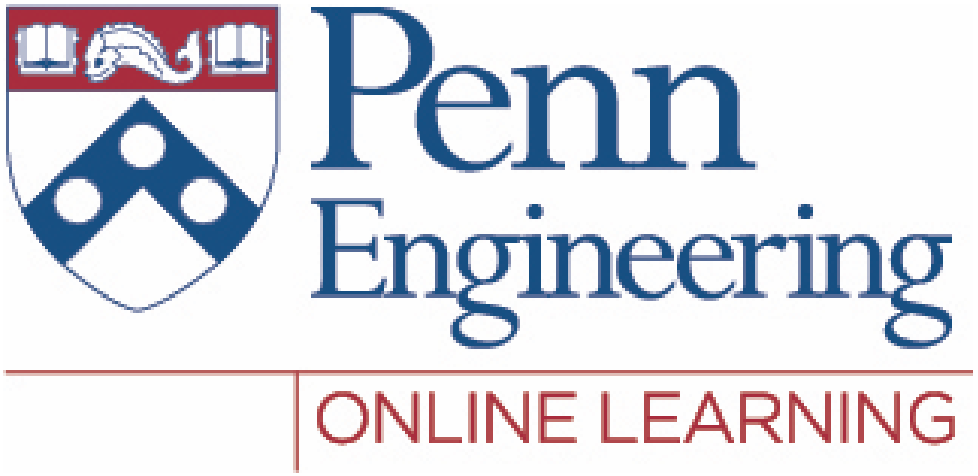

Video 2.1
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Convolutional Neural Networks (CNNs) +3

What is a convolutional neural network?

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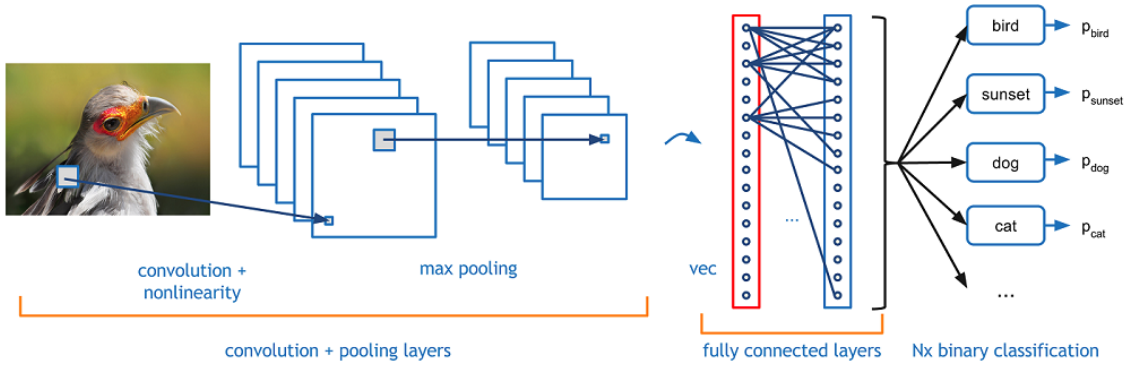
A Beginner's Guide To Understanding Convolutional Neural Networks



UNDERSTANDING CONVOLUTIONAL NEURAL NETWORKS FOR NLP

When we hear about Convolutional Neural Network (CNNs), we typically think of Computer Vision. CNNs were responsible for major breakthroughs in Image Classification and are the core of most Computer Vision systems today, from Facebook's automated photo tagging to self-driving cars.

patterns; with more than one, you could look for patterns of patterns. Take the case of image recognition, which tends to rely on a contraption called a "convolutional neural net." (These were elaborated in a seminal 1998 paper whose lead author, a Frenchman named Yann LeCun, did his postdoctoral research in Toronto under Hinton and now directs a huge A.I. endeavor at

digital input and send output to other nodes. Layers upon layers of these nodes make up so-called convolutional neural networks, which, with sufficient training data, have become better and better at identifying images.

Inside a Convolutional Neural Network

But how does this filtering work? The secret is in the addition of two new types of layers: convolutional and pooling layers. We'll break the process down below, using the example of a network designed to do just one thing: determine whether a picture contains a grandma or not.

Next, Zweig and co optimized their own deep-learning systems based on convolutional neural networks with varying number of layers, each of which processes a different aspect of speech. They then used the

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convolutional neural networks

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Convolution

$$(f \star g)(t) = \int_{-\infty}^{\infty} f(t')g(t-t')dt'$$

Is Convolution that convoluted?

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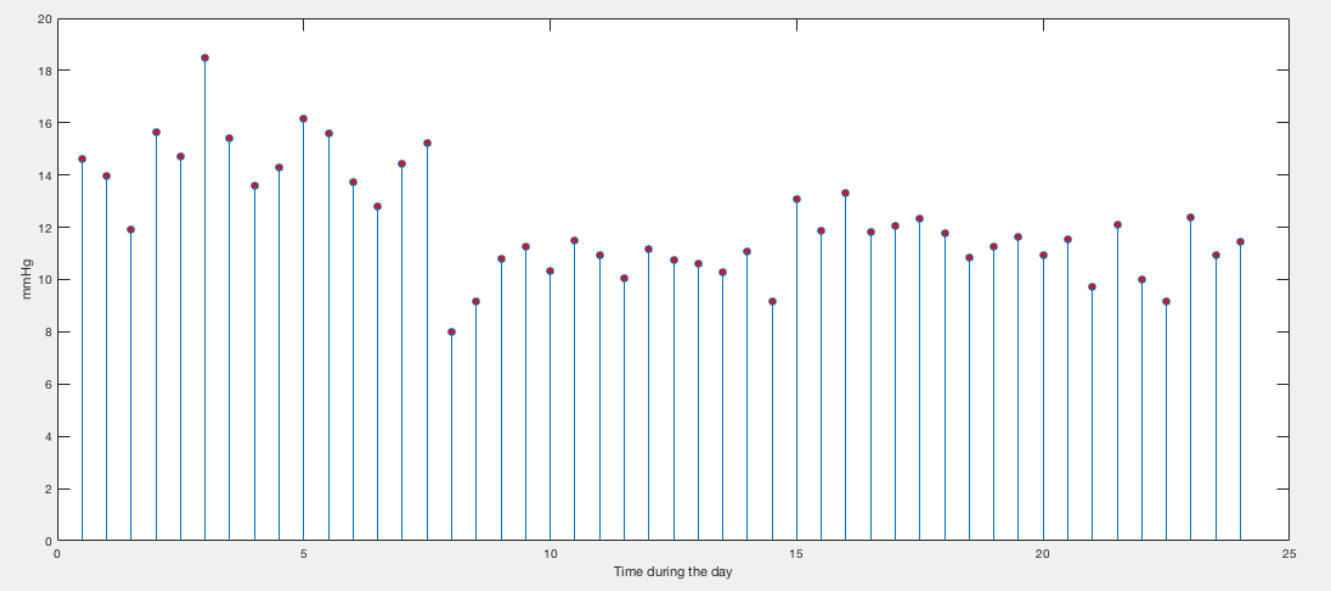
Let us start with discrete 1D signals !

For example, my systolic blood pressure measured with a wearable over 24 hours on 1/16/2016:



$s[n]$

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Interested in the largest drop?

```
 15   14   12   16   15   18   15   14   14   16   16   14   13   14   15    8    9   11   11   10   11
-1    1
     -1    1
          -1    1
               -1    1
```

We are moving a sliding window and at every position we take the scalar product between the mask and the signal.

```
-1   -2    4   -1    4   -4   -1    0    2    0   -2   -1    1    1   -7    1    2    0   -1
```


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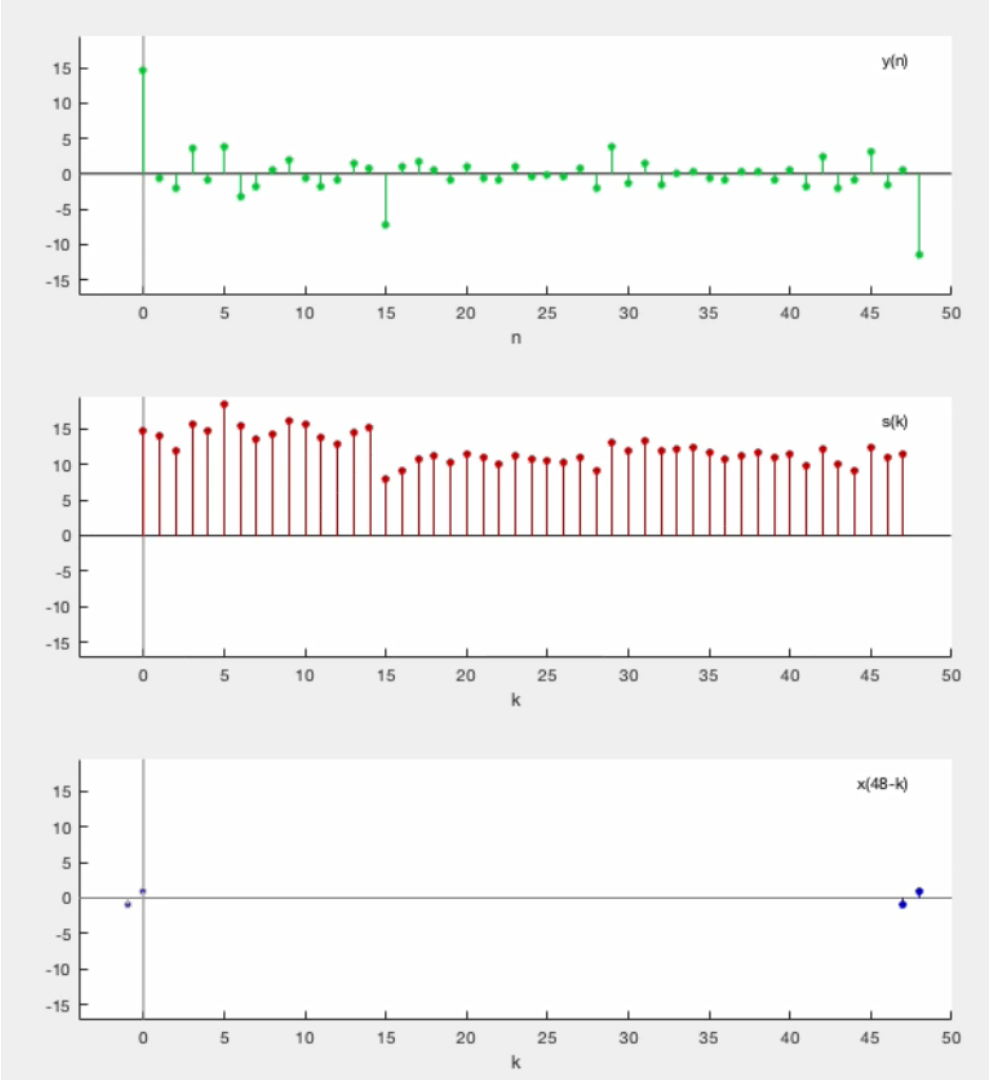
Let us visualize it while in action:

We will call this operation correlation between two discrete signals and we can write it as

$$y[n] = \sum_{k=1}^{N} s[k]\, h[k-n]$$



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Convolution differs from correlation by a reflection:

We take the "mask" d[k] −1 1

and reflect it to d[-k] 1 −1

and then we shift to
d[-(k-n)]=d[n-k]

$$y[n] = \sum_{k=1}^{N} s[k]\, h[n-k]$$

1 −1

15 14 12 16 15 18 15 14 14 16 16 14 13 14 15 8 9 11 11 10 11

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The informal term for the convolution mask is “filter mask”, its values are called “filter weights” or convolution weights.

Let us do another operation on the systolic pressure signal.

Taking the local average over a window of two hours.

The averaging mask would look like
[¼ ¼ ¼ ¼]

Does the convolution differ from correlation?

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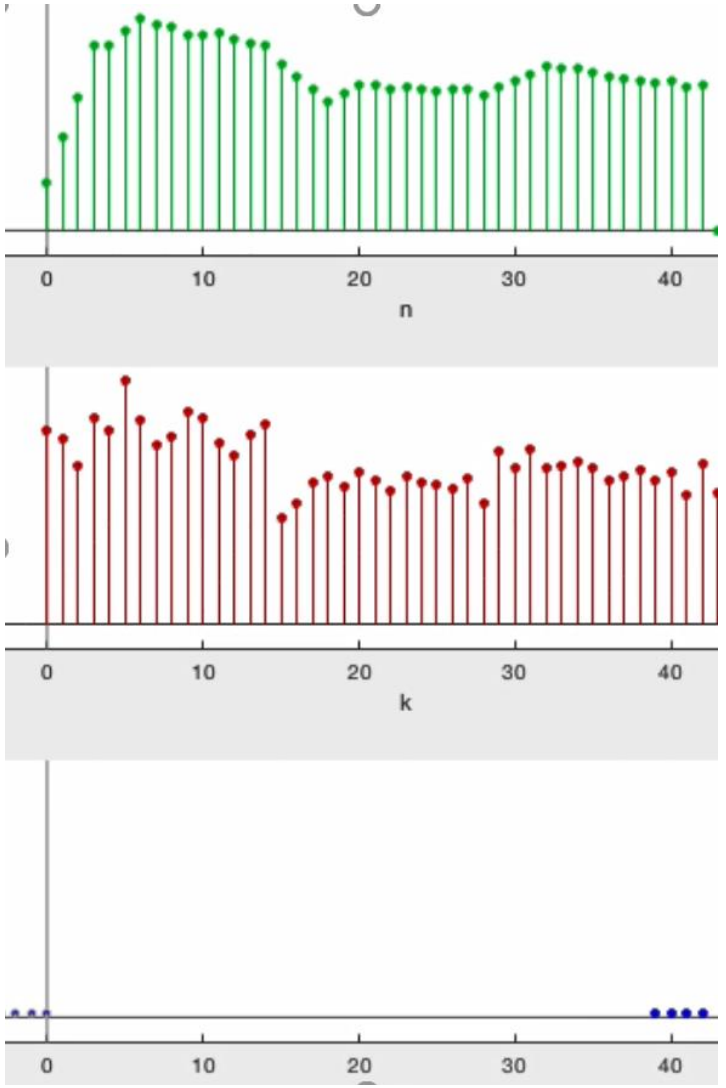

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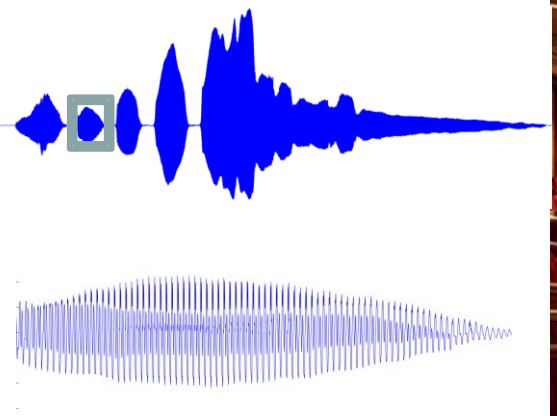
Let us move to the Academy of Music!



