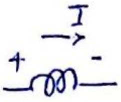
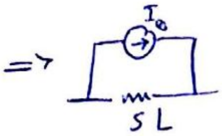
به نام خدا

تمرین 3 مدار

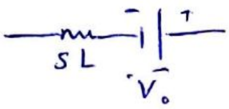
1- الف)



$$V = L\dot{I} \Rightarrow \mathcal{L}(V) = L\mathcal{L}(\dot{I}) = sL\mathcal{L}(I) - LI_{(0^-)}$$

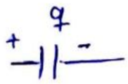


$$\Rightarrow I_0 sL = LI_{(0^-)} \Rightarrow \boxed{I_{0_I} = \frac{I_{(0^-)}}{s}}$$

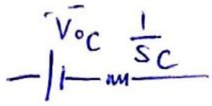


$$\boxed{V_{0_I} = LI_{(0^-)}} \quad \boxed{R = sL}$$

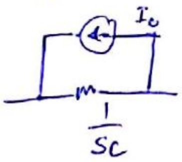
ب)



$$V = \frac{q}{C} \quad q = \int_{-\infty}^{t} I\,dt \Rightarrow \mathcal{L}(V) = \frac{1}{C}\mathcal{L}\left(\int_{-\infty}^{t} I\,dt\right) = \frac{1}{C}\times\left[\frac{1}{s}\mathcal{L}(I) + \frac{q_{(0^-)}}{s}\right]$$

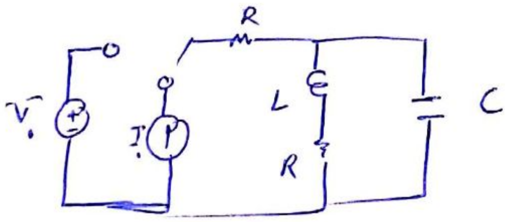


$$\Rightarrow V_{0_C} = \frac{q_{(0^-)}}{SC}$$



$$I_{0_C}\times\frac{1}{SC} = \frac{q_{(0^-)}}{SC} \Rightarrow \boxed{I_{0_C} = q_{(0^-)}}$$

ج) بار خازن و جریان سلف در $t<0$ ثابت شده‌اند.

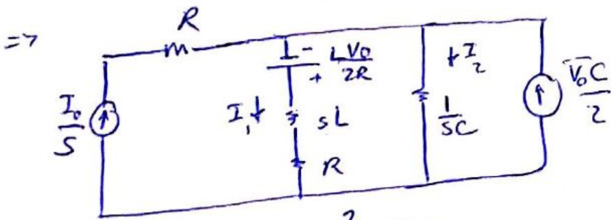


$$V_C = \frac{V_0}{2} \quad I_L = \frac{V_0}{2R} \quad q_{(0^-)} = \frac{V_0 C}{2}$$

$$I_1 + I_2 = \frac{V_0 C}{2} + \frac{I_0}{S}$$

$$-\frac{LV_0}{2R} + I_1(R+SL) = I_2\times\frac{1}{SC}$$

$\Rightarrow$



با توجه به مقادیر

$$\Rightarrow I_1 + I_2 = \frac{1}{2} + \frac{1}{S}$$

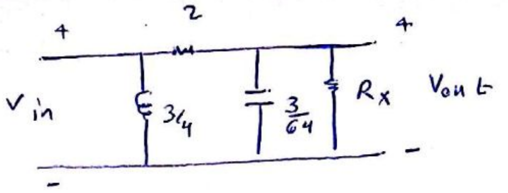
$$-\frac{1}{2} + I_1(1+S) = \frac{I_2}{S}$$

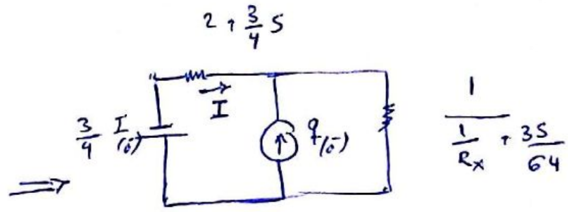
$$\Rightarrow I_2 = \frac{S^2+2S+2}{2(S^2+S+1)}$$

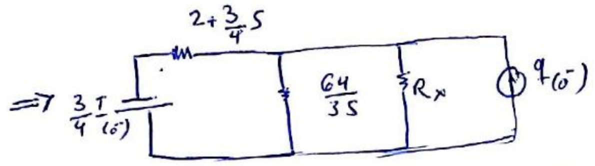
$$q = \frac{1}{2S} + \frac{I_2}{S}$$

$$\boxed{q = \frac{3}{2} - \frac{e^{-t/2}}{2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{6}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}$$

$$\boxed{I = \frac{\delta}{2} + \frac{1}{2}e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{6}e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}$$







$$-\frac{3}{4}I_{(0^-)} = \left(2+\frac{3}{4}s\right)[I] + \left(q_{(0^-)}+I\right)\times\frac{1}{\frac{1}{R_x}+\frac{3s}{64}}$$

$$\Rightarrow \quad I\times\left[2+\frac{3}{4}s + \frac{64R_x}{3sR_x+64}\right] = -\frac{3}{4}I_{(0^-)} - \frac{q_{(0^-)}\times 64\times R_x}{64+3sR_x}$$

$$I = \frac{-\frac{3}{4}I_{(0^-)}\times(64+3R_x s) - q_{(0^-)}64R_x}{\left[64R_x + \left(2+\frac{3}{4}s\right)(64+3R_x s)\right]}$$

$$V_{out} = I\times\frac{R_x 64}{64+3sR_x}$$

مخرج باید ریشه مضاعف داشته باشد.

