Aircraft Power, Drag, and Weight Modeling

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1. Nomenclature and Definitions

D	aircraft drag
L	aircraft lift
W	total aircraft weight
W_*	component weights
$V\,,\,V_\infty$	flight speed
$\dot{h}$	climb rate ($= V\sin\gamma$)
d	flight distance (range)
S	aircraft reference area (typ. wing area)
b	aircraft span
l	body length
c	chord length
AR	aspect ratio ($\equiv b^2/S$)
C_L	overall lift coefficient ($\equiv L/\frac{1}{2}\rho V^2 S$)
C_D	overall drag coefficient ($\equiv D/\frac{1}{2}\rho V^2 S$)
CDA_*	component's drag area ($\equiv D_*/\frac{1}{2}\rho V^2$)
T	propeller thrust
T_c	thrust coefficient ($\equiv T/\frac{1}{2}\rho V^2\,\pi R^2$)
P	power
E	energy expended
R	propeller radius
ρ	air density
γ	flight path angle
$\eta_{\rm motor}$	overall motor efficiency
$\eta_{\rm prop}$	prop efficiency ($\equiv TV/P_{\rm shaft}$)
$\eta_{\rm v}$	viscous efficiency (viscous loss)
$\eta_{\rm i}$	Froude efficiency (inviscid loss)
e	span efficiency
Re	Reynolds number
c_ℓ	airfoil profile lift coefficient
c_d	airfoil profile drag coefficient

2. Thrust Power

Generation of thrust in flight requires the expenditure of power. For a propeller or a jet-engine fan, the shaft power and the thrust are related by the definition of propeller efficiency.

$$P_{\rm shaft} \;=\; \frac{TV}{\eta_{\rm prop}} \qquad (1)$$

This $\eta_{\rm prop}$ can be treated as the product of a viscous efficiency $\eta_{\rm v}$ which accounts for the viscous profile drag on the blades, and an inviscid Froude efficiency $\eta_{\rm i}$ which accounts for the kinetic energy lost in the accelerated propwash.

$$\eta_{\rm prop} \;=\; \eta_{\rm v}\,\eta_{\rm i} \qquad (2)$$

For a propeller, $\eta_{\rm i}$ can be estimated by the modified actuator-disk relation

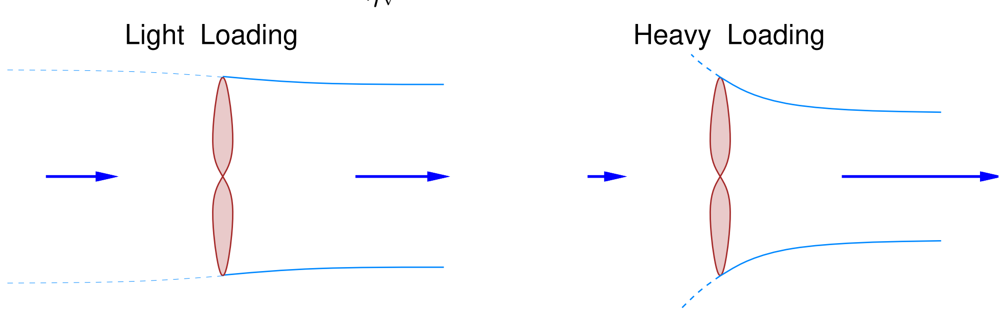
$$\eta_{\rm i} \;=\; \frac{2}{2+\left(\sqrt{1+T_c}-1\right)/\eta_{\rm add}} \qquad (3)$$

where the extra $\eta_{\rm add}$ factor accounts for additional losses from the swirl and nonuniformity in a real propwash. It's reasonable to assume $\eta_{\rm add} \simeq 0.7$ which is typical for most props.

