# Seismic Control of a 3-Story Frame Building using an MR Damper

### NUMERICAL MODEL FOR FRAME BUILDING

Consider an MDOF structure with 3 degrees of freedom, subjected to a base ground motion  $\ddot{x}_g$ . Assuming the structure performs within the elastic range, the equation of motion is expressed as:

$$M\ddot{x} + C\dot{x} + Kx = -M \mathbb{I} \ddot{x}_g + \Gamma f \tag{1}$$

where x,  $\dot{x}$ , and  $\ddot{x}$  are  $3 \times 1$  state vectors of relative displacements, velocities, and accelerations. M, C, and K are  $3 \times 3$  mass, damping and stiffness matrices respectively.  $\mathbb{I}$  is an  $3 \times 1$  vector of ones and  $\Gamma$  is an  $3 \times 1$  vector of input forces. f contains all potential external forces (e.g. added damper contribution).

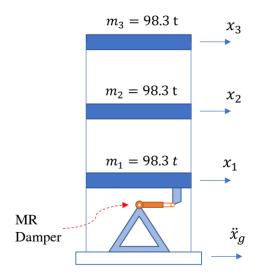


Fig. 1 Three-story building frame

The stiffness and damping properties for the frame shown in Fig. 1 are given below:

$$\mathbf{K} = 10^{7} \begin{bmatrix} 12.0 & -6.84 & 0 \\ -6.84 & 13.7 & -6.84 \\ 0 & -6.84 & 6.84 \end{bmatrix} \frac{N}{m}; \quad \mathbf{C} = 10^{4} \begin{bmatrix} 1.04 & -0.34 & -0.08 \\ -0.34 & 1.04 & -0.40 \\ -0.08 & -0.40 & 0.72 \end{bmatrix} \frac{Ns}{m}; \quad \mathbf{\Gamma} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$
(2)

A 1<sup>st</sup> story MR damper is envisioned in Fig. 1. The behavior and capacity of the MR damper are adjustable in real-time. This feature enables engineers to control the response of the structure through parametric

variations. In addition, force control of the MR damper can be through active, semi-active and passive control of damper parameters. In the next section, dynamic model for an MR damper is described.

## MR DAMPER MODEL

In this section, a *Bouc-Wen* model is used for characterization of an MR damper. Fig. 2 illustrates the dynamical features for modeling used herein. For a given input displacement x, a nonlinear output can be achieved for a restoring force F.

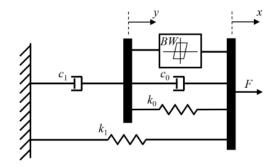


Fig. 2 Phenomenological model of MR Damper

The restoring force can be described by formulating the equation of motion for the setup in Fig. 2, and is described as:

$$F = \alpha z + c_0(\dot{x} - \dot{y}) + k_0(x - y) + k_1(x - x_0)$$
(3)

where z is an evolutionary variable used by the Bouc-Wen hysteretic element (Baber and Wn 1981). A more in-depth description of the parameters in (3) and (4) will be provided next.

$$\dot{z} = -\gamma z |\dot{x} - \dot{y}| |z|^{n-1} - \beta (\dot{x} - \dot{y}) |z|^n + A(\dot{x} - \dot{y})$$
(4)

The following table contains the modeling parameters described by Phillips and Spencer Jr. (2012) for the 200 kN MR damper at UIUC – Nathan Newmark Civil Engineering building, in Fig. 3. These parameters are next used to calculate model variables described in (5)-(10). In these equations,  $i_c$  is the current in Amps applied to the MR damper.

