

Task 1 – Accident prediction with Naïve Bayes

Instance: $X = (\text{Weather} = \text{Rain}, \text{Road} = \text{Good}, \text{Traffic} = \text{Normal}, \text{Engine} = \text{No})$

1. Priors

$$P(\text{Accident} = \text{Yes}) = 5/10 = 0.5$$

$$P(\text{Accident} = \text{No}) = 5/10 = 0.5$$

2. Likelihoods for Accident = Yes

$$P(\text{Rain} | \text{Yes}) = 1/5$$

$$P(\text{Good} | \text{Yes}) = 1/5$$

$$P(\text{Normal} | \text{Yes}) = 1/5$$

$$P(\text{Engine} = \text{No} | \text{Yes}) = 2/5$$

$$P(X | \text{Yes}) = (1/5) \times (1/5) \times (1/5) \times (2/5) = 2/625$$

$$P(\text{Yes} | X) \propto 0.5 \times 2/625 = 1/625$$

3. Likelihoods for Accident = No

$$P(\text{Rain} | \text{No}) = 2/5$$

$$P(\text{Good} | \text{No}) = 3/5$$

$$P(\text{Normal} | \text{No}) = 2/5$$

$$P(\text{Engine} = \text{No} | \text{No}) = 4/5$$

$$P(X | \text{No}) = (2/5) \times (3/5) \times (2/5) \times (4/5) = 48/625$$

$$P(\text{No} | X) \propto 0.5 \times 48/625 = 24/625$$

4. Normalised posterior probabilities

$$P(\text{Yes} | X) = (1/625) / (1/625 + 24/625) = 1/25 = 0.04$$

$$P(\text{No} | X) = (24/625) / (1/625 + 24/625) = 24/25 = 0.96$$

Decision: Since $0.96 > 0.04$, the Bayes classifier predicts

Accident = No for $X = (\text{Rain}, \text{Good}, \text{Normal}, \text{No})$.

Task 2 – Weather-Based Game Prediction (Naïve Bayes, no smoothing)

Dataset size: 14

$$\text{Play} = \text{yes} : 9 \rightarrow P(\text{Yes}) = 9/14$$

$$\text{Play} = \text{no} : 5 \rightarrow P(\text{No}) = 5/14$$

Question 1

$X = (\text{sunny}, \text{hot}, \text{high}, \text{false})$

Given Play = Yes (9 rows)

- $P(\text{outlook}=\text{sunny} | \text{Yes}) = 2/9$
- $P(\text{temp}=\text{hot} | \text{Yes}) = 2/9$
- $P(\text{humidity}=\text{high} | \text{Yes}) = 3/9$
- $P(\text{windy}=\text{false} | \text{Yes}) = 6/9$

$$P(X | \text{Yes}) = (2/9)(2/9)(3/9)(6/9) = 8/729$$

$$P(\text{Yes} | X) \propto P(\text{Yes})P(X|\text{Yes}) = (9/14)(8/729) = 12/1701$$

Given Play = No (5 rows)

- $P(\text{outlook}=\text{sunny} | \text{No}) = 3/5$
- $P(\text{temp}=\text{hot} | \text{No}) = 2/5$
- $P(\text{humidity}=\text{high} | \text{No}) = 4/5$
- $P(\text{windy}=\text{false} | \text{No}) = 2/5$

$$P(X | \text{No}) = (3/5)(2/5)(4/5)(2/5) = 48/625$$

$$P(\text{No} | X) \propto P(\text{No})P(X|\text{No}) = (5/14)(48/625) = 24/875$$

Normalising:

- $P(\text{Yes} | X) \approx 0.20$
- $P(\text{No} | X) \approx 0.80$

Bayes classifier decision for X: play = no.

Question 2

In Table 1, the row (sunny, hot, high, false) has play = no.
So yes, the Bayes classifier agrees with the table.

