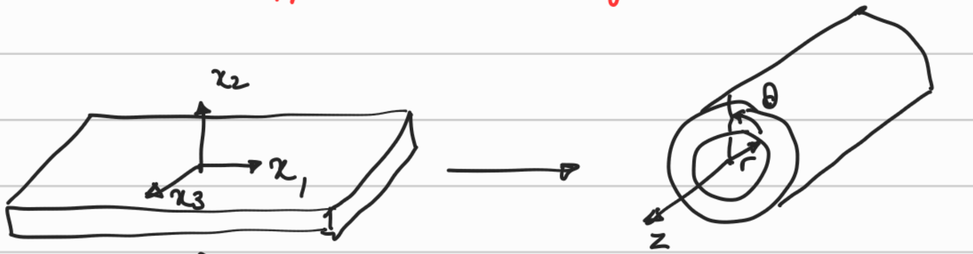
Deformation Tensors in Different Coordinate Systems:



Ref config. Current Config.

Reference Config.	Current Config.
Cartesian	Cartesian
Cylindrical	Cylindrical
Spherical	Spherical

Cylindrical Coordinate System for Both the Reference and Current Configuration:

$x_i = x_i(X_j, t)$ Cylindrical coordinates $\begin{cases} r = r(r_0, \theta_0, z_0, t) \\ \theta = \theta(r_0, \theta_0, z_0, t) \\ z = z(r_0, \theta_0, z_0, t) \end{cases}$

$[F] = ?$ $dx = F dX$ Cylindrical Coordinate $\begin{cases} d\vec{x} = dr\,\hat{e_r} + r d\theta\,\hat{e_\theta} + dz\,\hat{e_z} \\ d\vec{X} = dr_0\,\hat{\mathring{e_r}} + r_0 d\theta_0\,\hat{\mathring{e_\theta}} + dz_0\,\hat{\mathring{e_z}} \end{cases}$

$F_{ij} = \dfrac{\partial x_i}{\partial X_j}$ current (i), reference (j)

$$\begin{Bmatrix} dr \\ r d\theta \\ dz \end{Bmatrix} = \begin{bmatrix} F_{rr} & F_{r\theta} & F_{rz} \\ F_{\theta r} & F_{\theta\theta} & F_{\theta z} \\ F_{zr} & F_{z\theta} & F_{zz} \end{bmatrix} \begin{Bmatrix} dr_0 \\ r_0 d\theta_0 \\ dz_0 \end{Bmatrix}$$

current $\to \{e_i\}\{\mathring{e}_j\} \leftarrow$ reference

$$= \begin{Bmatrix} F_{rr}\,dr_0 + F_{r\theta}\,r_0 d\theta_0 + F_{rz}\,dz_0 \\ F_{\theta r}\,dr_0 + F_{\theta\theta}\,r_0 d\theta_0 + F_{\theta z}\,dz_0 \\ F_{zr}\,dr_0 + F_{z\theta}\,r_0 d\theta_0 + F_{zz}\,dz_0 \end{Bmatrix}$$

$$dr = dr_0\; e_r \cdot F\mathring{e}_r + r_0 d\theta_0\; e_r \cdot F\mathring{e}_\theta + dz_0\; e_r \cdot F\mathring{e}_z$$

$$r d\theta = dr_0\; e_\theta \cdot F\mathring{e}_r + r_0 d\theta_0\; e_\theta \cdot F\mathring{e}_\theta + dz_0\; e_\theta \cdot F\mathring{e}_z$$

$$dz = dr_0\, e_z \cdot \mathring{F} e_r + r_0 d\theta_0\, e_z \cdot \mathring{F} e_\theta + dz_0\, e_z \cdot \mathring{F} e_z$$

