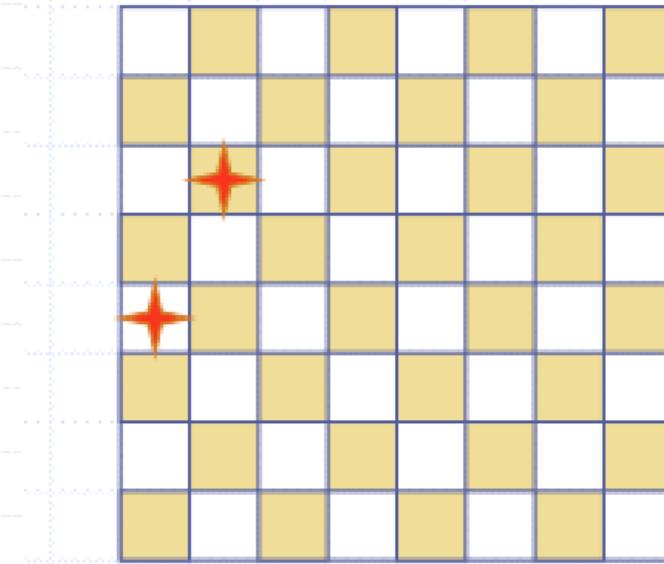


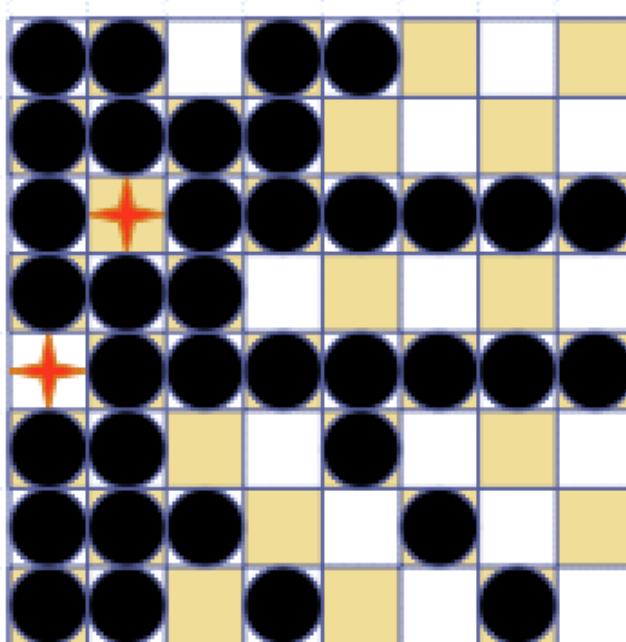
# **Constraint Satisfaction Problems**

# Intro Example: 8-Queens



Generate-and-test:  $8^8$  combinations

# Intro Example: 8-Queens



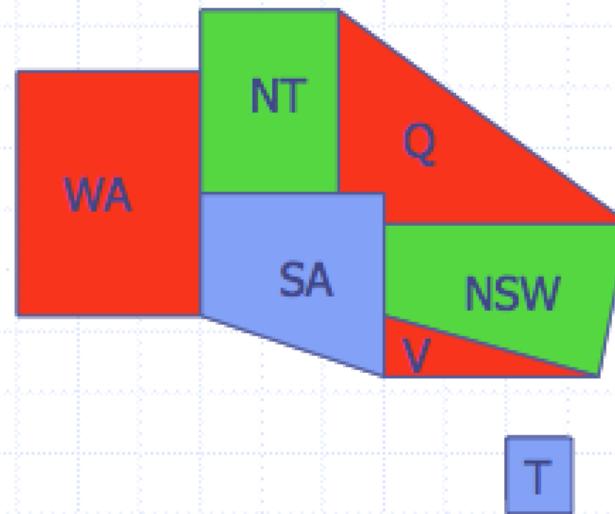
# Constraint Satisfaction Problem

- ◆ Set of variables  $\{X_1, X_2, \dots, X_n\}$
- ◆ Each variable  $X_i$  has a domain  $D_i$  of possible values
  - Usually  $D_i$  is discrete and finite
- ◆ Set of constraints  $\{C_1, C_2, \dots, C_p\}$ 
  - Each constraint  $C_k$  involves a subset of variables and specifies the allowable combinations of values of these variables
- ◆ Assign a value to every variable such that all constraints are satisfied

# Example: 8-Queens Problem

- ◆ 8 variables  $X_i$ ,  $i = 1$  to 8
- ◆ Domain for each variable  $\{1, 2, \dots, 8\}$
- ◆ Constraints are of the forms:
  - $X_i = k \rightarrow X_j \neq k$  for all  $j = 1$  to 8,  $j \neq i$
  - $X_i = k_i, X_j = k_j \rightarrow |i-j| \neq |k_i - k_j|$ 
    - ◆ for all  $j = 1$  to 8,  $j \neq i$

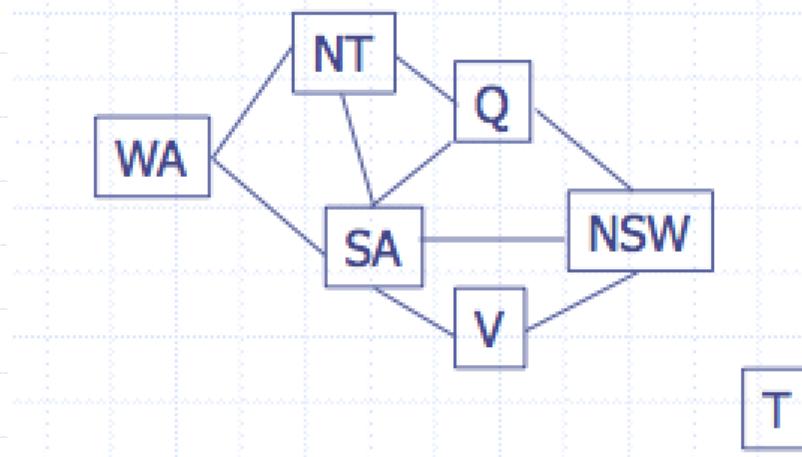
# Example: Map Coloring



- 7 variables {WA,NT,SA,Q,NSW,V,T}
- Each variable has the same domain {red, green, blue}
- No two adjacent variables have the same value:  
 $WA \neq NT, WA \neq SA, NT \neq SA, NT \neq Q, SA \neq Q, SA \neq NSW, SA \neq V, Q \neq NSW, NSW \neq V$