Equation (3) has the following limiting cases.

$$T_c \gg 1\,,\;\eta_{\rm i} \ll 1 \;:\quad P_{\rm shaft} \;\simeq\; \frac{T^{3/2}}{(2\pi\,\rho)^{1/2}}\,\frac{1}{R}\,\frac{1}{\eta_{\rm v}\,\eta_{\rm add}} \qquad \text{(heavy loading: takeoff or hover)} \qquad (4)$$

$$T_c \ll 1\,,\;\eta_{\rm i} \simeq 1 \;:\quad P_{\rm shaft} \;\simeq\; T\,V\,\frac{1}{\eta_{\rm v}} \qquad \text{(light loading: high-speed cruise)} \qquad (5)$$



For a given thrust or power, both T_c and $\eta_{\rm i}$ are seen to strongly depend on the thrust and flight speed, and also on the propeller radius. In contrast, $\eta_{\rm v}$ does not vary much and is often considered a constant for a given application. It falls in the range $0.80 < \eta_{\rm v} < 0.95$, depending mainly on the blade chord Reynolds number.

3. Force and Power Balance Relations

In straight-line flight at some flight path angle γ, the force balance perpendicular to the flight path is $L = W\cos\gamma$. Combining this with the definition of the lift coefficient C_L gives

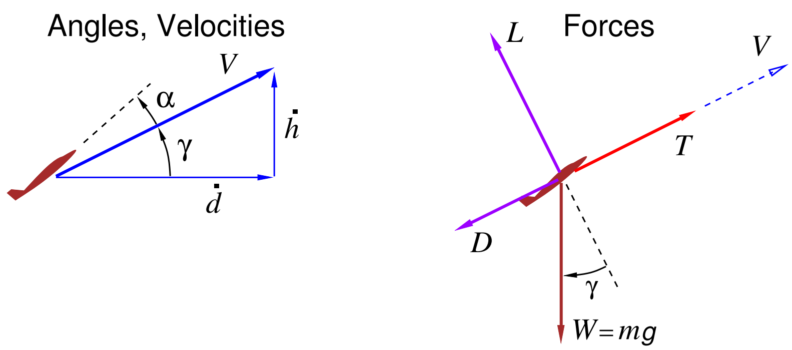
$$L \;=\; \frac{1}{2}\rho V^2\, S\, C_L \;=\; W\cos\gamma \tag{6}$$

which can be used to determine the required wing loading for some chosen flight speed and lift coefficient values (e.g. at the level-flight cruise design point at $\gamma = 0.$).

$$\boxed{\frac{W}{S} \;=\; \frac{1}{2}\rho V_{\rm design}^2\, (C_L)_{\rm design}} \tag{7}$$

The same relation also gives the flight speed for any other off-design W or C_L values.

$$\boxed{V \;=\; \left(\frac{2\, W\cos\gamma}{\rho\, S\, C_L}\right)^{1/2}} \tag{8}$$



The force balance along the flight path is $T = D + W\sin\gamma + m\dot{V}$, which gives the thrust and the thrust power in terms of aircraft parameters.

$$T \;=\; \frac{1}{2}\rho V^2 S\, C_D \;+\; W\sin\gamma \;+\; m\dot{V} \;=\; W\left(\cos\gamma\,\frac{C_D}{C_L} \;+\; \sin\gamma \;+\; \frac{\dot{V}}{g}\right) \tag{9}$$

$$TV \;=\; \frac{1}{2}\rho V^3 S\, C_D \;+\; WV\sin\gamma \;+\; mV\dot{V} \;=\; \left(\frac{2W^3\cos^3\!\gamma}{\rho\, S}\right)^{1/2}\frac{C_D}{C_L^{3/2}} \;+\; W\dot{h} \;+\; m\left(\frac{\dot{V^2}}{2}\right) \tag{10}$$

In the constant-speed flight case where $\dot{V} = 0$ we can also express the thrust coefficient and hence the prop Froude efficiency (3) in terms of the drag coefficient.

$$T_c \;\equiv\; \frac{T}{\frac{1}{2}\rho V^2\,\pi R^2} \;=\; \frac{D + L\tan\gamma}{\frac{1}{2}\rho V^2\,\pi R^2} \;=\; \frac{S}{\pi R^2}\,(C_D + C_L\tan\gamma) \tag{11}$$

$$\boxed{\eta_{\rm i} \;=\; \frac{2}{1 + \sqrt{1 + \dfrac{S}{\pi R^2}\,(C_D + C_L\tan\gamma)}}} \tag{12}$$

