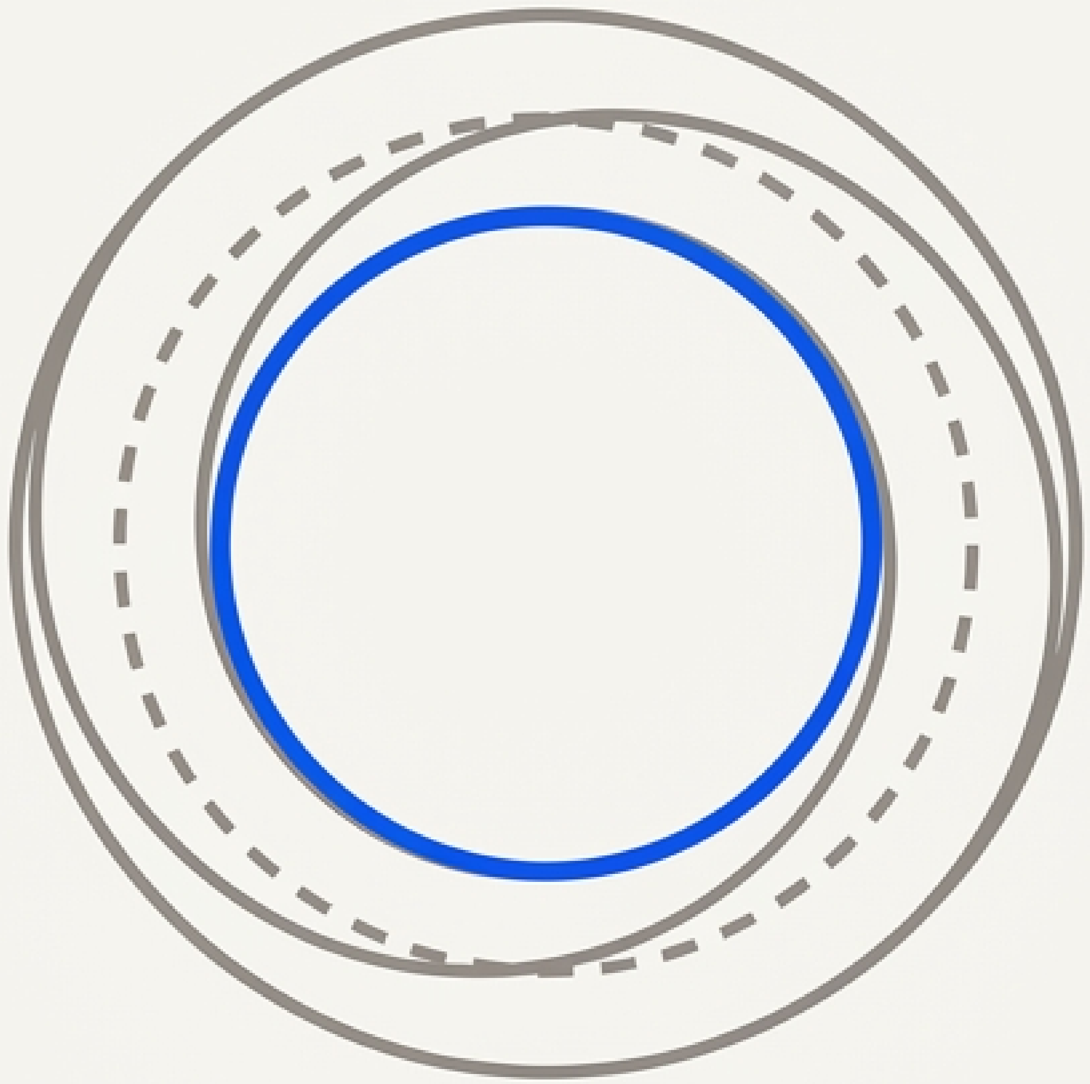


The Power of Context

A Strategist’s Guide to Conditional Probability

Let's Start with a Simple Universe

We begin with a complete dataset. Consider the following breakdown of 178 individuals:

	Pierced Ears (Yes)	Pierced Ears (No)	Total
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

From this table, we can calculate some basic probabilities for a randomly selected person:

P(Male) = **90** / 178
P(Pierced Ears) = **103** / 178
P(Male AND Pierced Ears) = **19** / 178
P(Male OR Pierced Ears) = P(Male) + P(Pierced) – P(Male AND Pierced)
= (**90**/178) + (**103**/178) - (**19**/178) = **174** / 178

Now, Let’s Add Context.

