To: Team Water Stakeholders

From: Amira Malik

Subject: Main Wing Memo

Date: Nov. 15 2022

Driving Constraints and Design of the Main Wing

Nomenclature

x,y	longitudinal, spanwise positions	V	velocity
S	reference area	ρ	air density
b	wingspan	L	total lift
c	average wing chord (=S/b)	D	total drag
$\mathcal{R}$	aspect Ratio	C_D	drag coefficient
C_L	lift coefficient	i_r	wing incidence angle, root
Υ	dihedral angle	i_t	wing incidence angle, tip
η_p	propeller efficiency	q	dynamic pressure
k	lift-induced drag constant	C_{Di}	$C_L^2 k$, the induced drag coefficient
L/D	Lift over Drag	η_s	solar panel efficiency
η_a	solar panel area per wing area	η_m	motor efficiency
Q	$\left(\frac{P}{S}\right)_{solar}$, solar irradiance	OML	Outer Mold Line
e	wing efficiency factor	AOA	angle of attack

1 Executive Summary

The SEAWAY aircraft's main selling point is that its range is tied to the sunlight it receives and that, in theory, if you chase good weather and have plenty of snacks, you could fly until you die. The plane does this with its array of solar panels tied to its wing area. This memo will determine a wing area that satisfies SEAWAY's ability to generate the power it requires to fly, that area being 25.5 m^2. Additionally, the memo will touch upon the characteristics of the wing's lift and drag as a consequence of this area and show a final OML. This wing will allow the aircraft to produce enough power to stay aloft in sunlight, have good flying characteristics, and be easy to build.

There is an interplay between wing area, drag, the power required, and power generated at different wing dimensions. So we will first explain the relationship between these variables, take some assumptions from the current wing design, analyze our variable relationships to find a wing area, and then determine a final 3D wing configuration and show its lift and drag characteristics. This design, while close to the final wing design, will need some refinement as other design memos get released. But the tools and method used in this memo makes any changes easy to implement. Additionally, this paper will not touch on the topic of control surfaces or stability, as those are other memos.

2 Wing Analysis Method

This section will pre-brief the analysis, giving a background of how the wing area is to be found.

2.1 Governing Relations

We will be designing SEAWAY for cruise, where L = W, D = T, and V is constant. We will not be touching upon takeoff as the CONOPS of the aircraft include a battery that will deliver the necessary additional power to take off and climb. At cruise, SEAWAY will be solar-sufficient if the power generated by its solar panels are equal or greater than the power required by its configuration. The equations relating these constraints are as follows:

$$L = \frac{1}{2} C_L \rho S V^2 \tag{1}$$

$$D = \frac{1}{2} C_D \rho S V^2 \tag{2}$$

$$P_{required} = \frac{S}{\eta_P} \sqrt{\frac{2}{\rho}} \left(\frac{C_D}{C_L^{\frac{3}{2}}} \right) \left(\frac{W}{S} \right)^{3/2} \tag{3}$$

$$P_{generated} = \eta_p \eta_m \eta_s \eta_a S Q \tag{4}$$

What drives each equation and this overall design is speed: speed must be minimized to minimize the power required, but not be so small that the resulting wing area, which is proportional to the power generated, is much larger than its mass budget, being around 100lbs. Therefore, we can parameterize these equations by speed, and by sweeping through speeds, we can find which wing areas will satisfy $\frac{P_{gen}}{P_{req}} > 1$ that is also as light weight as possible. We have a helpful equation, detailed in the appendix, that takes a constant speed at cruise and relates our drag / thrust to our lift / weight and area:

$$\frac{T}{W} = q C_D \left(\frac{1}{\frac{W}{S}} \right) + k \left(\frac{1}{q} \right) \left(\frac{W}{S} \right) \tag{5}$$

With our current vehicle estimated weight to be 256 kg, a desired cruise density altitude of 3000'(where altitude does not lower the estimated target Q [11]), and a slew of known variables and ranges from our propulsion team, we know:

Symbol	Known Variables	Value (SI units)
L, W	Weight-Force at Cruise [6]	2510 N
ρ	Density Altitude [6]	.91 kg/m^3
Q	Determined Value [2]	800 W/m^2
η_s	Determined Value [3]	22.4%
η_p	Determined Value [4]	76.5%
η_m	Determined Value [4]	96.22%
η_a	Determined Range [5]	[75%, 85%]

These variables alone can not "solve" the governing relationships for a wing area. It is apparent here that we need a starting point for our design from which to justify a few key assumptions of our variables so that we can search for a wing area.