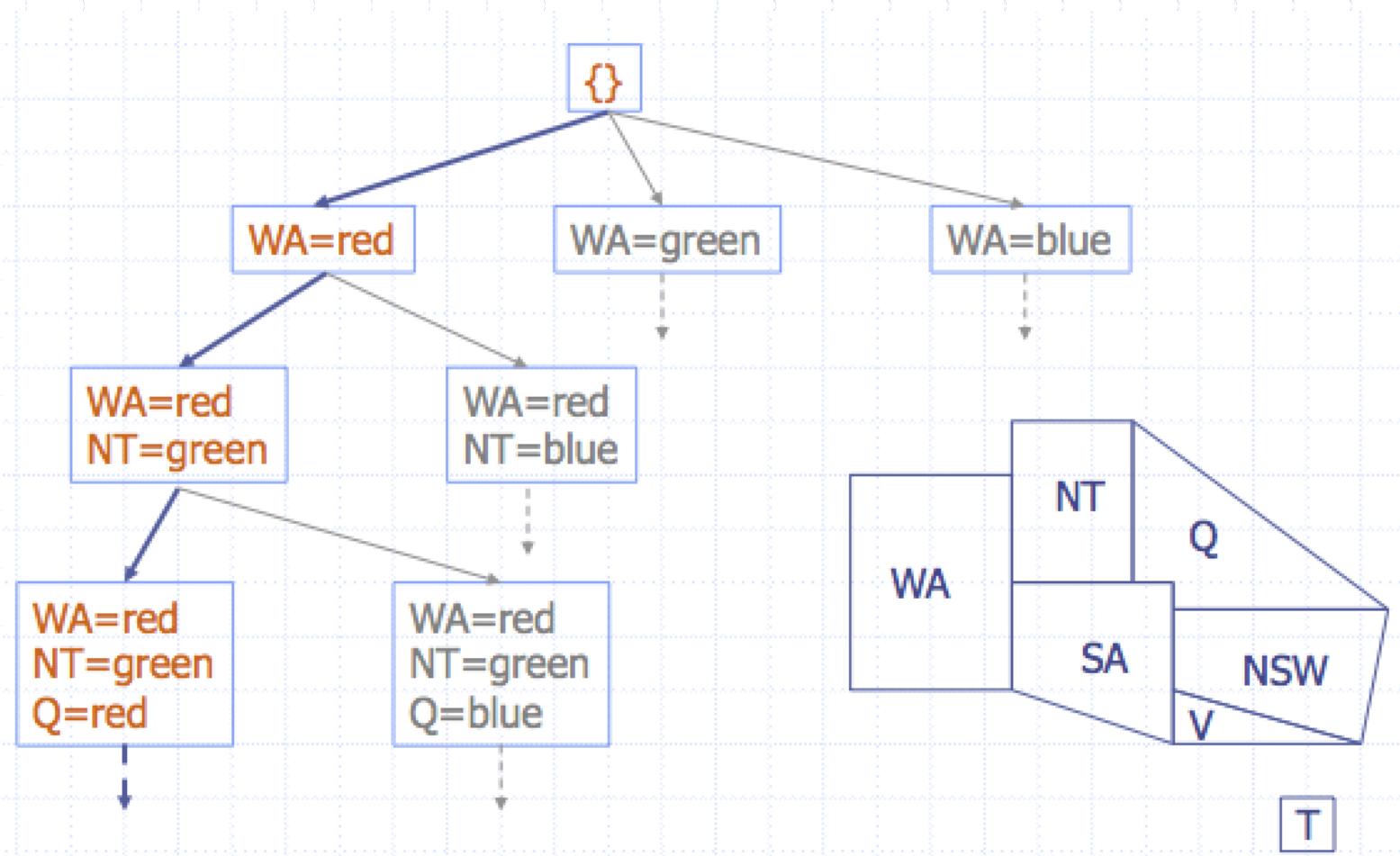
# Constraint Graph

Binary constraints



Two variables are adjacent or neighbors if they are connected by an edge or an arc

# Map Coloring



# Backtracking Algorithm

CSP-BACKTRACKING(PartialAssignment  $a$ )

- If  $a$  is complete then return  $a$
- $X \leftarrow$  select an unassigned variable
- $D \leftarrow$  select an ordering for the domain of  $X$
- For each value  $v$  in  $D$  do
  - ◆ If  $v$  is consistent with  $a$  then
    - Add  $(X = v)$  to  $a$
    - $\text{result} \leftarrow \text{CSP-BACKTRACKING}(a)$
    - If  $\text{result} \neq \text{failure}$  then return  $\text{result}$
- Return  $\text{failure}$

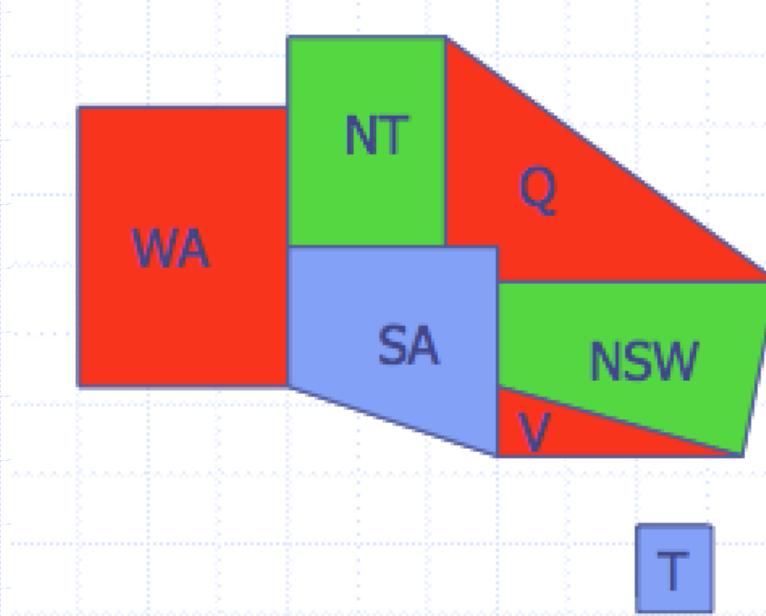
CSP-BACKTRACKING( $\{\}$ )

# Questions

1. Which variable  $X$  should be assigned a value next?
2. In which order should its domain  $D$  be sorted?
3. In which order should constraints be verified?