$e_r \cdot \mathring{F} e_r = \dfrac{\partial r}{\partial r_0}$	$e_r \cdot \mathring{F} e_\theta = \dfrac{\partial r}{r_0 \partial \theta_0}$	$e_r \cdot \mathring{F} e_z = \dfrac{\partial r}{\partial z_0}$
$e_\theta \cdot \mathring{F} e_r = \dfrac{r\partial \theta}{\partial r_0}$	$e_\theta \cdot \mathring{F} e_\theta = \dfrac{r\partial \theta}{r_0 \partial \theta_0}$	$e_\theta \cdot \mathring{F} e_z = \dfrac{r\partial \theta}{\partial z_0}$
$e_z \cdot \mathring{F} e_r = \dfrac{\partial z}{\partial r_0}$	$e_z \cdot \mathring{F} e_\theta = \dfrac{\partial z}{r_0 \partial \theta_0}$	$e_z \cdot \mathring{F} e_z = \dfrac{\partial z}{\partial z_0}$

$$\mathring{F} e_r = \frac{\partial r}{\partial r_0} \hat{e}_r \qquad \mathring{F} e_r = F_{jr} e_j = F_{rr} \hat{e}_r + F_{\theta r} \hat{e}_\theta + F_{zr} \hat{e}_z$$

$$\mathring{F} e_r = \frac{\partial \theta}{\partial r_0} \hat{e}_\theta \qquad \mathring{F} e_r = \frac{\partial r}{\partial r_0} \hat{e}_r + \frac{\partial \theta}{\partial r_0} \hat{e}_\theta + \frac{\partial z}{\partial r_0} \hat{e}_z$$

$$\mathring{F} e_r = \frac{\partial z}{\partial r_0} \hat{e}_z$$

$$\boxed{\begin{aligned}
\mathring{F} e_r &= \frac{\partial r}{\partial r_0} \hat{e}_r + \frac{r\partial \theta}{\partial r_0} \hat{e}_\theta + \frac{\partial z}{\partial r_0} \hat{e}_z \\
\mathring{F} e_\theta &= \frac{\partial r}{r_0 \partial \theta_0} \hat{e}_r + \frac{r\partial \theta}{r_0 \partial \theta_0} \hat{e}_\theta + \frac{\partial z}{r_0 \partial \theta_0} \hat{e}_z \\
\mathring{F} e_z &= \frac{\partial r}{\partial z_0} \hat{e}_r + \frac{r\partial \theta}{\partial z_0} \hat{e}_\theta + \frac{\partial z}{\partial z_0} \hat{e}_z
\end{aligned}}$$

$[F]^T = ?$

$$e_r \cdot \mathring{F} e_r = \mathring{F} e_r \cdot e_r = e_r \cdot \mathring{F}^T e_r = \frac{\partial r}{\partial r_0} = F^T_{rr}$$

$$e_r \cdot \mathring{F} e_\theta = \mathring{F} e_\theta \cdot e_r = e_\theta \cdot \mathring{F}^T e_r = \frac{\partial r}{r_0 \partial \theta_0} = F^T_{\theta r}$$

$$e_r \cdot \mathring{F} e_z = \mathring{F} e_z \cdot e_r = e_z \cdot \mathring{F}^T e_r = \frac{\partial r}{\partial z_0} = F^T_{zr}$$

$[F]^T =$

$\dfrac{\partial r}{\partial r_0}$	$\dfrac{\partial r}{r_0 \partial \theta_0}$	$\dfrac{\partial r}{\partial z_0}$
$\dfrac{r\partial \theta}{\partial r_0}$	$\dfrac{r\partial \theta}{r_0 \partial \theta_0}$	$\dfrac{r\partial \theta}{\partial z_0}$
$\dfrac{\partial z}{\partial r_0}$	$\dfrac{\partial z}{r_0 \partial \theta_0}$	$\dfrac{\partial z}{\partial z_0}$

$$F^T e_r = F^T_{jr} e_j^\circ = F^T_{rr} e_r^\circ + F^T_{\theta r} e_\theta^\circ + F^T_{zr} e_z^\circ$$

$$\boxed{F^T e_r = \frac{\partial r}{\partial r_0} e_r^\circ + \frac{\partial r}{r_0 \partial \theta_0} e_\theta^\circ + \frac{\partial r}{\partial z_0} e_z^\circ}$$