2.2 Starting Point: Current Design

The starting point consists of as few key parameters we can get from the current aircraft that allows us to use the governing relations above to find a wing area that generates as much or more power as it requires. The parameters to serve as assumptions in our analysis are:

Variable	Assumed Value
airfoil	Eppler 603
L/D, aircraft	22
L/D, wing	36
Æ	15
e	.8
c_L	1.2

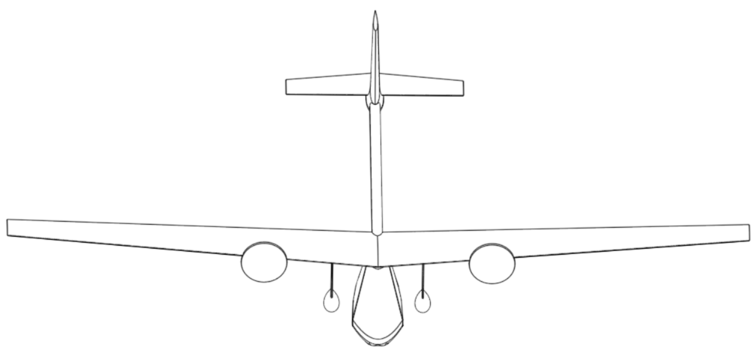
The rest of this sections details why we chose these particular parameters. From our current design we want to take over as an initial guess as few assumptions as we need. From our governing relationships, we can see that simply knowing the relative lift and drag of our configuration as well as the induced drag coefficient, we can make a direct connection between our power, drag, lift, and area based on speed. To get these quantities, we can take a look at the drag estimation for the current vehicle:

Speed (19 $\frac{m}{s}$)
Air Density (1.225 $\frac{kg}{m^3}$)

Source	Wetted Area (m^2)	Total C_d	Total Drag (N)	Percent Total Drag
Fuselage	03.40	.010	7.50	8.8%
Sponson	00.19	.020	0.84	1%
Sponson Strut	00.16	.005	0.18	0.2%
Horizontal Tail	02.26	.008	4.00	4.7%
Vertical Tail	01.45	.008	2.70	3%
Wing	16.68	.019	70.00	82.3%

An estimation of the vehicle drag given the old wing

This drag is based on the current vehicle, which is:



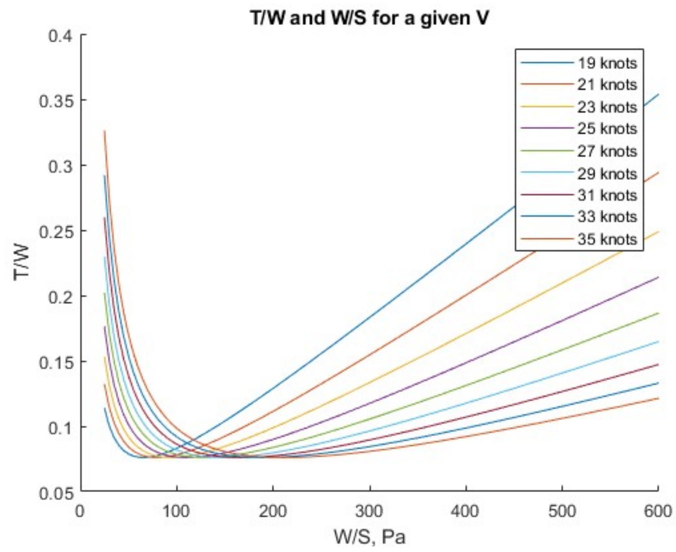
The plane from which the drag is built-up

From the drag build-up, we can see that the major drag contribution comes from the wing. Since the tail surfaces' drag is low and the fuselage, floats, and float struts are all part of a fixed volume, we know that through our wing iteration, the wing drag is most important and that with our wing dimensions sizing, if we keep a similar Æ and wing efficiency, that our L/D will remain mostly unchanged. For some cushion, we can assume a 25% hit in our L/D from the non-linearity of our analysis, idealization of drag, and the likelihood that an amateur built ultralight will likely have higher than normal skin friction from poor materials. This means that from our starting point, we can take the assumptions shown above.