The question changes when we are given a piece of information.
What is the probability a person is male, *GIVEN* that we know they have pierced ears?

	Pierced Ears (Yes)	Pierced Ears (No)	Total
Pierced Ears (No)	71	4	75
Pierced Ears (Yes)	19	84	**103**
	Male	Female	Total
	90	88	~~178~~

Intuitively, if we *know* the person has pierced ears, our world of possibilities shrinks. We are no longer considering all 178 people.

We are only looking at the 103 people in the “Pierced Ears (Yes)” category.

$$P(\text{Male} \mid \text{Pierced Ears}) = \frac{19}{103}$$

The Question's Direction Shapes the Answer

Does the order matter? Let's ask the reverse question: What is the probability a person has pierced ears, *GIVEN* we know they are male?

	Pierced Ears (Yes)	Pierced Ears (No)	Total
Male	19	71	90
Female	84	4	88
Total	103	75	~~178~~

This time, our given information is that the person is male. Our **universe** of possibilities shrinks to the 90 males in the dataset.

$$P(\text{Pierced Ears} \mid \text{Male}) = \frac{19}{90}$$

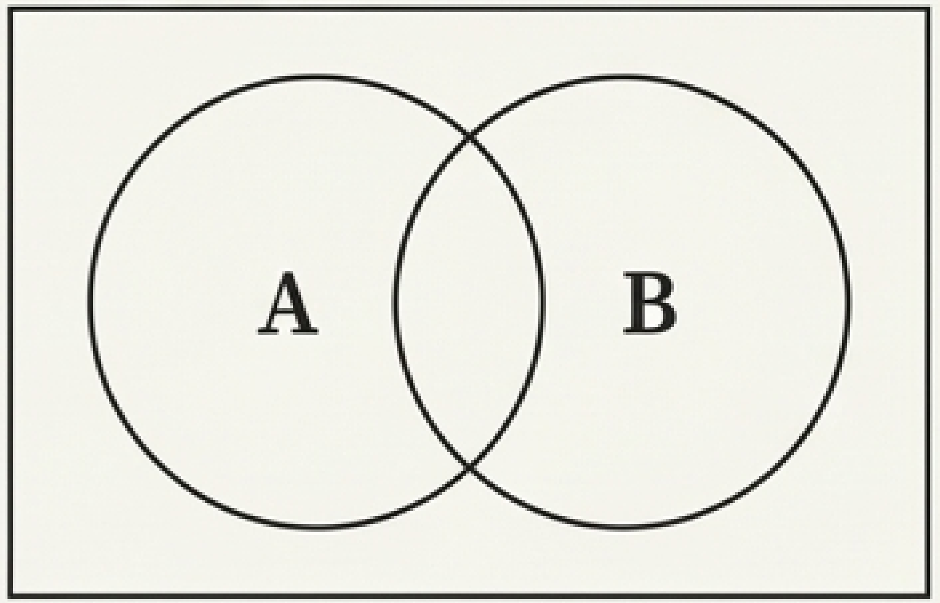
Key Insight*: 19/90 ≠ 19/103. The 'given' condition defines the sample space.

