Q1 $X \sim U(1,5)$ $\quad Y = 3^X$

random variable values = $\{1,2,3,4,5\}$

1) $E(X) = \dfrac{a+b}{2} = \dfrac{1+5}{2} = 3$

2) $E[Y] = E[3^X] = \sum_{j=1}^{k} 3^x \cdot Pr(X=x)$

$= \dfrac{1}{5} \cdot (3^1 + 3^2 + 3^3 + 3^4 + 3^5)$

$= \dfrac{1}{5} \cdot (3 + 9 + 27 + 81 + 243) = 72,6$

3) $Pr(X=x) = \dfrac{1}{b-a+1} = \dfrac{1}{5-1+1} = \dfrac{1}{5}$

Q2

$\lambda = 2$

1) $P_2(X = x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$ (PMF)

$P_2(X=0) = \dfrac{e^{-2} 2^{0}}{0!} = e^{-2} = 0,1353$

2) $P_2(X \geq 4) = 1 - P(X<4)$

$P_2(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$P(X=0) = 0,1353$

$P(X=1) = \dfrac{e^{-2} 2^{1}}{1!} = 2e^{-2} \approx 0,2707$

$P(X=2) = 0,2707$

$P(X=3) = 0,1804$

$P(X<4) = 0,1353 + 0,2707 + 0,2707$
$+ 0,1808 = 0,8571$

$P(X \geq 4) = 1 - P(X<4) =$
$= 1 - 0,8571 = 0,1429$

$14,29\%$

Q3 Prove $E[g(X)] \neq g(E[X])$

Let's take as an example 6-sided die roll as the random variable X and apply transformation $g(x) = x^2$

$P(X=x) = \frac{1}{6} \quad X \in \{1,2,3,4,5,6\}$

$$E[X] = \sum_{x=1}^{6} x \cdot P(X=x) = \sum_{x=1}^{6} x \cdot \frac{1}{6}$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3,5$$

$$g(E[X]) = (3,5)^2 = 12,25$$

$$E[g(X)] = E[X^2]$$

$$E[X^2] = \sum_{x=1}^{6} x^2 \cdot \frac{1}{6} \approx 15,17$$

$$12,25 \neq 15,17$$

Q4

1) Its a binomial distribution since here we have fixed number of independent trials (10) with same probability of success (0,25)

$X \sim Bin(10, 0.25)$

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

where $x \in \{1,2,3 \dots 10\}$

2) 55% prob of sucess is $P(X \geq 6)$

$P(X \geq 6) = P(X=6) + P(X=7) \dots P(X=10)$

$= 0,1972$

Q5 ① Loan = 10 000

$P(d/H) = 0{,}1$ $\quad$ $P(d/L) = 0{,}025$

$i = 0{,}16$ $\quad$ $i = 0{,}5$

a) high risk

$$E[X] = Loan \times (i+1) \times (1 - P(d/H))$$

$$= 10000 \cdot 1{,}16 \cdot 0{,}9$$

$$= 10\,360$$

b) Low risk

$$E[X] = 10000 \cdot 1{,}05 \cdot 0{,}975$$

$$= 10237{,}5$$

Solution: expected value of high risk customer is higher

Q5 2

$P(def|Low) = 0{,}025$ $\quad$ $P(def|High) = 0{,}1$

$P(Loan|low) = 0{,}8$ $\quad$ $P(Loan|high) = 0{,}2$

$$\text{Mean}/E[x] = \frac{1}{p}$$

$$p = P(def|Low) \times P(Loan|Low) + P(def|High) \times P(Loan|High)$$

$$= 0{,}025 \times 0{,}8 + 0{,}1 \times 0{,}2 = 0{,}04$$

$$\text{Mean} = \frac{1}{0{,}04} = 25$$

$\mu = 25$ loans until first default

$$Var(D) = \frac{1-p}{p^2} = \frac{1-0{,}04}{0{,}04^2} = 600$$

$$P(D \geq 4) = (1-p)^3 = (1-0{,}04)^3$$

$$= 0{,}88$$