For this analysis, we will not explore different airfoils. The justification for the Eppler 603, being a a purpose-designed airfoil for gliders used in prominent aircraft like the Grob 103, which has been studied and shown to perform well at low speeds generating high lift [7], and has a generous amount of internal area, which will make wiring and mounting solar panels easier since the hardware can be tucked inside, is still the justification for continuing use of the airfoil. Further analysis or optimization of the airfoil used is certainly welcome and will follow this memo, but the framework to pick a wing area as detailed by this report will still be applicable. This Eppler 603 is still satisfactory to the design.

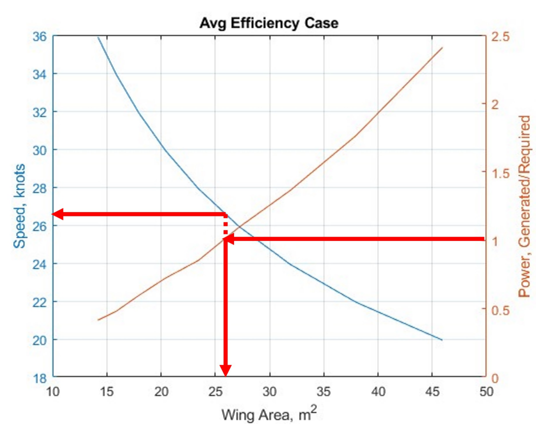
3 Analysis and Final Design

Using the above relationships, the known variables, and these assumed variables, we can perform a fixed lift analysis to find combinations of wing areas and cruise speeds that correspond to particular ratios of power generated over power required, from which we can choose a final wing area of 25.5 m^2. To show how we obtained this area, we can first see the relationship between wing loading and the thrust to weight ratio:



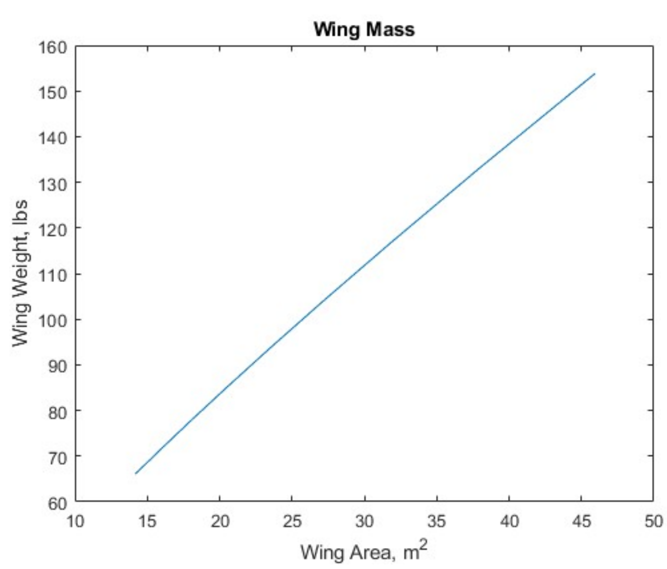
The relationship between thrust to weight and wing loading

We can see clear troughs, or optimization points for each speed where the thrust is minimized. Since we want to minimize the power required, we will take a further look at those points. For a given speed, we can then find the power as a ratio of generated over required:



The power ratio and speed for a given area

Here you can see we picked where the power ratio is exactly 1, which corresponds to a wing area of 25.5 m^2, which means the aircraft must cruise at at least 27 knots. Using the mass estimation of the wing [8], we can see that $25.5m^2$ is just below the 100 lbs wing mass budget. Picking a lower ratio than this means SEAWAY can't fly indefinitely, and picking any number much higher than 1 means our wing will be roughly over 100 lbs.