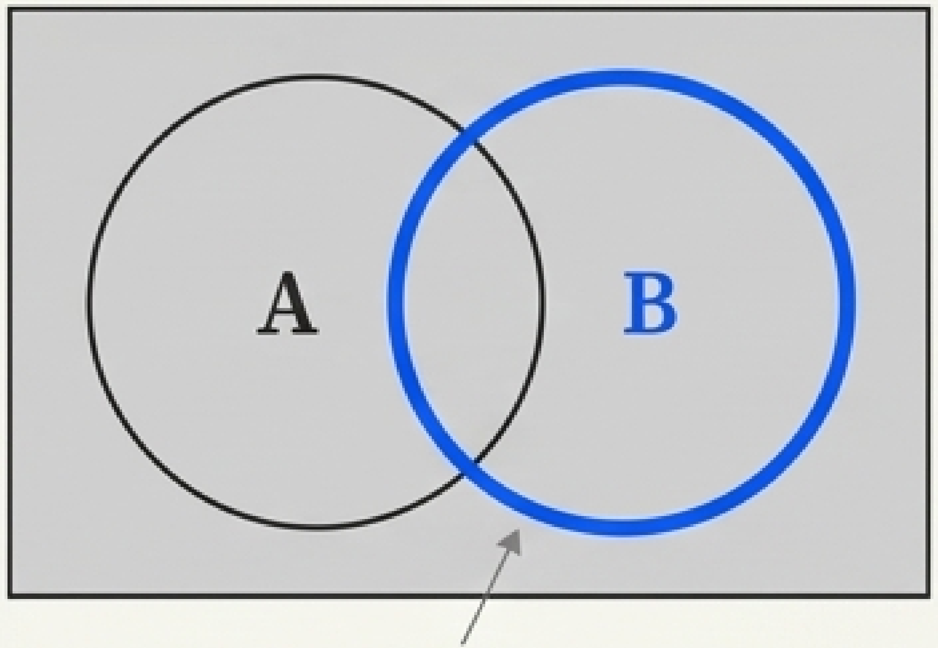
Giving Our Intuition a Language

The process we just performed is called **conditional probability.** It's the probability that an event A happens *given* that event B has already occurred.

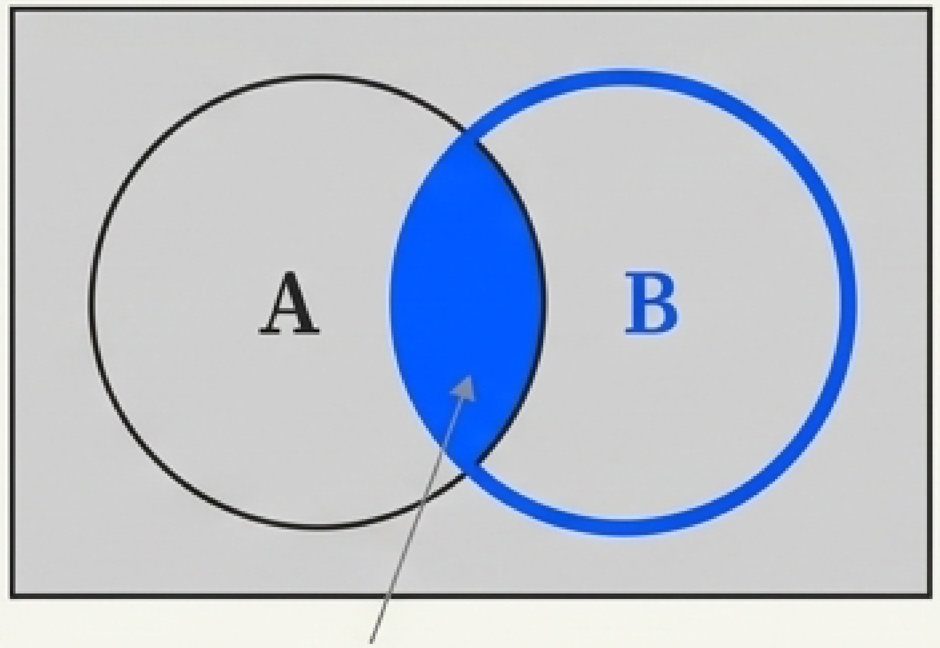
We write this as **P(A|B).**

Sample Space





Given B has occurred, our universe shrinks to the set of outcomes in B.



The successful outcomes are those in A that are also in our new universe, B. This is the
This is the intersection A ∩ B.

The Universal Formula for Conditional Probability

Our intuitive action of shrinking the sample space and focusing on the intersection is captured by a single formula.

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

Validation: Let's apply this to our first question: P(Male | Pierced Ears).

A = Male, B = Pierced Ears

From our original table (Slide 2), we know:

$P(\text{Male} \cap \text{Pierced Ears}) = 19 / 178$
$P(\text{Pierced Ears}) = 103 / 178$

$P(\text{Male} \mid \text{Pierced Ears}) = (19 / 178) / (103 / 178) = 19 / 103$

Key Insight: The formula perfectly matches our intuitive result from Slide 3. It's a formal language for the logic we've already used.

