The Language of Chance

Probability is the model we use to measure and navigate a world of uncertainty.

This is the mathematics of chance, a fundamental tool for making informed decisions when outcomes are not guaranteed. It applies to everything from biology and finance to the very foundations of machine learning.

The Core Vocabulary: Experiments, Sample Spaces, and Events

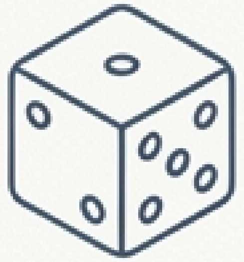
To speak the language of chance, we need three key terms:

Experiment: A repeatable process with one or more well-defined outcomes. (e.g., rolling a die, flipping a coin).

Sample Space (Ω): The set of all possible outcomes of an experiment.

Event (E): A specific set of outcomes; a subset of the sample space.

Visual Example: Rolling a Die



Experiment: Rolling a standard six-sided die.

Sample Space (Ω): {1, 2, 3, 4, 5, 6}

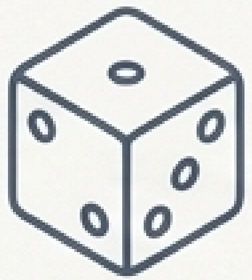
$$\{1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6\}$$

$$\{2 \quad 4 \quad 6\}$$

Event (E): Rolling an even number.
E = {2, 4, 6}

Defining the Universe of Possibilities

Let's map the sample space for three common experiments. The sample space contains every possible outcome, and it is impossible to get any outcome not in this set.



Experiment 1: Roll a common die

Sample Space $\Omega = \{1, 2, 3, 4, 5, 6\}$

Experiment 2: Flip a single coin

Sample Space $\Omega = \{\text{Heads, Tails}\}$

Experiment 3: Flip two coins

Sample Space $\Omega = \{\text{Heads Heads, Heads Tails, Tails Heads, Tails Tails}\}$

Assigning a Value to Chance

The probability of an event is the sum of the individual probabilities of the outcomes within that event. We denote these probability values $P_1, P_2, \ldots, P_n$. These values must follow two essential properties:

1. They are numbers between 0 and 1: $0 \leq P_i \leq 1$

2. Their sum equals 1: $\sum P_i = 1$

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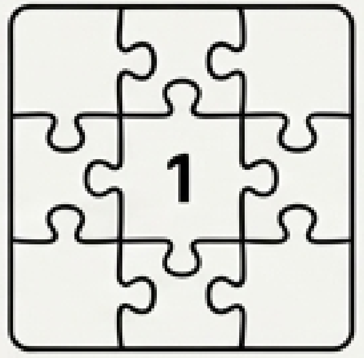
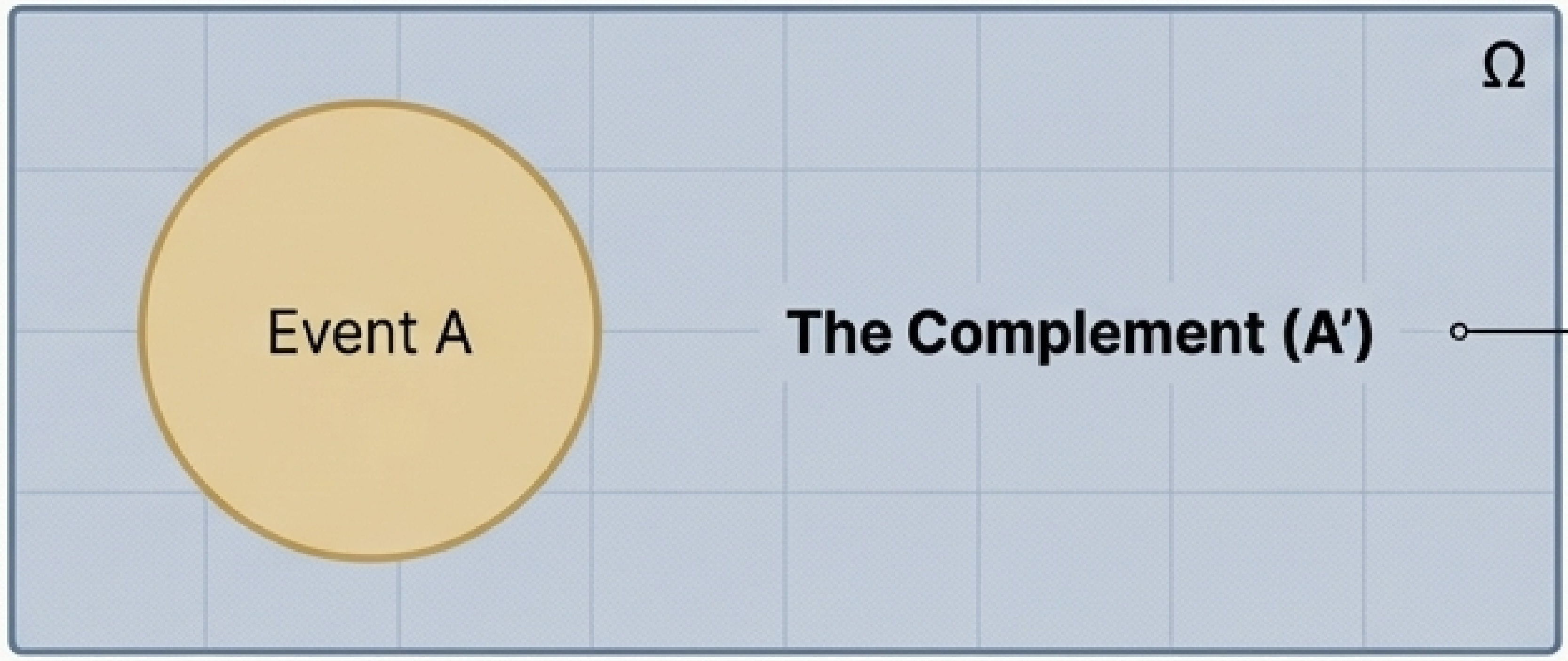


Theoretical vs. Experimental Probability

Theoretical	Experimental
For a fair coin, we assume P(Heads) = P(Tails) = 0.5. For two coins, the probability of each outcome is 1/4. This is based on the ideal model. **Two Fair Coins** P(HH) = 0.25 P(HT) = 0.25 P(TH) = 0.25 P(TT) = 0.25	Imagine flipping a coin 1000 times. What if it lands on its side once? Your observed probabilities might be different. This is derived from data. **1000 Flips** P(Heads) = 0.4995 P(Tails) = 0.4995 P(Side) = 0.001

The Elegant Shortcut: Using the Complement

Sometimes, calculating the probability of what you *don't* want is much easier than calculating the probability of what you *do* want.



the set of all outcomes in the sample space that are **not** in A.

Example: Rolling a Die
Sample Space Ω = {1, 2, 3, 4, 5, 6}
Let Event A be rolling a 1 or a 5. A = {1, 5}
The complement, A', is rolling any other number. A' = {2, 3, 4, 6}

The Power Formula: The probability of an event and its complement always sum to 1.