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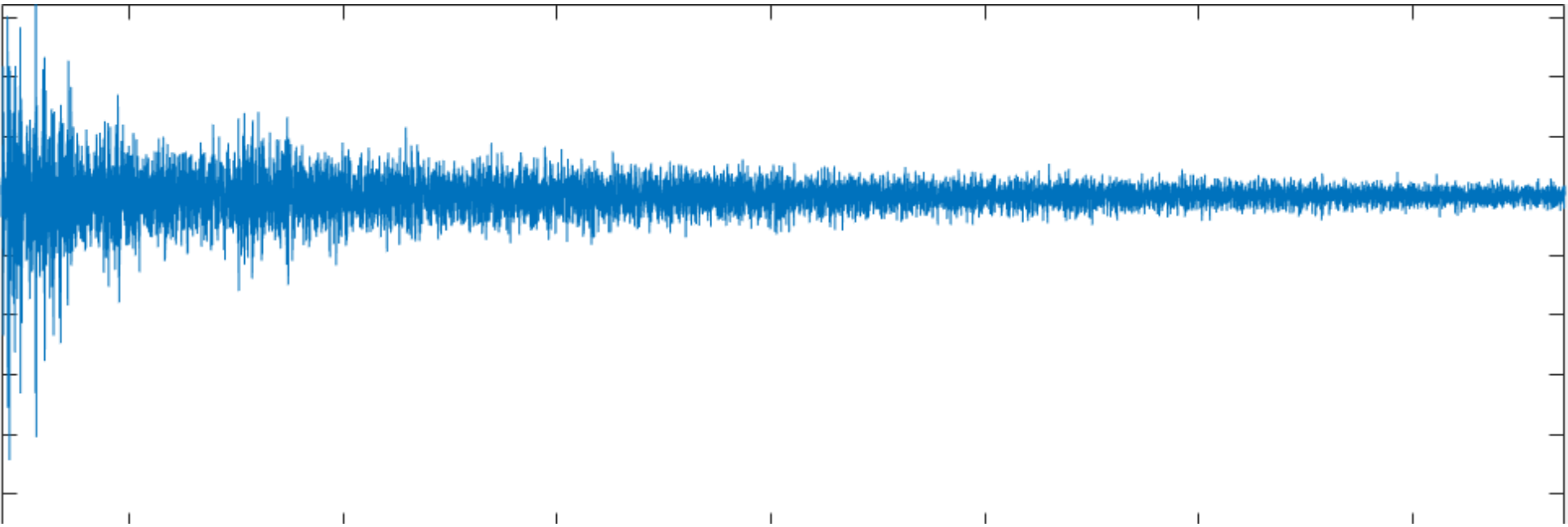
Sound is reflected creating multiple echos



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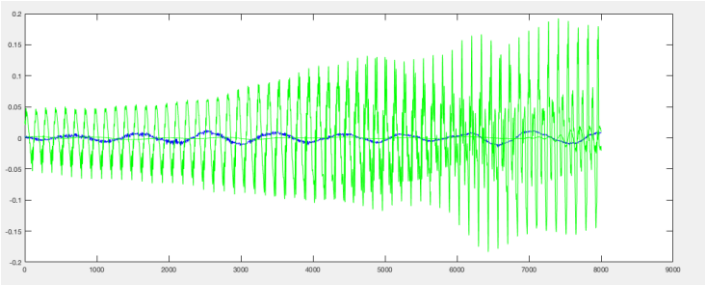
Now let us take the song s[n] and convolve it with the pistol h[n]

$$y[n] = \sum_{k=1}^{N} s[k]\, h[n-k]$$



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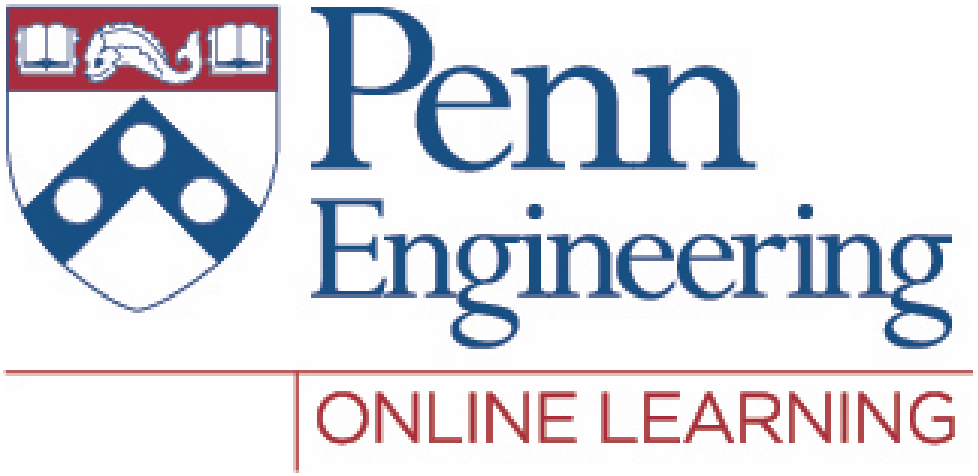

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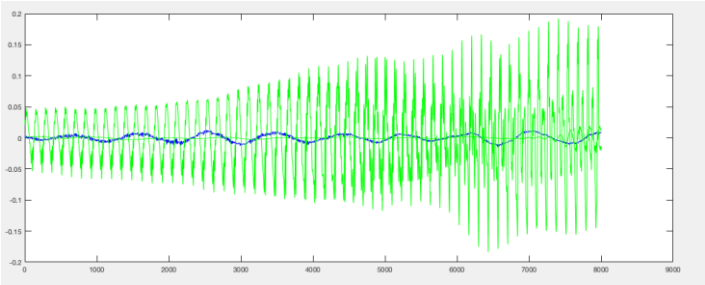
In the last lecture the song s[n] and convolve it with the pistol h[n]

$$y[n] = \sum_{k=1}^{N} s[k]\, h[n-k]$$



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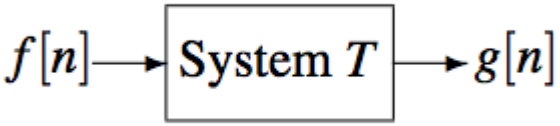
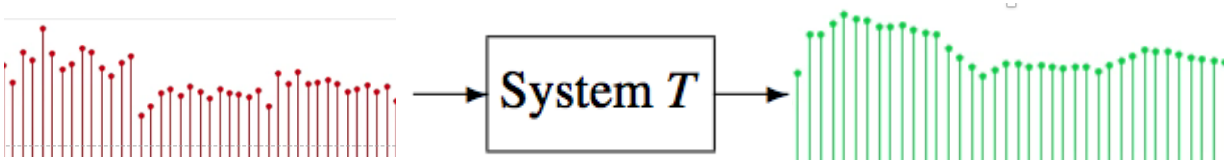
The systems' view

Consider it as a system with input and output. In the previous examples, input might have been

- Audio like a song
- Blood pressure measurements
- In general any 1D signal





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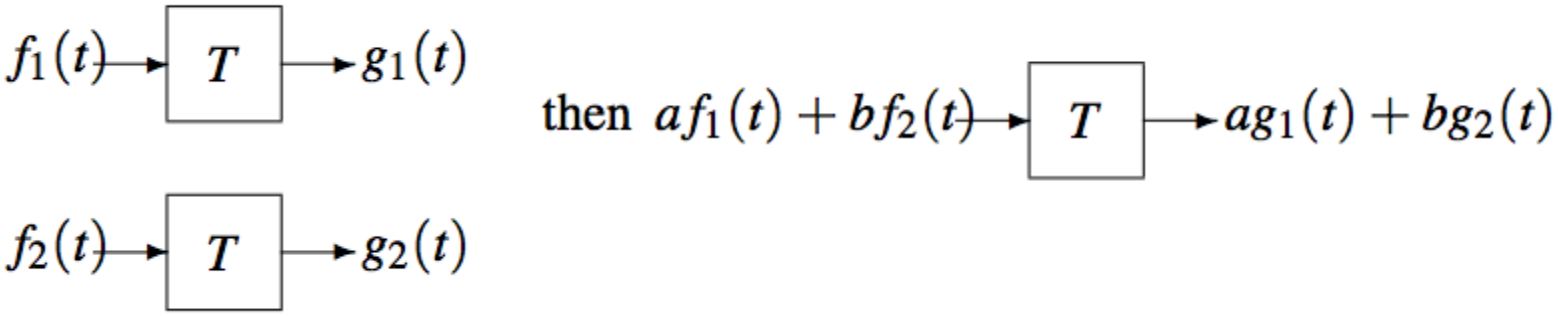
Linear System

- We will say that a system is linear if



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Example

$T\{f\}(t) = f(t) - f(t-1)$ is linear:

$$
\begin{aligned}
g_1(t) &= f_1(t) - f_1(t-1) \\
g_2(t) &= f_2(t) - f_2(t-1) \\
T\{af_1 + bf_2\}(t) &= [af_1(t) + bf_2(t)] - [af_1(t-1) - bf_2(t-1)] \\
&= a(f_1(t) - f_1(t-1)) + b(f_2(t) - f_2(t-1)) \\
&= aT\{f_1\}(t) + bT\{f_2\}(t)
\end{aligned}
$$

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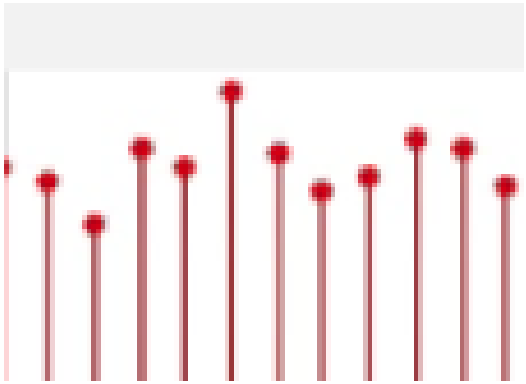
What is non-linear?

$$g(t) = \max(f(t), f(t-1), f)t-2))$$



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Shift-invariant (or time-invariant) system

Examples:

- $T\{f\}(t) = f(t) - f(t-1)$ is shift-invariant.

$$\begin{aligned} T\{f(t-t_0)\} &= f(t-t_0) - f(t-t_0-1) \\ &= g(t-t_0) \end{aligned}$$

- $g(t) = tf(t)$ is not shift-invariant.

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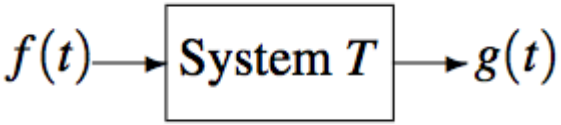
Is there a formula for describing linear shift-invariant systems?



Yes, the convolution! Discrete or continuous

$$g(t) = \int_{-\infty}^{\infty} f(t')h(t-t')dt' \qquad g[n] = \sum_{k=-\infty}^{\infty} f[k]\,h[n-k]$$

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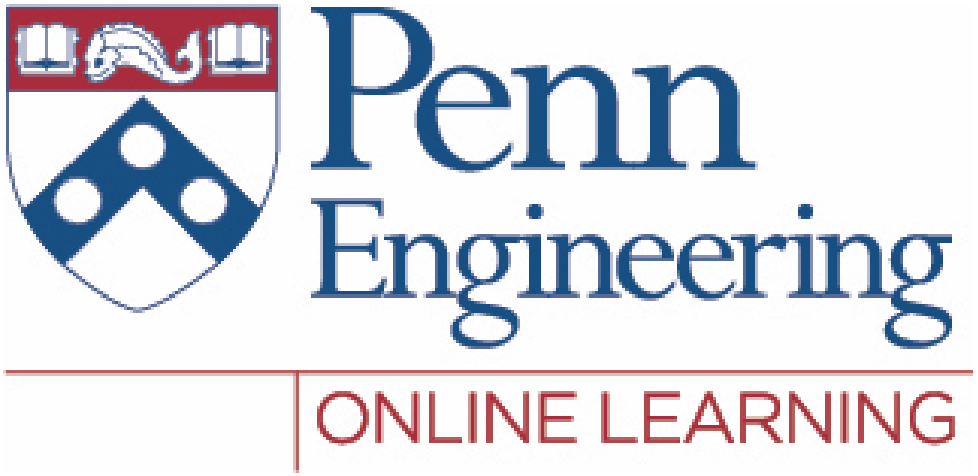

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What is a filter?