Putting the Formula to Work

We roll two standard six-sided dice. What is the probability that the sum is 5, given that the product of the two dice is 6?

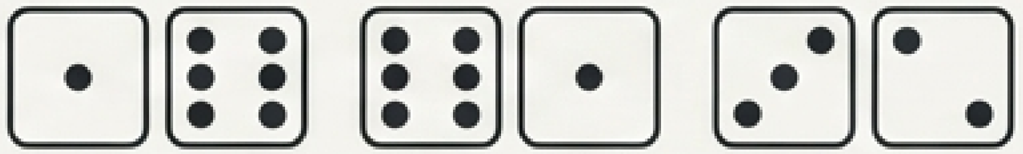
1. Define Events

A: The sum is 5.
B: The product is 6.

We want to find $P(A|B)$.

2. Identify the New Sample Space (Event B)

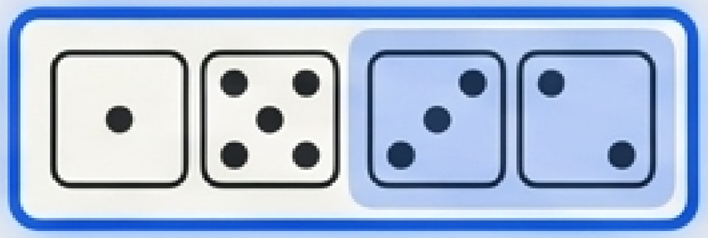
The outcomes where the product is 6 are: $\{(1, 6), (6, 1), (2, 3), (3, 2)\}$.



The total sample space for two dice has 36 outcomes. Therefore, $P(B) = 4/36$.

3. Identify the Intersection ($A \cap B$)

Which of the outcomes in B also have a sum of 5? $\{(2, 3), (3, 2)\}$.



Therefore, $P(A \cap B) = 2/36$.

4. Apply the Formula

$P(A|B) = P(A \cap B) / P(B) = (2/36) / (4/36) = 2/4 = 1/2$. **1/2.**

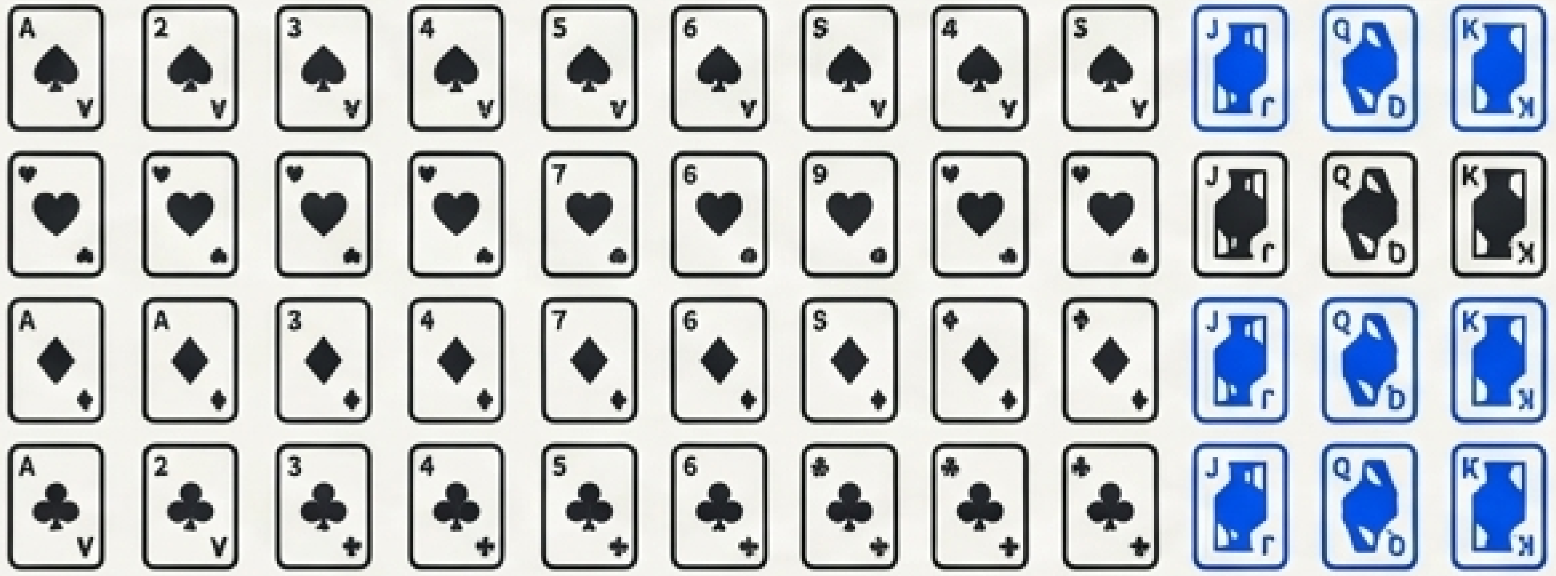
When Context Doesn't Matter: Independence