The required shaft power can then be written as

$$\boxed{P_{\text{shaft}} \;=\; \frac{1}{\eta_{\text{v}}}\,\frac{1}{\eta_{\text{i}}}\left(\frac{2W^3\cos^3\gamma}{\rho\, S}\right)^{1/2}\frac{C_D}{C_L^{3/2}}} \tag{13}$$

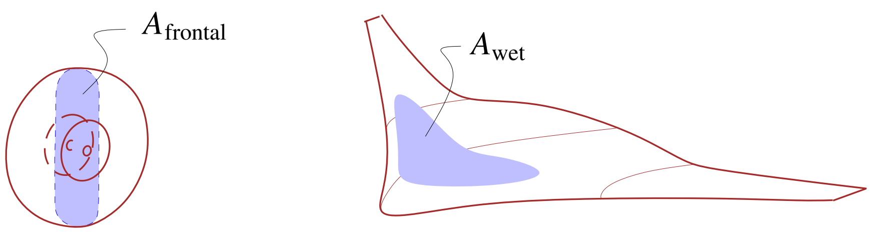
where $\eta_{\text{i}(C_D,C_L,\gamma,S,R)}$ is given by equation (12). For preliminary estimates it is often adequate to simply assume an overall propeller efficiency $\eta_{\text{prop}} = \eta_{\text{v}}\,\eta_{\text{i}}$, say $0.65 < \eta_{\text{prop}} < 0.85$.

4. Drag Breakdown and Estimation

To obtain the required shaft power via (10) or (13), we need the overall aircraft drag coefficient C_D, which can be broken down into three profile-drag parts plus the induced drag.

$$\boxed{C_D \;\equiv\; \frac{D}{\frac{1}{2}\rho V^2\, S} \;=\; \frac{\sum CDA}{S} \;+\; c_{d_{\text{w}}(C_L,Re_c)} \;+\; c_{d_{\text{t}}(Re)}\,\frac{S_{\text{t}}}{S} \;+\; \frac{C_L^2}{\pi\, AR\, e}} \tag{14}$$

The first term in equation (14) gives the profile drag of all the non-surface components, such as the fuselage, landing gear, external stores, etc., via the sum of their individual drag areas CDA. The drag area of each component can be estimated via to the component's drag or friction coefficient and corresponding reference area.



The drag area of a relatively bluff object like a wheel, antenna, etc., is typically estimated in terms of the object's frontal area and a frontal-area drag coefficient.

$$CDA \;=\; A_{\text{frontal}}\, C_{D_{\text{frontal}}} \qquad \text{(bluff component)} \tag{15}$$

The $C_{D_{\text{frontal}}}$ value can often be obtained from handbooks such as Hoerner's *Fluid Dynamic Drag*. The drag area of streamlined shapes such as odd-shaped surfaces, pods, and slender nacelles is usually estimated via the object's wetted area A_{wet}.

$$CDA \;=\; A_{\text{wet}}\bar{C}_f\, K_f \qquad \text{(streamlined component)} \tag{16}$$

The average skin friction coefficient $\bar{C}_{f(Re_\ell,Re_{x_{\text{tr}}})}$ is assumed to correspond to that on a flat plate with some mix of laminar and turbulent flow values $\bar{C}_{f_l}, \bar{C}_{f_t}$, and depends on the length-based Reynolds number Re_ℓ, and the transition-length Reynolds number $Re_{x_{\text{tr}}}$. Approximate expressions are given in many references, e.g. Schlichting's *Boundary Layer Theory*,

