Question 1

Let $X \sim U(1,5)$ be a discrete uniform random variable, and define $Y = 3^X$.

1. What is the expected value of X?
2. What is the expected value of Y?
3. What is the probability mass function of Y?

Question 2

Consider a business that conducts a certain number of transactions per weekday. The number of transactions per weekday is a Poisson random variable, with a mean of 2.

1. What is the probability that on a given day there will be no transactions?
2. What is the probability that at least four transactions will be performed on a given day?

Question 3

Give a simple example of a random variable X and function g to show that in general, $E[g(X)] \neq g(E[X])$.

Question 4

In a multiple-choice exam, there are 10 questions. Each has four possible answers (with only one correct). Alice didn't prepare for the exam, so she guessed all her answers. Let X denote the number of her correct answers.

1. What is the distribution of X? Write its PMF.
2. To pass the test, a student should get 55%. What is the probability that Alice passed the test?

Question 5

You are advising an institution planning to offer a \$10,000 loan. There are two applicants for the loan: a low-risk applicant and a high-risk applicant. The probability of a high-risk applicant defaulting on the loan is 0.1, while the probability of a low-risk applicant defaulting is 0.025. However, the company can offer a high-risk applicant an interest rate of 15%, while it can only offer a low-risk applicant an interest rate of 5%.

1. For each applicant, calculate the expectation and variance of the revenue. Is it necessarily better to choose the option with the higher expected revenue?
2. Please refer to this YouTube video to learn about the geometric distribution. You can also refer to the lecture notes and Wikipedia.

   Following the success of the first loan, the institution started offering further loans, one after the other. Each loan is offered to a low-risk applicant with probability 0.8 and to a high-risk applicant with probability 0.2, independently of the previous loans.

   Denote by D the number of loans until the first default. For example, $D = 10$ means nine loans did not default and the tenth one did. Calculate the expectation and variance of D. What is the probability that $D \geq 4$?