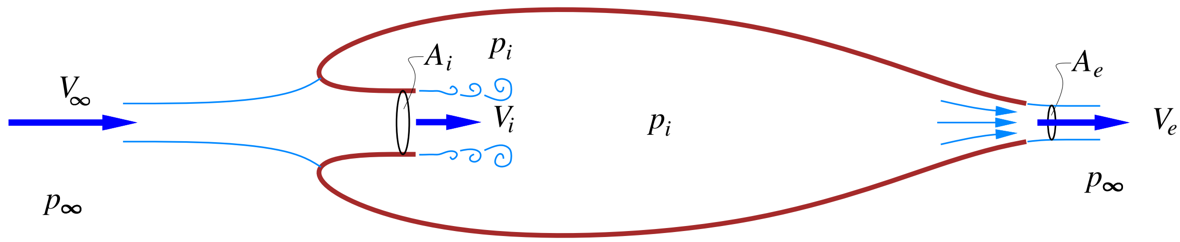
Internal Flow Losses

15 Sep 2020

The diagram below shows an idealized aerodynamic body with some interior flow. The flow enters with velocity V_i through an inlet area A_i, and mixes out in in the interior plenum which has some pressure p_i. The flow then exits with a velocity V_e, through an effective exit area A_e.



Assuming incompressible flow, the mass flow relation gives the velocity ratio in terms of the area ratios.

$$\frac{V_e}{V_i} = \frac{A_i}{A_e} \tag{1}$$

The interior pressure and inflow velocity are related by the Bernoulli equation applied from the freestream to the interior jet, before it mixes out.

$$p_\infty + \tfrac{1}{2}\rho V_\infty^2 = p_i + \tfrac{1}{2}\rho V_i^2 \tag{2}$$

Applying Bernoulli from the nearly-still interior to the exit jet also relates the interior pressure to the exit jet, which is at the ambient p_∞.

$$p_i = p_\infty + \tfrac{1}{2}\rho V_e^2 \tag{3}$$

Combining the above equations gives the inflow and exit velocity ratios in terms of only the area ratio.

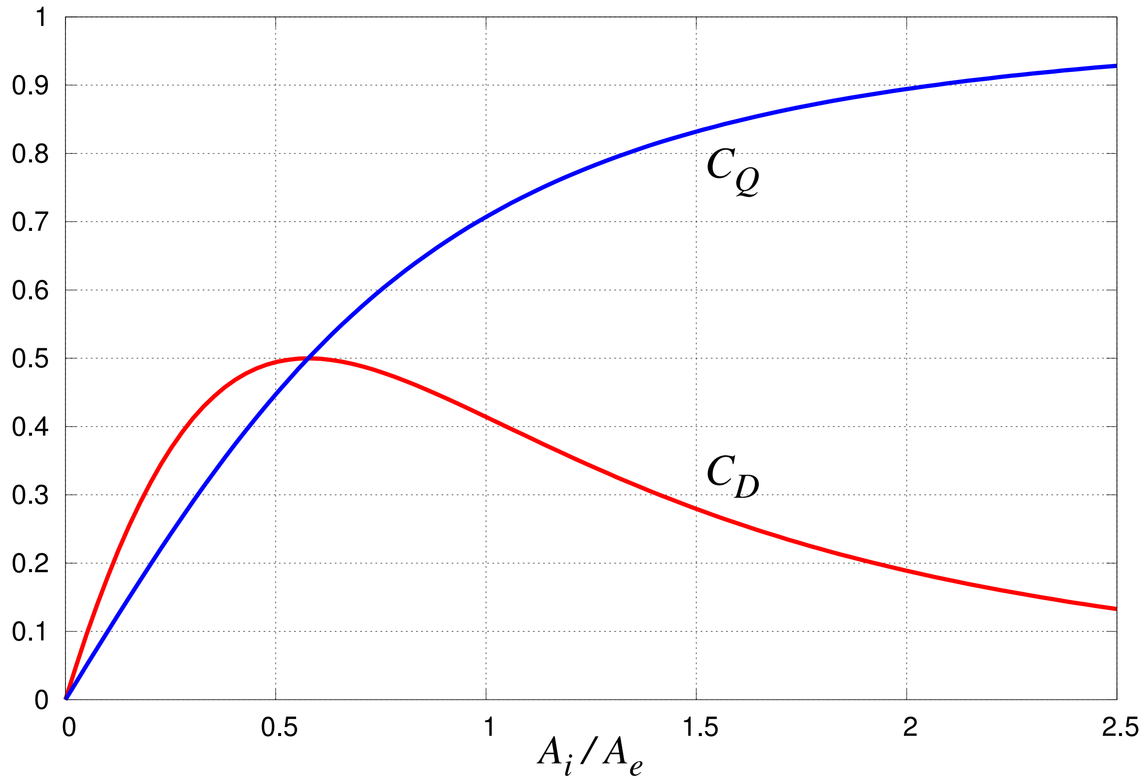
$$\frac{V_i}{V_\infty} = \frac{1}{\sqrt{1+(A_i/A_e)^2}} \quad , \qquad \frac{V_e}{V_\infty} = \frac{A_i/A_e}{\sqrt{1+(A_i/A_e)^2}} \tag{4}$$

Of particular interest is the mass flow and the interior mixing drag, both conveniently quantified in terms of their coefficients based on the exit area.