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Last lecture's main result: Linear shift-invariant systems can be written as convolutions!

$$g(t) = \int_{-\infty}^{+\infty} f(t')h(t-t')dt' = f(t) \star h(t)$$

But what exactly is this h(t) or h[n] that we call filter?

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Box function

$$\text{rect}(t) = \begin{cases} 1\,, & |t| \le 1/2 \\ 0\,, & \textit{otherwise} \end{cases}$$

$$\text{rect}(t/a) = \begin{cases} 1/a\,, & |t| \le a/2 \\ 0\,, & \textit{otherwise} \end{cases}$$

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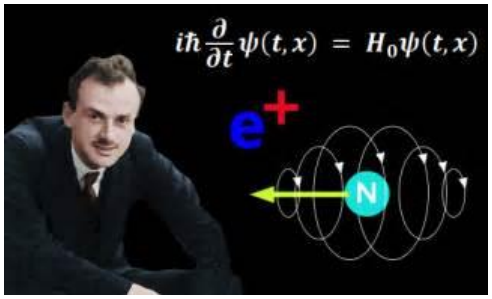
Dirac function



$$\delta(t) \doteq \lim_{a\to 0} \frac{1}{a} \operatorname{rect}(t/a)$$

It follows from definition of the Dirac function that

$$\delta(t) = 0 \quad \text{for all } t \neq 0$$

and

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

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Absorption property

$$\int_{-\infty}^{+\infty} \delta(t) f(t) dt = f(0)$$

or in a more general form:

$$\int_{-\infty}^{+\infty} \delta(t - t_0) f(t) dt = f(t_0)$$

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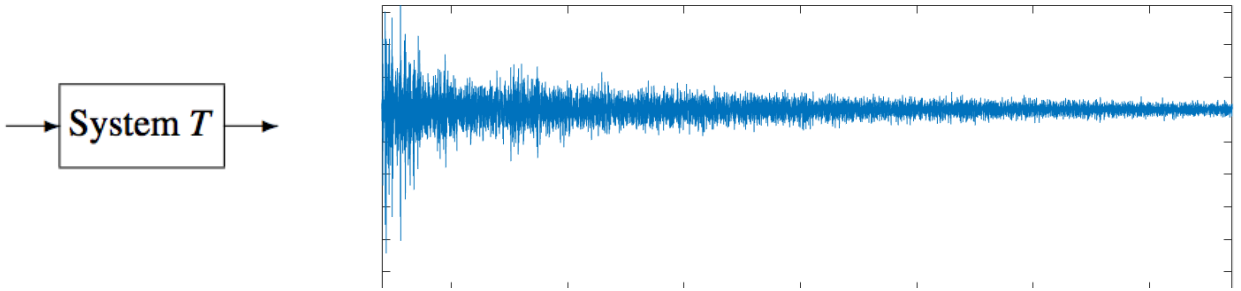
What happens when the input to an LSI is a Dirac?

$$\int_{-\infty}^{+\infty} \delta(t')h(t-t')dt' = h(t)$$



A filter *h(t) or h[n]* is the response of the system to the Dirac impulse.

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Example. Let an LSI system with impulse response defined as

$$h(t) = (1/2)(\delta(t+1) - \delta(t-1))$$

$$\begin{aligned}
g(t) &= \int_{-\infty}^{+\infty} f(t')h(t-t')dt' \\
&= \int_{-\infty}^{+\infty} f(t-u)h(u)du \\
&= \int_{-\infty}^{+\infty} f(t-u)(1/2)(\delta(u+1) - \delta(u-1))du \\
&= (1/2)\int_{-\infty}^{+\infty} f(t-u)\delta(u+1)du - (1/2)\int_{-\infty}^{+\infty} f(t-u)\delta(u-1)du \\
&= (1/2)(f(t+1) - f(t-1))
\end{aligned}$$

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Dirac description of sampling

- What do we get discrete signals out of analog?

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Sampling is multiplication with the comb

Sampling is a multiplication of the signal by the comb function $\mathrm{III}_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. T is the sampling interval:

$$f_S(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

(S for "sampled") After sampling, we forget about the T: we just obtain a sequence of numbers, that we note $f_s[k]$.

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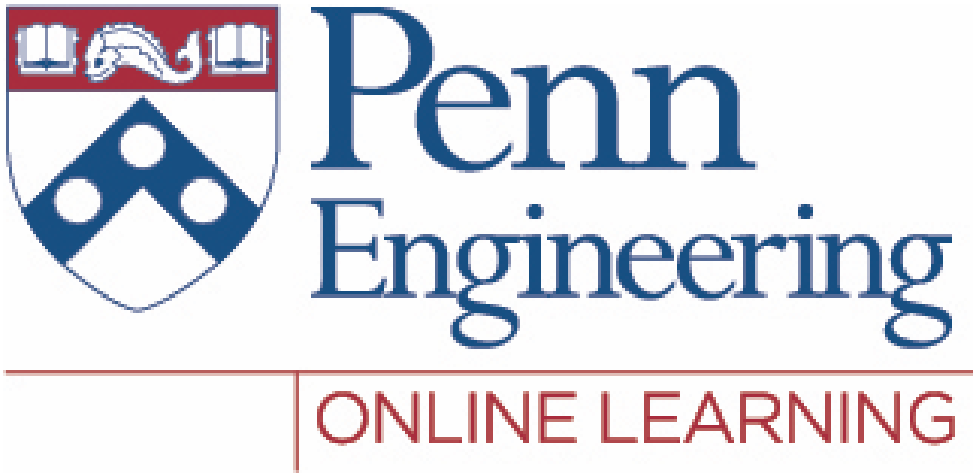

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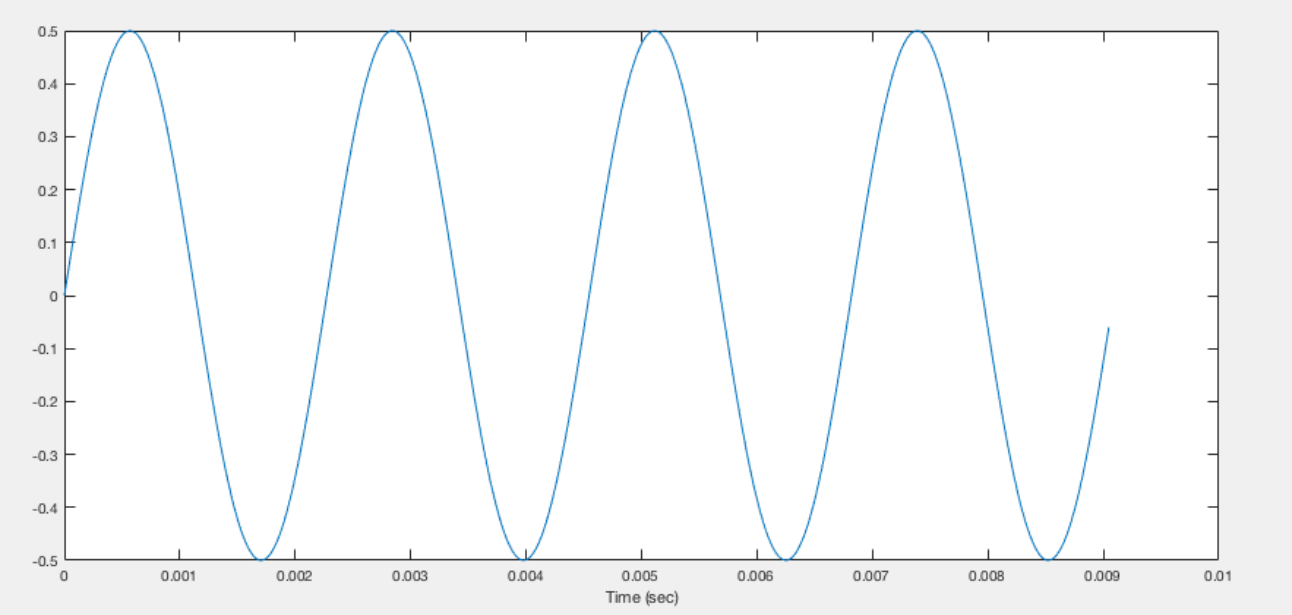
Back to music: sine wave 440Hz



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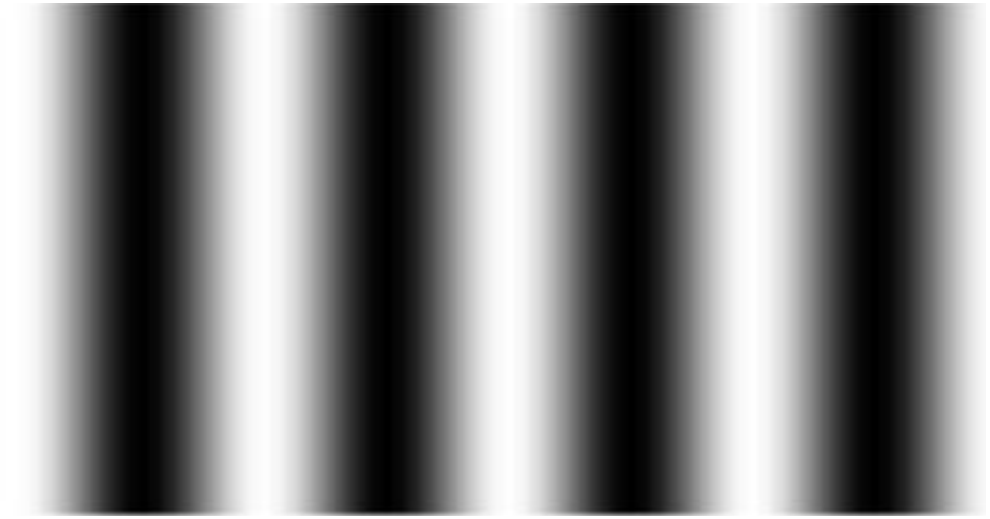
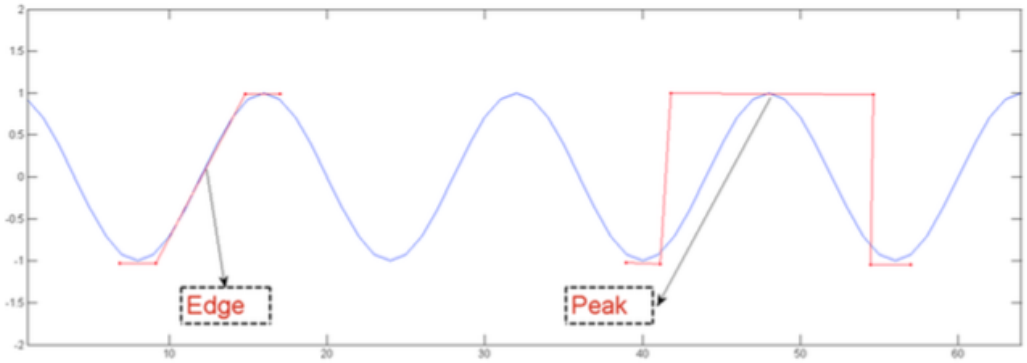
Or images….



2D Signal $f(x, y) = \cos\left(\frac{2\pi x}{16}\right)$



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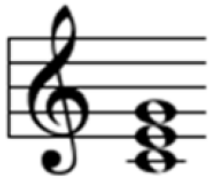
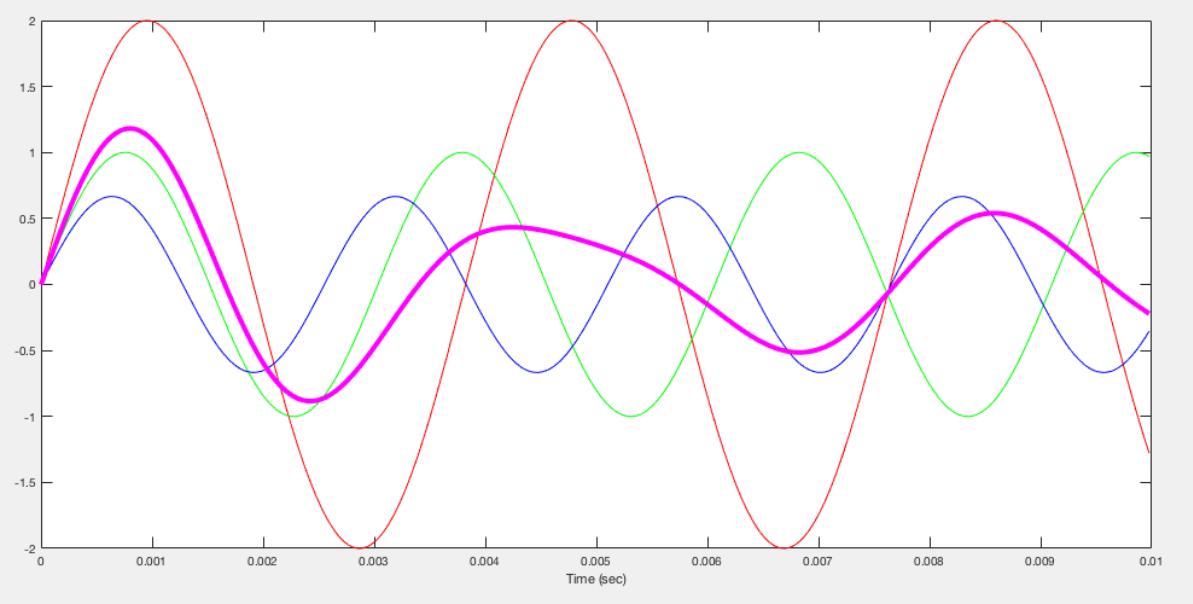
Play three notes together: C-major chord consists of a superposition of C (262Hz), E(330HZ) and G(392Hz)





$$A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t) + A_3 \sin(2\pi f_3 t)$$