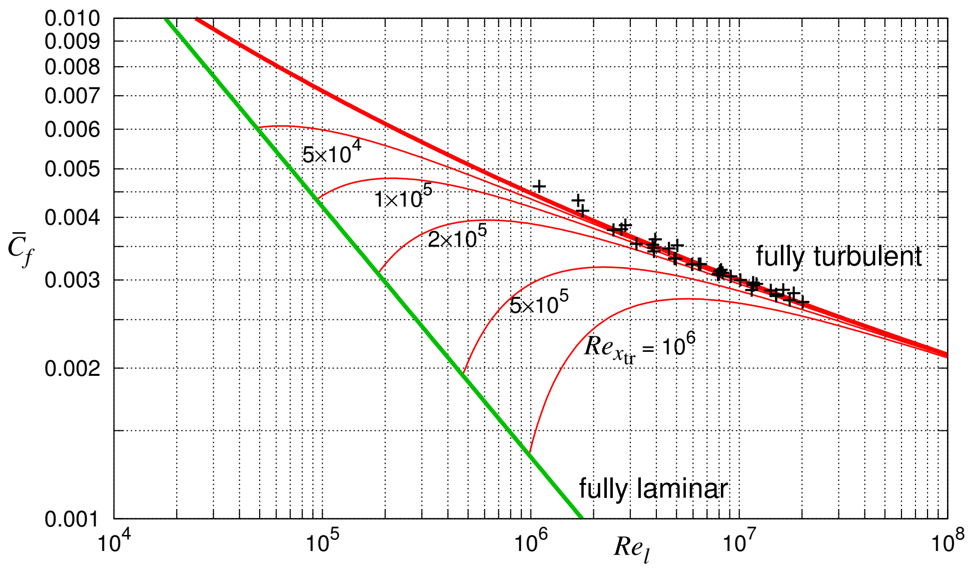
$$\bar{C}_{f_l} \;=\; \frac{1.328}{Re_\ell^{1/2}} \qquad \text{(fully laminar)} \tag{17}$$

$$\bar{C}_{f_t} \;=\; \frac{0.455}{(\,\log_{10} Re_\ell\,)^{2.58}} \qquad \text{(fully turbulent)} \tag{18}$$

$$\bar{C}_f \;=\; \max\!\left(\bar{C}_{f_l}\;,\;\bar{C}_{f_t} - \frac{Re_{x_{\text{tr}}}/320-39}{Re_\ell}\right) \qquad \text{(transition at } x_{\text{tr}}\text{)} \tag{19}$$

$$Re_\ell = \frac{\rho_\infty V_\infty \ell}{\mu_\infty} \quad , \qquad Re_{x_{tr}} = \frac{\rho_\infty V_\infty x_{tr}}{\mu_\infty} \tag{20}$$

These $\bar{C}_f$ functions are shown in the plot below, along with fully-turbulent data.

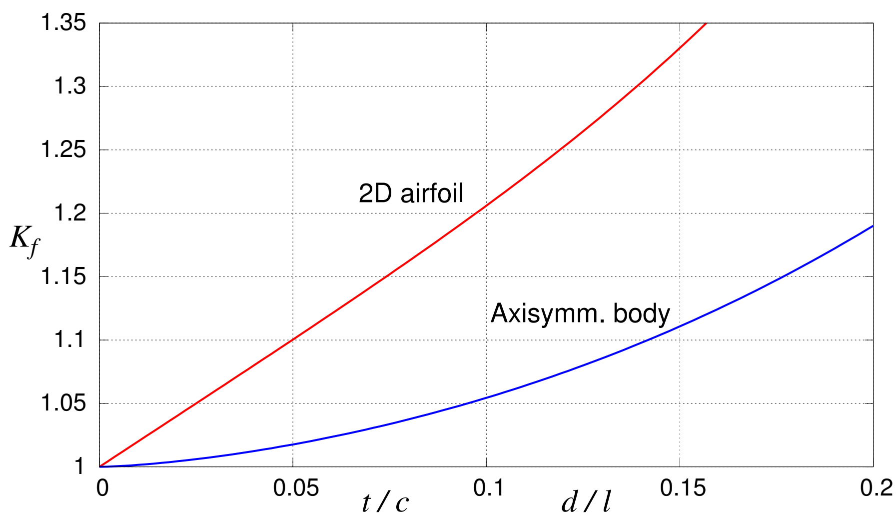


The form factor K_f accounts for the body shape, and falls in the range $1.0 < K_f < 1.4$ for most streamlined shapes. Hoerner gives the following empirical expressions in terms of airfoil thickness/chord ratio and body of revolution diameter/length ratio.

$$K_f = 1 + 2.0\, t/c + 60(t/c)^4 \qquad \text{(2D airfoil)} \tag{21}$$

$$K_f = 1 + 1.5\,(d/\ell)^{3/2} + 7(d/\ell)^3 \qquad \text{(Axisymm. body)} \tag{22}$$

These are shown in the figure below. It should be noted that the axisymmetric body corresponds to a "teardrop" shape. A body which has a center cylindrical portion will have a greater form factor than indicated by expression (22). A conservative estimate is to assume that the overall length ℓ excludes the straight cylindrical section in the d/ℓ ratio (but not in the overall Reynolds number Re_ℓ).