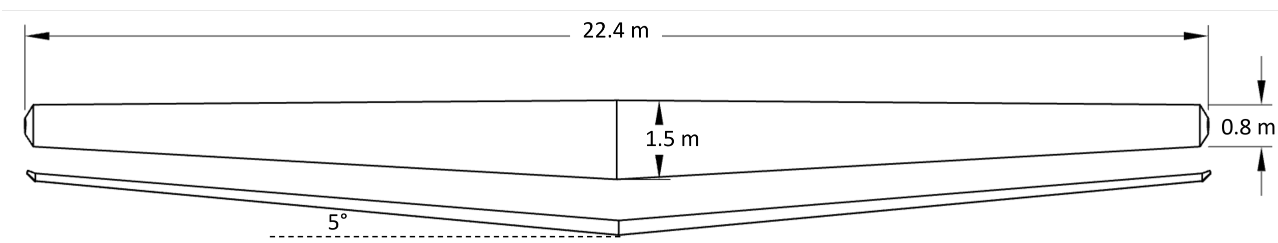


The weight of the wing proportioned to area and span; roughly linear

From this wing area, we get:

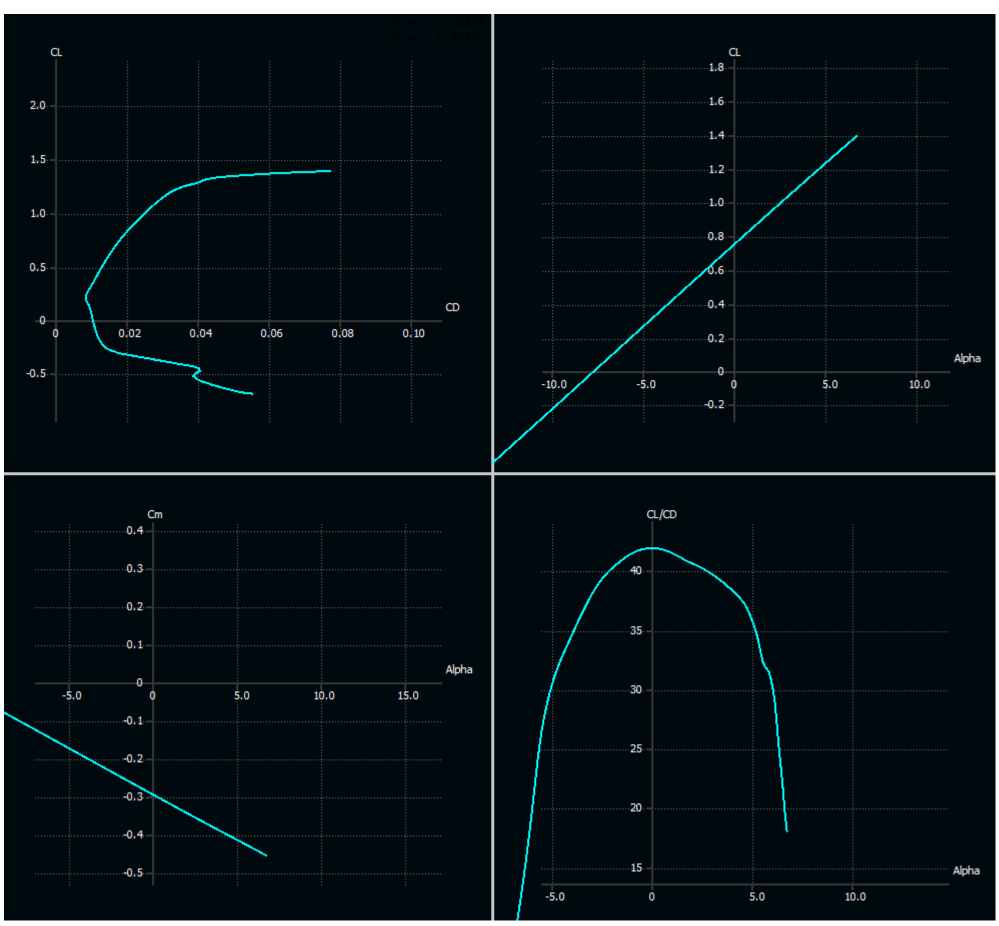
Variable	Value	Units
S	25.5	m^2
Æ	15	-
b	19.8	m
c	1.3	m
V_{cruise}	27	knots
V_{stall}	23	knots
Reynolds #	1,100,000	-
Airfoil	Eppler 603	-

