Problem 11.19 page 477

$$y''(x) = \frac{-bx^2}{\left(1-\frac{x^2}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}\ a^4} - \frac{b}{\sqrt{(1-x^2/a^2)}\ a^2}$$

$$= \frac{-bx^2}{a(a^2-x^2)\sqrt{a^2-x^2}} - \frac{b}{a\sqrt{a^2-x^2}}$$

$$= \frac{-b}{a\sqrt{a^2-x^2}}\left[\underbrace{\frac{x^2}{a^2-x^2}+1}_{\frac{a^2}{a^2-x^2}}\right] = \frac{-a^2 b}{a(a^2-x^2)\sqrt{a^2-x^2}}$$

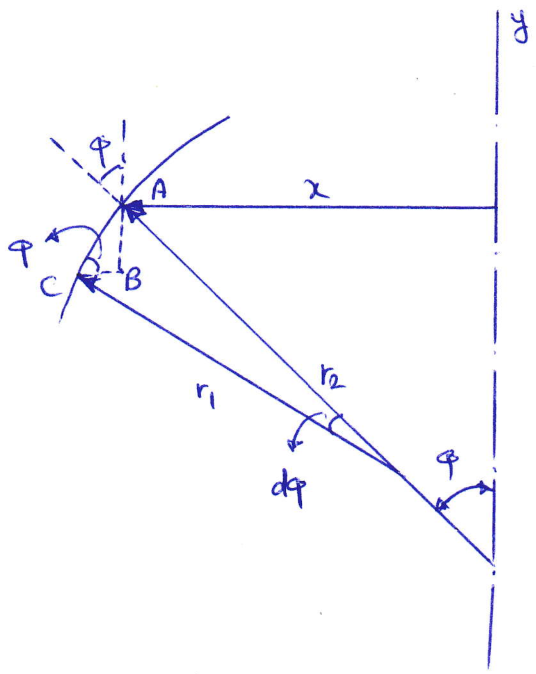
substituting $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow y = \pm\frac{b}{a}\sqrt{a^2-x^2}$ we have

$$= \frac{\mp \cancel{a^2} b}{\cancel{a}\left(\frac{y^2a^2}{b^2}\right)\left(\frac{\pm \cancel{a} y}{b}\right)} = \frac{b^4}{a^2y^3}$$

$$y'(x) = \frac{-bx}{\sqrt{1-x^2/a^2}\ a^2} = \frac{-bx}{a\sqrt{a^2-x^2}} = \frac{\mp bx}{a(\pm y\, a/b)} = \frac{b^2}{a^2}\frac{x}{y}$$

$$R_1 = \frac{(1+y'^2)^{3/2}}{y''} = \frac{\left(1+\frac{b^4}{a^4}\frac{x^2}{y^2}\right)^{3/2}}{\frac{b^4}{a^2y^3}} = \frac{\left(\frac{a^4y^2+b^4x^2}{a^4\cancel{y^2}}\right)\frac{\sqrt{a^4y^2+b^4x^2}}{\cancel{a^2}\cancel{y}}}{\frac{b^4}{\cancel{a^2}\cancel{y^3}}}$$

$$= \frac{(a^4y^2+b^4x^2)\sqrt{a^4y^2+b^4x^2}}{a^4b^4}$$



using the figure of problem 11.20 and changing variables

$\overset{\Delta}{ABC}$: $\tan\varphi = \dfrac{dy}{dx}$ (1)

we have $y = \pm\dfrac{b}{a}\sqrt{a^2 - x^2}$ (*)

from (1) $\tan\varphi = \dfrac{-bx}{a\sqrt{a^2-x^2}}$ (2)

solving eq(2) in Maple gives $x = \pm\dfrac{\tan\varphi\, a^2}{\sqrt{a^2\tan^2\varphi + b^2}}$

$$= \pm\frac{a^2\sin\varphi}{\sqrt{a^2\sin^2\varphi + b^2\cos^2\varphi}}$$

substituting x in (*) gives y

$$y = \pm\frac{b}{a}\sqrt{a^2 - \frac{\tan^2\varphi\, a^4}{a^2\tan^2\varphi + b^2}} = \pm b^2\frac{\cos\varphi}{\sqrt{a^2\sin^2\varphi + b^2\cos^2\varphi}}$$

Since, in the following, we always have an even (squared) power of x and y, I use positive values of x and y in equations.

Substituting x and y in R_1

$$R_1 = \frac{(a^4y^2 + b^4x^2)\sqrt{a^4y^2+b^4x^2}}{a^4b^4}$$

$$= \frac{\left(\dfrac{a^4b^4\cos^2\varphi}{a^2\sin^2\varphi+b^2\cos^2\varphi} + \dfrac{b^4a^4\sin^2\varphi}{a^2\sin^2\varphi+b^2\cos^2\varphi}\right)^{3/2}}{a^4b^4}$$

$$= \frac{\dfrac{\cancel{a^4b^4}(a^2b^2)}{(a^2\sin^2\varphi+b^2\cos^2\varphi)^{3/2}}}{\cancel{a^4b^4}} = \frac{a^2b^2}{(a^2\sin^2\varphi+b^2\cos^2\varphi)^{3/2}}$$

for R_2 we know $\tan\varphi = \frac{x}{h} = y'(x) = \frac{b^2}{a^2}\frac{x}{y} \Rightarrow h = \frac{a^2y}{b^2}$

From Pythagorean Theorem: $R_2^2 = x^2 + h^2$

$$\Rightarrow R_2^2 = x^2 + \frac{a^4}{b^4}y^2$$

$$= \frac{a^4\sin^2\varphi}{a^2\sin^2\varphi+b^2\cos^2\varphi} + \frac{a^4}{\cancel{b^4}}\frac{\cancel{b^4}\cos^2\varphi}{a^2\sin^2\varphi+b^2\cos^2\varphi}$$

$$= \frac{a^4}{a^2\sin^2\varphi+b^2\cos^2\varphi}$$

$$\Rightarrow R_2 = \frac{a^2}{\sqrt{a^2\sin^2\varphi+b^2\cos^2\varphi}}$$

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> restart:
> y := x -> b / a * sqrt(a^2 - x^2)
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$$y := x \mapsto \frac{b\sqrt{a^2 - x^2}}{a} \qquad (1)$$

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> tan(phi) = diff(y(x), x)
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$$\tan(\phi) = -\frac{b\,x}{a\sqrt{a^2 - x^2}} \qquad (2)$$

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> x := solve(tan(phi) = diff(y(x), x), x)
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$$x := -\frac{\tan(\phi)\,a^2}{\sqrt{a^2\tan(\phi)^2 + b^2}}, \frac{\tan(\phi)\,a^2}{\sqrt{a^2\tan(\phi)^2 + b^2}} \qquad (3)$$

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> y(x)
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$$\frac{b\sqrt{a^2 - \frac{\tan(\phi)^2\,a^4}{a^2\tan(\phi)^2 + b^2}}}{a} \qquad (4)$$