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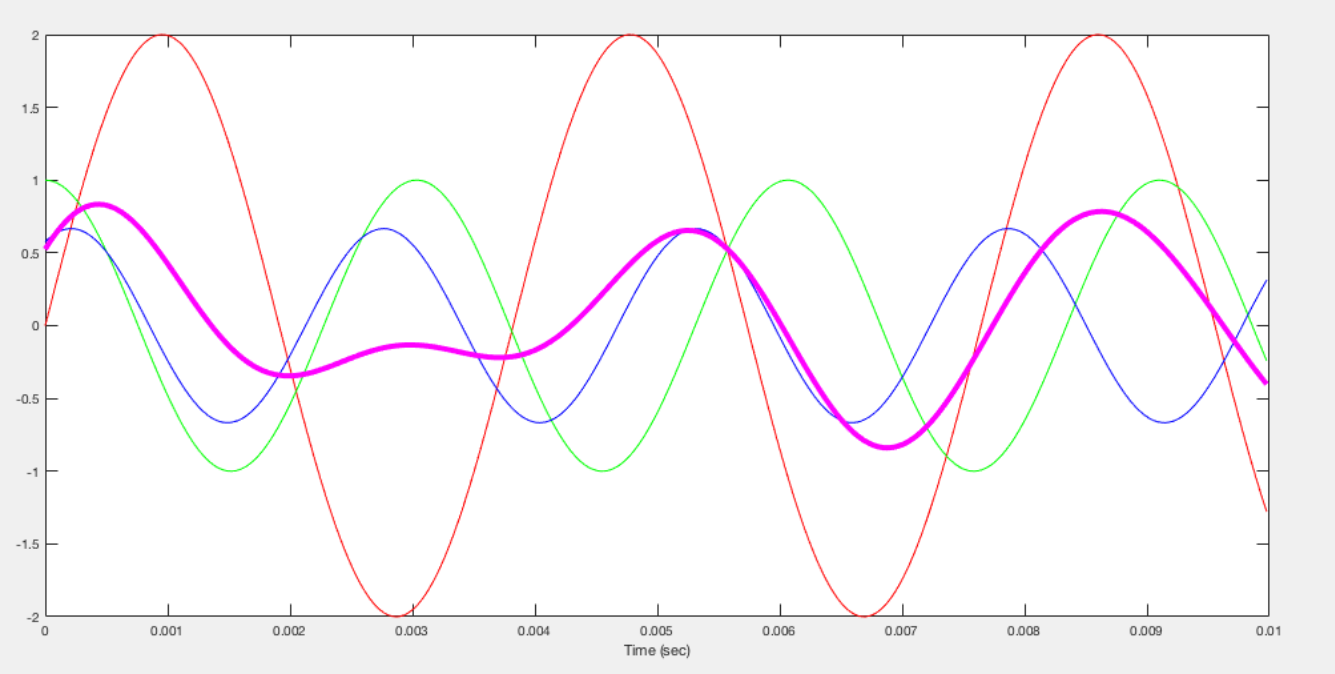
We could also change the phases:



$$A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t + \frac{\pi}{2}) + A_3 \sin(2\pi f_3 t + \frac{\pi}{3})$$

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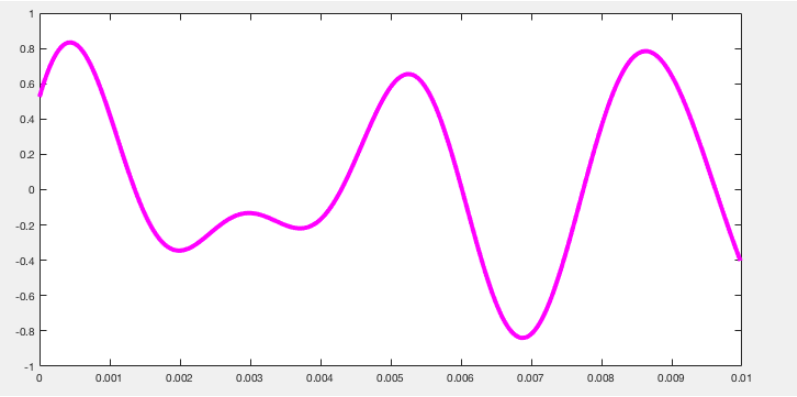
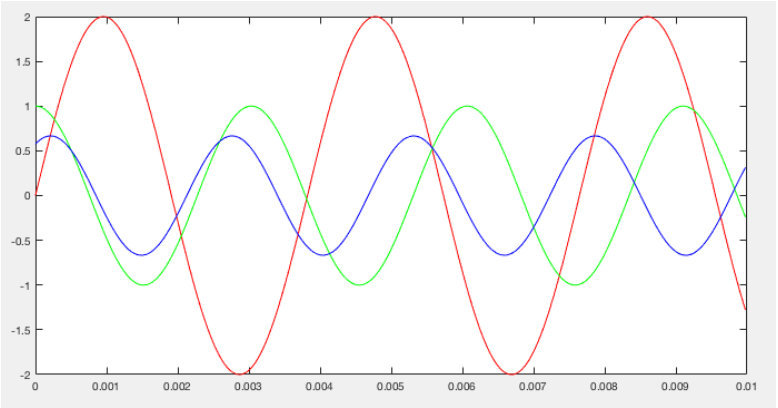
Can we solve the inverse problem?





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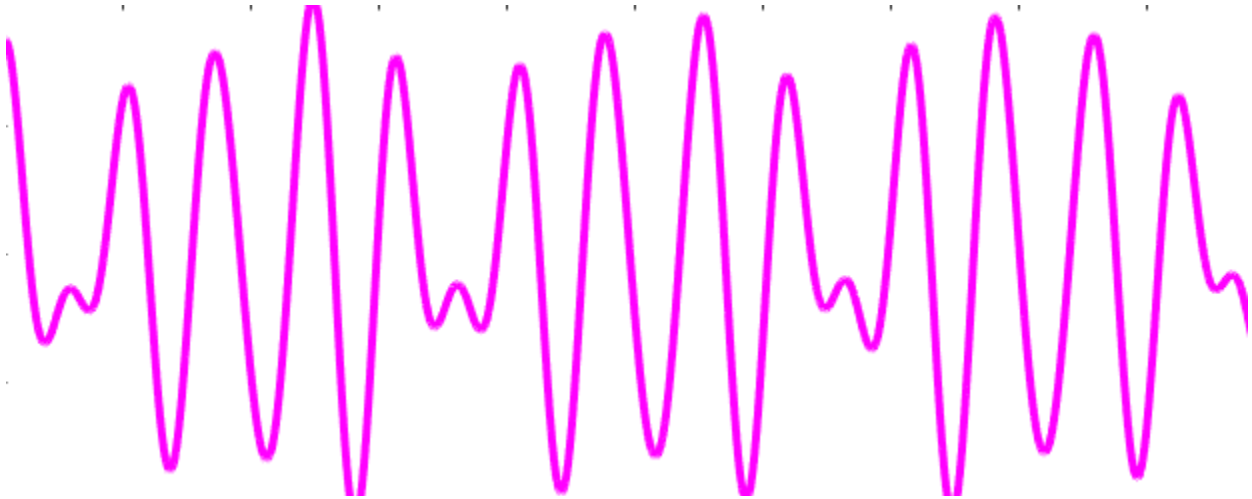
Yes, we can decompose a function into "sinusoids" if it is periodic: Fourier series



$$f(t) = \sum_{n=-\infty}^{\infty} F[n] e^{j2\pi \frac{n}{T} t}$$

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The complex Fourier coefficients F[n] can be found with the Fourier Transform

$$F[n] = \int_{-T/2}^{T/2} f(t) e^{-j2\pi \frac{n}{T} t} dt$$

$j^2=-1$

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Quick reminder on complex numbers:

- $a + jb \in \mathbb{C}$, $j^2 = -1$
- $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$.
- $a + jb = re^{j\theta}$ with $r = \sqrt{a^2 + b^2}$, $\theta = atan2(b, a)$

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Why we need complex numbers?

To account for the phase

$$f(t) = \sum_{n=-\infty}^{\infty} F[n] e^{j2\pi \frac{n}{T} t}$$

or

$$f(t) = \sum_{n=-\infty}^{\infty} A_k(\cos(2\pi \frac{n}{T} t + \phi_k)$$

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Let us agree on a notation:

$$F[n] = \int_{-T/2}^{T/2} f(t) e^{-j2\pi \frac{n}{T} t} dt$$

Definition of the Fourier Transform:

$$f(t) \circ\!\!-\!\!\bullet F(\omega) = \mathcal{F}\{f(t)\}$$

$$F : \mathbb{R} \to \mathbb{C}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt,$$

where ω denotes the frequency. This definition is sometimes called non-unitary Fourier transform, with angular frequency (ω is referred to as angular frequency, and $s = \frac{n}{T}$ is the ordinary frequency in Hz such that $\omega = 2\pi s$).

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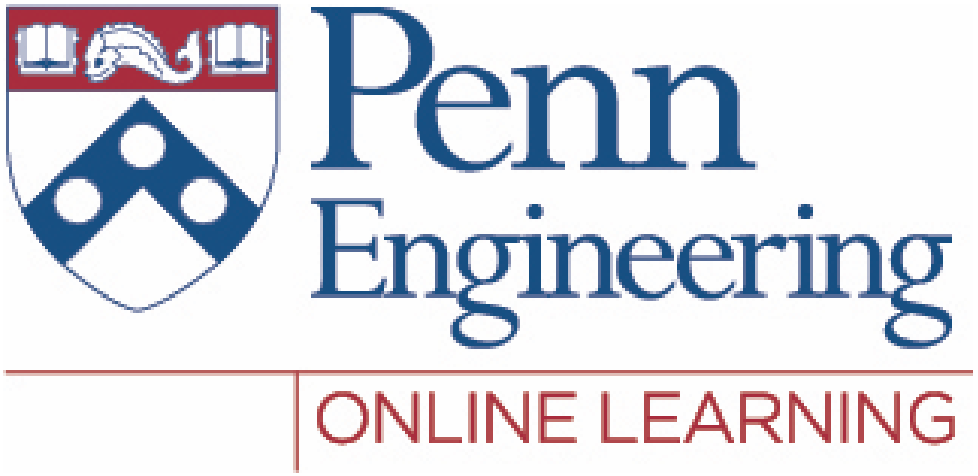

Video 2.5
Kostas Daniilidis

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Some facts about the Fourier Transform

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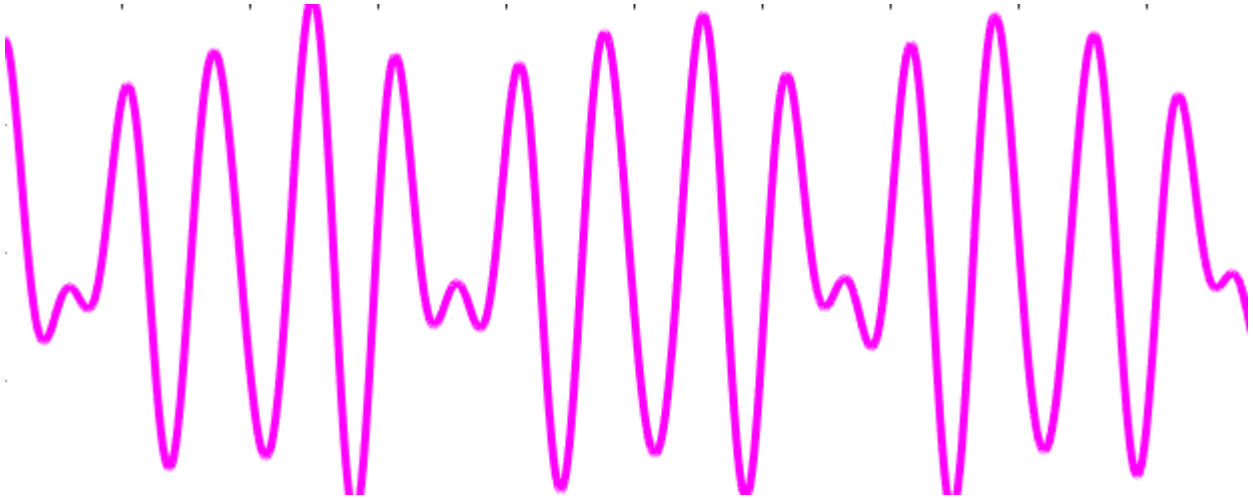
We learnt about the Fourier series of periodic signals