Question 3

$X' = (\text{overcast}, \text{cool}, \text{high}, \text{true})$

Given Play = Yes

- $P(\text{outlook}=\text{overcast} | \text{Yes}) = 4/9$
- $P(\text{temp}=\text{cool} | \text{Yes}) = 3/9$
- $P(\text{humidity}=\text{high} | \text{Yes}) = 3/9$
- $P(\text{windy}=\text{true} | \text{Yes}) = 3/9$

$$P(X' | \text{Yes}) = (4/9)(3/9)(3/9)(3/9) = 4/243$$

$$P(\text{Yes} | X') \propto (9/14)(4/243) = 6/567$$

Given Play = No

No “overcast” cases in the 5 “no” rows, so

$$P(\text{outlook}=\text{overcast} | \text{No}) = 0 \rightarrow P(X' | \text{No}) = 0 \rightarrow P(\text{No} | X') = 0$$

Therefore the Bayes classifier chooses:

play = yes for $X' = (\text{overcast}, \text{cool}, \text{high}, \text{true})$.

Task 3 – Loan Approval (Naïve Bayes)

Dataset

- 5 records
- LoanApproved = Yes → 3 times
- LoanApproved = No → 2 times

$$P(\text{Yes}) = 3/5$$

$$P(\text{No}) = 2/5$$

1. Applicant: Employed, Good, Medium

Given “Yes” rows:

(Employed, Good, High)

(Employed, Good, Medium)

(Unemployed, Good, Low)

- $P(\text{Employed} \mid \text{Yes}) = 2/3$
- $P(\text{Good} \mid \text{Yes}) = 3/3$
- $P(\text{Medium} \mid \text{Yes}) = 1/3$

$$P(X \mid \text{Yes}) = (2/3)(1)(1/3) = 2/9$$

$$P(\text{Yes} \mid X) \propto (3/5)(2/9)$$

Given “No” rows:

(Unemployed, Bad, Low)

(Employed, Bad, Medium)

- $P(\text{Good} \mid \text{No}) = 0 \rightarrow \text{probability becomes } 0$

Prediction:

LoanApproved = YES

2. Applicant: Unemployed, Bad, Low

Given “No” rows:

Unemployed, Bad, Low

Employed, Bad, Medium

- $P(\text{Unemployed} \mid \text{No}) = 1/2$
- $P(\text{Bad} \mid \text{No}) = 2/2$
- $P(\text{Low} \mid \text{No}) = 1/2$

$$P(X | \text{No}) = (1/2)(1)(1/2) = 1/4$$

$$P(\text{No} | X) \propto (2/5)(1/4)$$

Given “Yes” rows:

No “Bad credit” → probability = 0

Prediction:

LoanApproved = NO

3. Effect of Scoring System

If the bank assigns scores (e.g., Employed = 3, Unemployed = 1), then:

- Naïve Bayes will treat features like numbers instead of categories.
- Bigger scores → higher probability for approval.
- It changes how probabilities are calculated but the idea is the same.

Task 4 – Disease Diagnosis

Dataset counts:

Positive = 3

Negative = 2

$$P(\text{Pos}) = 3/5$$

$$P(\text{Neg}) = 2/5$$

1. $X = (\text{Fever}=\text{Yes}, \text{Cough}=\text{No}, \text{Fatigue}=\text{Yes}, \text{Travel}=\text{No})$

Positive rows: 3

- Fever Yes = 3/3
- Cough No = 2/3
- Fatigue Yes = 3/3
- Travel No = 2/3

$$P(X | \text{Pos}) = (1)(2/3)(1)(2/3)$$

Negative has Fever=Yes = 0 → $P(X | \text{Neg}) = 0$

Prediction: Positive

2. $X = (\text{Fever}=\text{No}, \text{Cough}=\text{Yes}, \text{Fatigue}=\text{No}, \text{Travel}=\text{No})$

Negative rows: 2

- Fever No = 2/2

- Cough Yes = 2/2
- Fatigue No = 2/2
- Travel No = 1/2

$$P(X | \text{Neg}) = (1)(1)(1)(1/2)$$

Positive has Fever=No = 0 $\rightarrow P(X | \text{Pos}) = 0$

Prediction: Negative

3. Effect of travel weighting

If travel doubles the probability:

- TravelHistory contributes more to $P(X | \text{Class})$
- A patient with recent travel becomes more likely to be classified Positive

Conclusion: Weighted travel increases Positive prediction.