The wing's geometry, however, is not determined with this analysis but by iteratively tapering and modifying the wing until it meets the assumptions of the analysis, namely that the wing L/D is >36, Æ 15, $C_L = 1.2$, and the e is > .8. We want to meet and not exceed these two parameters because there would be no end to how much we can taper and twist the wing until the wingspan and efficiency reach infinite, but then the manufacturing and solar cell area packing efficiency would surely suffer with a complex wing. All the while, we must maintain or exceed the wing area, keep the $\frac{1}{4}$ of the wing straight (to easily accomodate structural elements such as spars), and twist our wing such that the wing at 0° is most efficient (since we want a zero angle of the fuselage in cruise to reduce its drag). Applying the minimal amount of modification to a baseline rectangular wing, and after a dozen or so iterations, we get a final OML of:

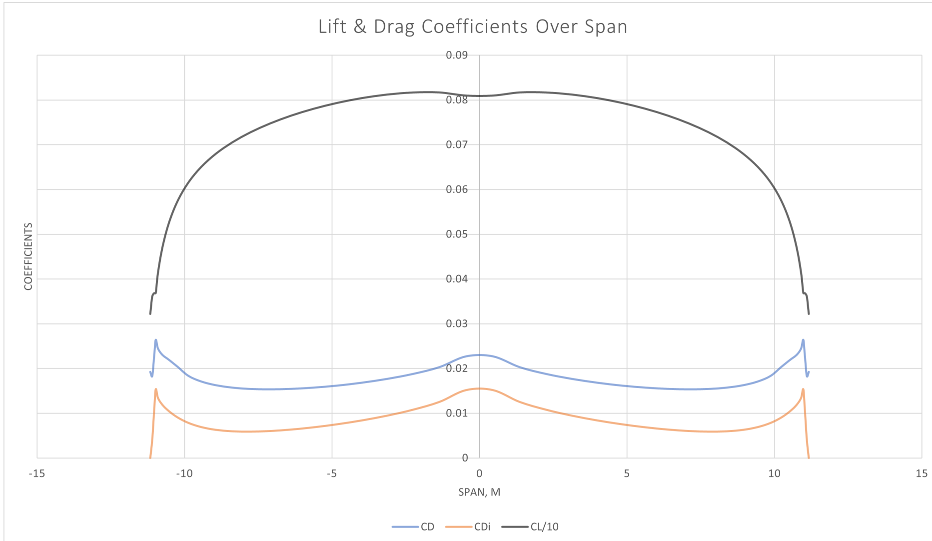


The final wing dimensions

Whose characteristics are:



Wing Polars



Lift & Drag Coefficients

As shown, the wing has a -2.5° twist so that the wing "washes out" and that at stall, the air separates at the root and travels to the tips, lessening any chance of a violent asymmetric stall. Additionally, preliminary rounded wingtips are added. There is also an angle of incidence of the root is 5° and the tip is 2.5°. The wingtips contribute to .1m span each and have a dihedral angle of 20^{circ} and the wing's dihedral of 5° is taken from the starting point design, but the dihedral will be set by further analysis into the dynamic and stability modes of the wing.

4 Implications for Future Design

This memo and preliminary analysis has outlined a satisfying design for the SEAWAY aircraft. With this wing with no further modifications, we are confident that SEAWAY can be covered in solar panels with a respective packing efficiency, and once at cruise, can stay in the air as long as the solar irradiance is greater than 800 $\frac{W}{m^2}$ at a density altitude of less than 3000'. However, this doesn't mean the wing can not be further optimized. The airfoil and wing geometry as shown has best glide speed and stall speed at a mere 15% difference in speed. This is not uncommon in some gliders where the minimum sink speed is close to the stall speed [9], but this plane will surely not be fun to fly if it flies most optimally on the verge of stall. Additionally, there could certainly be further optimization in either picking or designing a low reynolds number airfoil.