Sometimes, knowing that event B occurred gives us no new information about the probability of A.
This is called **independence.**

Events A and B are independent if **P(A|B) = P(A).**

Baseline Probability

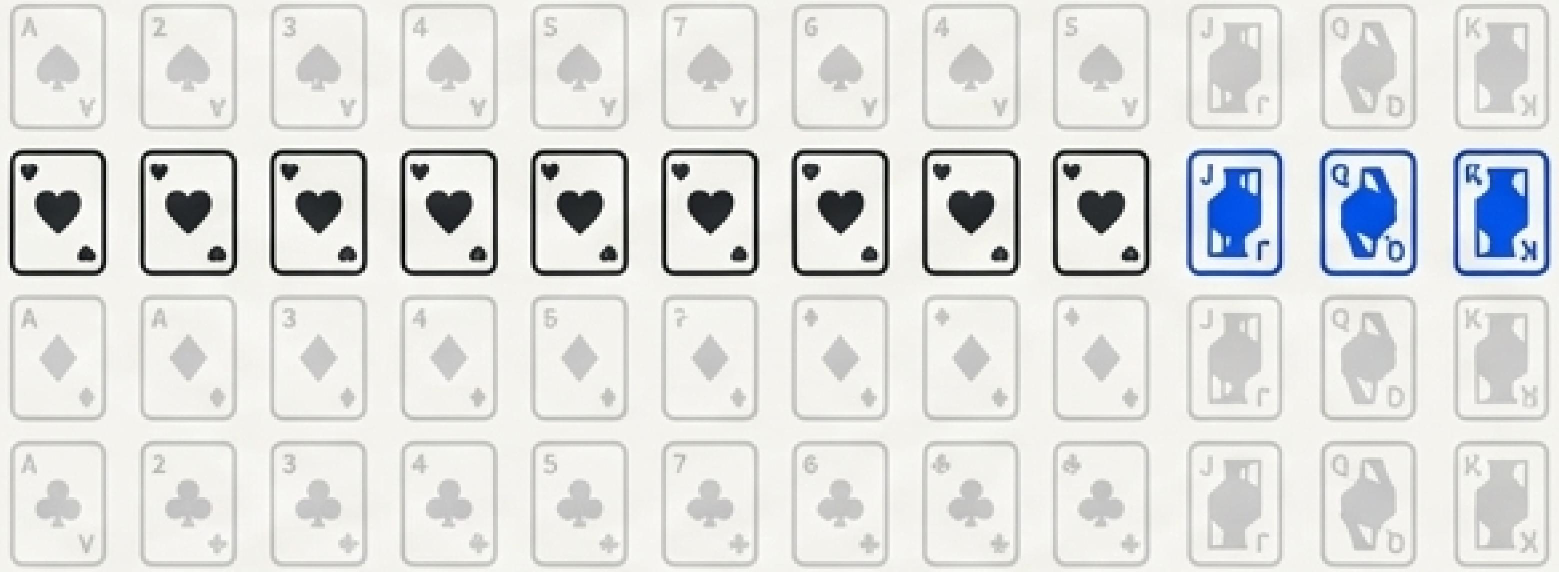
Event A:
Pick a Jack, Queen, or King.



There are 12 face cards in a 52-card deck.
P(A) = 12/52 = 3/13.

Conditional Probability

Event B: Pick a Heart.
Question: What is P(A|B)?