$$e_\theta \cdot F e_r^\circ = F e_r^\circ \cdot e_\theta = e_r^\circ \cdot F^T e_\theta = \frac{r \partial \theta}{\partial r_0} = F_{r\theta}$$

$$e_\theta \cdot F e_\theta^\circ = F e_\theta^\circ \cdot e_\theta = e_\theta^\circ \cdot F^T e_\theta = \frac{r \partial \theta}{r_0 \partial \theta_0} = F_{\theta\theta}$$

$$e_\theta \cdot F e_z^\circ = F e_z^\circ \cdot e_\theta = e_z^\circ \cdot F^T e_\theta = \frac{r \partial \theta}{\partial z_0} = F_{z\theta}$$

$$F^T e_\theta = F_{j\theta} e_j^\circ = F_{r\theta} e_r^\circ + F_{\theta\theta} e_\theta^\circ + F_{z\theta} e_z^\circ$$

$$\boxed{F^T e_\theta = \frac{r \partial \theta}{\partial r_0} e_r^\circ + \frac{r \partial \theta}{r_0 \partial \theta_0} e_\theta^\circ + \frac{r \partial \theta}{\partial z_0} e_z^\circ}$$

$$e_z \cdot F e_r^\circ = F e_r^\circ \cdot e_z = e_r^\circ \cdot F^T e_z = \frac{\partial z}{\partial r_0} = F_{rz}$$

$$e_z \cdot F e_\theta^\circ = F e_\theta^\circ \cdot e_z = e_\theta \cdot F^T e_z = \frac{\partial z}{r_0 \partial \theta_0} = F_{\theta z}$$

$$e_z \cdot F e_z^\circ = F e_z^\circ \cdot e_z = e_z^\circ \cdot F^T e_z = \frac{\partial z}{\partial z_0} = F_{zz}$$

$$\boxed{F^T e_z = \frac{\partial z}{\partial r_0} e_r^\circ + \frac{\partial z}{r_0 \partial \theta_0} e_\theta^\circ + \frac{\partial z}{\partial z_0} e_z^\circ}$$

$[B] = ?$ w.r.t current configuration (x)

$$B_{rr} = e_r \cdot B e_r \overset{B = FF^T}{=\!=} e_r \cdot (FF^T) e_r = e_r \cdot F F^T e_r$$

$$= e_r \cdot F \left(\frac{\partial r}{\partial r_0} e_r^\circ + \frac{\partial r}{r_0 \partial \theta_0} e_\theta^\circ + \frac{\partial r}{\partial z_0} e_z^\circ \right)$$

$$= \frac{\partial r}{\partial r_0} e_r \cdot F e_r^\circ + \frac{\partial r}{r_0 \partial \theta_0} e_r \cdot F e_\theta^\circ + \frac{\partial r}{\partial z_0} e_r \cdot F e_z^\circ$$

$$= \left(\frac{\partial r}{\partial r_0} \right)^2 + \left(\frac{\partial r}{r_0 \partial \theta_0} \right)^2 + \left(\frac{\partial r}{\partial z_0} \right)^2$$

$$B_{z\theta} = e_z \cdot B e_\theta = e_z \cdot F F^T e_\theta$$

$$= e_z \cdot F \left(\frac{r\partial\theta}{\partial r_0} \overset{\circ}{e_r} + \frac{r\partial\theta}{r_0 \partial\theta_0} \overset{\circ}{e_\theta} + \frac{r\partial\theta}{\partial z_0} \overset{\circ}{e_z} \right)$$

$$= \frac{r\partial\theta}{\partial r_0} e_z \cdot F\overset{\circ}{e_r} + \frac{r\partial\theta}{r_0\partial\theta_0} e_z \cdot F\overset{\circ}{e_\theta} + \frac{r\partial\theta}{\partial z_0} e_z \cdot F\overset{\circ}{e_z}$$

$$= \left(\frac{r\partial\theta}{\partial r_0}\right)\left(\frac{\partial z}{\partial r_0}\right) + \left(\frac{r\partial\theta}{r_0\partial\theta_0}\right)\left(\frac{\partial z}{r_0\partial\theta_0}\right) + \left(\frac{r\partial\theta}{\partial z_0}\right)\left(\frac{\partial z}{\partial z_0}\right)$$