With the release of other design memos, the dimensions of this wing will finalize other aspects of the aircraft such as tail sizing, propeller sizing, and structural weights, and more accurate weight estimations will be taken into account to refine the script used in this memo to see if a larger wing with more power margin is feasible while still being under 100 lbs.

The wing OML, is ready to, at best, be broken down into its structural components and start to be built immediately, and it will result in a flying aircraft that meets or exceeds the intended operation. With this wing, SEAWAY will be "the new way to see new sights." (10).

Appendix A: Works Cited

(1) Gudmundsson, Snorri. General Aviation Aircraft Design: Applied Methods and Procedures. 1st ed., Butterworth-Heinemann, 2014.

(2) Design Decision Memo, Andrew Manwaring

(3) Design Decision Memo, Lauren Carethers

(4) Design Decision Memo, Matthew McGillick

(5) Design Decision Memo, Blake Shepherd

(6) Preliminary Design Review of the Water Team and Design Decision Memo, Josh Malone

(7) https://ntrs.nasa.gov/api/citations/19790011865/downloads/19790011865.pdf

(8) Team Water Briefing 11/10/2022, slide 15

(9) https://utahsoaring.org/grobn8485w/

(10) Slogan, from Team Water PDR

(11) https://www.sciencedirect.com/science/article/pii/S0038092X10002410

Appendix B: Derivation of Governing Relations

Equation 2, Power Required Relationship

$$at\,cruise,\ L = W,\ V = \sqrt{\frac{2W/S}{\rho C_L}} \tag{6}$$

$$\eta_P P = \frac{1}{2}\rho V^3 S C_D \tag{7}$$

Combining these equations gave us our power relationship, equation 2.

Equation 5, Cruise Speed-Parametrized T/W & W/S

$$T = D = \frac{1}{2}\rho V^2 S C_D = \frac{1}{2}\rho V^2 S \left(C_{D_{min}} + \frac{C_L^2}{\pi Æ e}\right)$$

$$C_L = \frac{2W}{\rho V^2 S}$$

Combining, we get:

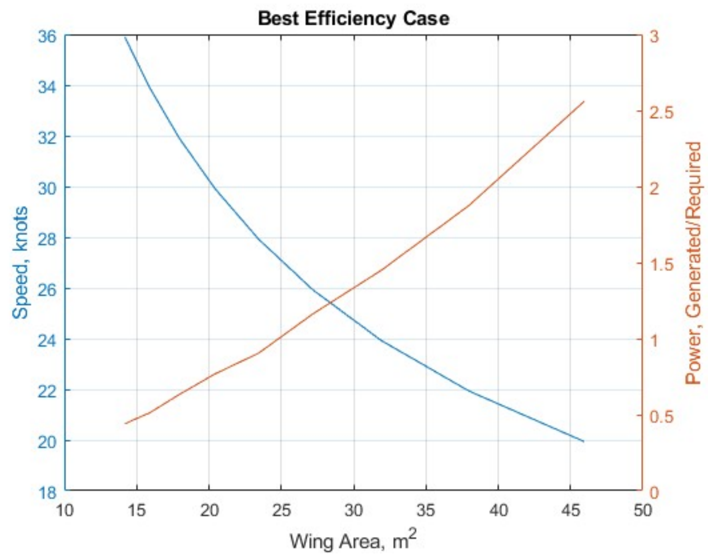
$$T = \frac{1}{2}\rho V^2 S C_{D_{min}} + \frac{2\rho V^2 S}{\pi Æ e \rho^2 V^4}\left(\frac{W}{S}\right)^2$$

