Planing Flow Lift and Drag Modeling

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1. Nomenclature and Definitions

D	hydrodynamic drag	V	speed
L	hydrodynamic lift	g	gravity acceleration
S	reference area for lift and drag coefficients	ρ	water density
b	submerged span (of planing surface)	α	planing surface incidence angle
ℓ	submerged length	AR	aspect ratio ($\equiv b^2/S$)
$\mathcal{V}$	submerged volume	Fr	Froude number ($\equiv V/\sqrt{g\ell}$)
W	weight	Re	Reynolds number ($\equiv \rho V \ell/\mu$)

2. Force Coefficients and Parameters

The hydrostatic buoyancy lift, and the hydrodynamic lift and drag forces on a partially-submerged object in motion, such as a boat or pontoon, can be decomposed as

$$L = \frac{1}{2}\rho V^2 S\, C_{L(Fr,\text{shape})} + \rho g \mathcal{V} \qquad (1)$$

$$D = \frac{1}{2}\rho V^2 S \left[C_{D_w}(Fr,\text{shape}) + C_{D_f}(Re) \right] \qquad (2)$$

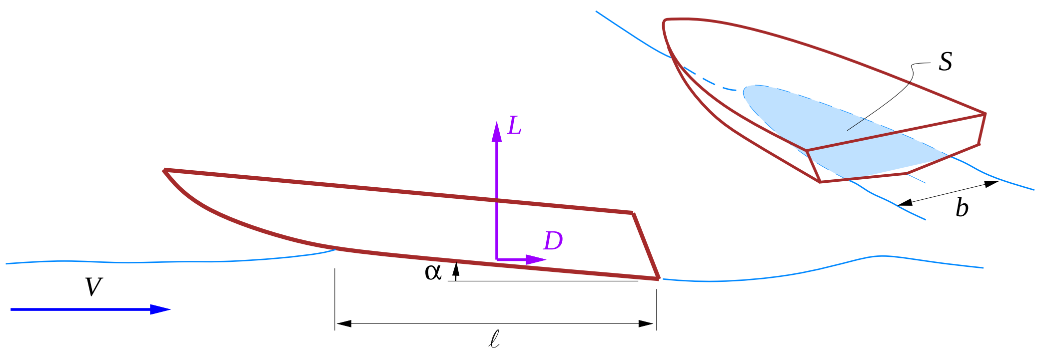
where the wave drag coefficient C_{D_w} depends primarily on the Froude number and the submerged body shape, while the friction drag coefficient C_{D_f} depends primarily on the Reynolds number as indicated. If S is defined as the water-wetted area, then a reasonable estimate for the friction component is

$$C_{D_f} \simeq C_{f(Re_\ell, Re_{\text{tr}})} \qquad (3)$$

where C_f is the flat-plate skin friction coefficient which is well characterized by skin friction charts. The focus here will be on C_{D_w} and C_L.

3. Planing Regime

In general, free-surface flows are extremely complex, and C_{D_w} and C_L strongly depend on the Froude number as well as the submerged shape of the body. However, when the Froude number exceeds $Fr > 1$ or so, the flow enters the *planing* regime which is easier to characterize.



Here the contact with the water is limited to a flat or nearly flat planing surface of area S and span b, set at some small angle α to the flow. For $Fr > 2$ or so, both C_{D_w} and C_L become nearly independent of Fr, and buoyancy also typically becomes relatively negligible. We then have

$$L \simeq \frac{1}{2}\rho V^2 S\, C_{L(\alpha, AR)} \qquad (4)$$

$$D \simeq \frac{1}{2}\rho V^2 S \left[C_{D_w(\alpha, AR)} + C_{D_f(Re)} \right] \qquad (5)$$

where AR is the aspect ratio of the wetted surface. Hoerner gives reasonable estimates for the lift coefficient at the limits of large and small aspect ratios,

$$C_L \simeq \begin{cases} \dfrac{\pi}{2} \dfrac{AR}{1+AR} \alpha & , \quad AR \gg 1 \\ \dfrac{\pi}{4} AR\, \alpha \; + \; 0.88\, \alpha^2 & , \quad AR \ll 1 \end{cases} \tag{6}$$

where α is in radians. For the small α values typical of efficient planing hulls, the changeover occurs at $AR \simeq 1$ where the two expressions above are roughly equal. In practice, most seaplane floats are relatively slender, with $AR < 0.25$ or less, to reduce aerodynamic drag in flight. The actual AR can also vary during the takeoff run as the hull rises with increasing speed, and the wetted length ℓ decreases faster than the wetted span b.

The wave drag coefficient is estimated by the fact that inviscid pressure forces must be normal to the planing surface. Assuming small angles this gives

$$C_{D_w} = C_L\, \alpha \tag{7}$$

where α is again in radians. It's also useful to note the planing drag/lift ratio,

$$\frac{D}{L} = \frac{C_{D_w} + C_{D_f}}{C_L} \simeq \alpha + \frac{C_{D_f}}{C_L} \tag{8}$$

whose minimum gives the theoretically optimum planing α and corresponding C_L. This minimum depends on the magnitude of C_{D_f} and hence on the Reynolds number, and also to some extent on AR via the C_L relation (6). For most applications, this optimum falls between $C_L = 0.05 \ldots 0.10$. The corresponding α depends considerably on AR, but typically falls in the range $\alpha = 3° \ldots 10°$.

3. Planing Body Sizing

On a seaplane it's important to provide enough available planing area (i.e. make the hull or pontoons sufficiently large) so that planing can begin at a sufficiently low speed during the takeoff run acceleration, otherwise the seaplane will be "stuck" on the water at the $Fr < 1$ regime where C_{D_w} is quite large and cannot be overcome by the available engine thrust. This minimum planing area can be estimated by requiring that the planing lift given by equation (4) is equal to the weight at the lowest speed (lowest Froude number) where planing is possible.

$$V = Fr_{\min} \sqrt{g\ell} \tag{9}$$

$$C_L \simeq C_{L_{\max}} \tag{10}$$

$$S_{\min} = \frac{W}{\frac{1}{2}\rho V^2 C_L} \tag{11}$$

Reasonable choices are $Fr_{\min} = 1.5$ and $C_{L_{\max}} = 0.1$, although these may differ somewhat for specialized planing hull shapes. Another requirement is that the pontoon buoyancy must exceed the weight,

$$\mathcal{V}_{\min} \geq k \frac{W}{\rho g} \tag{12}$$

where $k > 1$ is the buoyancy margin factor. Either the minimum area requirement (11) or the minimum volume requirement (12) will typically dictate the overall dimensions of the pontoons or hull.