$[B^{-1}] = ?$ w.r.t current configuration (x)

$$B^{-1} = (FF^T)^{-1} = (F^T)^{-1} F^{-1} = (F^{-1})^T F^{-1} \quad \begin{cases} F^{-1} \\ (F^{-1})^T \end{cases}$$

$$F^{-1}_{ij} = \frac{\partial X_i}{\partial x_j} \; , \quad dX = F^{-1} dx \; , \quad X_i = X_i(x_i, t) \quad \begin{cases} r_0 = r_0(r, \theta, z, t) \\ \theta_0 = \theta_0(r, \theta, z, t) \\ z_0 = z_0(r, \theta, z, t) \end{cases}$$

reference (X_i), current (x_j)

Cylindrical Coordinate
$$\begin{cases} \vec{dx} = dr\,\hat{e_r} + r d\theta\,\hat{e_\theta} + dz\,\hat{e_z} \\ \vec{dX} = dr_0\,\hat{\overset{\circ}{e_r}} + r_0 d\theta_0\,\hat{\overset{\circ}{e_\theta}} + dz_0\,\hat{\overset{\circ}{e_z}} \end{cases}$$

$$\begin{Bmatrix} dr_0 \\ r_0 d\theta_0 \\ dz_0 \end{Bmatrix} = \begin{bmatrix} F^{-1}_{rr} & F^{-1}_{r\theta} & F^{-1}_{rz} \\ F^{-1}_{\theta r} & F^{-1}_{\theta\theta} & F^{-1}_{\theta z} \\ F^{-1}_{zr} & F^{-1}_{z\theta} & F^{-1}_{zz} \end{bmatrix} \begin{Bmatrix} dr \\ r d\theta \\ dz \end{Bmatrix}$$

reference $\rightarrow \{\overset{\circ}{e_i}\} \{e_j\} \leftarrow$ current

$$= \begin{Bmatrix} F^{-1}_{rr} dr + F^{-1}_{r\theta} r d\theta + F^{-1}_{rz} dz \\ F^{-1}_{\theta r} dr + F^{-1}_{\theta\theta} r d\theta + F^{-1}_{\theta z} dz \\ F^{-1}_{zr} dr + F^{-1}_{z\theta} r d\theta + F^{-1}_{zz} dz \end{Bmatrix}$$

$$\begin{cases} dr_0 = \overset{\circ}{e_r} \cdot F^{-1} e_r\, dr + \overset{\circ}{e_r} \cdot F^{-1} e_\theta\, r d\theta + \overset{\circ}{e_r} \cdot F^{-1} e_z\, dz \\ dr_0 = \frac{\partial r_0}{\partial r} dr + \frac{\partial r_0}{\partial \theta} d\theta + \frac{\partial r_0}{\partial z} dz \end{cases}$$

$\overset{\circ}{e_r} \cdot \overset{-1}{F} e_r = \frac{\partial r_\circ}{\partial r}$	$\overset{\circ}{e_r} \cdot \overset{-1}{F} e_\theta = \frac{\partial r_\circ}{r \partial \theta}$	$\overset{\circ}{e_r} \cdot \overset{-1}{F} e_z = \frac{\partial r_\circ}{\partial z}$
$\overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_r = \frac{r_\circ \partial \theta_\circ}{\partial r}$	$\overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_\theta = \frac{r_\circ \partial \theta_\circ}{r \partial \theta}$	$\overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_z = \frac{r_\circ \partial \theta_\circ}{\partial z}$
$\overset{\circ}{e_z} \cdot \overset{-1}{F} e_r = \frac{\partial z_\circ}{\partial r}$	$\overset{\circ}{e_z} \cdot \overset{-1}{F} e_\theta = \frac{\partial z_\circ}{r \partial \theta}$	$\overset{\circ}{e_z} \cdot \overset{-1}{F} e_z = \frac{\partial z_\circ}{\partial z}$