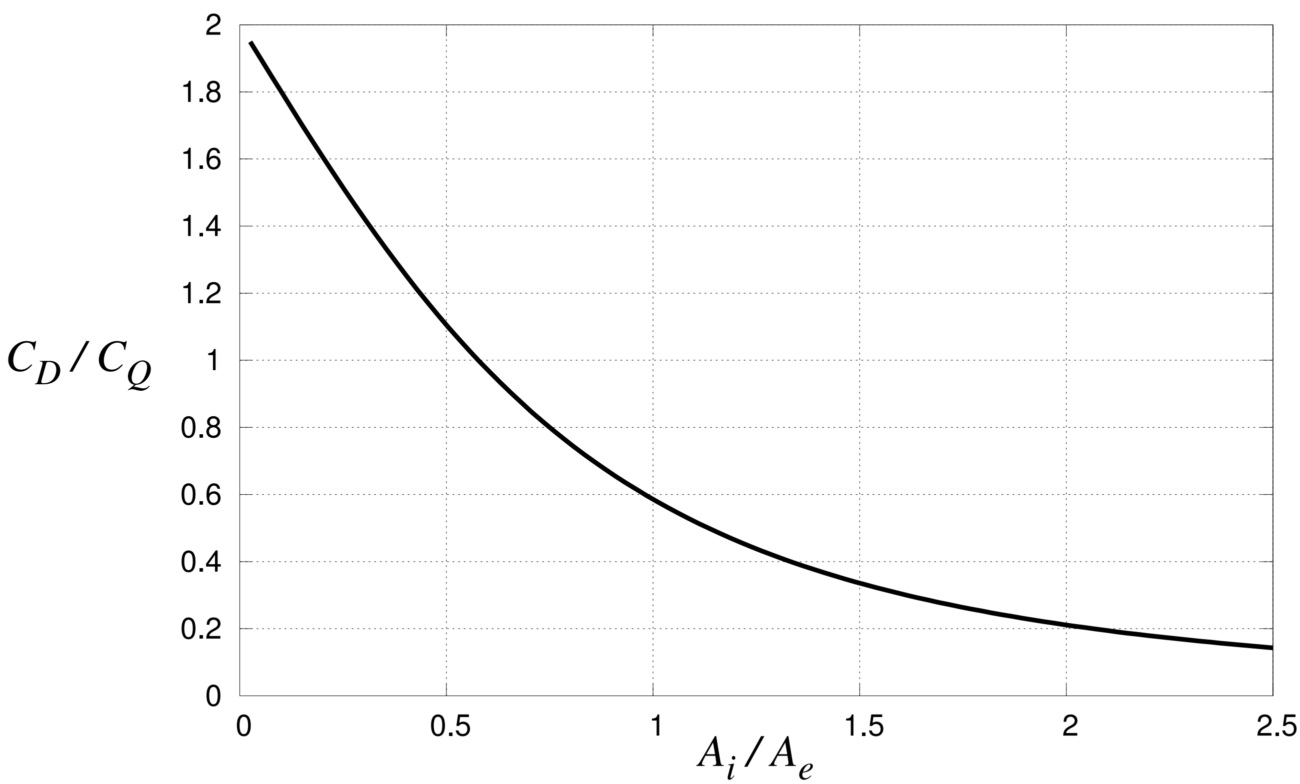
$$\frac{\rho V_e A_e}{\rho V_\infty A_e} = \frac{\dot m}{\rho V_\infty A_e} \equiv C_Q = \frac{A_i/A_e}{\sqrt{1+(A_i/A_e)^2}} \tag{5}$$

$$\frac{\dot m (V_\infty - V_e)}{\frac{1}{2}\rho V_\infty^2 A_e} = \frac{D}{\frac{1}{2}\rho V_\infty^2 A_e} \equiv C_D = \frac{A_i/A_e}{\sqrt{1+(A_i/A_e)^2}} \left[1 - \frac{A_i/A_e}{\sqrt{1+(A_i/A_e)^2}} \right] \tag{6}$$

These are shown versus the area ratio in the plots below. It is evident that the least drag is obtained with a relatively large inlet area. The physical reason is that drag is the result of loss rate of flow kinetic energy via mixing, which here is equal to $\frac{1}{2}\dot m V_i^2$. Hence, for a fixed required mass flow, a large A_i gives a small V_i, and hence small mixing loss and low drag.



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
C_Q
C_D
0
0.5
1
1.5
2
2.5
A_i / A_e



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
C_D / C_Q
0
0.5
1
1.5
2
2.5
A_i / A_e