The second term in equation (14) is the wing profile drag, estimated from the wing's 2D airfoil profile drag coefficient function $c_{d(c_\ell, Re_c)}$, where these coefficients are based on the wing surface planform area as usual. This wing profile drag estimate term assumes that 1) the wing carries almost the entire aircraft lift, and 2) the wing planform area is also the aircraft reference area, $S_w = S$, so that the typical wing airfoil and overall aircraft lift coefficient are equal, $c_\ell \simeq C_L$. A more general form of this term would be

$$c_{d\,w}(c_\ell, Re_c)\,\frac{S_w}{S} \qquad , \qquad c_\ell \;=\; C_L \frac{S}{S_w}\frac{L_w}{L}$$

where L_w/L is the lift fraction carried by the wing surface. This more elaborate model is rarely needed in practice.

The third term in equation (14) is the tail profile drag, where c_{d_t} is the tail airfoil's drag coefficient and S_t is the total tail planform area. Separate horizontal and vertical tail terms could also be used. The tails typically operate at close to $c_\ell = 0$, so their profile drag will not significantly depend on the aircraft C_L as indicated. Numerical values for $c_{d}(c_\ell, Re_c)$ can be obtained from wind tunnel airfoil data or from calculation methods such as XFOIL.

The last term in equation (14) is the induced drag coefficient C_{D_i}, which depends on C_L, the aspect ratio AR, and the span efficiency e. The latter depends on the shape of the spanwise lift distribution. It is $e = 1$ for an elliptical loading, and $e < 1$ for other loadings. Non-planar wings and tip devices such as winglets can give $e > 1$.

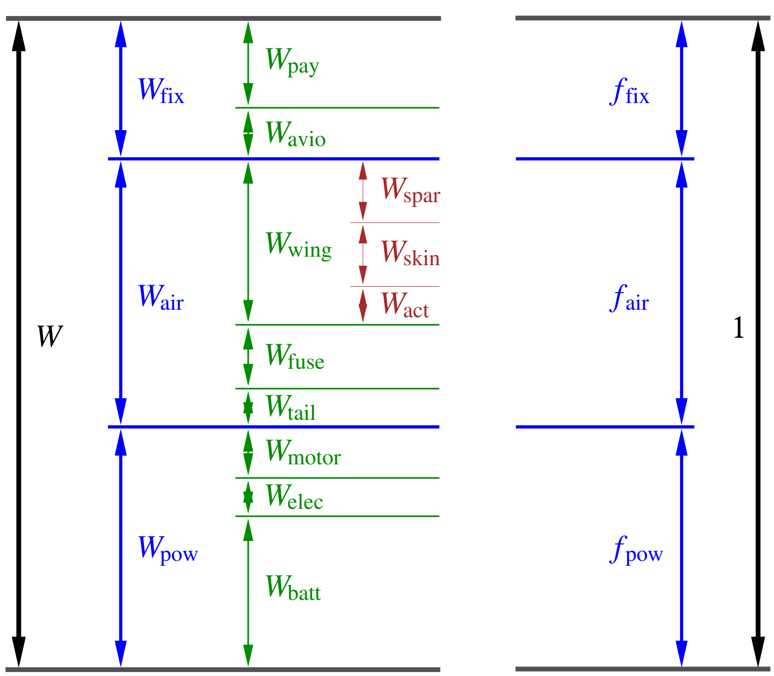
5. Weight Breakdown

A suitable high-level weight breakdown is

$$W \;=\; W_{fix} + W_{air} + W_{pow} \tag{23}$$

where W_{fix} is the fixed weight, W_{air} is the airframe weight, and W_{pow} is the power-system weight. Typically these would represent the following aircraft components.