$$\begin{cases} r_\circ d\theta_\circ = \overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_r \, dr + \overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_\theta (r d\theta) + \overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_z \, dz \\ d\theta_\circ = \frac{\partial \theta_\circ}{\partial r} dr + \frac{\partial \theta_\circ}{\partial \theta} d\theta + \frac{\partial \theta_\circ}{\partial z} dz \end{cases}$$

$$\begin{cases} dz_\circ = \overset{\circ}{e_z} \cdot \overset{-1}{F} e_r \, dr + \overset{\circ}{e_z} \cdot \overset{-1}{F} e_\theta \, r d\theta + \overset{\circ}{e_z} \cdot F e_z \, dz \\ dz_\circ = \frac{\partial z_\circ}{\partial r} dr + \frac{\partial z_\circ}{\partial \theta} d\theta + \frac{\partial z_\circ}{\partial z} dz \end{cases}$$

$$\overset{-1}{F} e_r = \overset{-1}{F}_{jr} \overset{\circ}{e_j} = \overset{-1}{F}_{rr} \overset{\circ}{e_r} + \overset{-1}{F}_{\theta r} \overset{\circ}{e_\theta} + \overset{-1}{F}_{zr} \overset{\circ}{e_z}$$

$$\boxed{\overset{-1}{F} e_r = \frac{\partial r_\circ}{\partial r} \overset{\circ}{e_r} + \frac{r_\circ \partial \theta_\circ}{\partial r} \overset{\circ}{e_\theta} + \frac{\partial z_\circ}{\partial r} \overset{\circ}{e_z}}$$

$$\overset{-1}{F} e_\theta = \overset{-1}{F}_{j\theta} \overset{\circ}{e_j} = \overset{-1}{F}_{r\theta} \overset{\circ}{e_r} + \overset{-1}{F}_{\theta\theta} \overset{\circ}{e_\theta} + \overset{-1}{F}_{z\theta} \overset{\circ}{e_z}$$

$$\boxed{\overset{-1}{F} e_\theta = \frac{\partial r_\circ}{r \partial \theta} \overset{\circ}{e_r} + \frac{r_\circ \partial \theta_\circ}{r \partial \theta} \overset{\circ}{e_\theta} + \frac{\partial z_\circ}{r \partial \theta} \overset{\circ}{e_z}}$$

$$\overset{-1}{F} e_z = \overset{-1}{F}_{jz} \overset{\circ}{e_j} = \overset{-1}{F}_{rz} \overset{\circ}{e_r} + \overset{-1}{F}_{\theta z} \overset{\circ}{e_\theta} + \overset{-1}{F}_{zz} \overset{\circ}{e_z}$$

$$\boxed{\overset{-1}{F} e_z = \frac{\partial r_\circ}{\partial z} \overset{\circ}{e_r} + \frac{r_\circ \partial \theta_\circ}{\partial z} \overset{\circ}{e_\theta} + \frac{\partial z_\circ}{\partial z} \overset{\circ}{e_z}}$$

$(\overset{-1}{F})^T = ?$

$$\overset{\circ}{e_r} \cdot \overset{-1}{F} e_r = \overset{-1}{F} e_r \cdot \overset{\circ}{e_r} = e_r \cdot (\overset{-1}{F})^T \overset{\circ}{e_r} = \frac{\partial r_\circ}{\partial r} = (\overset{-1}{F})^T_{rr}$$

$$\overset{\circ}{e_\theta} \cdot \overset{-1}{F} e_r = \overset{-1}{F} e_r \cdot \overset{\circ}{e_\theta} = e_r \cdot (\overset{-1}{F})^T \overset{\circ}{e_\theta} = \frac{r_\circ \partial \theta_\circ}{\partial r} = (\overset{-1}{F})^T_{r\theta}$$

$$e_z^\circ \cdot F^{-1} e_r = F^{-1} e_r \cdot e_z^\circ = e_r \cdot (F^{-1})^T e_z^\circ = \frac{\partial z_\circ}{\partial r} = (F^{-1})^T_{rz}$$