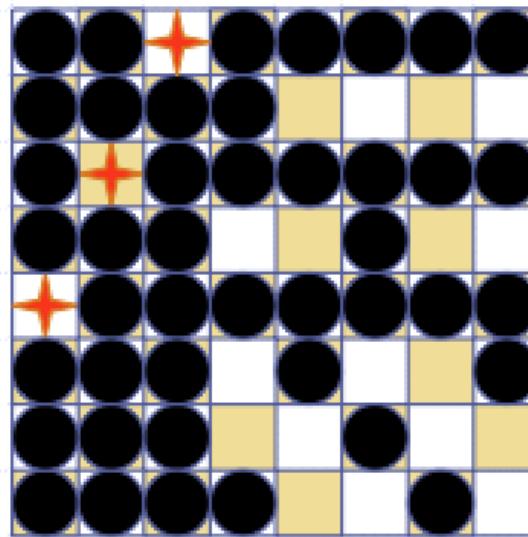
# Choice of Variable

## ◆ Map coloring



# Choice of Variable

◆ 8-queen



# Choice of Variable

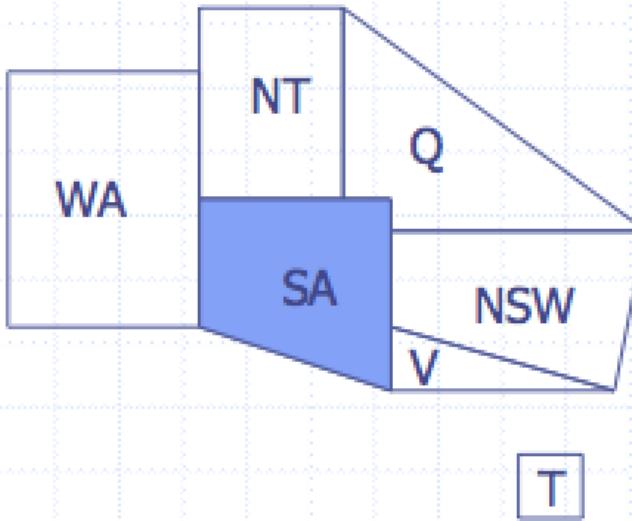
Most-constrained-variable heuristic:

Select a variable with the fewest remaining values

متغیری را اول انتخاب می کنیم که زودتر ما را به fail می رساند.  
به این خاطر که می دانیم تعداد بسیار زیادی شاخه داریم که fail می شود.

= Fail First Principle

# Choice of Variable



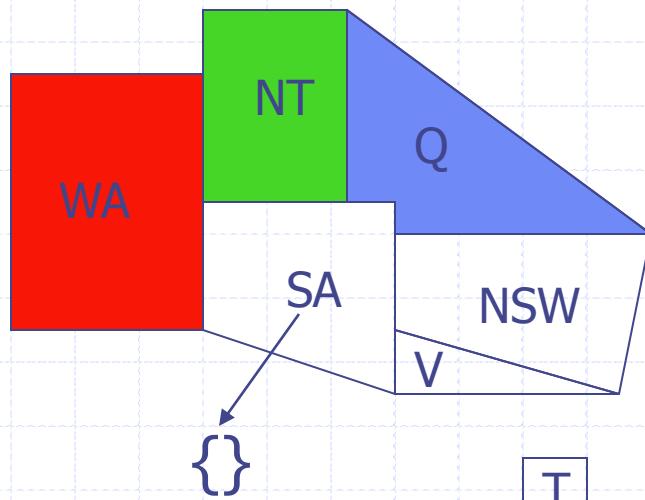
به متغیری مقدار دهیم که دامنه تعداد زیادی از متغیر ها را کوچک می کند  
بنابراین فضای جستجوی ما بسیار کوچکتر می شود.

Most-constraining-variable heuristic:

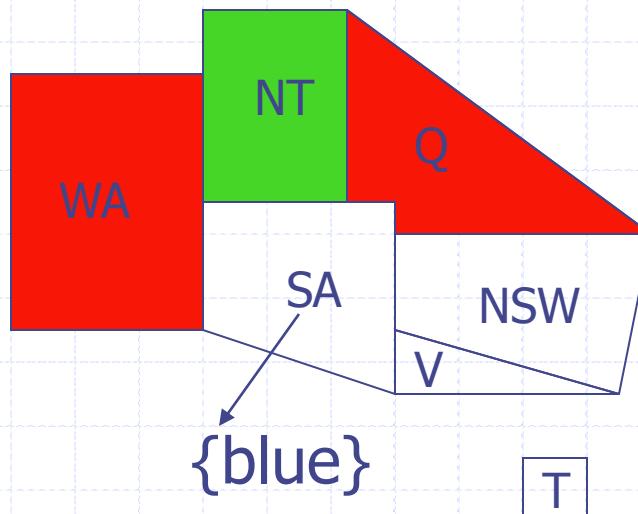
Select the variable that is involved in the largest number of constraints on other unassigned variables

= Fail First Principle again

# Choice of Value



# Choice of Value



Least-constraining-value heuristic:

Prefer the value that leaves the largest subset of legal values for other unassigned variables

# Choice of Constraint to Test

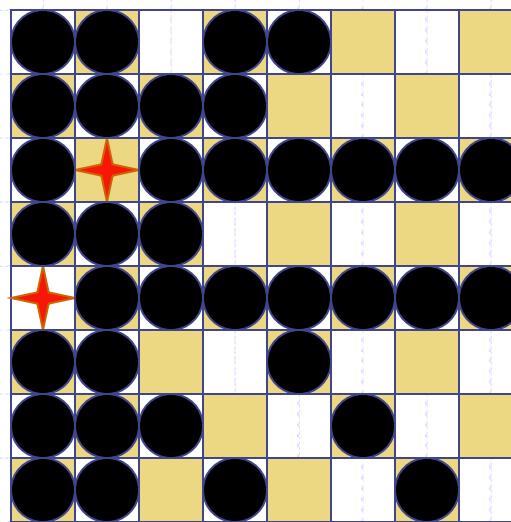
Most-constraining-Constraint:

Prefer testing constraints that are more difficult to satisfy

= Fail First Principle

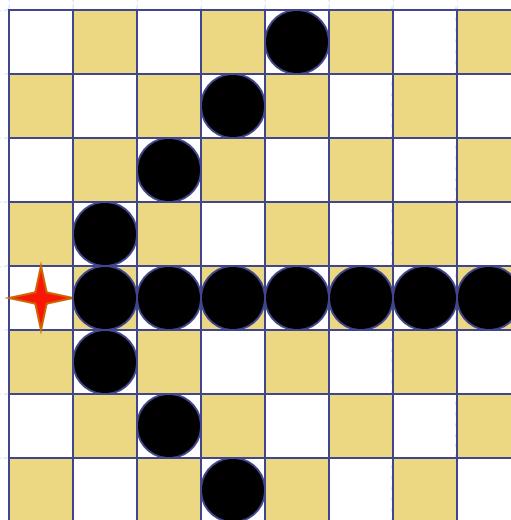
# Constraint Propagation ...