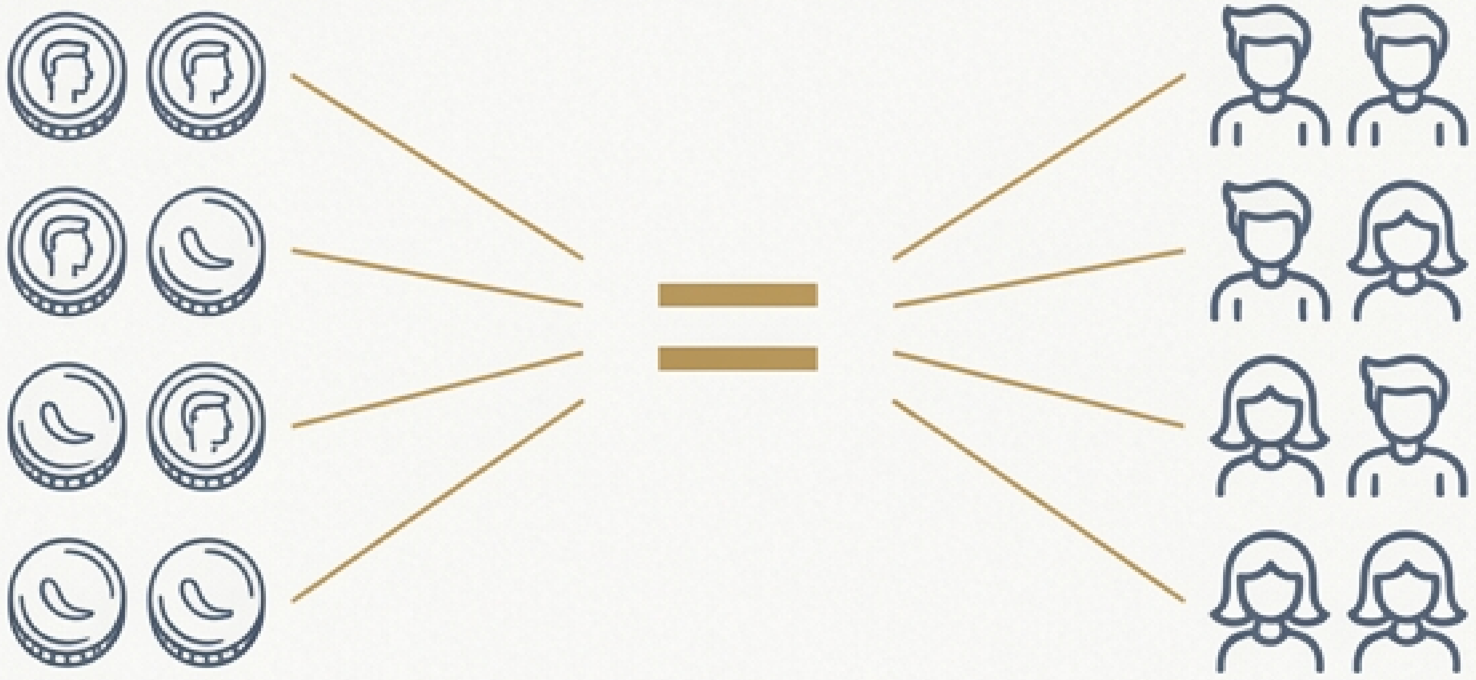
$$P(A) + P(A') = 1$$

This gives us a powerful shortcut: $P(A) = 1 - P(A')$

Application I: Modeling a Family

The Scenario: A couple plans to have two children. How can we model the possible outcomes and their probabilities?

This problem is structurally identical to flipping two coins. We can map the outcomes directly.



The Sample Space (Ω):
{Boy Boy, Boy Girl, Girl Boy, Girl Girl}

Assuming the probability of a boy or girl is equal (like a fair coin), each outcome has a probability of 1/4.

- P(BB) = **0.25**
- P(BG) = **0.25**
- P(GB) = **0.25**
- P(GG) = **0.25**

Calculating Family Probabilities

What is the probability they will have two boys?

This corresponds to a single outcome, BB.

P(Two Boys) = P(BB) = **1/4**

What is the probability they will have two boys?

This corresponds to a single outcome, BB.

What is the probability they will have one boy and one girl?

P(One Boy, One Girl) = P(BG) + P(GB) = 1/4 + 1/4 = **1/2**

This event includes two outcomes: Boy Girl and Girl Boy.

P(One Boy, One Girl) = P(BG) + P(GB) = 1/4 + 1/4 = **1/2**

What is the probability they will have two girls?

This event includes two outcomes: Boy Girl and Girl Boy.