$$e_r^\circ \cdot F^{-1} e_\theta = F^{-1} e_\theta \cdot e_r^\circ = e_\theta \cdot (F^{-1})^T e_r^\circ = \frac{\partial r_\circ}{r \partial \theta} = (F^{-1})^T_{\theta r}$$

$$e_\theta^\circ \cdot F^{-1} e_\theta = F^{-1} e_\theta \cdot e_\theta^\circ = e_\theta \cdot (F^{-1})^T e_\theta^\circ = \frac{r_\circ \partial \theta_\circ}{r \partial \theta} = (F^{-1})^T_{\theta\theta}$$

$$e_z^\circ \cdot F^{-1} e_\theta = F^{-1} e_\theta \cdot e_z^\circ = e_\theta \cdot (F^{-1})^T e_z^\circ = \frac{\partial z_\circ}{r \partial \theta} = (F^{-1})^T_{\theta z}$$

$$e_r^\circ \cdot F^{-1} e_z = F^{-1} e_z \cdot e_r^\circ = e_z \cdot (F^{-1})^T e_r^\circ = \frac{\partial r_\circ}{\partial z} = (F^{-1})^T_{zr}$$

$$e_\theta^\circ \cdot F^{-1} e_z = F^{-1} e_z \cdot e_\theta^\circ = e_z \cdot (F^{-1})^T e_\theta^\circ = \frac{r_\circ \partial \theta_\circ}{\partial z} = (F^{-1})^T_{z\theta}$$

$$e_z^\circ \cdot F^{-1} e_z = F^{-1} e_z \cdot e_z^\circ = e_z \cdot (F^{-1})^T e_z^\circ = \frac{\partial z_\circ}{\partial z} = (F^{-1})^T_{zz}$$

$$(F^{-1})^T = \begin{bmatrix} (F^{-1})^T_{rr} & (F^{-1})^T_{\theta r} & (F^{-1})^T_{zr} \\ (F^{-1})^T_{r\theta} & (F^{-1})^T_{\theta\theta} & (F^{-1})^T_{z\theta} \\ (F^{-1})^T_{rz} & (F^{-1})^T_{\theta z} & (F^{-1})^T_{zz} \end{bmatrix}$$

$$(F^{-1})^T e_r^\circ = (F^{-1})^T_{jr} e_j = (F^{-1})^T_{rr} e_r + (F^{-1})^T_{\theta r} e_\theta + (F^{-1})^T_{zr} e_z$$

$$\boxed{(F^{-1})^T e_r^\circ = \frac{\partial r_\circ}{\partial r} e_r + \frac{\partial r_\circ}{r \partial \theta} e_\theta + \frac{\partial r_\circ}{\partial z} e_z}$$

$$(F^{-1})^T e_\theta^\circ = (F^{-1})^T_{j\theta} e_j = (F^{-1})^T_{r\theta} e_r + (F^{-1})^T_{\theta\theta} e_\theta + (F^{-1})^T_{z\theta} e_z$$

$$\boxed{(F^{-1})^T e_\theta^\circ = \frac{r_\circ \partial \theta_\circ}{\partial r} e_r + \frac{r_\circ \partial \theta_\circ}{r \partial \theta} e_\theta + \frac{r_\circ \partial \theta_\circ}{\partial z} e_z}$$

$$(F^{-1})^T e_z^\circ = (F^{-1})^T_{jz} e_j = (F^{-1})^T_{rz} e_r + (F^{-1})^T_{\theta z} e_\theta + (F^{-1})^T_{zz} e_z$$

$$\boxed{(F^{-1})^T e_z^\circ = \frac{\partial z_\circ}{\partial r} e_r + \frac{\partial z_\circ}{r \partial \theta} e_\theta + \frac{\partial z_\circ}{\partial z} e_z}$$

$$B^{-1}_{r\theta} = e_r \cdot B^{-1} e_\theta = e_r \cdot (FF^T)^{-1} e_\theta = e_r \cdot (F^{-1})^T F e_\theta$$

$$= e_r \cdot (F^{-1})^T \left(\frac{\partial r_0}{r \partial \theta} \mathring{e_r} + \frac{r_0 \partial \theta_0}{r \partial \theta} \mathring{e_\theta} + \frac{\partial z_0}{r \partial \theta} \mathring{e_z} \right)$$