... is the process of determining how the possible values of one variable affect the possible values of other variables

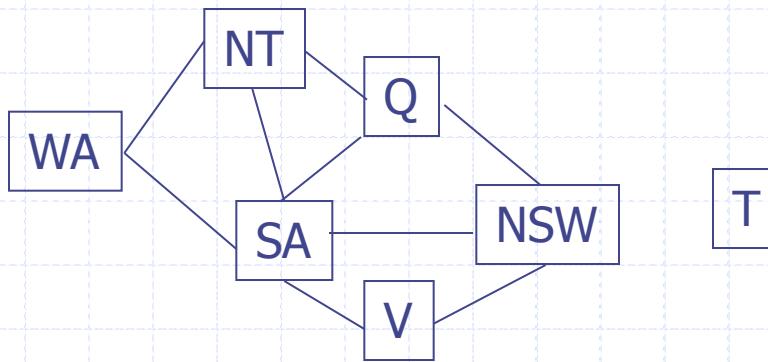


# Forward Checking

After a variable  $X$  is assigned a value  $v$ , look at each unassigned variable  $Y$  that is connected to  $X$  by a constraint and deletes from  $Y$ 's domain any value that is inconsistent with  $v$

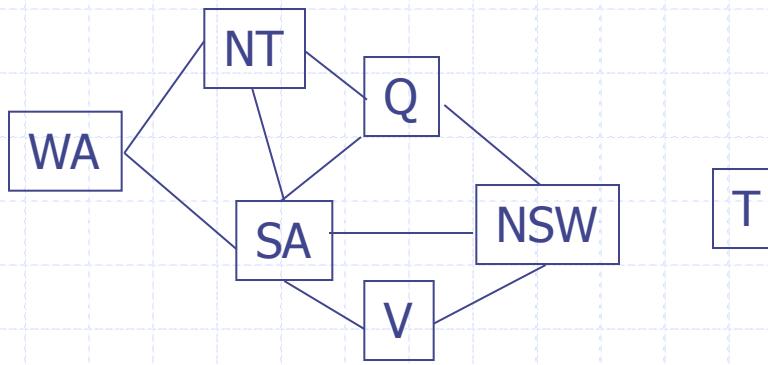


# Map Coloring



WA	NT	Q	NSW	V	SA	T
RGB						

# Map Coloring



WA	NT	Q	NSW	V	SA	T
RGB						
R	G	B	R	G	B	R

# Map Coloring

از سطر آخر می شد به این موضوع پی برد که

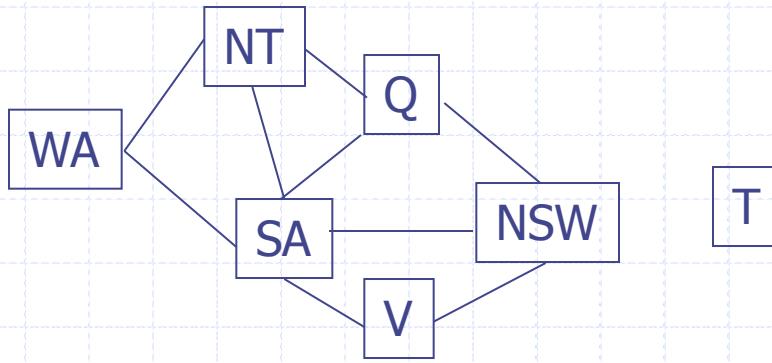
این حالت مقدار دهی به جواب نمی رسد

زیرا برای NT و SA فقط یک رنگ آبی باقی مانده است

در صورتی که این دو همسایه یک دیگر هستند پس نمی توانند یک رنگ باشند

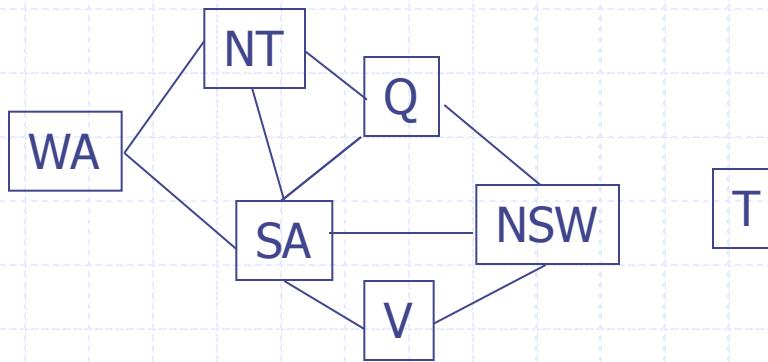
در واقع به حالتی می رسیم که هر طور به یک متغیر مقدار بدھیم دامنه یکی دیگر

از متغیر ها تهی می شود که این خلاف constraint های ماست !!!



