(OR$eventno)),]
> OR.expand<-ddply(OR.expand,c("dyadid","year"),mutate,
                eventrisk=cumsum(one))
> OR.expand$dispute<-ifelse(OR.expand$eventno==OR.expand$eventrisk
                    & OR.expand$dispute==1,1,0)
> dim(OR.expand)
[1] 163584     17

> head(OR.expand,9)
  dyadid year start stop futime dispute allies contig trade growth
1   2020 1951     0    1     35       0      1      1 0.014 0.0085
2   2020 1951     0    1     35       0      1      1 0.014 0.0085
3   2020 1951     0    1     35       0      1      1 0.014 0.0085
4   2020 1951     0    1     35       0      1      1 0.014 0.0085
5   2020 1951     0    1     35       0      1      1 0.014 0.0085
6   2020 1951     0    1     35       0      1      1 0.014 0.0085
7   2020 1951     0    1     35       0      1      1 0.014 0.0085
8   2020 1951     0    1     35       0      1      1 0.014 0.0085
9   2020 1952     1    2     35       0      1      1 0.015 0.0259
  democracy capratio one eventno altstart altstop eventrisk
1         1     0.20   1       1        0       1         1
2         1     0.20   1       1        0       1         2
3         1     0.20   1       1        0       1         3
4         1     0.20   1       1        0       1         4
5         1     0.20   1       1        0       1         5
6         1     0.20   1       1        0       1         6
7         1     0.20   1       1        0       1         7
8         1     0.20   1       1        0       1         8
9         1     0.19   1       1        1       2         1
```

# WLW Model

```
> OR.expand.S<-Surv(OR.expand$altstart,OR.expand$altstop,
                    OR.expand$dispute)
> OR.Cox.WLW<-coxph(OR.expand.S~allies+contig+capratio+growth+
                    democracy+trade+strata(eventno)+
                    cluster(dyadid),data=OR.expand,
                    method="efron")

> OR.Cox.WLW
Call:
coxph(formula = OR.expand.S ~ allies + contig + capratio + growth +
    democracy + trade + strata(eventno) + cluster(dyadid), data = OR.expand,
    method = "efron")


            coef exp(coef) se(coef) robust se       z       p
allies    -0.230    0.7947   0.1122    0.1248  -1.841 6.6e-02
contig     0.852    2.3435   0.1223    0.1297   6.568 5.1e-11
capratio  -0.160    0.8524   0.0471    0.0609  -2.621 8.8e-03
growth    -3.508    0.0300   1.2370    1.1671  -3.005 2.7e-03
democracy -0.271    0.7625   0.1037    0.1055  -2.570 1.0e-02
trade     -2.656    0.0702   9.2807    9.6144  -0.276 7.8e-01

Likelihood ratio test=129  on 6 df, p=0  n= 163584, number of events= 405
```

# Models of Repeated Events

|                  | First   | AG      | PWP-E   | PWP-G   | WLW     |
|------------------|---------|---------|---------|---------|---------|
| Allies           | −0.45   | −0.41   | −0.24   | −0.33   | −0.23   |
|                  | (0.16)  | (0.17)  | (0.13)  | (0.12)  | (0.12)  |
| Contiguity       | 1.07    | 1.21    | 0.87    | 0.89    | 0.85    |
|                  | (0.18)  | (0.18)  | (0.13)  | (0.13)  | (0.13)  |
| Capability Ratio | −0.20   | −0.21   | −0.16   | −0.17   | −0.16   |
|                  | (0.08)  | (0.08)  | (0.06)  | (0.06)  | (0.06)  |
| Growth           | −2.20   | −3.23   | −3.63   | −3.46   | −3.51   |
|                  | (1.90)  | (1.32)  | (1.20)  | (1.21)  | (1.17)  |
| Democracy        | −0.42   | −0.44   | −0.27   | −0.28   | −0.27   |
|                  | (0.13)  | (0.12)  | (0.11)  | (0.10)  | (0.11)  |
| Trade            | −6.73   | −13.16  | −2.51   | −4.29   | −2.66   |
|                  | (13.90) | (13.82) | (9.94)  | (10.46) | (9.61)  |
| AIC              | 2538.02 | 5015.77 | 3892.77 | 4103.47 | 5597.54 |
| Num. events      | 205     | 405     | 405     | 405     | 405     |

# Parameter Change Across Events

- Values of $\beta$ differ from $k$ to $k + 1$

- Again: Institutionalization, learning, etc.

- Addressed using *strata by covariate interactions*

# Parameter Change Example

```
> OR$capXevent<-OR$capratio*OR$eventno
> OR.Cox.BVary<-coxph(OR.PWPGS~allies+contig+growth+democracy+
                      trade+capratio+capXevent+strata(eventno)+
                      cluster(dyadid),data=OR,
                      method="efron")

> OR.Cox.BVary
Call:
coxph(formula = OR.PWPGS ~ allies + contig + growth + democracy +
    trade + capratio + capXevent + strata(eventno) + cluster(dyadid),
    data = OR, method = "efron")


            coef exp(coef) se(coef) robust se      z       p
allies    -0.349    0.7053   0.1120    0.1177 -2.967 3.0e-03
contig     0.897    2.4517   0.1221    0.1254  7.150 8.7e-13
growth    -3.519    0.0296   1.2196    1.2129 -2.901 3.7e-03
democracy -0.305    0.7374   0.1037    0.0972 -3.135 1.7e-03
trade     -3.297    0.0370   9.7624   10.1869 -0.324 7.5e-01
capratio  -0.340    0.7117   0.0997    0.1054 -3.227 1.2e-03
capXevent  0.135    1.1443   0.0631    0.0581  2.321 2.0e-02

Likelihood ratio test=143  on 7 df, p=0  n= 20448, number of events= 405
```

# Conclusions / Recommendations

As a practical matter, estimating these models is simply a function of:

- Setting up the data correctly (so as to define the right risk sets),
- Stratifying when appropriate, and
- Calculating / using robust standard errors...