Question 1

A mobile app developer claims their new feature improves user retention rates. Historically, 60% of users return after one week. The developer proposes a test: sample 20 users and conclude the feature improves retention if at least 15 users return.

1. Formally define the problem, the parameter of interest, and the null and alternative hypotheses.
2. Compute the significance level of the test.
3. Compute the power of the test, assuming the true retention rate is 80%.

Question 2

An online store wants to decide if they should offer free shipping based on its impact on sales. Sales data from a random sample of 50 days with free shipping showed an average daily revenue of \$1,050 with a standard deviation of \$200. Historically, without free shipping, the average revenue is \$1,000 per day.

1. Devise a hypothesis test to decide whether free shipping significantly increases average daily revenue.
2. Compute the test statistic and the p-value.
3. Determine the required sample size to detect an increase of \$100 in average daily revenue with 90% power at a 5% significance level.

Question 3

An e-commerce company is testing two versions of a landing page (Layout A and Layout B) to increase customer engagement. They measure the click-through rate (CTR, proportion of visitors who click) for each layout in an A/B test:

- Layout A: $n_A = 300$, $x_A = 45$ clicks (CTR = 15%).
- Layout B: $n_B = 350$, $x_B = 70$ clicks (CTR = 20%).

The company wants to determine whether Layout B significantly improves CTR compared to the existing Layout A.

1. Compute the p-value for a two-tailed z-test to determine if the click-through rates are significantly different.
2. Construct a 95% confidence interval for the difference in click-through rates between the two layouts.
3. How would the above answers change if the company used a one-sided test? Would you consider using a one-sided test in this context?

Question 4

A study investigates whether customers' purchase decisions are independent of their browsing time. The data is summarized in the table below:

Browsing Time	Purchased	Did Not Purchase
< 5 minutes	30	70
5−15 minutes	50	50
> 15 minutes	40	60

(1) State the null and alternative hypotheses.

(2) Perform a chi-square test of independence. Compute the test statistic and p-value, and interpret the result at a 5% significance level.

(3) Suppose the expected proportions of purchases for each browsing category are known to be $(0.3, 0.5, 0.2)$. Perform a goodness-of-fit test to verify this claim.