$$\Rightarrow \frac{9}{4}R_x s^2 + s\times[6R_x+48] + 64\times[R_x+2] = 0$$

$$\Rightarrow [6R_x+48]^2 = 4\times\frac{9}{4}R_x\times 64[R_x+2] = 6^2[R_x+8]^2$$

$$16R_x[R_x+2] = [R_x+8]^2 \Rightarrow \boxed{R_x = \frac{8}{\sqrt{15}}}$$

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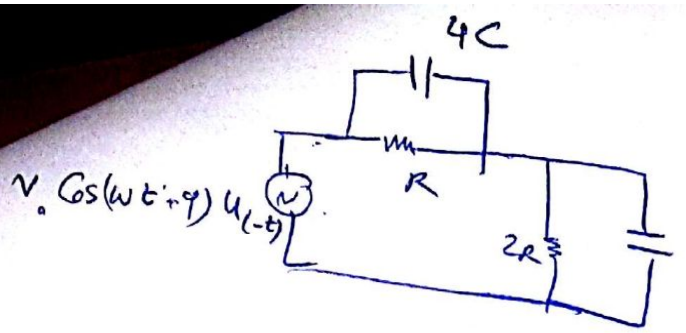
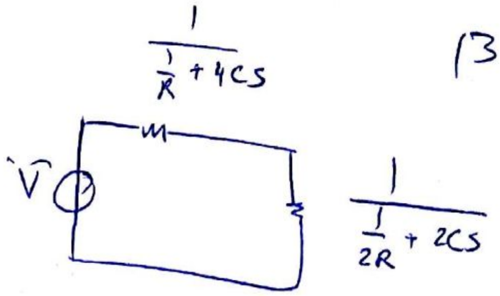
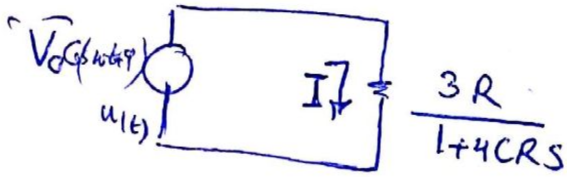






برای نداشتن جمله گذرا $I_{(c)}$ باید صفر شود.

$$Z(\cos(\omega t+\varphi)) = Z(\cos(\omega t)\cos\varphi - \sin(\omega t)\sin\varphi) = \frac{s\cos\varphi}{(s^2+\omega^2)} - \frac{\omega\sin\varphi}{(s^2+\omega^2)}$$

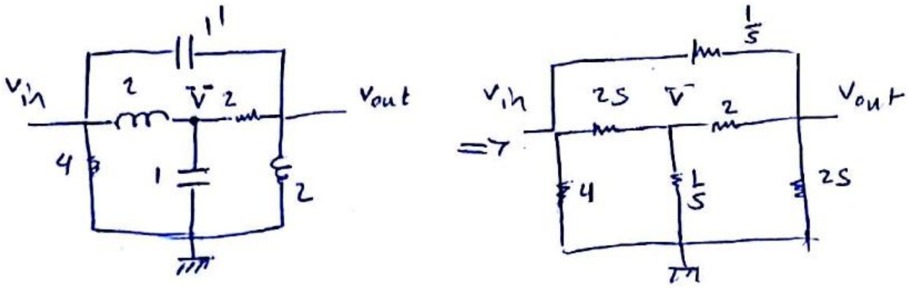
$$\Rightarrow I = Z(V) \times \frac{1+4CRS}{3R}$$

$$\Rightarrow I = \left[\frac{s\cos\varphi}{s^2+\omega^2} - \frac{\omega\sin\varphi}{(s^2+\omega^2)}\right] \times \frac{1+4CRS}{3R}$$

باید ضریب جمله کسینوسی صفر شود.

$$\Rightarrow s\cos\varphi - \omega 4CRs\sin\varphi = 0 \Rightarrow \boxed{4RC\omega = ctg\varphi}$$

$$\boxed{\Rightarrow \varphi = tg^{-1}\left(\frac{1}{4RC\omega}\right)}$$



$$\begin{cases} \dfrac{V_{in}-V}{2S} + \dfrac{V_{out}-V}{2} = \dfrac{V}{\frac{1}{S}} \\[2ex] \dfrac{V_{in}-V_{out}}{\frac{1}{S}} + \dfrac{V-V_{out}}{2} = \dfrac{V_{out}}{2S} \end{cases} \Longrightarrow \begin{array}{l} \dfrac{V_{in}}{2S} + \dfrac{V_{out}}{2} = V \times \left(S+\frac{1}{2}+\frac{1}{2S}\right) \\[2ex] V_{in} \times S + \dfrac{V}{2} = V_{out} \times \left(S+\frac{1}{2S}+\frac{1}{2}\right) \end{array}$$