P(Two Girls) = P(GG) = **1/4**

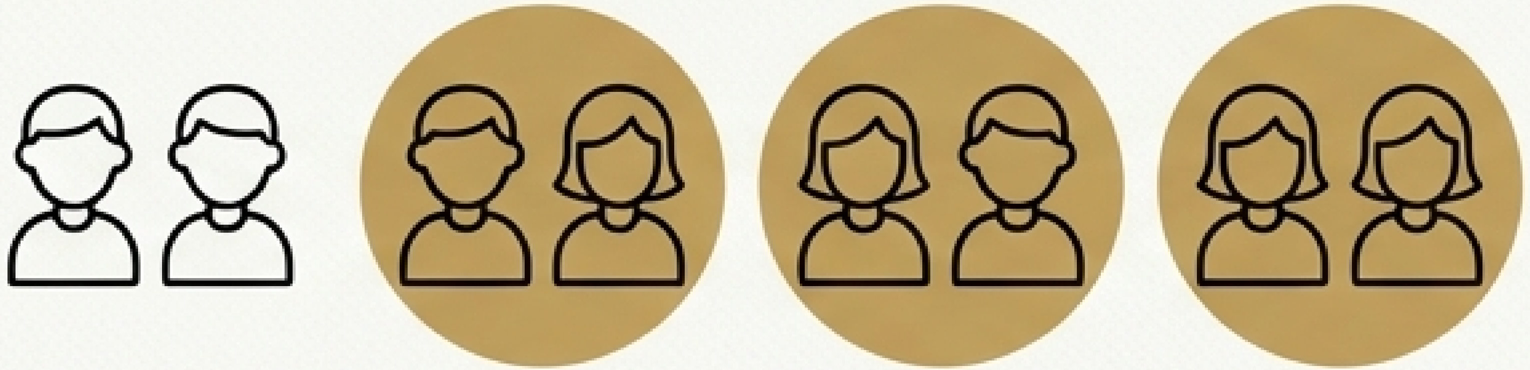
What is the probability they will have two girls?

This corresponds to a single outcome, GG.

The Shortcut in Action: “At Least One”

Question: What is the probability they will have **at least one girl**?

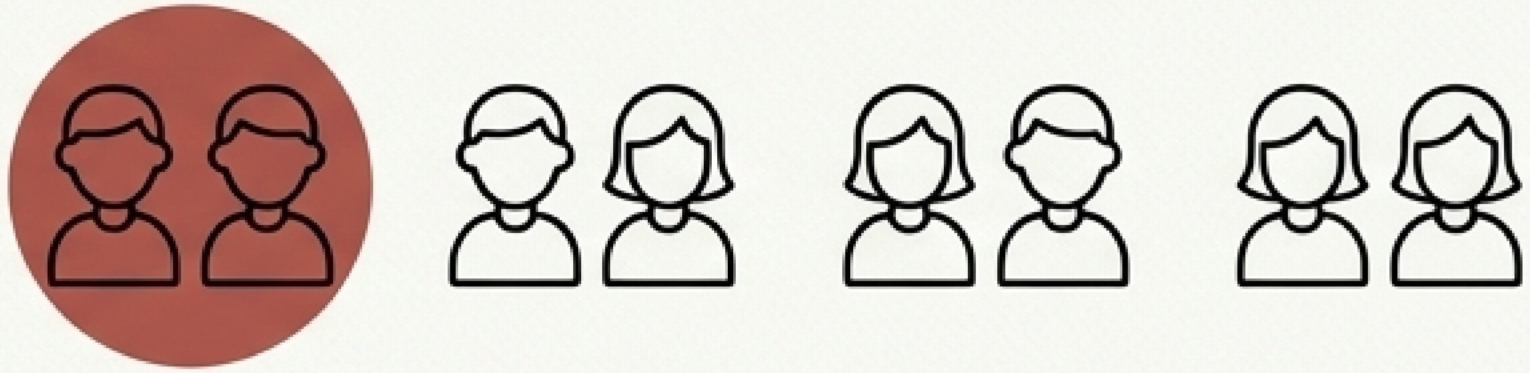
Method 1: Direct Calculation



The event “at least one girl” consists of three outcomes: {Boy Girl, Girl Boy, Girl Girl}.

P(at least one girl) = P(BG) + P(GB) + P(GG)
= 1/4 + 1/4 + 1/4 = **3/4**.