WA	NT	Q	NSW	V	SA	T
RGB						
R	GB	RGB	RGB	RGB	GB	RGB
R	B	G	RB	RGB	B	RGB

# Map Coloring

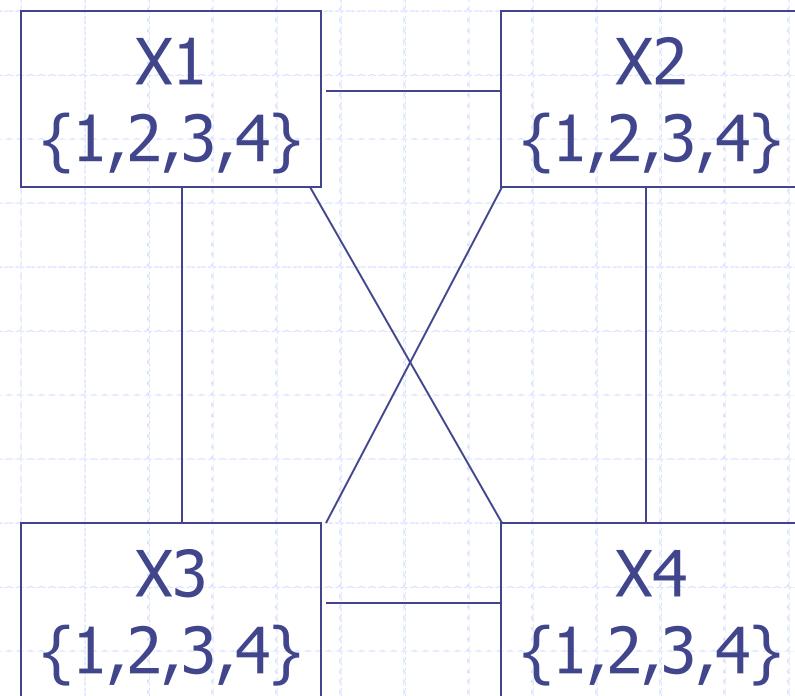


Impossible assignments that forward checking do not detect

WA	NT	Q	SA	NSW	V	T
RGB						
R	GB	RGB	RGB	RGB	GB	RGB
R	B	G	RB	RGB	B	RGB
R	B	G	R	B		RGB

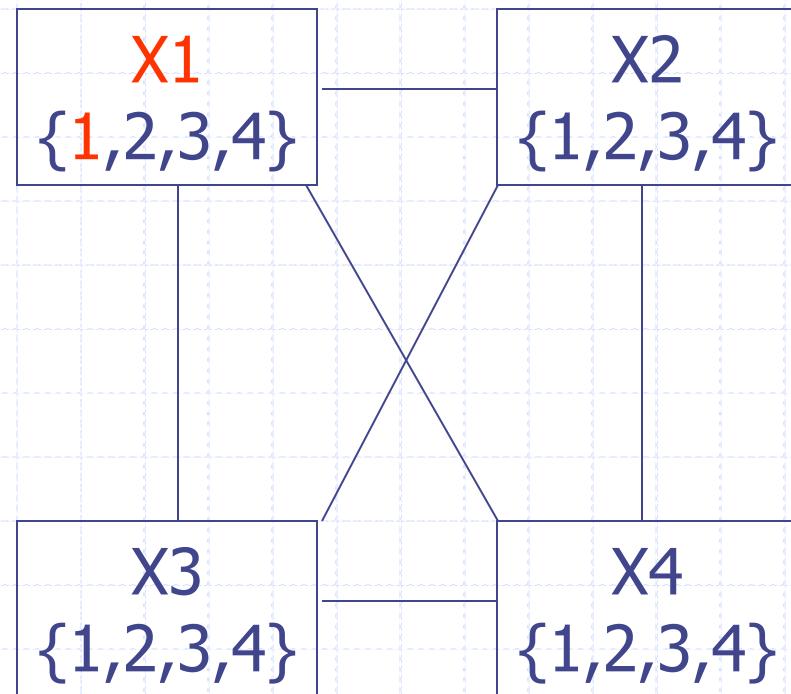
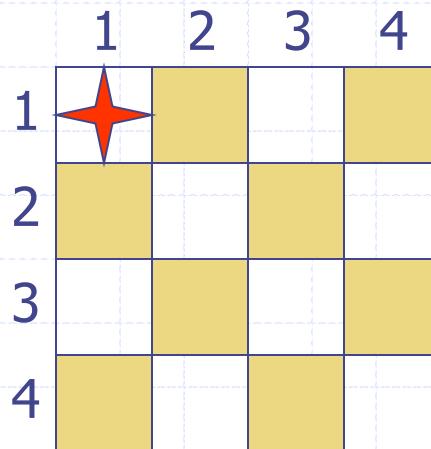
# Example: 4-Queens Problem

	1	2	3	4
1				
2				
3				
4				

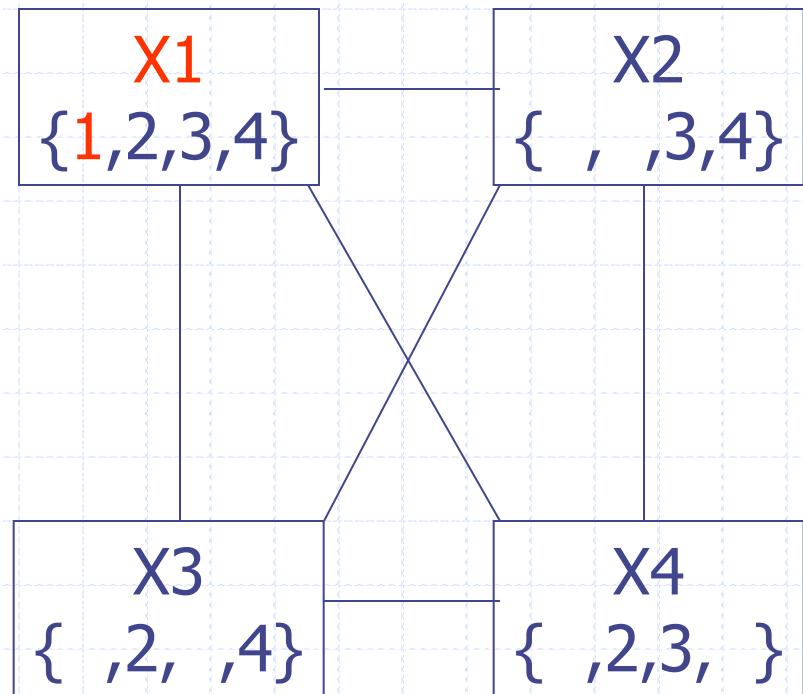
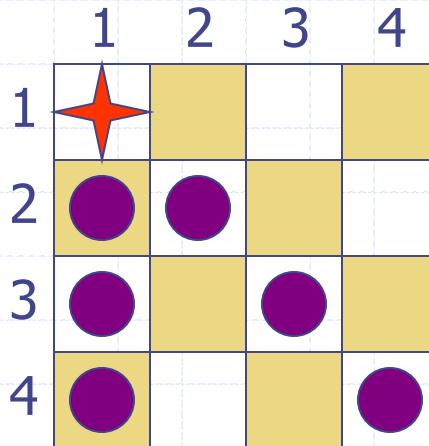