$$f(t) = \sum_{n=-\infty}^{\infty} F[n] e^{j2\pi \frac{n}{T} t}$$

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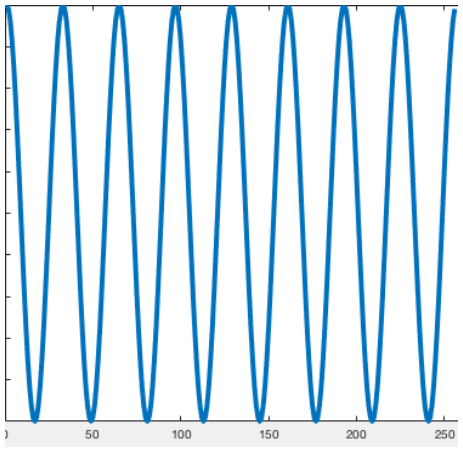
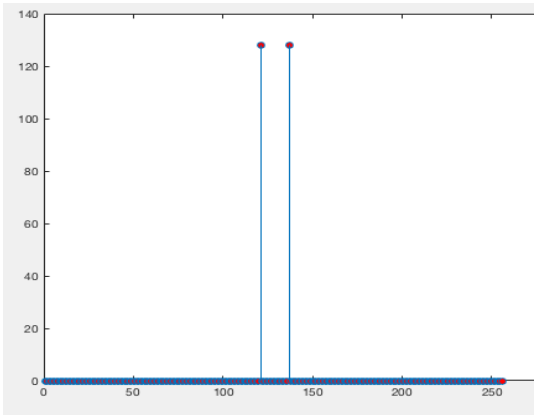
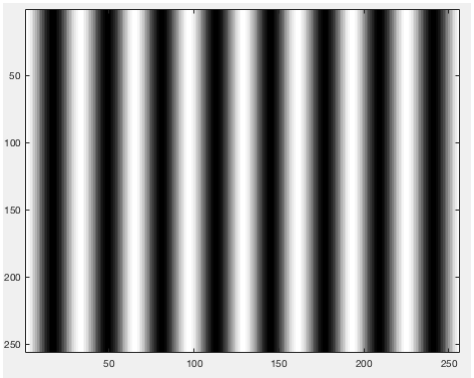
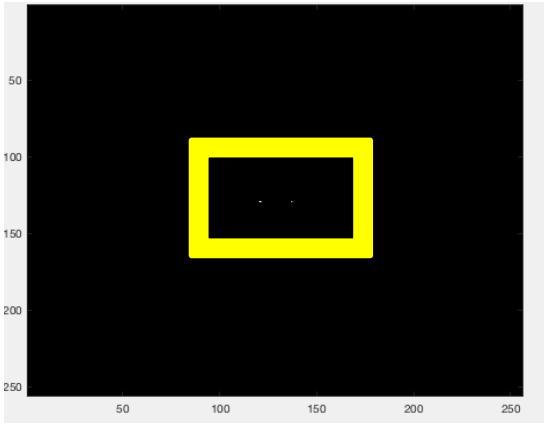
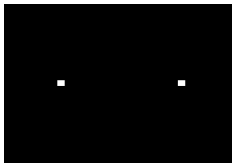
Fourier Transform of a Cosine











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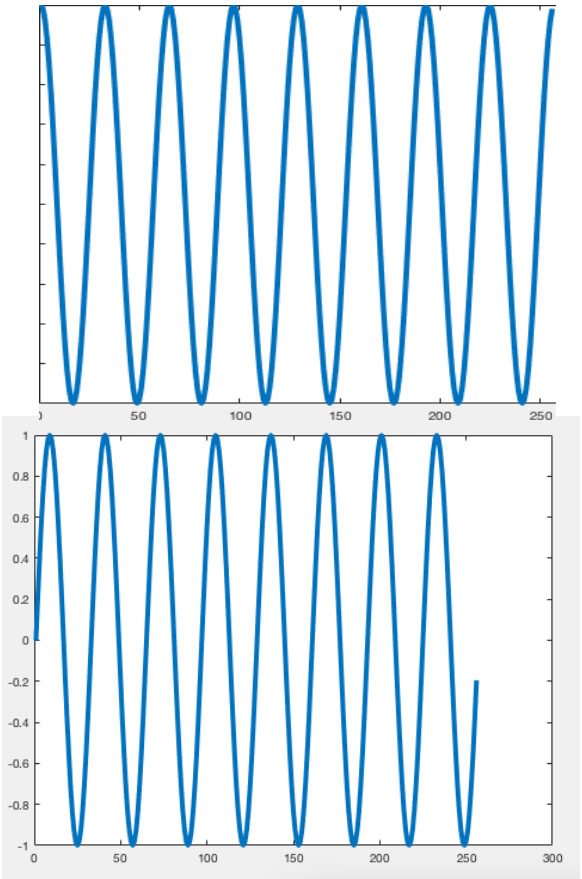
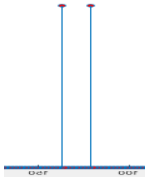
Do sine and cosine have the same Fourier transform



Only in the magnitude |F[n]|!



They differ in the phase !

$$\cos(\omega_0 t) \quad \multimap\bullet \quad 2\pi.\frac{1}{2}(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$

$$\sin(\omega_0 t) \quad \multimap\bullet \quad 2\pi.\frac{1}{2j}(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$$

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Fourier of the harmonic exponential

$$e^{j\omega_0 t} \circ\!\!-\!\!\bullet\; 2\pi\delta(\omega - \omega_0)$$

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Two phase-related facts

Shift theorem

$$f(t-t_0) \multimap F(\omega)e^{-j\omega t_0}$$

Example: for $f(t)$ even, $f(t-\frac{T}{2}) \multimap F(\omega)e^{-j\omega\frac{T}{2}}$, where $F(\omega)$ is real.

Modulation theorem

$$f(t)e^{j\omega_0 t} \multimap F(\omega-\omega_0)$$

Multiplying by a complex exponential causes a shift in the frequency domain.

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Let us look at some more Fourier transforms: Dirac

Recall the absorption property

$$\int_{-\infty}^{\infty} \delta(t-t_0) f(t) dt = f(t_0)$$

Then we have:

$$\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$$

We will remember the two following results:

$$\delta(t) \quad \circ\!\!-\!\!\bullet \quad 1$$

$$1 \quad \circ\!\!-\!\!\bullet \quad 2\pi\delta(t) \quad \text{(DC-component)}$$

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Fourier of the comb function $\text{Ш}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \multimap \frac{1}{|T|} \sum_{n=-\infty}^{\infty} \delta(s - \frac{n}{T})$$

(or $\frac{2\pi}{|T|} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T})$ when using the non-unitary Fourier Transform with angular frequency).

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Step and sgn functions

Preliminary: The step function is defined such that $\frac{d}{dt}u(t) = \delta(t)$

The sign function comes handy: $\text{sgn}(t) = 2u(t) - 1,\ u(0) = 1/2$, or equivalently $u(t) = \frac{1}{2} + \frac{1}{2}\text{sgn}(t)$.

We have the following straightforward Fourier transforms:

$$u(t) \circ\!\!-\!\!\bullet\, \mathcal{U}(\omega) = \frac{1}{2}\delta(\omega) + \frac{1}{j\omega}$$

$$\frac{d}{dt}\text{sgn}(t) = 2\delta(t) \circ\!\!-\!\!\bullet\, j\omega\mathcal{F}(\text{sgn}(t)) = 2.$$

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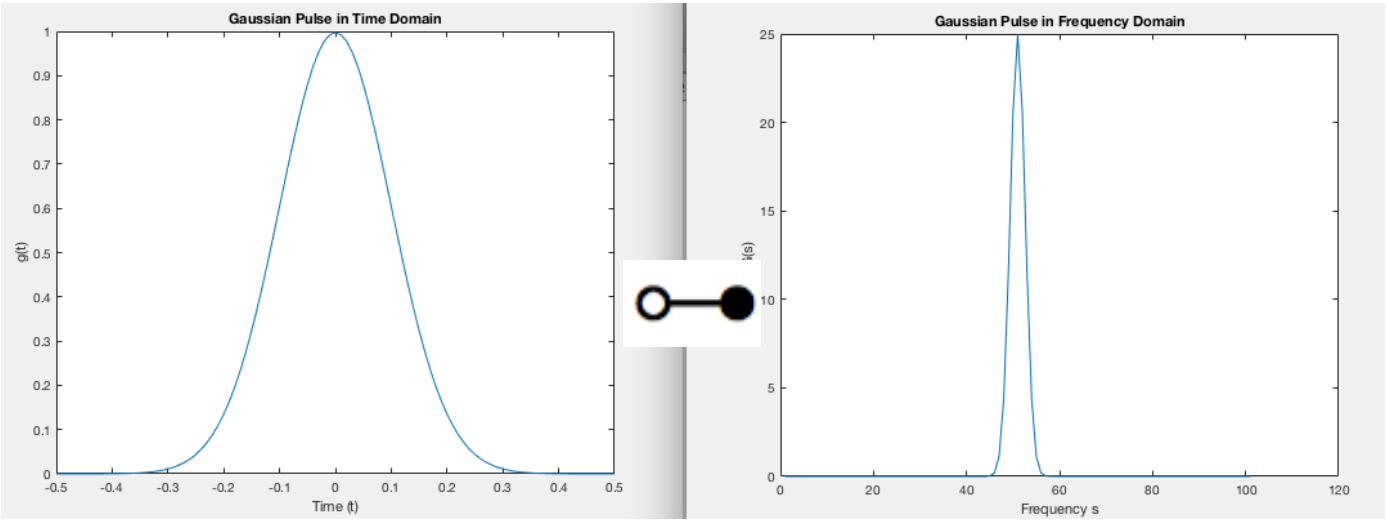
Fourier Transform of the Gaussian Function



$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \quad \multimap \quad F(s) = \frac{1}{\sigma\sqrt{2\pi}} e^{-2\pi^2\sigma^2 s^2}$$

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Convolution Theorem

Convolution in frequency means multiplication in time or space

$$f(t) \rightarrow \boxed{h(t)} \rightarrow g(t) = \int_{-\infty}^{\infty} g(t')h(t-t')dt' = f(t) * h(t)$$

$$F(\omega) \qquad H(\omega) \qquad G(\omega)?$$

$$f(t) * h(t) \multimap F(\omega)H(\omega)$$

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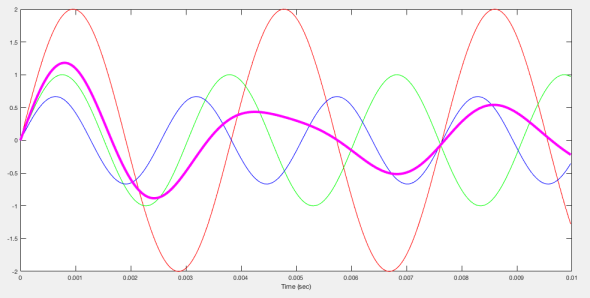
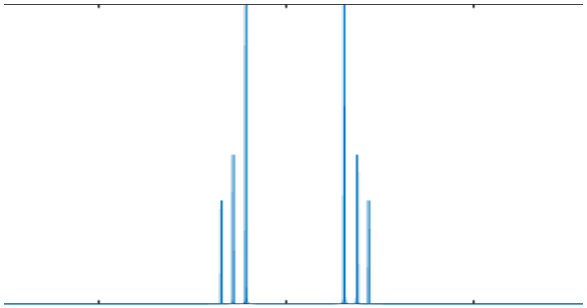
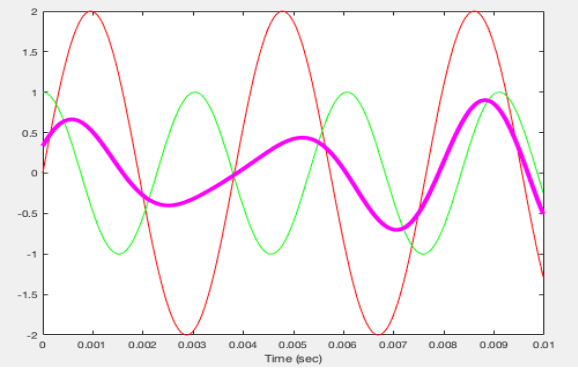
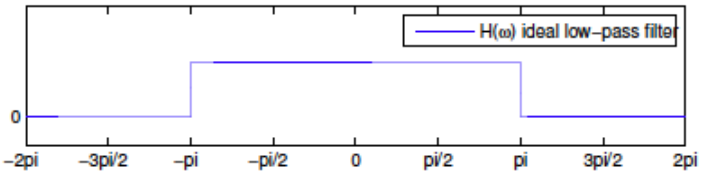
Convolution Theorem: Low Pass Filtering

$$f(t) * h(t) \multimap\!\bullet\; F(\omega)H(\omega)$$









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Convolution Theorem II

Convolution in frequency means multiplication in time or space

$$f(t)h(t) \circ\!\!-\!\!\bullet\; F(\omega) * H(\omega)$$

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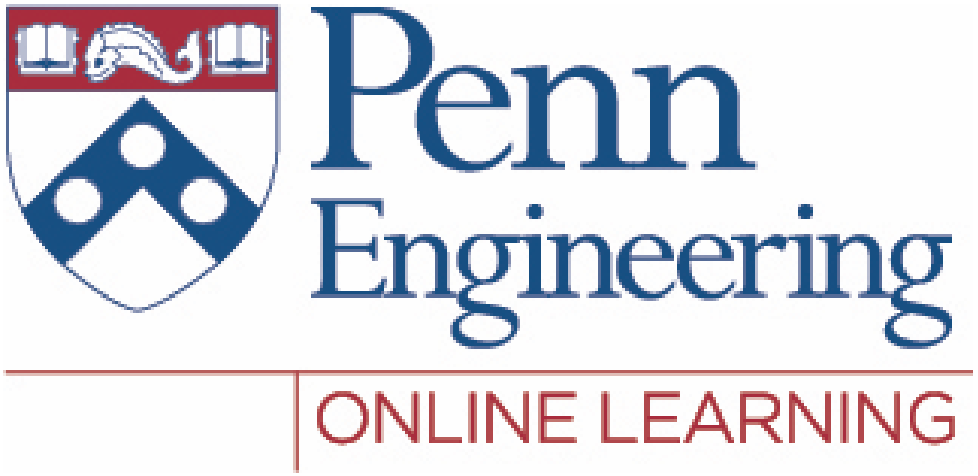

Video 2.6
Kostas Danillidis

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Sampling: Converting analog time or space to digital

Optical Illusion: Wagon Wheel - YouTube

https://www.youtube.com/watch?v=tMk14AEpRd8

Jan 14, 2016 - Uploaded by Cognic

Optical **Illusions** and Mind Tricks to make people understand something. We see and feel and smell and touch ...