$$\Longrightarrow V_{in} S + \frac{1}{2\left(S+\frac{1}{2}+\frac{1}{2S}\right)} \times \left[\frac{V_{in}}{2S} + \frac{V_{out}}{2}\right] = V_{out} \times \left(S+\frac{1}{2}+\frac{1}{2S}\right)$$

$$\Longrightarrow V_{in} \times \left[\frac{1}{4S\left(S+\frac{1}{2}+\frac{1}{2S}\right)} + S\right] = V_{out} \times \left[\left(S+\frac{1}{2}+\frac{1}{2S}\right) - \frac{1}{4\left(S+\frac{1}{2}+\frac{1}{2S}\right)}\right]$$

$$\Longrightarrow \frac{V_{out}}{V_{in}} = \frac{4S^2\left(S+\frac{1}{2}+\frac{1}{2S}\right)+1}{S \times \left[4\left(S+\frac{1}{2}+\frac{1}{2S}\right)^2 - 1\right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{4S^3+2S^2+2S+1}{S \times \left[\left(2S+\frac{1}{S}\right)\left(2S+2+\frac{1}{S}\right)\right]} = \frac{(2S^2+1)(2S+1)}{(2S^2+1)(2S^2+2S+1)}$$

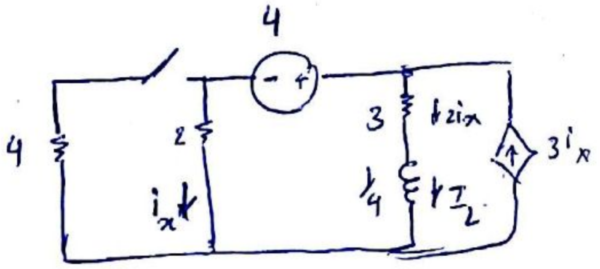
$$\frac{V_{out}}{V_{in}} = \frac{2S+1}{2S^2+2S+1}$$

$$2S^2+2S+1=0$$

$$\Longrightarrow S = \frac{-2 \pm \sqrt{4-8}}{4} = \frac{-1 \pm i}{2}$$

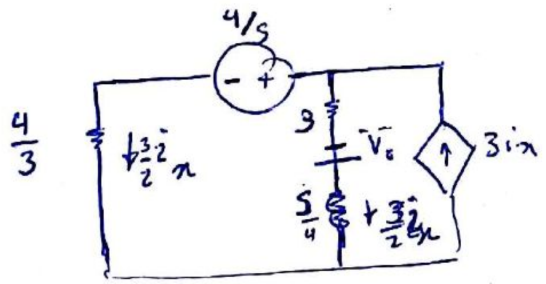
⇐ پایدار است.

5- در $t<0$ جریان ها ثابت شده اند.



$$\Rightarrow \quad 4 + 2i_x = 3 \times 2i_x \Rightarrow \boxed{i_x = 1}$$

$$\Rightarrow \quad I_L(0^-) = 2 \qquad \frac{1}{\frac{1}{2}+\frac{1}{4}} = \frac{4}{3}$$



$$V_0 = L\, I_{(0^-)} = \frac{1}{4} \times 2 = 1/2$$

$$\Rightarrow \quad \frac{4}{3} \times \frac{3}{2} i_x + \frac{4}{s} + \frac{1}{2} = \left(3 + \frac{s}{4}\right) \times \frac{3}{2} i_x$$

$$\Rightarrow \quad \rightarrow \quad i_x \times \left[\frac{3}{8}s + \frac{9}{2} - 2\right] = \frac{1}{2}\left(\frac{8}{s} + 1\right)$$

$$i_x \times \left[\frac{3}{8}s + \frac{5}{2}\right] = \frac{1}{2}\left(\frac{8}{s} + 1\right) \Rightarrow i_x = \frac{(8+s)}{s\left[\frac{3}{4}s + 5\right]}$$

$$\Rightarrow i_x = \frac{A}{s} + \frac{B}{\left(s + \frac{20}{3}\right)} \qquad \left\{ s\, i_x \Big|_{s=0} = A \right. \qquad \left\{ \left(s + \frac{20}{3}\right) i_x \Big|_{s=-\frac{20}{3}} = B \right.$$

$$\Rightarrow A = \frac{8}{5} \qquad B = -\frac{4}{15}$$

$$\Rightarrow i_x = \frac{8}{5} \times \frac{1}{s} - \frac{4}{15} \times \frac{1}{\left(s + \frac{20}{3}\right)} \Rightarrow \cancel{i_x = \frac{8}{5} e^{-3t} - \frac{4}{5} e^{-\frac{20}{3}t}}$$

$$\Rightarrow \boxed{i_x = \frac{8}{5} - \frac{4}{15} e^{-\frac{20}{3}t}}$$

$$i_L = \frac{3}{2}(i_x) \qquad \Rightarrow \boxed{i_L = \frac{12}{5} - \frac{2}{5} e^{-\frac{20}{3}t}}$$

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