# Example: 4-Queens Problem

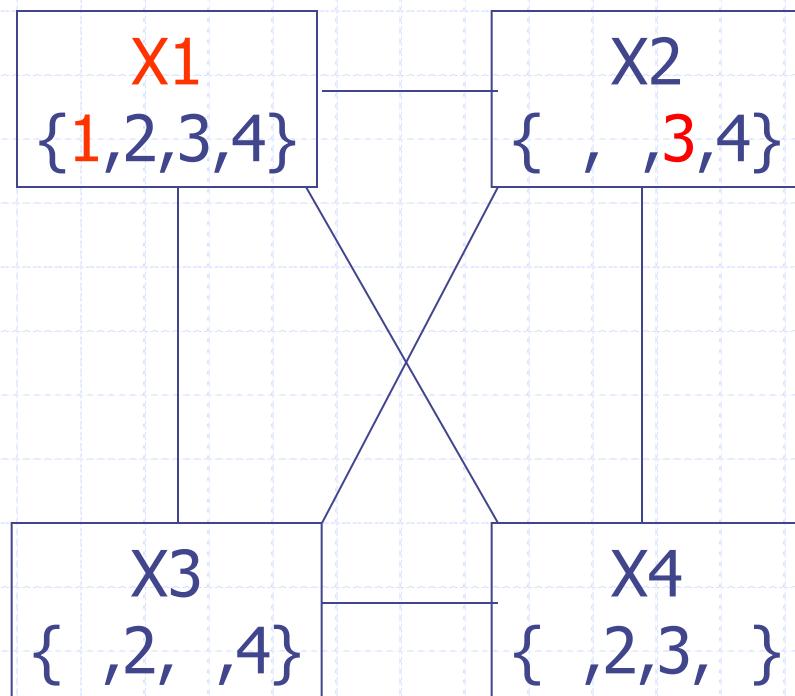
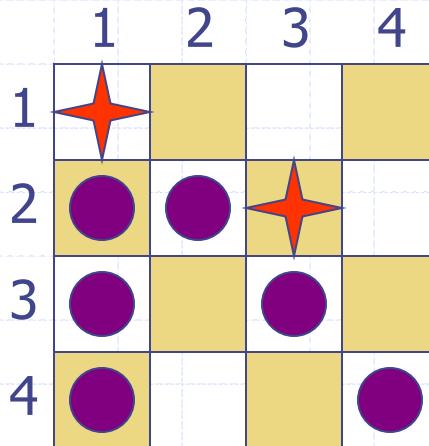
متغیرها را به ترتیب شماره سطر ها مقدار دهی می کنیم



# Example: 4-Queens Problem

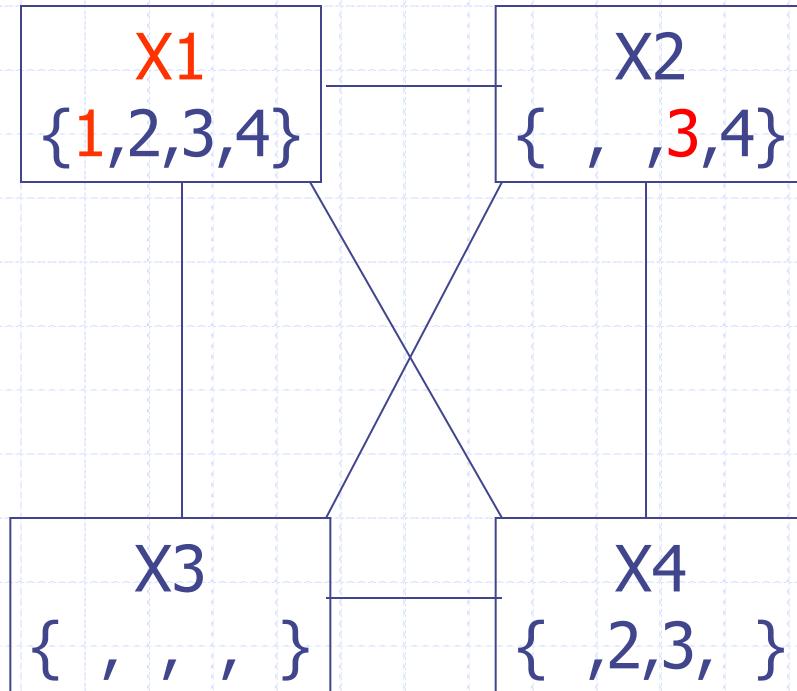
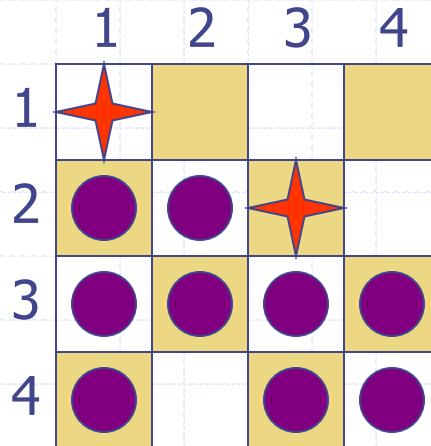


# Example: 4-Queens Problem



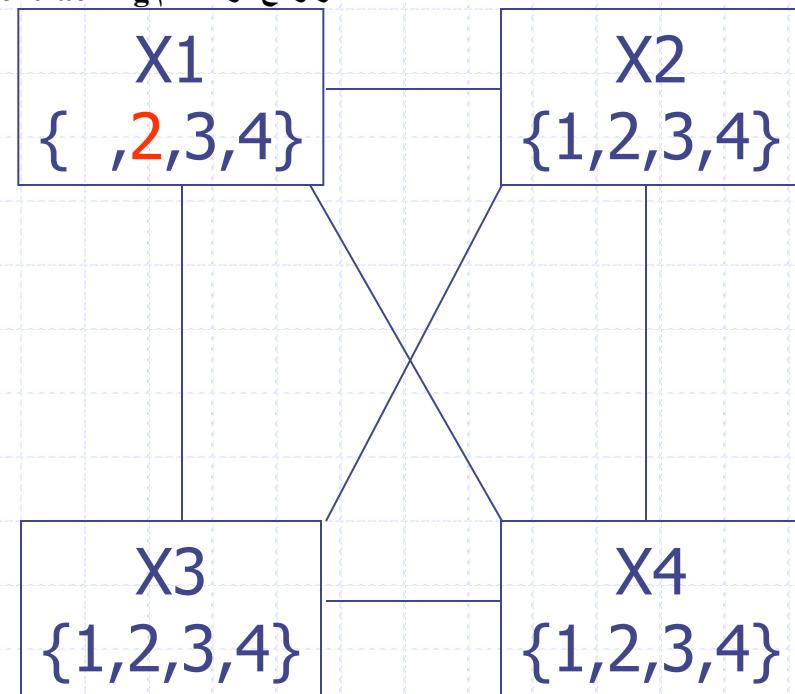
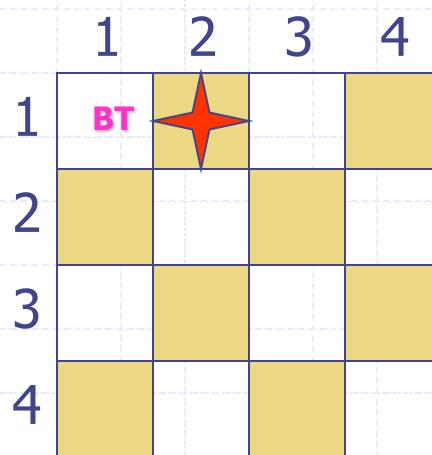
# Example: 4-Queens Problem