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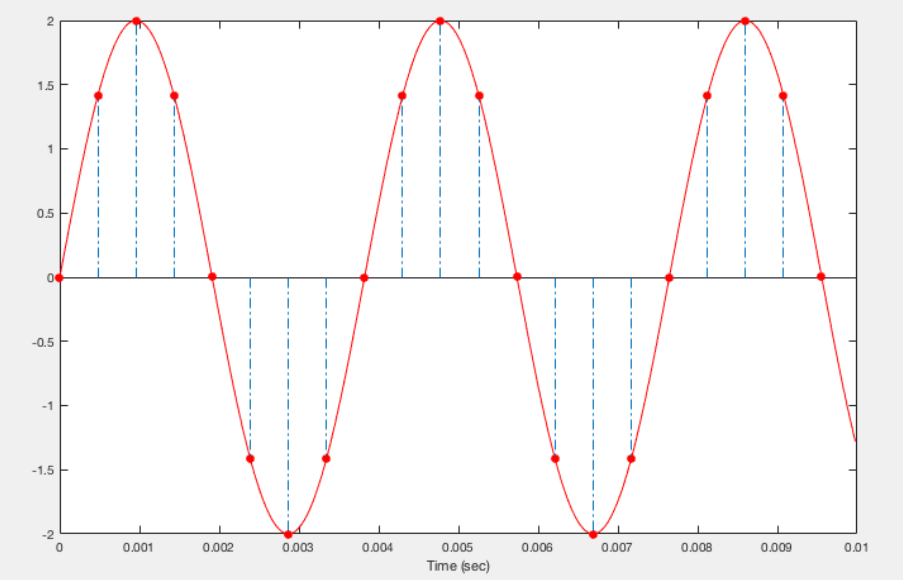
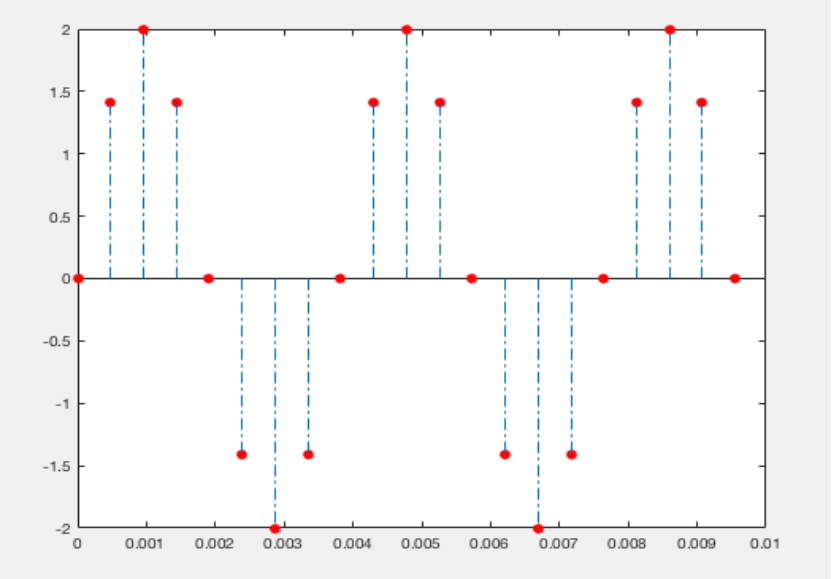
Back to the sine-wave: sampling interval 1/8 of wavelength





For note A (440 Hz), sampling frequency of 44.1kHz means 100 samples per wavelength !

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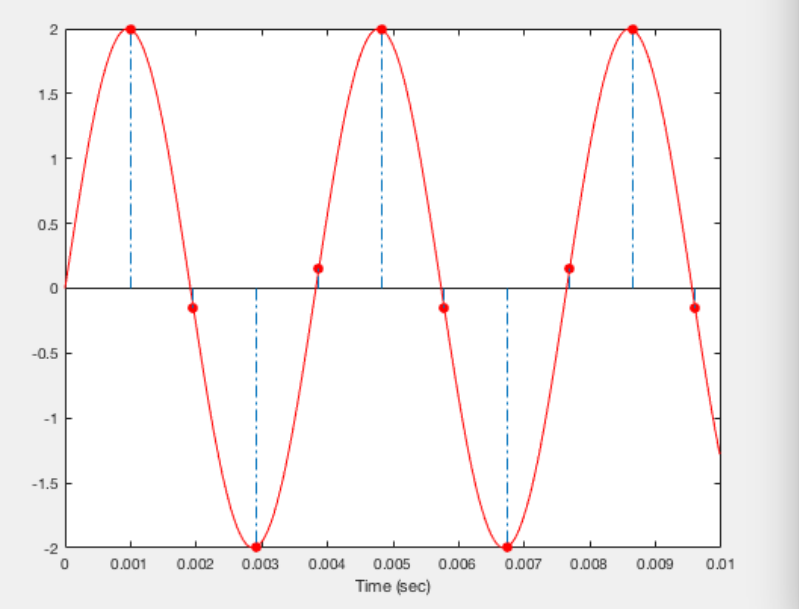
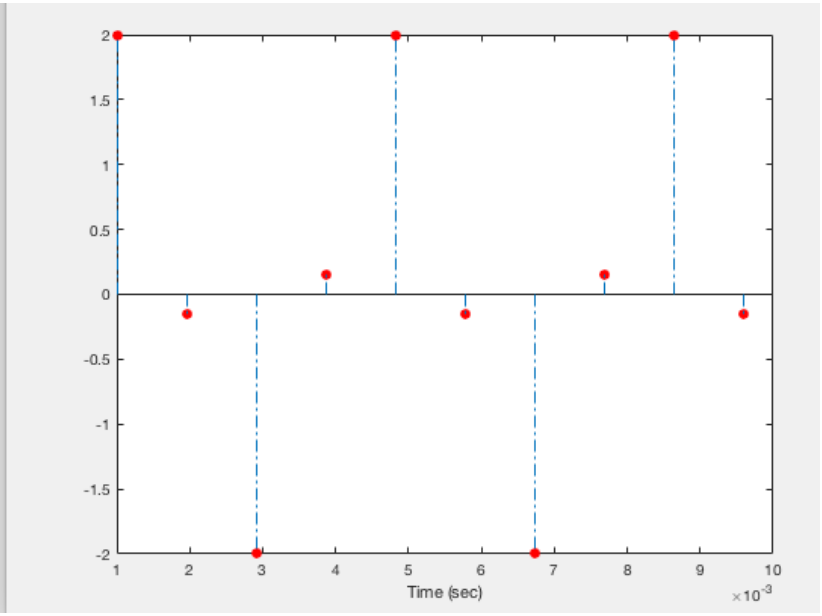
Increasing sampling interval to 1/4 of wavelength





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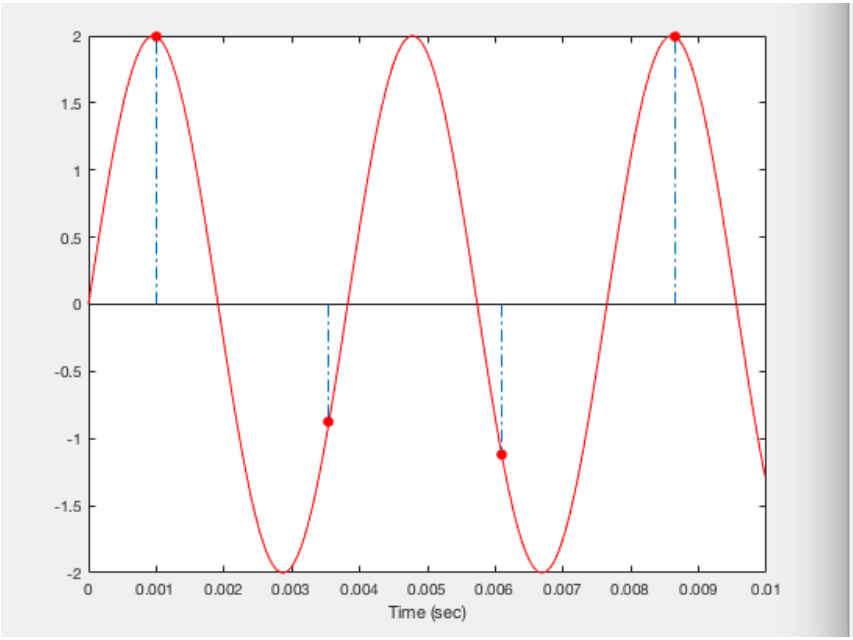
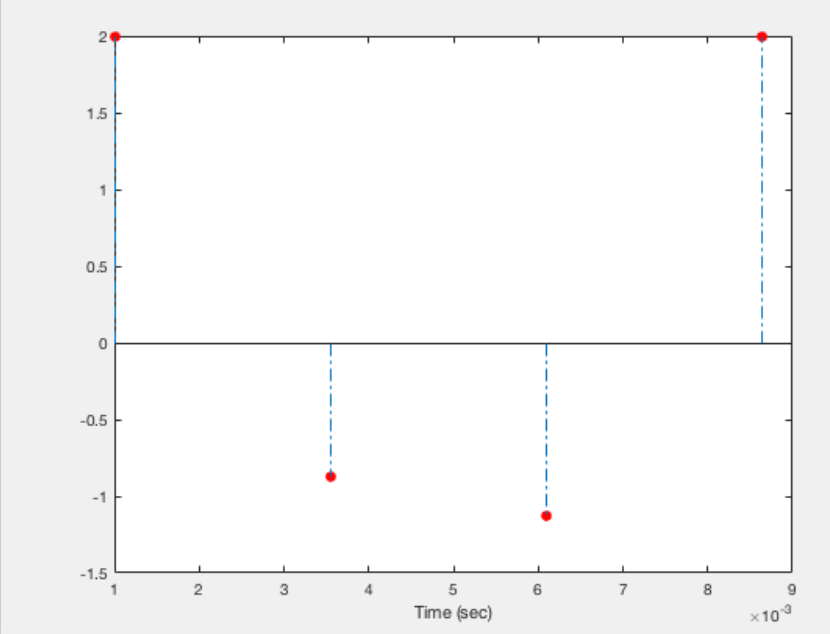
Increasing sampling interval to more than ½ of wavelength





We cannot reconstruct the original wave!

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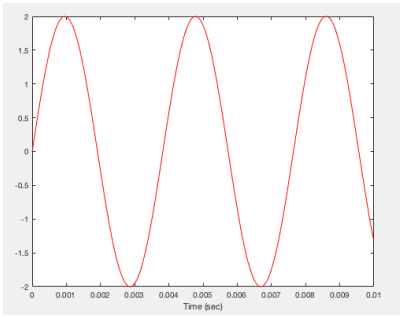
What does sampling mean?

Sampling is a multiplication of the signal by the comb function $\text{Ш}_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. T is the sampling interval:

$$f_S(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

(S for "sampled") After sampling, we forget about the T: we just obtain a sequence of numbers, that we note $f_s[k]$.



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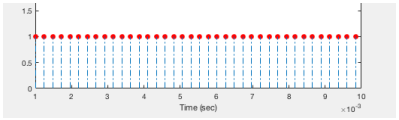

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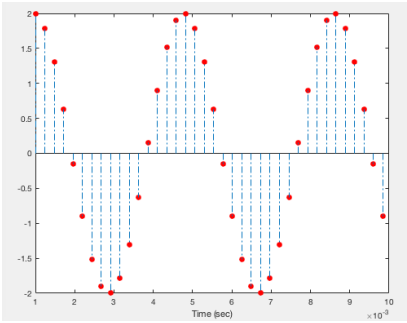

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Multiplication in time means convolution in frequency!

Question 1: what is the Fourier transform of the comb ?

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \circ\!\!-\!\!\bullet \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T}) = \sum_{n=-\infty}^{\infty} \delta(s - \frac{n}{T})$$

A comb with inverse sampling interval!

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Question 2: What means convolution with a comb?

$$F_S(\omega) = F(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T})$$

It is creating copies of the ”spectrum” at inverse time intervals!

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Question 3: Can we reconstruct the original spectrum from the "xeroxed" spectrum?

Only if the copies do not overlap ! Aliasing!

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Shannon-Nyquist Theorem

An analog signal can be reconstructed from sampling only if the sampling freqency is at least twice the maximum frequency; equivalently, if the sampling interval is less equal the minimum half the minimum wavelength.

Obviously the analog signal has to be bandlimited!

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How can the reconstruction be realized?