$$= \frac{\partial r_0}{r \partial \theta} e_r \cdot (F^{-1})^T \mathring{e_r} + \frac{r_0 \partial \theta_0}{r \partial \theta} e_r \cdot (F^{-1})^T \mathring{e_\theta} + \frac{\partial z_0}{r \partial \theta} e_r \cdot (F^{-1})^T \mathring{e_z}$$

$$= \left(\frac{\partial r_0}{r \partial \theta} \right) \left(\frac{\partial r_0}{\partial r} \right) + \left(\frac{r_0 \partial \theta_0}{r \partial \theta} \right) \left(\frac{r_0 \partial \theta_0}{\partial r} \right) + \left(\frac{\partial z_0}{r \partial \theta} \right) \left(\frac{\partial z_0}{\partial r} \right)$$

$[C] = ?$ w.r.t reference configuration X

$x_i = x_i(X_j, t)$ Cylindrical coordinates

$$\begin{cases} r = r(r_0, \theta_0, z_0, t) \\ \theta = \theta(r_0, \theta_0, z_0, t) \\ z = z(r_0, \theta_0, z_0, t) \end{cases}$$

$$C_{r_0\theta_0} = \mathring{e_r} \cdot C \mathring{e_\theta} = \mathring{e_r} \cdot (F^T F) \mathring{e_\theta} = \mathring{e_r} \cdot F^T F \mathring{e_\theta}$$

$$= \mathring{e_r} \cdot F^T \left(\frac{\partial r}{r_0 \partial \theta_0} e_r + \frac{r \partial \theta}{r_0 \partial \theta_0} e_\theta + \frac{\partial z}{r_0 \partial \theta_0} e_z \right)$$

$$= \frac{\partial r}{r_0 \partial \theta_0} \mathring{e_r} \cdot F^T e_r + \frac{r \partial \theta}{r_0 \partial \theta_0} \mathring{e_r} \cdot F^T e_\theta + \frac{\partial z}{r_0 \partial \theta_0} \mathring{e_r} \cdot F^T e_z$$

$$= \left(\frac{\partial r}{r_0 \partial \theta_0} \right) \left(\frac{\partial r}{\partial r_0} \right) + \left(\frac{r \partial \theta}{r_0 \partial \theta_0} \right) \left(\frac{r \partial \theta}{\partial r_0} \right) + \left(\frac{\partial z}{r_0 \partial \theta_0} \right) \left(\frac{\partial z}{\partial r_0} \right)$$

$[C^{-1}] = ?$ w.r.t reference configuration X

$$C^{-1}_{r_0\theta_0} = \mathring{e_r} \cdot C^{-1} \mathring{e_\theta} = \mathring{e_r} \cdot (F^T F)^{-1} \mathring{e_\theta} = \mathring{e_r} \cdot F^{-1} (F^{-1})^T \mathring{e_\theta}$$

$$= \overset{\circ}{e_r} \cdot F^{-1}\left(\frac{r_0 \partial\theta_0}{\partial r} e_r + \frac{r_0 \partial\theta_0}{r\partial\theta} e_\theta + \frac{r_0 \partial\theta_0}{\partial z} e_z\right)$$

$$= \frac{r_0 \partial\theta_0}{\partial r} \overset{\circ}{e_r} \cdot F^{-1} e_r + \frac{r_0 \partial\theta_0}{r\partial\theta} \overset{\circ}{e_r} \cdot F^{-1} e_\theta + \frac{r_0 \partial\theta_0}{\partial z} \overset{\circ}{e_r} \cdot F^{-1} e_z$$

$$= \left(\frac{r_0 \partial\theta_0}{\partial r}\right)\left(\frac{\partial r_0}{\partial r}\right) + \left(\frac{r_0 \partial\theta_0}{r\partial\theta}\right)\left(\frac{\partial r_0}{r\partial\theta}\right) + \left(\frac{r_0 \partial\theta_0}{\partial z}\right)\left(\frac{\partial r_0}{\partial z}\right)$$