طراحی و تحلیل الگوریتم ها

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تحلیل الگوریتم ها

Example 1.

```
/*return Position of largest value in "A" */
int largest(int[] A, int n) {
  int currlarge = 0;  // Position of largest
  for (int i=1; i<n; i++)
    if (A[currlarge] < A[i])
         currlarge = i; // Remember pos
  return currlarge;   // Return largest pos
}
```

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Example 2.

```
a = b;
```

This assignment takes constant time, so it is $\Theta(1)$.

Example 3.

```
sum = 0;
for (i=1; i<=n; i++)
   sum += n;
```

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Example 4.

```
sum = 0;
for (i=1; i<=n; i++)
   for (j=1; j<=n; j++)
      sum++;
}
```

i	j	
۱	1..n	n بار
۲	1..n	n بار
...		
n	1..n	n بار

تحلیل الگوریتم ها

Example 5.

```
sum2 = 0;
for (j=1; j<=n; j++)
   for (i=1; i<=j; i++)
      sum2++;
```

j	i	
1	1..1	1 بار
2	1..2	2 بار
…		
n-1	1..n-1	n-1 بار
n	1..n	n بار

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

تحلیل الگوریتم ها

Example 6.

```
sum = 0;
for (j=1; j<=n; j++)
   for (i=1; i<=j; i++)
      sum++;
for (k=0; k<n; k++)
   A[k] = k;
```

تحلیل الگوریتم ها

Example 7.

```
sum1 = 0;
for (k=1; k<=n; k*=2)
   for (j=1; j<=n; j++)
      sum1++;
```

تحلیل الگوریتم ها

3-2 *Relative asymptotic growths*

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

تحلیل الگوریتم ها

3-3 Ordering by asymptotic growth rates

a. Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{30}$ of the functions satisfying $g_1 = \Omega(g_2)$, $g_2 = \Omega(g_3)$, $\ldots$, $g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

$\lg(\lg^* n)$	$2^{\lg^* n}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$	$(\lg n)!$
$(\frac{3}{2})^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\ln \ln n$	$\lg^* n$	$n \cdot 2^n$	$n^{\lg \lg n}$	$\ln n$	1
$2^{\lg n}$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$\lg^*(\lg n)$	$2^{\sqrt{2 \lg n}}$	n	2^n	$n \lg n$	$2^{2^{n+1}}$

تحلیل الگوریتم ها

3-4 ***Asymptotic notation properties***

Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.

a. $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.

b. $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.

c. $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.

d. $f(n) = O(g(n))$ implies $2^{f(n)} = O\left(2^{g(n)}\right)$.

e. $f(n) = O\left((f(n))^2\right)$.

f. $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.

g. $f(n) = \Theta(f(n/2))$.

h. $f(n) + o(f(n)) = \Theta(f(n))$.