Our new sample space is the 13 Hearts. **P(B)** = 13/52.
The intersection (J, Q, or K of Hearts) consists of 3 cards. **P(A ∩ B)** = 3/52.
P(A|B) = P(A ∩ B) / P(B) = (3/52) / (13/52) = 3/13.

Since **P(A|B) = P(A)** = 3/13, the events are independent. Knowing the card is a Heart doesn't change the probability of it being a face card.

From Conditioning to Intersection: The Multiplication Rule

By rearranging the conditional probability formula, we get a powerful tool for finding the probability of an intersection, especially for sequential events.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiply both sides by P(B):

$$\mathbf{P(A \cap B) = P(B) * P(A|B)}$$

This is the general way to find the probability of two events both happening. It reads: "The probability of B happening, multiplied by the probability of A happening *given that* B already happened."

Connection to Independence

If A and B are independent, $P(A|B)$ simplifies to $P(A)$. This gives us the special case for independent events:

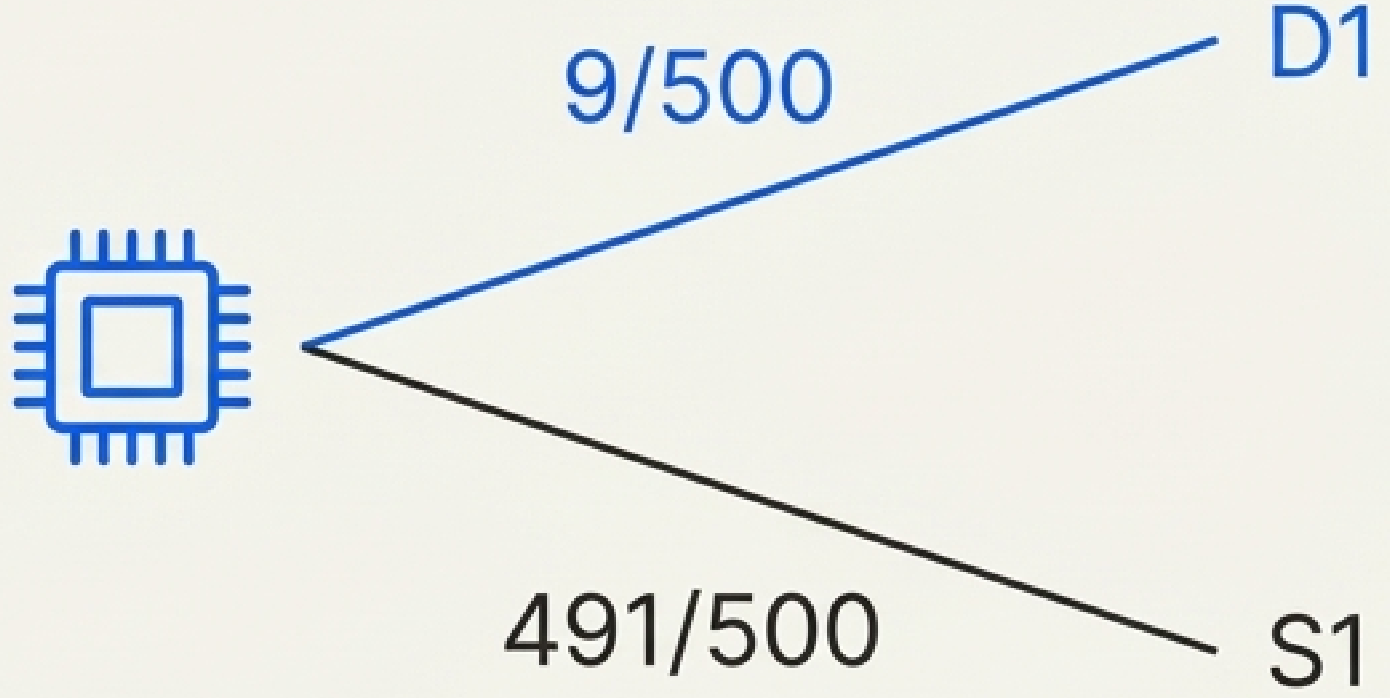
$$\mathbf{P(A \cap B) = P(B) * P(A)}$$

Case Study: Sampling Without Replacement

A box contains 500 microchips. 9 are defective (D) and 491 are satisfactory (S). We sample 3 chips at random, *without replacement*. What is the probability of choosing three defective chips?