Multiplying the spectrum of the sampled signal with the rectangle-function

$$\Pi(t) = \begin{cases} \frac{1}{2\pi} & -\pi \leq \omega \leq \pi \\ 0 & \text{anywhere else} \end{cases}$$

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Which function has the box as Fourier-transform?

$$\begin{aligned}\mathcal{F}^{-1}\{\mathrm{rect}(\omega)\} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{rect}(\omega)e^{j\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{-\frac{1}{2}}^{\frac{1}{2}}e^{j\omega t}d\omega \\ &= \frac{1}{2\pi}\frac{1}{jt}[e^{j\omega t}]_{-\frac{1}{2}}^{\frac{1}{2}} \\ &= \frac{1}{2\pi}\mathrm{sinc}\left(\frac{t}{2}\right)\end{aligned}$$

$$\frac{1}{2\pi}\mathrm{sinc}\left(\frac{t}{2}\right) \circ\!\!-\!\!\bullet\, \mathrm{rect}(\omega)$$

$$\frac{1}{2\pi}\mathrm{sinc}(\pi t) \circ\!\!-\!\!\bullet\, \frac{1}{2\pi}\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$$

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Multiplication in frequency means convolution in time (space)

Reconstruction means convolution of the sampled signal with sinc

$$f_{reconstr}(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) * \frac{1}{2\pi}\text{sinc}(\pi t)$$

$$= \sum_{n=-\infty}^{\infty} f[nT]\frac{1}{2\pi}\text{sinc}(\pi(t - nT))$$

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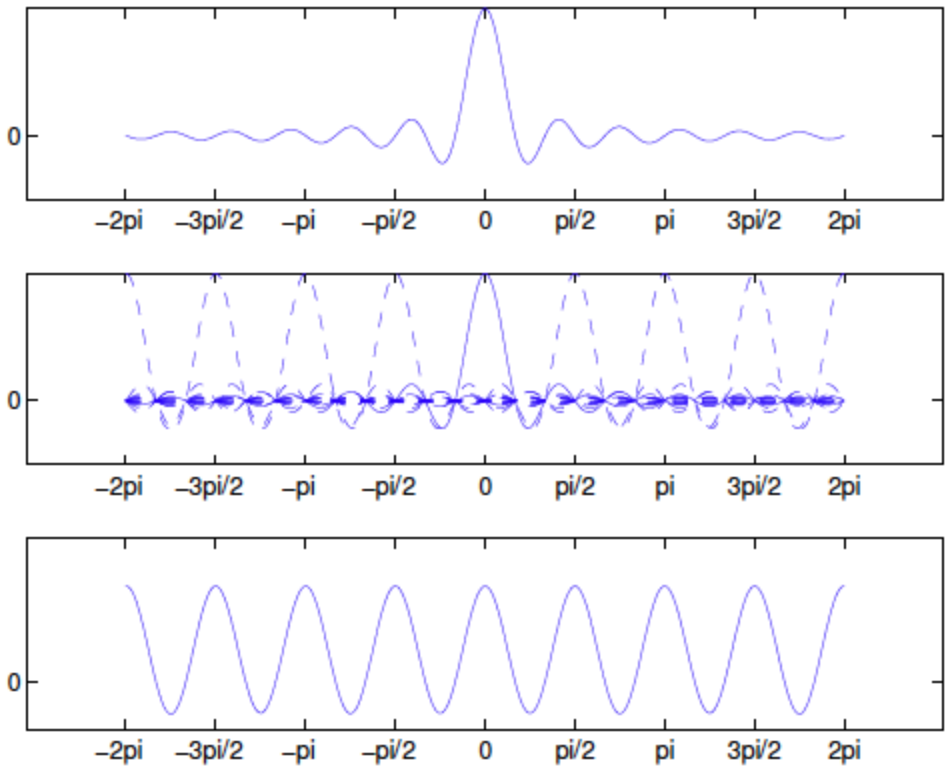

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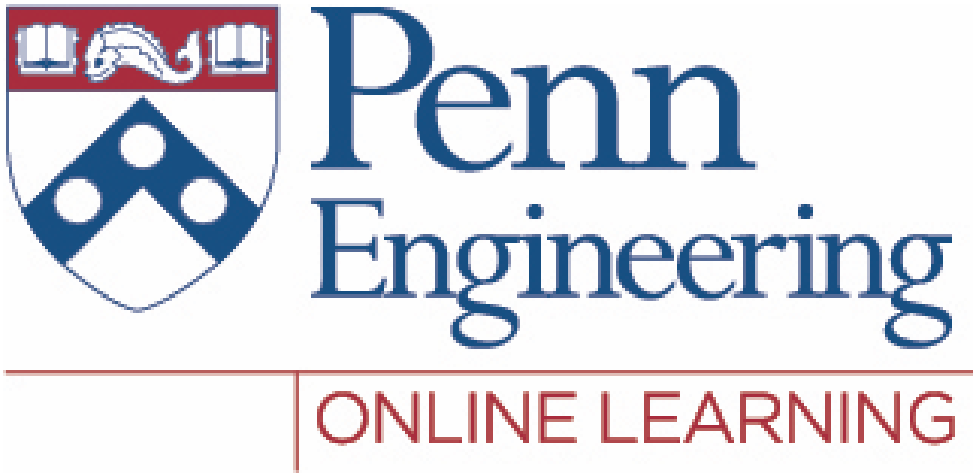

Video 2.7
Kostas Daniilidis

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Discrete time signals have still continuous Fourier transforms (CFT)!

$$f[n] \multimap \sum_{n=0}^{L-1} f[n] e^{-j\omega n}$$

Discrete time signals have continuous and periodic Fourier transforms with period 2π!
That's why we consider only the interval $(-\pi,\pi]$ when we talk about the CFT of a discrete signal!

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Example. Let $f[n]$ a discrete time signal and $h[n]$ a discrete time signal defined as:

$$h[n] = \begin{cases} 1, & n = 0 \\ -1, & n = 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$H(\omega) = \sum_n h[n]e^{-j\omega n} = h[0]e^{-j\omega \cdot 0} + h[1]e^{-j\omega \cdot 1} = 1 - e^{-j\omega}$$

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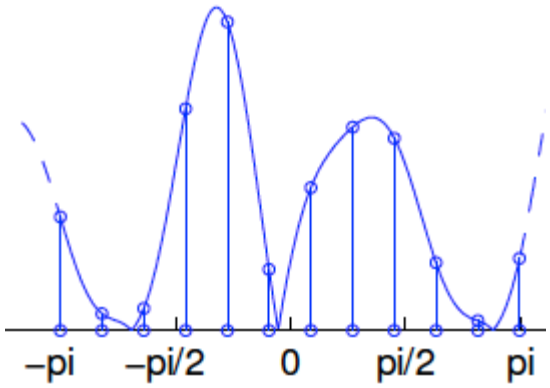
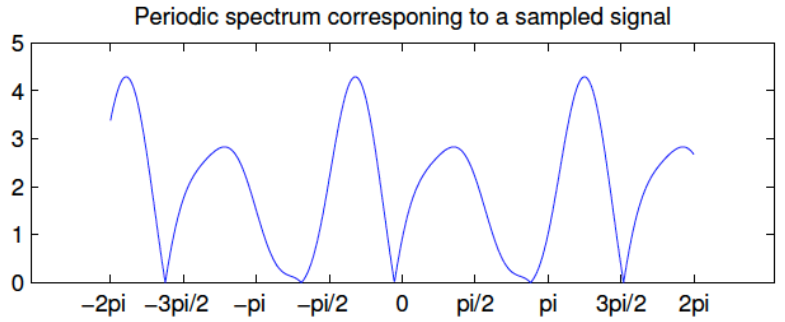
What does discrete frequency mean?





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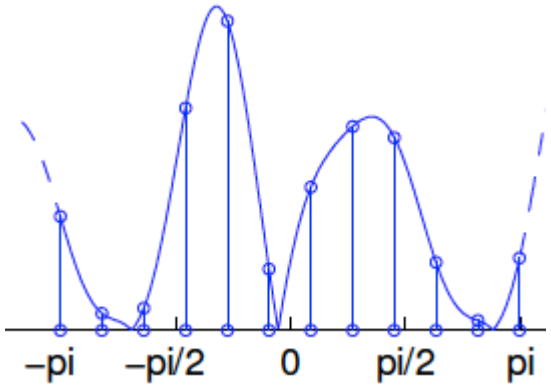
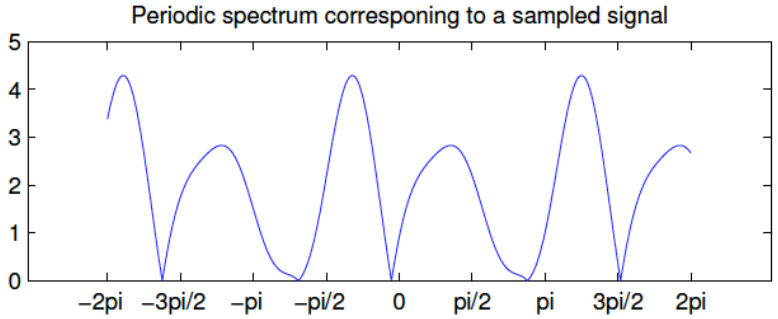
What does sampling in the frequency domain correspond to in the time/space domain? It corresponds to a **convolution**:

- FREQUENCY DOMAIN: multiplication with $\sum \delta(\omega - \frac{2\pi k}{L})$
- TIME DOMAIN: convolution with with $\sum \delta[n - kL]$, equivalent to **replication of the signal at multiples of its length.**





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Why is the DFT of a discrete time finite signal same as the CFT?

Because sampling the frequency domain only means replicating the original signal.

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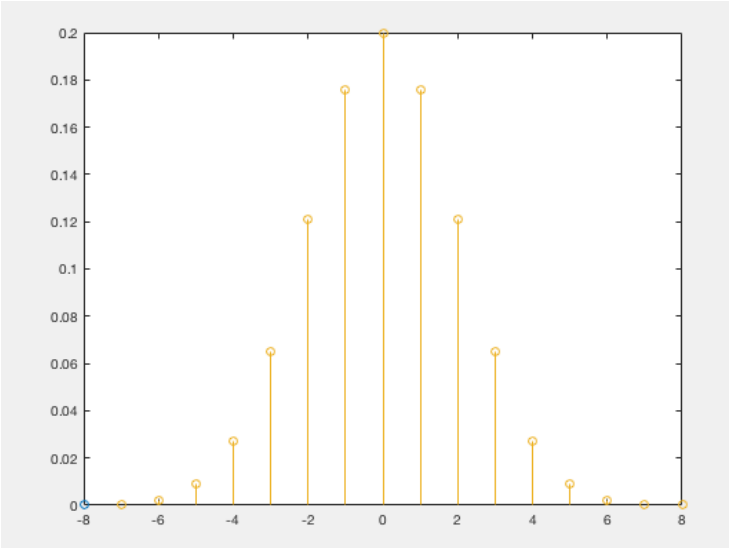
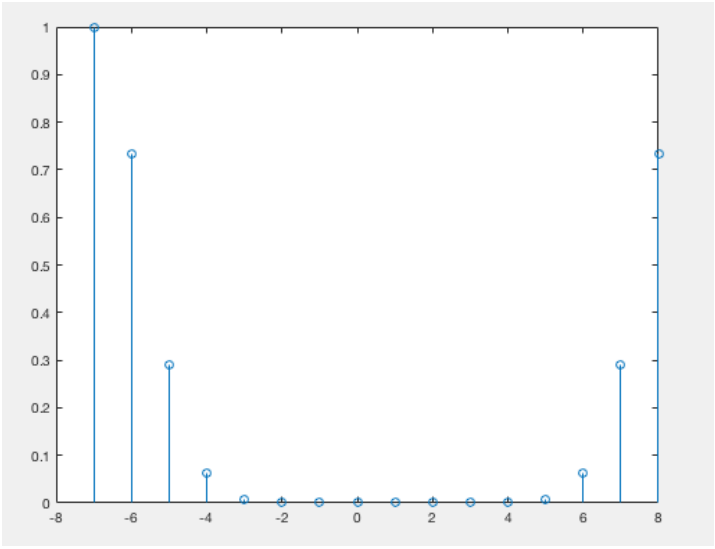
Some MA

```
support = 16;
sigma = 2;
d = -floor(support/2)+1:floor(support/2);
g = (1/(sigma*sqrt(2*pi)))*exp(-d.^2/(2*sigma^2));

figure(1)
stem(d,g);

figure;
stem(d,abs(fft(g)));
```





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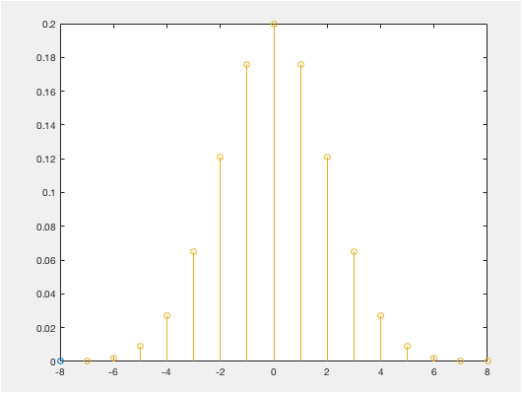
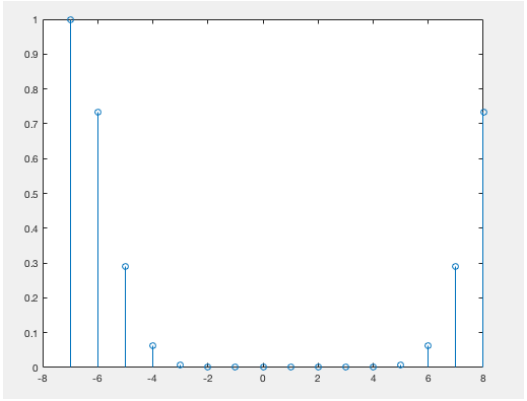
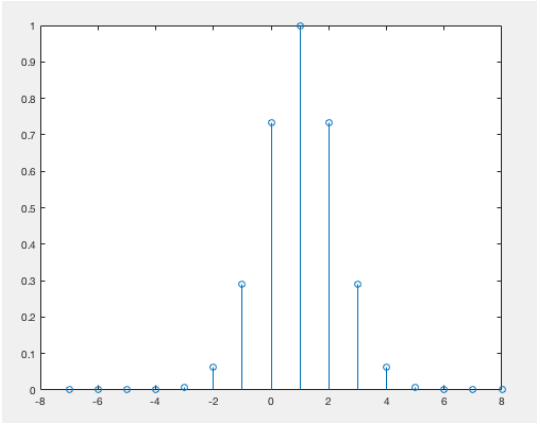
fftshift…