W_{fix}	payload, avionics, power for payload and avionics
W_{air}	structure, landing gear, fittings, actuators, electrical or hydraulic power
W_{pow}	powerplant, propeller, fuel and fuel tanks, or batteries



The intent of this weight breakdown is to enable aircraft sizing. Specifically, W_{fix} is defined by the mission requirements. $W_{\rm air}$ can typically be related to wing area, span, etc. and also to the gross weight W itself, via suitable structural and weight models. $W_{\rm pow}$ would be related to the specified range or duration, as well as the gross weight W and the various engine and aerodynamic performance parameters as derived above.

It's also useful to define and examine corresponding weight-component fractions.

$$f_{\rm fix} \equiv \frac{W_{\rm fix}}{W} \quad , \qquad f_{\rm air} \equiv \frac{W_{\rm air}}{W} \quad , \qquad f_{\rm pow} \equiv \frac{W_{\rm pow}}{W} \tag{24}$$

In a well-balanced aircraft design, each of these fractions will fall into some reasonable range of values. A fraction which is much smaller or much larger than on other similar aircraft usually indicates a sub-optimal design.

Weight fractions are also useful for initial total weight estimation. For example, $W_{\rm fix}$ is typically known from the requirements. If we now assume some estimated value for its fraction $f_{\rm fix}$, then the corresponding initial total weight estimate is

$$W \simeq \frac{W_{\rm fix}}{f_{\rm fix}} \tag{25}$$

which can then be used to estimate the necessary wing area, power, structural gauges, etc. These can then be used to obtain a more refined overall weight estimate, and the sizing process can then be repeated until (hopefully) convergence occurs.

For the more detailed weight estimates, each weight component is typically further broken down into sub-components, as diagrammed in the figure above. For example, the airframe weight might be broken down as follows.

$$W_{\rm air} = W_{\rm wing} + W_{\rm fuse} + W_{\rm tail} + \ldots \tag{26}$$

$$W_{\rm wing} = W_{\rm skin} + W_{\rm spar} + W_{\rm act} + \ldots \tag{27}$$

A suitable weight model might be to assume $W_{\rm skin}$ scales with wing area, $W_{\rm spar}$ depends on the span and airfoil thickness according to a beam model, the actuator weight $W_{\rm act}$ scales with the maximum control-surface hinge torques, etc.

Similarly, the power-system weight might be broken down as

$$W_{\rm pow} = W_{\rm motor} + W_{\rm elec} + W_{\rm batt} + W_{\rm wiring} \tag{28}$$

where the weights of the motor and power-control electronics, $W_{\rm motor}$ and $W_{\rm elec}$, would scale with the maximum required power (e.g. for takeoff or dash speed), while the battery weight $W_{\rm batt}$ would depend on the required range or duration, etc.

Most of these weight-component models involve the total weight W itself, so iteration is required to converge to a closed design where the assumed W is equal to the sum of all component weights, and which meets the various force balance relations and power requirements.

6. Power System Requirements

Assuming that the total aircraft weight does not change appreciably during flight, and the flight is level and steady with $\gamma = 0$ and $\dot{V} = 0$, the time t and shaft energy $E_{\rm shaft}$ required to fly a distance d is

$$t = \frac{d}{V} \tag{29}$$

$$E_{\rm shaft} = P_{\rm shaft}\, t = \frac{T\, d}{\eta_{\rm prop}} \tag{30}$$

Combining the relations and assumptions above, we have the following summary relations for a sustained level flight of distance d.