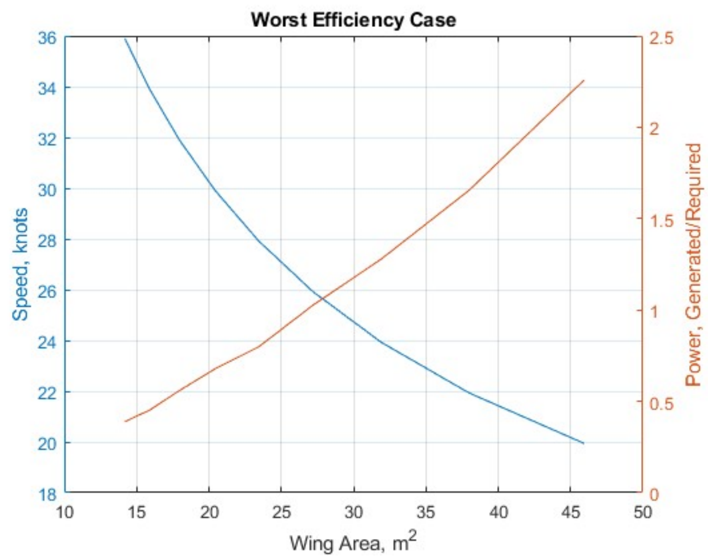
With some massaging of the equation (to see the full derivation, page 63 of General Aviation Design [1]):

$$\frac{T}{W} = qC_D\left(\frac{1}{\frac{W}{S}}\right) + k\left(\frac{1}{q}\right)\left(\frac{W}{S}\right)$$

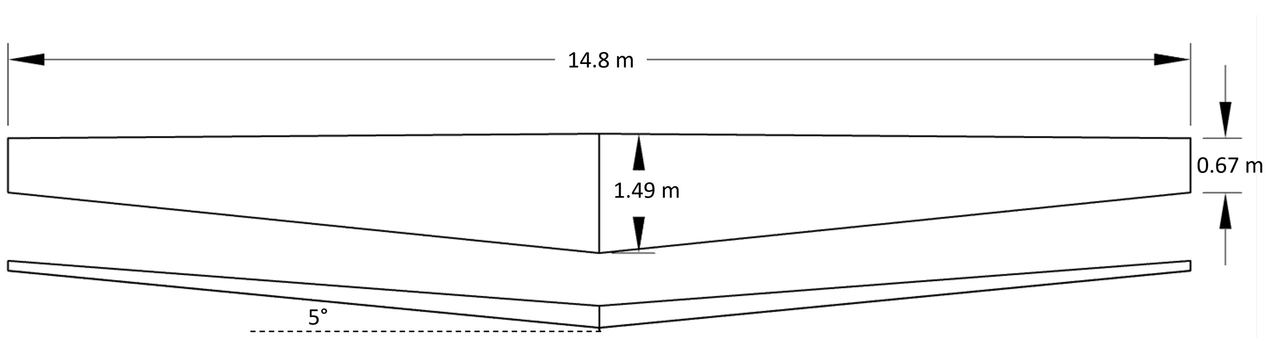
Appendix C: Best & Worst Case Efficiencies

These graphs were much more relevant when the propulsion team still had wide ranges for their 4 different efficiency metrics. Now, all but one are discrete values, the range being the solar cell area packing efficiency. Nonetheless, the power ratio and speed for a given wing area for the average η_a is used in the report, but the best and worst case scenarios, based on the high and low of the packing efficiency range are detailed here:

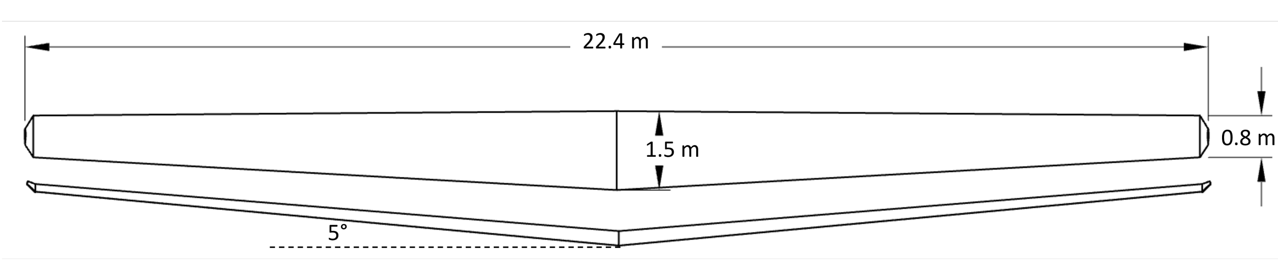




Appendix D: Comparison of New & Old Wing



The dimensions of the wing dimensions from which we start



The final wing dimensions