Table 1 Model parameters for 200 kN MR Damper

| Parameter                    | Value               | Parameter          | Value               | Parameter                          | Value                   |
|------------------------------|---------------------|--------------------|---------------------|------------------------------------|-------------------------|
| $\mathcal{C}_{0,\mathrm{a}}$ | 0.080 kN·sec/mm     | $c_{\mathrm{l,a}}$ | 3.0 kN·sec/mm       | $\gamma_{\rm a}$ , $\beta_{\rm a}$ | 0.050 mm <sup>-2</sup>  |
| $\mathcal{C}_{0,\mathrm{b}}$ | 0.32 kN·sec/mm      | $c_{\mathrm{l,b}}$ | 15.0 kN·sec/mm      | $\gamma_{\rm b}$ , $\beta_{\rm b}$ | 0.0020 mm <sup>-2</sup> |
| $\mathcal{C}_{0,\mathrm{c}}$ | 1.5 A <sup>-1</sup> | $c_{\mathrm{l,c}}$ | 2.0 A <sup>-1</sup> | $\gamma_{\rm c}$ , $\beta_{\rm c}$ | 5.2 A <sup>-1</sup>     |
| $k_0$                        | 0.0 kN/mm           | $\alpha_{\rm a}$   | 0.11 kN/mm          | A                                  | 300                     |
| $k_1$                        | 0.0 kN/mm           | $\alpha_{\rm b}$   | 0.55 kN/mm          | n                                  | 2.0                     |
| $x_0$                        | 0.0 mm              | $\alpha_{ m c}$    | 1.0 A <sup>-1</sup> |                                    |                         |



Fig. 3 200 kN MR Damper @ UIUC

$$\alpha = \alpha_b + (\alpha_a - \alpha_b) \times \exp(-\alpha_c i_c)$$
(5)

$$c_0 = c_{0,b} + (c_{0,a} - c_{0,b}) \times \exp(-c_{0,c}i_c)$$
(6)

$$c_1 = c_{1,b} + (c_{1,a} - c_{1,b}) \times \exp(-c_{1,c}i_c)$$
(7)

$$(8)$$

$$\beta = \beta_{b} + (\beta_{a} - \beta_{b}) \times \exp(-\beta_{c} i_{c})$$
(9)

$$\gamma = \gamma_{\rm b} + (\gamma_{\rm a} - \gamma_{\rm b}) \times \exp(-\gamma_{\rm c} i_{\rm c})$$
(10)

## SIMULINK MODEL

The equation of motion in (1) is next converted to state-space format and discretized for digital implementation. The state-space model of the 3-story frame building and the MR damper model are implemented in the SIMULINK model "MRsim", and the corresponding MATLAB script "MRscript".

$$\dot{\mathbf{z}} = A\mathbf{z} + B_1 f + B_2 \ddot{\mathbf{x}}_a \tag{11}$$

$$y = Cz + Df \tag{12}$$

where  $\mathbf{A}$  is a 6 × 6 system matrix,  $\mathbf{B_1}$  is the 6 × 1 force input vector,  $\mathbf{B_2}$  is a 6 × 1 ground acceleration input vector,  $\mathbf{C}$  is a 6 × 6 output matrix and  $\mathbf{D}$  is a 6 × 1 feedthrough vector. These state-space matrices are next described in terms of  $\mathbf{K}$ ,  $\mathbf{C}$ , and  $\mathbf{M}$ :

$$A = \begin{bmatrix} \mathbf{0} & I \\ \mathbf{M}^{-1} K & \mathbf{M}^{-1} C \end{bmatrix}_{6 \times 6}; B_{1} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \Gamma \end{bmatrix}_{6 \times 1}; B_{2} = \begin{bmatrix} \mathbf{0} \\ -\mathbb{I} \end{bmatrix}_{6 \times 1}; C = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{M}^{-1} K & \mathbf{M}^{-1} C \end{bmatrix}_{6 \times 1};$$

$$D = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \Gamma \end{bmatrix}_{6 \times 1}$$
(13)

### REAL-TIME HYBRID SIMULATION

The real-time hybrid simulation (RTHS) methodology is a substructuring technique for dynamic testing of structures. It offers a cost-effective solution and is highly practical in confined laboratory settings. In this section, the general architecture of an RTHS experiment is described where the three-story building is modeled as a numerical component and the MR damper is tested experimentally. The general architecture for RTHS testing is illustrated in Fig. 4. RTHS testing requires the use of a tracking controller to compensate for actuator dynamic, disturbance, and computational lag.

