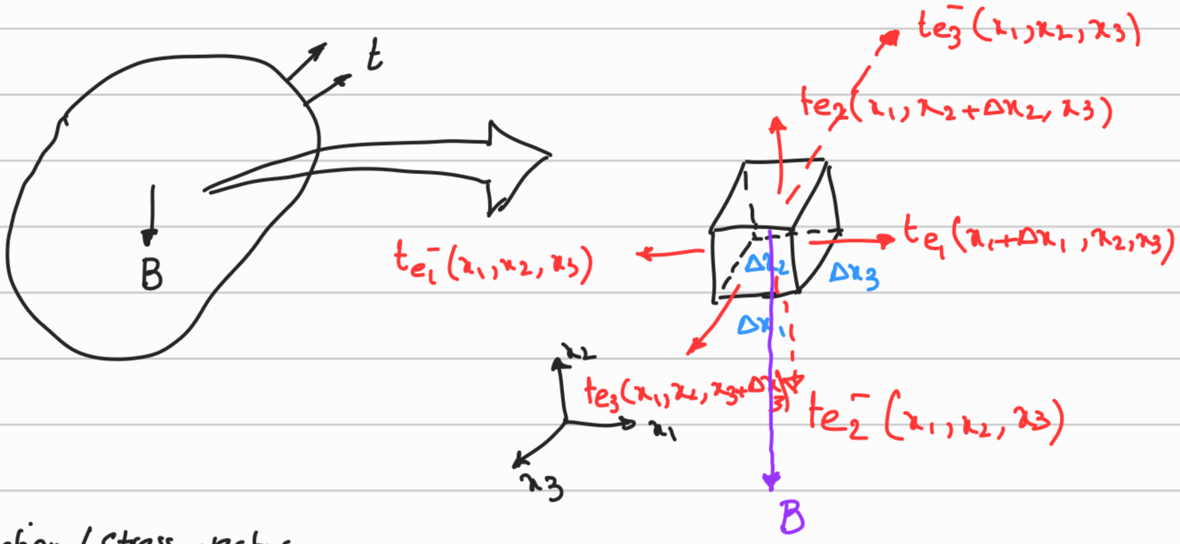
Principle of Linear Momentum (Equation of Motion):



t: traction / stress vector (force/unit area)

(weight/unit mass)

$$te_1(x_1+\Delta x_1,x_2,x_3)\,\Delta x_2\Delta x_3 + te_1^-(x_1,x_2,x_3)\,\Delta x_2\Delta x_3$$

$$+te_2(x_1,x_2+\Delta x_2,x_3)\,\Delta x_1\Delta x_3 + te_2^-(x_1,x_2,x_3)\,\Delta x_1\Delta x_3$$

$$+te_3(x_1,x_2,x_3+\Delta x_3)\,\Delta x_1\Delta x_2 + te_3^-(x_1,x_2,x_3)\,\Delta x_1\Delta x_2$$

$$+\rho B(\Delta x_1\Delta x_2\Delta x_3) = \rho(\Delta x_1\Delta x_2\Delta x_3)\,a$$

$te_1^-(x_1,x_2,x_3) = -te_1(x_1,x_2,x_3)$, ...

$$te_1(x_1+\Delta x_1,x_2,x_3)\,\Delta x_2\Delta x_3 - te_1(x_1,x_2,x_3)\,\Delta x_2\Delta x_3$$

$$+te_2(x_1,x_2+\Delta x_2,x_3)\,\Delta x_1\Delta x_3 - te_2(x_1,x_2,x_3)\,\Delta x_1\Delta x_3$$

$$+te_3(x_1,x_2,x_3+\Delta x_3)\,\Delta x_1\Delta x_2 - te_3(x_1,x_2,x_3)\,\Delta x_1\Delta x_2$$

$$+\rho B(\Delta x_1\Delta x_2\Delta x_3) = \rho(\Delta x_1\Delta x_2\Delta x_3)\,a$$

$$\underbrace{t_{e_1}(x_1+\Delta x_1, x_2, x_3)\,\frac{1}{\Delta x_1} \quad - t_{e_1}(x_1, x_2, x_3)\,\frac{1}{\Delta x_1}}$$

$$+ t_{e_2}(x_1, x_2+\Delta x_2, x_3)\,\frac{1}{\Delta x_2} \quad - t_{e_2}(x_1, x_2, x_3)\,\frac{1}{\Delta x_2}$$

$$+ t_{e_3}(x_1, x_2, x_3+\Delta x_3)\,\frac{1}{\Delta x_3} \quad - t_{e_3}(x_1, x_2, x_3)\,\frac{1}{\Delta x_3}$$

$$+\rho B = \rho a$$

$$\frac{t_{e_1}(x_1+\Delta x_1, x_2, x_3) - t_{e_1}(x_1, x_2, x_3)}{\Delta x_1} = \frac{\partial t_{e_1}}{\partial x_1}, \ldots$$

$$\begin{cases} \dfrac{\partial t_{e_1}}{\partial x_1} + \dfrac{\partial t_{e_2}}{\partial x_2} + \dfrac{\partial t_{e_3}}{\partial x_3} + \rho B = \rho a \\[2ex] \dfrac{\partial t_{e_i}}{\partial x_i} + \rho B_i = \rho a_i \end{cases}$$

$$\left.\begin{aligned} &\frac{\partial t_{e_i}}{\partial x_i} + \rho B_i = \rho a_i \\ &t_{e_i} = t_{\hat{e}_i} = T_{ji}\hat{e}_j \end{aligned}\right\} \quad \frac{\partial (T_{ji}\hat{e}_j)}{\partial x_i} + \rho B_i = \rho a_i$$

$$\downarrow$$

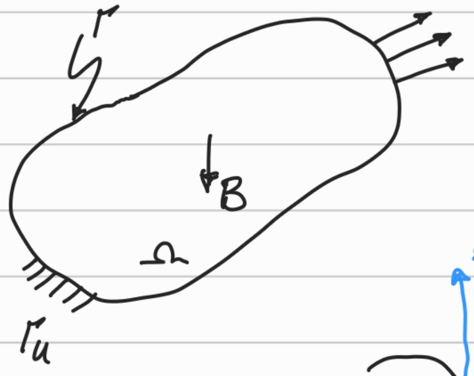
$$\frac{\partial T_{ji}}{\partial x_i}\hat{e}_j + T_{ji}\cancel{\frac{\partial \hat{e}_j}{\partial x_i}}^{\,0} + \rho B_i = \rho a_i$$

$$\boxed{\frac{\partial T_{ji}}{\partial x_i}\hat{e}_j + \rho B_j \hat{e}_j = \rho a_j \hat{e}_j}$$

Equation of motion:

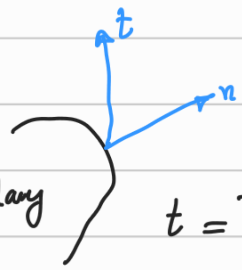
$$\boxed{\begin{aligned} &\frac{\partial T_{ij}}{\partial x_j} + \rho B_i = \rho a_i \\[2ex] &\frac{\partial T_{ij}}{\partial x_j} + \rho B_i = 0 \end{aligned}}$$

Boundary Condition (B.C):



$\Gamma_t = t$ (Natural B.C)

(essential/geometrical B.C)



$t = T n$

free surface : $t = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$

$$t = \begin{bmatrix} 0 & 0 & 0 \\ 0 & T_{22} & T_{23} \\ 0 & T_{23} & T_{33} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$