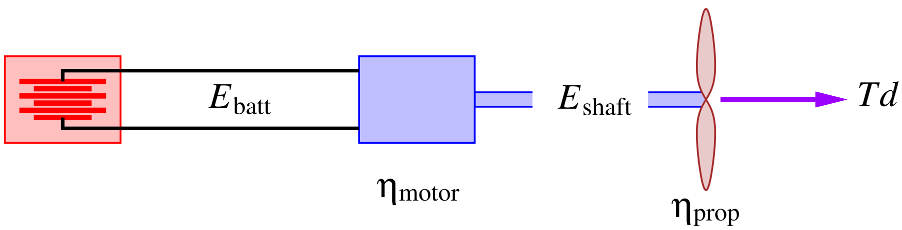
$$\boxed{E_{\text{shaft}} \;=\; \frac{1}{\eta_{\text{v}}}\frac{1}{\eta_{\text{i}}}\, W \,\frac{C_D}{C_L}\, d} \tag{31}$$

For an electrical power system, we can also define a motor efficiency η_{motor}, which relates the motor shaft power and the battery electrical power (current $\times$ voltage).

$$\eta_{\text{motor}} \;\equiv\; \frac{P_{\text{shaft}}}{P_{\text{batt}}} \;=\; \frac{E_{\text{shaft}}}{E_{\text{batt}}} \tag{32}$$

$$P_{\text{batt}} \;=\; \frac{TV}{\eta_{\text{prop}}\,\eta_{\text{motor}}} \tag{33}$$

$$E_{\text{batt}} \;=\; \frac{T\,d}{\eta_{\text{prop}}\,\eta_{\text{motor}}} \;=\; \frac{E_{\text{shaft}}}{\eta_{\text{motor}}} \tag{34}$$



The weight of the batteries can be estimated from this energy requirement E_{batt}, and a known or assumed specific energy $(E/m)_{\text{batt}}$ (e.g. in Joules/kg).

$$\boxed{W_{\text{batt}} \;=\; \frac{E_{\text{batt}}}{(E/m)_{\text{batt}}}\, g} \tag{35}$$

Some allowance for extra energy use in takeoff, climb, and maneuvers will typically be required.

The motor size and weight can often be estimated from the maximum required power P_{max}, and a known or assumed specific power $(P/m)_{\text{motor}}$ (e.g. in Watts/kg) for the chosen class of power plant.

$$W_{\text{motor}} \;=\; \frac{P_{\text{max}}}{(P/m)_{\text{motor}}}\, g \tag{36}$$

P_{max} itself it typically set by a takeoff distance requirement, or a climb rate requirement, or a maximum speed requirement. Continuous maximum power output of electric motors is typically limited by resistive heat rejection rate. A high power output which is of sufficiently short duration may be limited by other factors, such as the maximum current capacity of the batteries or control electronics. Regardless of how the maximum power is estimated, equation (36) can then be used for motor weight estimation.

An alternative estimate for electric motor weight is in terms of the required maximum motor torque Q_{max} and a known or assumed specific torque $(Q/m)_{\text{motor}}$ for the chosen motor class and architecture.

$$W_{\text{motor}} \;=\; \frac{Q_{\text{max}}}{(Q/m)_{\text{motor}}}\, g \tag{37}$$

Since electric motor weight tends to scale more closely with torque than with power, an assumed $(Q/m)_{\mathrm{motor}}$ value is generally more universal than an assumed $(P/m)_{\mathrm{motor}}$ value and hence equation (37) is preferable over (36). However, since in general we have $P = Q\Omega$, obtaining the flight power and corresponding aircraft performance from $Q_{\max}$ also requires involving the rotation speed Ω which is strongly constrained by the propeller's demands.

One example of a rotation speed constraint is the requirement that the prop tip Mach number must be below some maximum value

$$\frac{\Omega R}{a} \leq M_{\max} \tag{38}$$

$$\text{or equivalently} \qquad \Omega \leq \frac{M_{\max}\, a}{R} \tag{39}$$

where the limit might be $M_{\max} \simeq 0.8$ for acceptable performance in cruise, an as low as $M_{\max} \simeq 0.4$ for acceptably low noise on takeoff. If such a rotation speed constraint is active, for a given required flight power $P = Q\Omega$ the result is an increase in Q which then increases W_{motor} via (37). Hence, motor weight estimation based on torque also involves propeller design and modeling, and hence brings in some added overall design complexity. It should be noted that one possible way to circumvent the $M_{\max}$ constraint is by the use of a gearbox between the motor and propeller. However, a gearbox adds losses, cost, weight, and reliability concerns, and hence is not necessarily a practical solution.