طراحی و تحلیل الگوریتم ها

دکتر امیر لکی زاده

استادیار گروه مهندسی کامپیوتر دانشگاه قم

ضرب ماتریس ها(۱)

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SQUARE-MATRIX-MULTIPLY(A, B)
n = A.rows
let C be a new n × n matrix
for i = 1 to n
    for j = 1 to n
        c_ij = 0
        for k = 1 to n
            c_ij = c_ij + a_ik · b_kj
return C
```

ضرب ماتریس ها (۲)

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}, \quad C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},$$

so that we rewrite the equation $C = A \cdot B$ as

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.$$

Equation (4.10) corresponds to the four equations

$$\begin{aligned} C_{11} &= A_{11} \cdot B_{11} + A_{12} \cdot B_{21} , \\ C_{12} &= A_{11} \cdot B_{12} + A_{12} \cdot B_{22} , \\ C_{21} &= A_{21} \cdot B_{11} + A_{22} \cdot B_{21} , \\ C_{22} &= A_{21} \cdot B_{12} + A_{22} \cdot B_{22} . \end{aligned}$$

ضرب ماتریس ها (۲)

```
SQUARE-MATRIX-MULTIPLY-RECURSIVE(A, B)
n = A.rows
let C be a new n × n matrix
if n == 1
    c11 = a11 · b11
else partition A, B, and C as in equations (4.9)
    C11 = SQUARE-MATRIX-MULTIPLY-RECURSIVE(A11, B11)
          + SQUARE-MATRIX-MULTIPLY-RECURSIVE(A12, B21)
    C12 = SQUARE-MATRIX-MULTIPLY-RECURSIVE(A11, B12)
          + SQUARE-MATRIX-MULTIPLY-RECURSIVE(A12, B22)
    C21 = SQUARE-MATRIX-MULTIPLY-RECURSIVE(A21, B11)
          + SQUARE-MATRIX-MULTIPLY-RECURSIVE(A22, B21)
    C22 = SQUARE-MATRIX-MULTIPLY-RECURSIVE(A21, B12)
          + SQUARE-MATRIX-MULTIPLY-RECURSIVE(A22, B22)
return C
```

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 , \\ 8T(n/2) + \Theta(n^2) & \text{if } n > 1 . \end{cases}$$

ضرب ماتریس ها (۳)
Strassen Alg

$$
\begin{aligned}
S_1 &= B_{12} - B_{22} ,\\
S_2 &= A_{11} + A_{12} ,\\
S_3 &= A_{21} + A_{22} ,\\
S_4 &= B_{21} - B_{11} ,\\
S_5 &= A_{11} + A_{22} ,\\
S_6 &= B_{11} + B_{22} ,\\
S_7 &= A_{12} - A_{22} ,\\
S_8 &= B_{21} + B_{22} ,\\
S_9 &= A_{11} - A_{21} ,\\
S_{10} &= B_{11} + B_{12} .
\end{aligned}
$$

$$
\begin{aligned}
P_1 &= A_{11} \cdot S_1 &&= A_{11} \cdot B_{12} - A_{11} \cdot B_{22} ,\\
P_2 &= S_2 \cdot B_{22} &&= A_{11} \cdot B_{22} + A_{12} \cdot B_{22} ,\\
P_3 &= S_3 \cdot B_{11} &&= A_{21} \cdot B_{11} + A_{22} \cdot B_{11} ,\\
P_4 &= A_{22} \cdot S_4 &&= A_{22} \cdot B_{21} - A_{22} \cdot B_{11} ,\\
P_5 &= S_5 \cdot S_6 &&= A_{11} \cdot B_{11} + A_{11} \cdot B_{22} + A_{22} \cdot B_{11} + A_{22} \cdot B_{22} ,\\
P_6 &= S_7 \cdot S_8 &&= A_{12} \cdot B_{21} + A_{12} \cdot B_{22} - A_{22} \cdot B_{21} - A_{22} \cdot B_{22} ,\\
P_7 &= S_9 \cdot S_{10} &&= A_{11} \cdot B_{11} + A_{11} \cdot B_{12} - A_{21} \cdot B_{11} - A_{21} \cdot B_{12} .
\end{aligned}
$$

ضرب ماتریس ها (۳)
Strassen Alg

$$C_{11} = P_5 + P_4 - P_2 + P_6 .$$

$$C_{12} = P_1 + P_2 ,$$

$$C_{21} = P_3 + P_4$$

$$C_{22} = P_5 + P_1 - P_3 - P_7 ,$$

ضرب ماتریس ها (۳) Strassen Alg

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1\,, \\ 7T(n/2) + \Theta(n^2) & \text{if } n > 1\,. \end{cases}$$

$$T(n) = \Theta(n^{\lg 7}).$$

ضرب دو عدد n بیتی(۱)

$Multiply\ (u.v)\{$

$n = \max(number\ of\ digits\ in\ u\ and\ v)\ ;$

$else\ if\ (n \leq threshold)\ return\ (u \times v); \rightarrow$ یعنی اگر دو عدد کوچک بودند

$else\ \{M = \lfloor n/2 \rfloor;$

رخ می دهد $\theta\ (n) \leftarrow \begin{cases} x = u\ divide\ 10^M; y = u\ rem\ 10^M; \\ w = V\ divide\ 10^M; z = V\ rem\ 10^M; \end{cases}$

$return\ \big(multiply\ (x, w) \times 10^{2M} + Multiply(x, z) \times 10^M + \ Multiply\ (y, w)$

$\times 10^M + Multiply\ (y, z)\big);$

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$T(n) = 4T(n/2) + \ddot{4}\ T(n/2) + \theta(n) \rightarrow T(n) = \theta(n^2)$

ضرب دو عدد n بیتی(۲)

ضرب دو عدد n بیتی(۲)