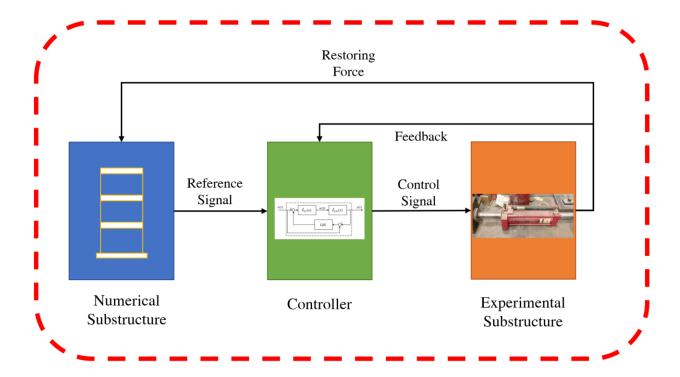


Fig. 4 RTHS architecture

### RTHS IMPLEMENTATION

An actuator transfer function model was selected to mimic the behavior of a hydraulic actuator to be attached to the MR damper. The following dynamical model was adopted from Phillips and Spencer Jr. (2012):

$$G(s) = \frac{1.600 \times 10^7}{(s + 151.7)(s^2 + 250.4s + 1.061 \times 10^5)}$$
(14)

The model in (14) along with dynamical models of the 3-story building and MR damper are implemented on the MATLAB using the RTHS framework. The 3-story building substructured numerical and the MR damper is considered to be experimental through incorporation of actuator dynamics. Two scenarios are considered: (i) 3-story building with MR damper, and (ii) 3-story building without MR damper. For case (i) two different MR damper current levels are studied. Fig. 5-7 illustrate the displacement, acceleration and hysteretic behaviors under a current level of 0*A*. Reductions are observed in both displacement and acceleration behaviors due to passive action of the MR damper.

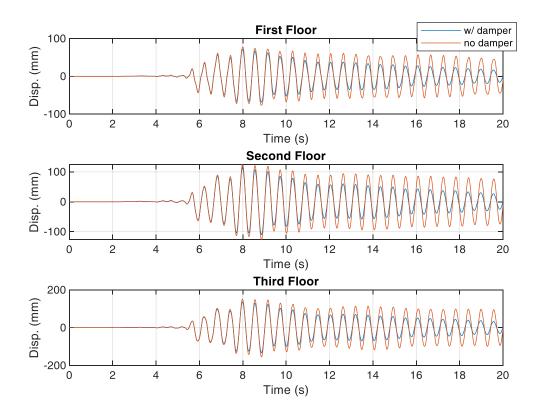


Fig. 5 Floor displacement results – 0A

A current level of 1A is next applied to the MR damper. Noticeable reductions are observed in the displacement and acceleration behaviors of the floor in the frame building per Fig. 8-9. A critical component

for numerical stability of the MR damper model is the sampling time  $f_s$  of the simulation. Due to the large nonlinearities present in this model, a sampling rate of  $f_s = 1/1000$  sec was used to compile simulations.