Method 2: The Elegant Shortcut (Using the Complement)



The complement of “at least one girl” is “no girls” (i.e., both are boys).

P(no girls) = P(BB) = 1/4.
P(at least one girl) = 1 - P(no girls) = 1 - 1/4 = **3/4**.

Application II: Software Reliability

The Scenario: A new software product is tested. Historical data provides the following probabilities for errors found in the code:

Probability of 0 errors: **0.05**
Probability of 1 error: **0.08**
Probability of 2 errors: **0.35**

Question 1: What is the probability of finding **no more than two errors**?

This is the event A = {0 errors, 1 error, 2 errors}.

$$P(A) = P(0) + P(1) + P(2)$$

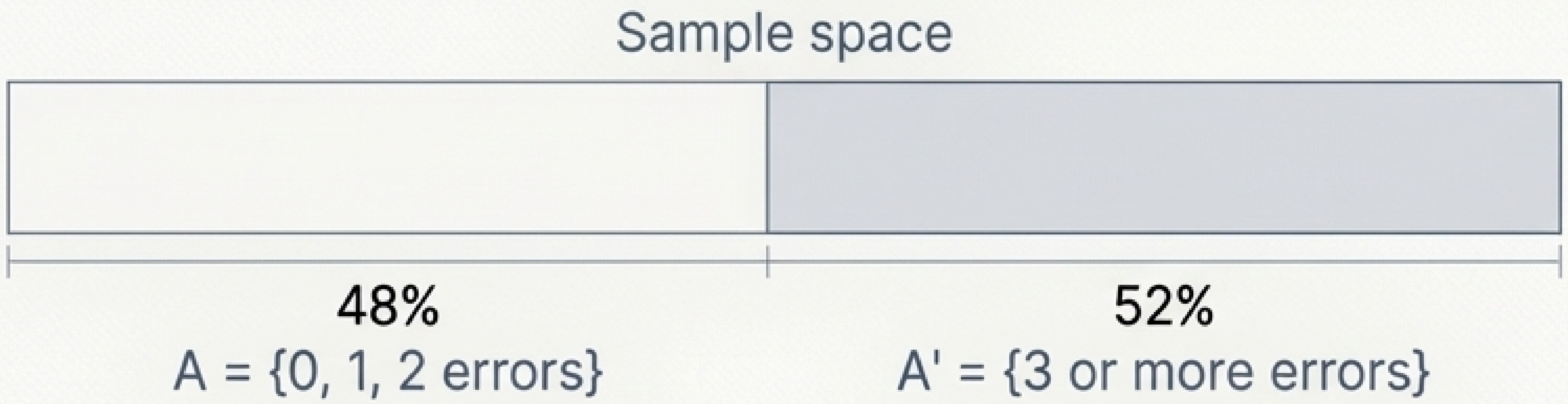
$$P(A) = 0.05 + 0.08 + 0.35 = \boxed{0.48}$$

Assessing Major Risk in Software

The Scenario (cont.): We know the probability of finding no more than two errors is 0.48.

Question 2: What is the probability of finding **three or more errors**?

Calculating this directly would require knowing P(3), P(4), etc., which we don't have.



The Shortcut

The event "three or more errors" is the complement of "no more than two errors" (which is Event A from the previous slide).

$P(\text{3 or more errors}) = P(A')$

$P(A') = 1 - P(A)$

$P(A') = 1 - 0.48 =$ **0.52**

Application III: Selecting a Leader

The Scenario: A company is choosing a new CEO. There are four candidates: Adam (A), Bill (B), Chris (C), and David (D). Based on internal analysis, the subjective probabilities of each candidate being chosen are:

Adam (A)
P(A) = 0.1

Bill (B)
P(B) = 0.2

Chris (C)
P(C) = 0.5

David (D)
P(D) = 0.2

The Key Information: Adam and Chris are internal candidates.

The Question: What is the probability that an internal candidate is chosen?

Why ‘2 out of 4’ is the Wrong Answer

A Common Pitfall

One might assume that since 2 of the 4 candidates are internal, the probability is...

$$2/4 \cancel{=} 0.5$$

Why This is Incorrect: This assumes all outcomes are equally likely. In this case, they are not. Candidate C is much more likely to be chosen than Candidate A.