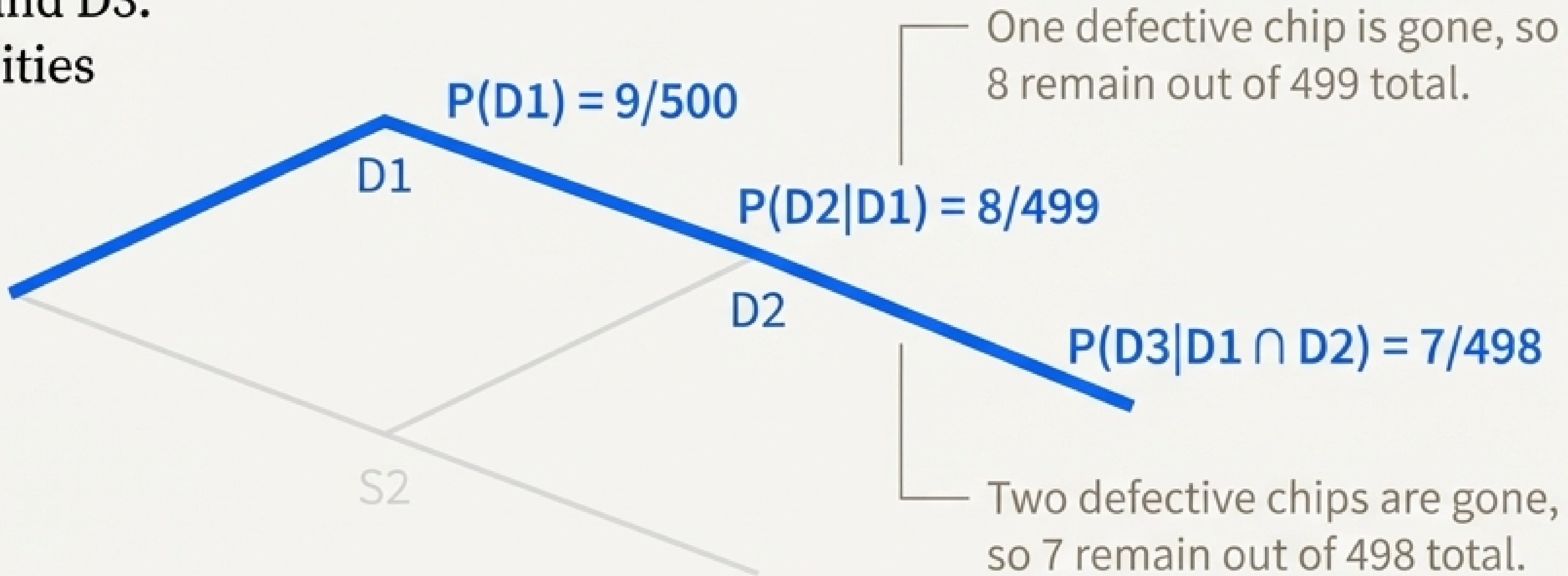
This is a sequence of events where the outcome of each draw depends on the previous ones.

The ideal way to model this is with a **probability tree.**



Chaining Probabilities Along a Path

We use the Multiplication Rule to find the probability of the sequence D1, D2, and D3. This means we multiply the probabilities along the desired path in the tree.



$$P(D1 \cap D2 \cap D3) = P(D1) * P(D2|D1) * P(D3|D1 \cap D2)$$

$$= (9/500) * (8/499) * (7/498)$$

$$\approx 4.05 \times 10^{-6} \text{ (a highly unlikely event)}$$

The Conditional Probability Playbook

Context is King

New information shrinks the sample space, updating our beliefs about probability. Think of it as a focusing lens.

Independence is a Special Case

When context is irrelevant, $P(A|B) = P(A)$. The new information provides no leverage.

The Master Formula

$$\frac{[A \cap B]}{[B]}$$

$P(A|B) = P(A \cap B) / P(B)$ is your fundamental tool for calculating probability given new context.

Model Sequences with Trees

Use the Multiplication Rule, $P(A \cap B) = P(A) * P(B|A)$, to calculate the probability of paths of dependent, sequential events.

Thank You.

Questions & Discussion