منتظر نمی مانیم از همین مرحله backtrack می کنیم

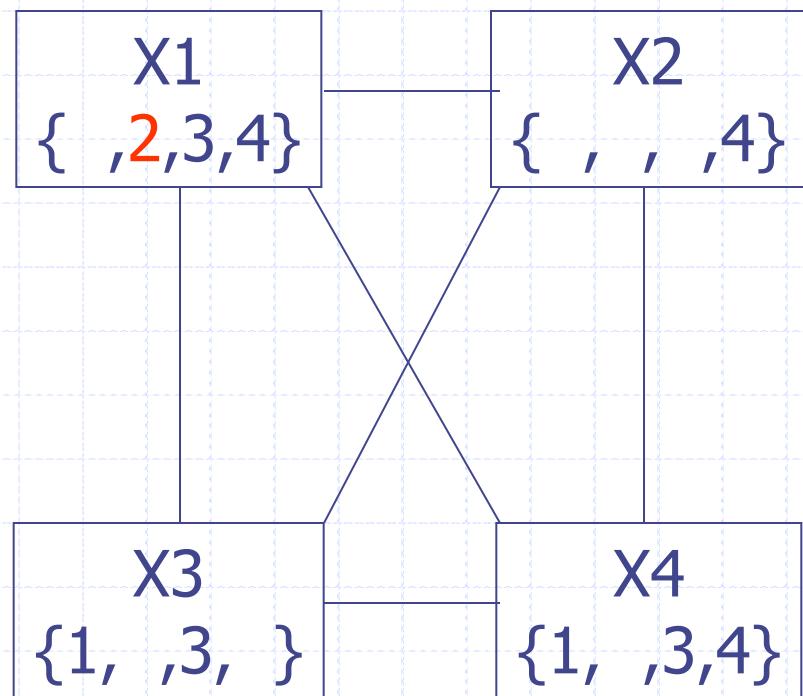
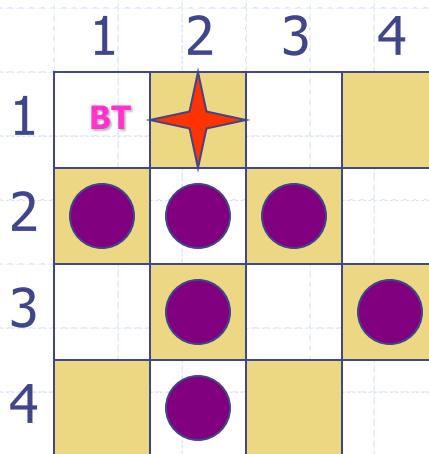


# Example: 4-Queens Problem

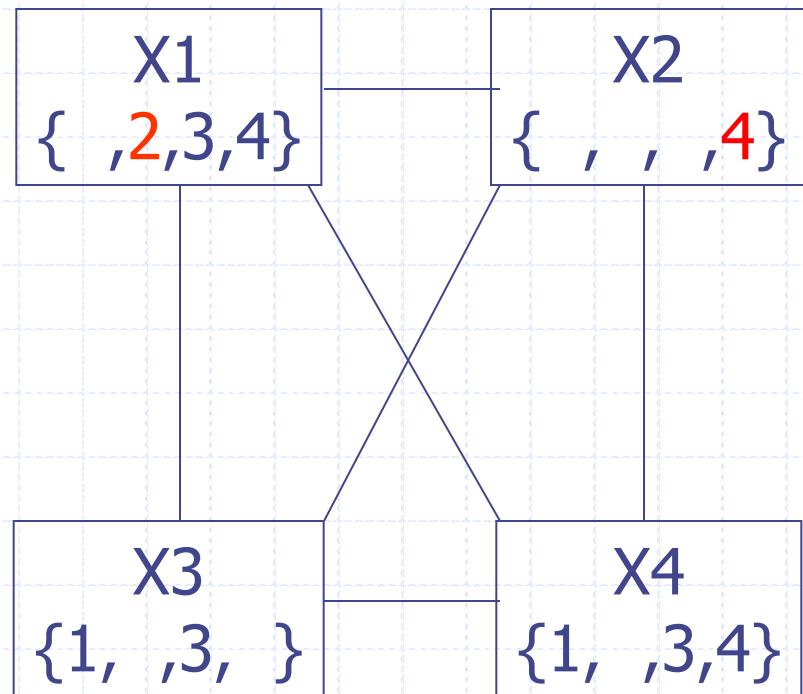
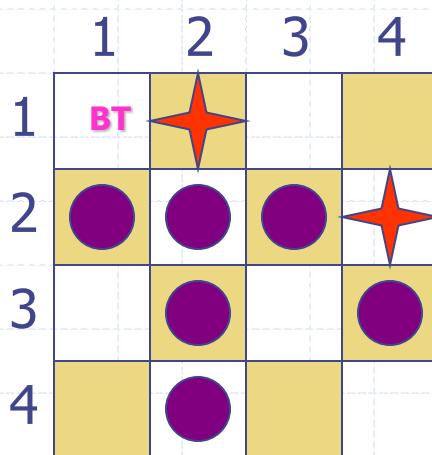
توجه: در این مرحله ما ابتدا جای وزیر دوم را عوض می کنیم (که به جواب نمی رسیم)  
و بعد باید به سراغ عوض کردن جای وزیر اول برویم  
در واقع در هنگام backtracking به ترتیبی که پیمایش را انجام دادیم backtrack می کنیم