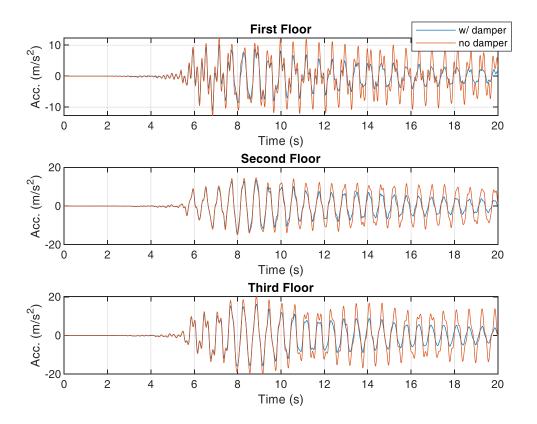


Fig. 6 Floor acceleration results – 0A

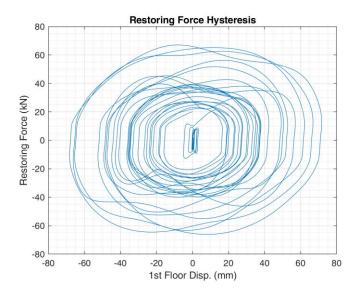


Fig. 7 Restoring force hysteresis of the MR damper – 0A

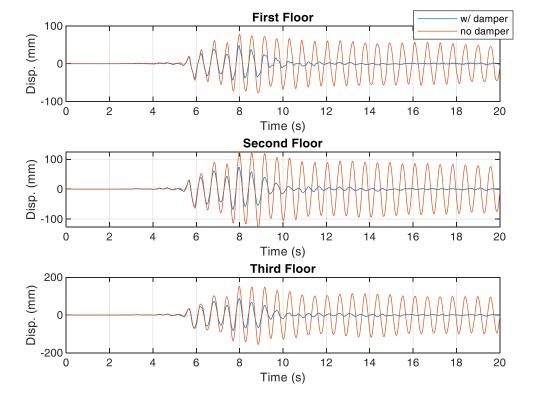


Fig. 8 Floor displacement results – 1A

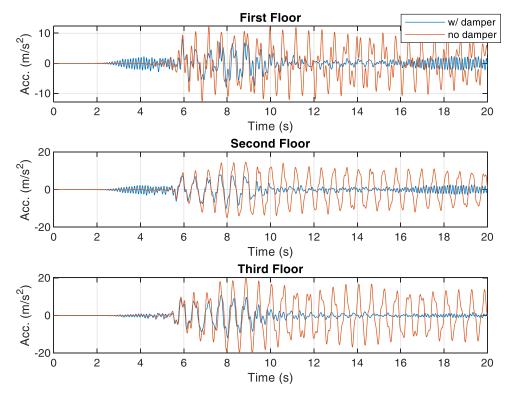


Fig. 9 Floor acceleration results – 1A

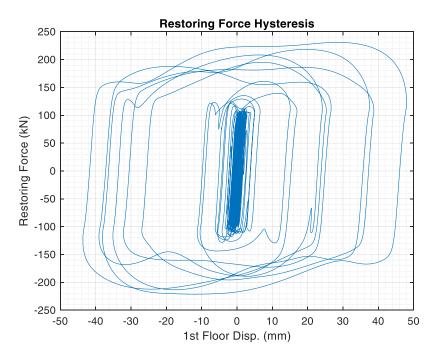


Fig. 10 Restoring force hysteresis of the MR damper – 1A

### **SUMMARY**

The 3-story model described in this document may be replaced by any other structure. The fundamental concepts to the RTHS framework is the input and output from the experimental substructure. A numerical substructure must provide the experimental, with an input displacement stroke. The MR damper in return will provide the numerical substructure with a restoring force. The restoring force is equal and opposite to the force demonstrated in Fig. 2. In this SIMULINK implementation, a controller (i.e. feedback controller) is not implemented as the aim was to demonstrate the effect of the actuator dynamics. A real experimental implementation necessitates the use of a controller.

#### References:

Baber, T. T., and Wn, Y.-K. (1981). Random Vibration of Hysteretic Degrading Systems.

Phillips, B. M., and Spencer Jr., B. F. (2012). "Model-Based Framework for Real-Time Dynamic Structural Performance Evaluation." NSEL Report Series, University of Illinois Urbana-Champaign.