The Correct Method

We must **sum** the probabilities of the individual outcomes that make up the event “an internal candidate is chosen.”

Event (Internal) = {Adam is chosen, Chris is chosen}

$$P(\text{Internal}) = P(A) + P(C)$$

$$P(\text{Internal}) = 0.1 + 0.5 = \boxed{\mathbf{0.6}}$$

The Rules of Chance: A Summary

Core Concepts

- **Experiment:** A repeatable process with defined outcomes.
- **Sample Space (Ω):** The set of all possible outcomes.
- **Event (E):** A subset of the sample space.

Fundamental Properties of Probability (P)

$0 \leq P(\text{outcome}) \leq 1$

The sum of probabilities for all outcomes in Ω is 1.

Calculating Event Probability

For equally likely outcomes:

$$P(E) = \frac{\text{(Number of favorable outcomes)}}{\text{(Total number of outcomes)}} = \frac{|E|}{|\Omega|}$$

General formula:

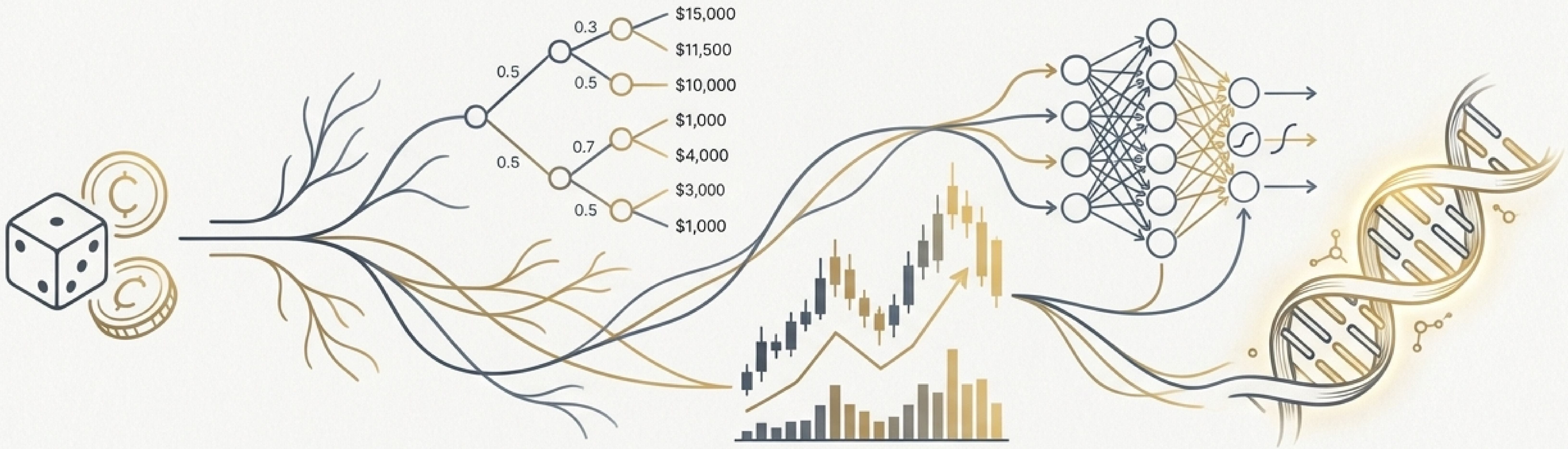
$P(E)$ = **Sum of probabilities** of all outcomes in E.

The Complement Rule

The complement of **A** is **A′** (all outcomes not in A).

$P(A) = 1 - P(A')$

From Simple Rules to Strategic Advantage



The principles of probability—from sample spaces to the complement rule—are more than just mathematical exercises.

They form a rigorous language for modeling uncertainty and quantifying risk.

Mastering this language provides a strategic advantage, enabling better decision-making in complex domains like software engineering, financial markets, scientific research, and machine learning. The mathematics of chance is the foundation for navigating an uncertain future.