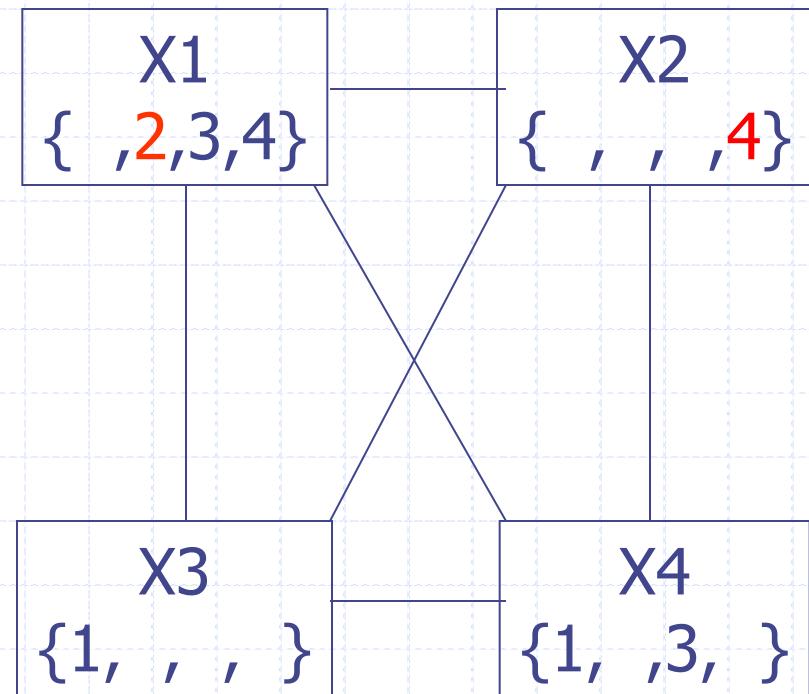
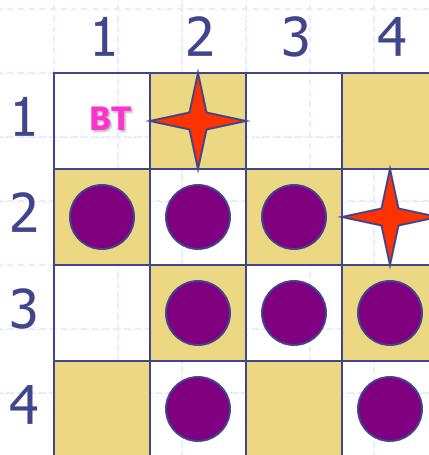
# Example: 4-Queens Problem



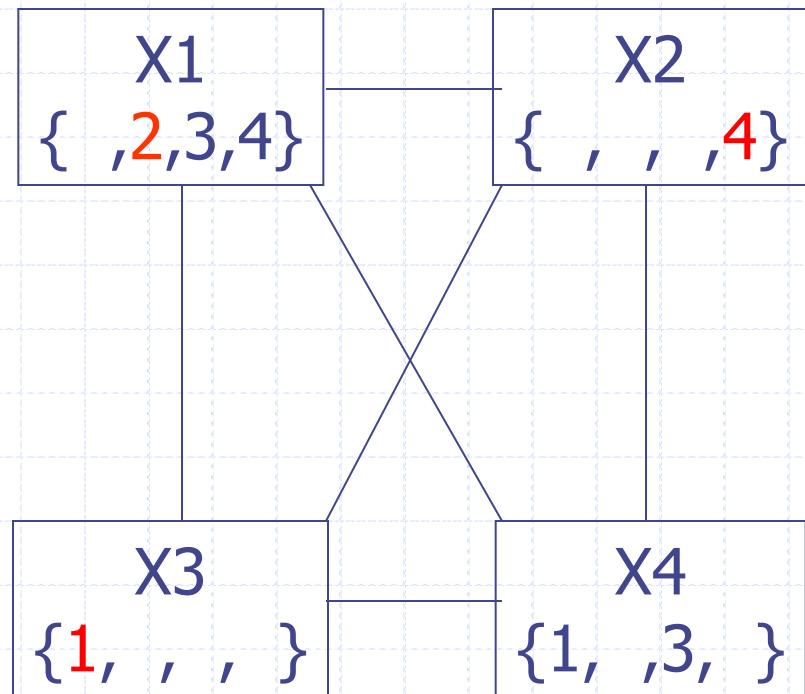
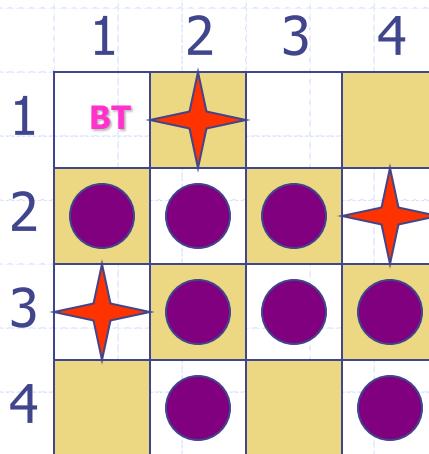
# Example: 4-Queens Problem



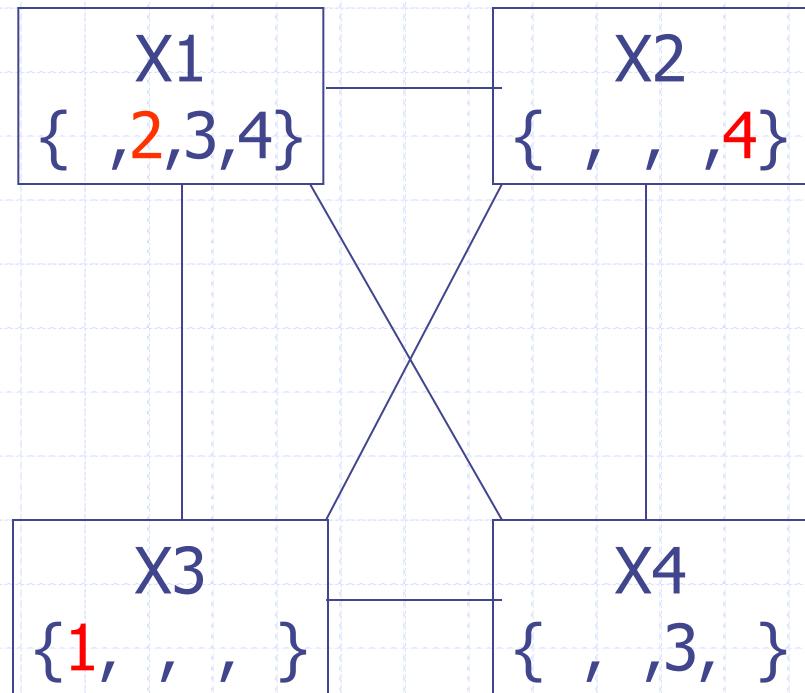
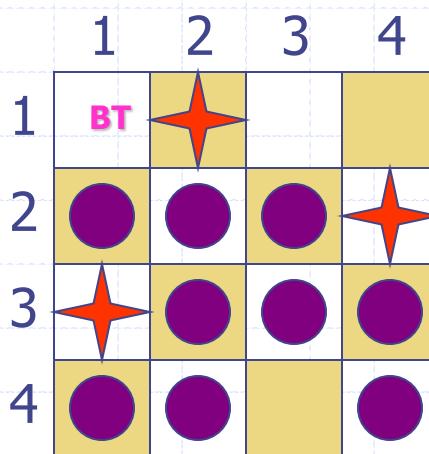
# Example: 4-Queens Problem



# Example: 4-Queens Problem



# Example: 4-Queens Problem



# Example: 4-Queens Problem