`fftshift(fft(f))` performs a shift by $\frac{L}{2}$

(modulation of the signal by $e^{\frac{-j2\pi L/2}{L}} = e^{-j\pi} = -1$)







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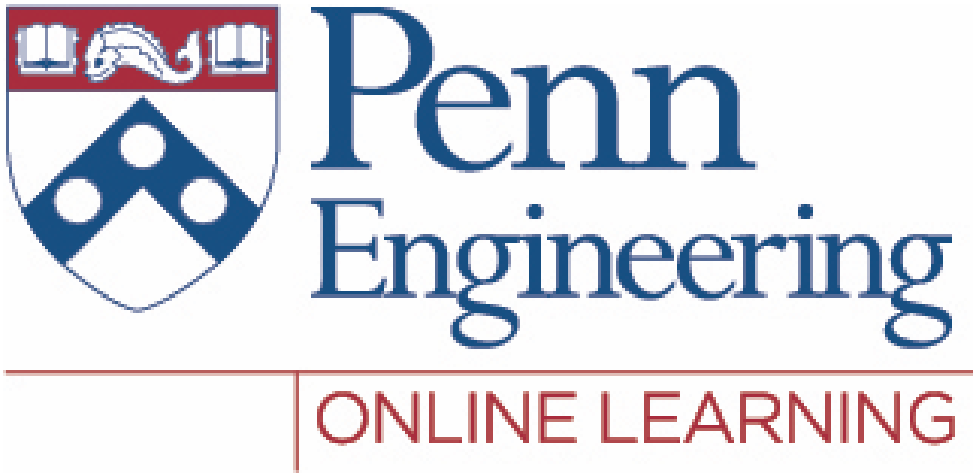

Video 2.8
Kostas Daniilidis

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

2D Fourier Transform

$$f(x,y) \circ\!\!-\!\!\bullet \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j\omega_x x+\omega_y y}dxdy$$

$$F(\omega_x,\omega_y) \bullet\!\!-\!\!\circ \left(\frac{1}{2\pi}\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\omega_x,\omega_y)e^{\cdots}d\omega_x d\omega_y$$

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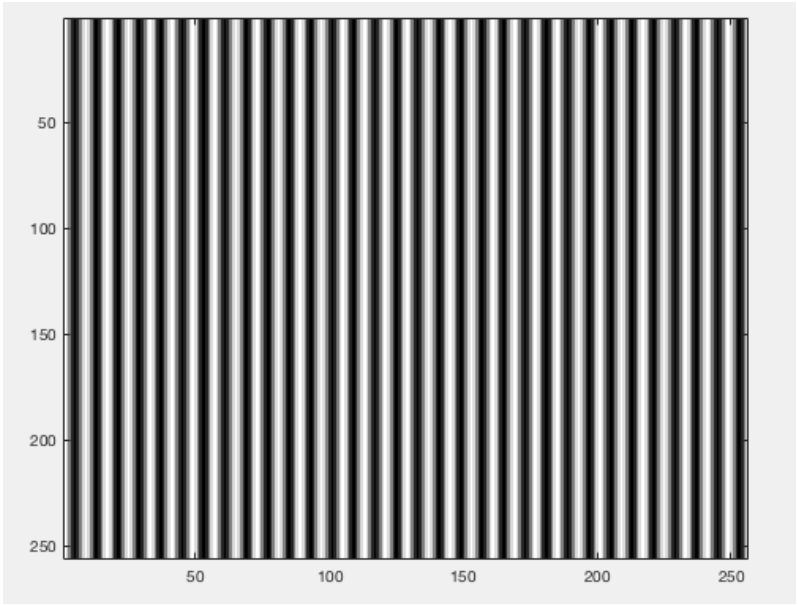
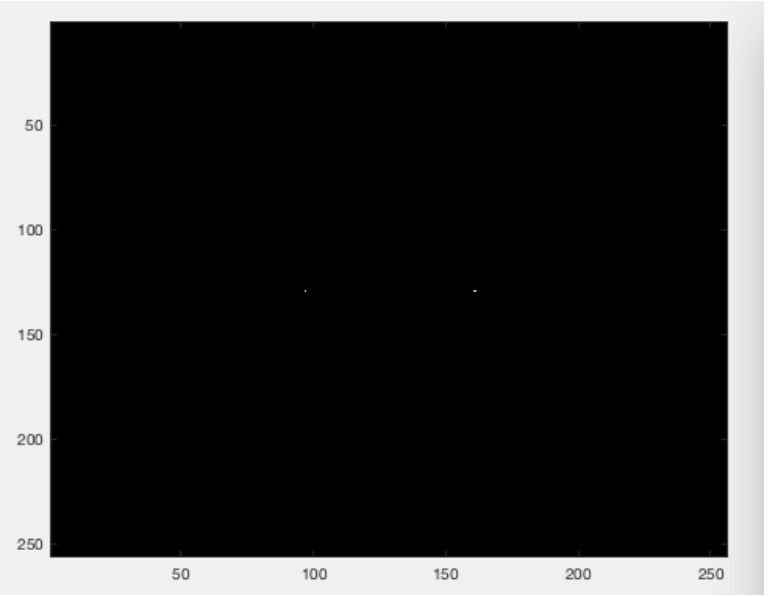
Example: 1D-cosine as an image





$$f(x,y) = \cos(\omega_0 x) \qquad f(x,y) \multimap\!\bullet \frac{1}{2}(\delta(\omega_x - \omega_0) + \delta(\omega_x + \omega_0)).\delta(\omega_y)$$

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Separable functions

$$f(x,y) = f_1(x)f_2(y) \multimap \int_{-\infty}^{\infty} f_1(x)e^{-j\omega_x x}dx \int_{-\infty}^{\infty} f_2(y)e^{-j\omega_y y}dy = F_1(\omega_x)F_2(\omega_y)$$

$$f(x,y) = \cos(\omega_1 x)\cos(\omega_2 y)$$

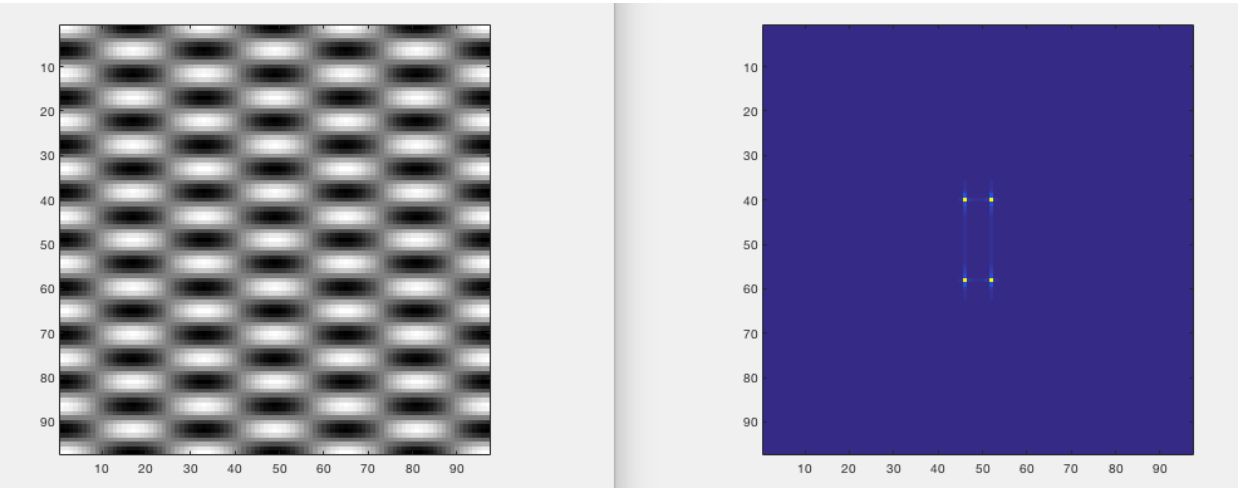
$$\frac{1}{2}(\delta(\omega_x - \omega_1) + \delta(\omega_x + \omega_1))\frac{1}{2}(\delta(\omega_y - \omega_2) + \delta(\omega_y + \omega_2))$$

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Separable functions

$$f(x,y) = \cos(\omega_1 x)\cos(\omega_2 y)$$

$$\frac{1}{2}(\delta(\omega_x - \omega_1) + \delta(\omega_x + \omega_1))\frac{1}{2}(\delta(\omega_y - \omega_2) + \delta(\omega_y + \omega_2))$$



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Shift Theorem in 2D

$$f(x-x_0, y-y_0) \multimap\!\bullet\; F(\omega_x, \omega_y) e^{-j(\omega_x x_0 + \omega_y y_0)}$$

If we know the phases of two 1D signals we can recover how their relative displacement? But can we do that for 2D images?

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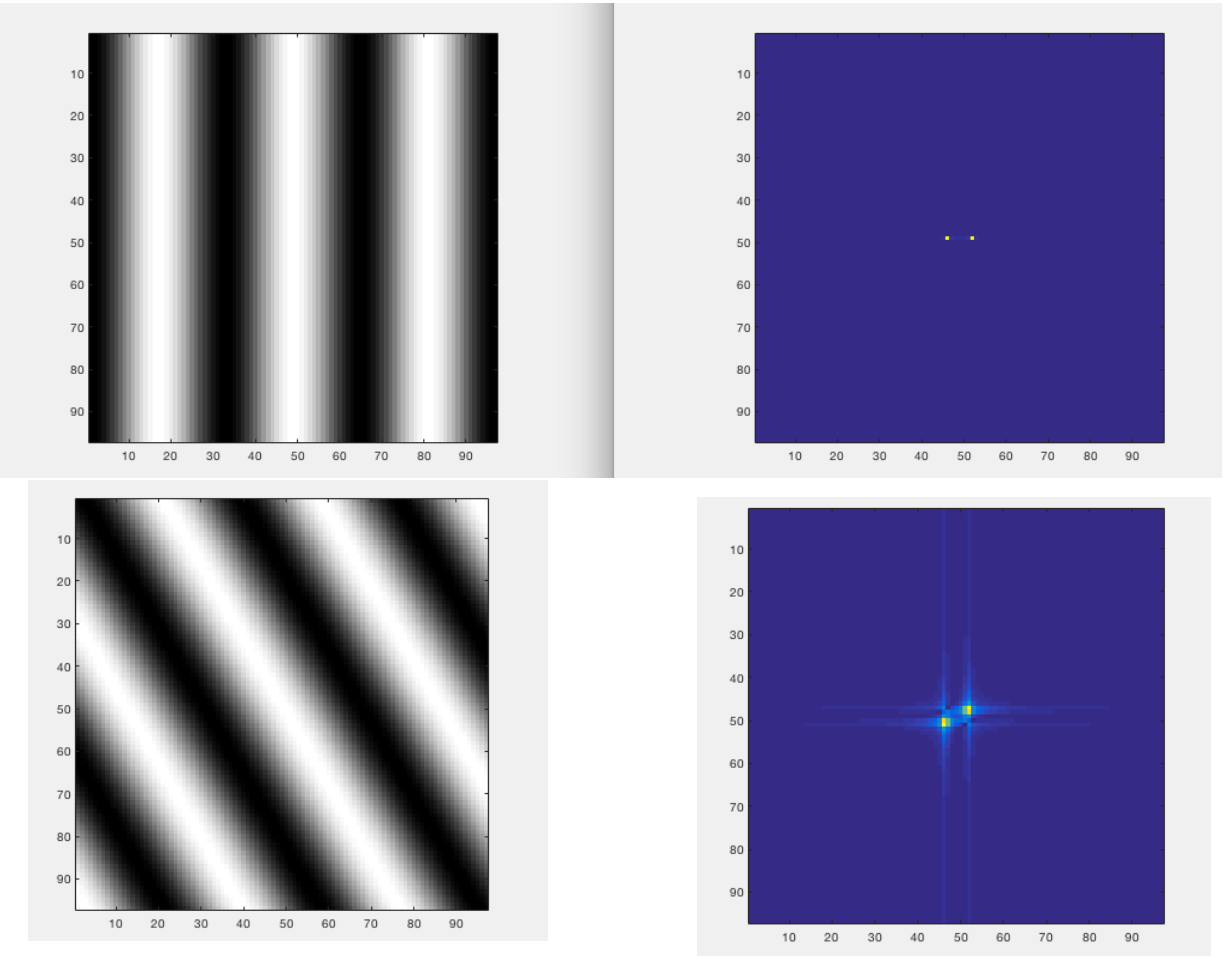
2D rotation



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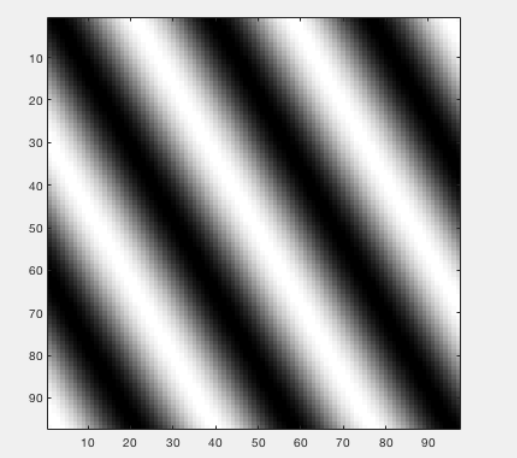
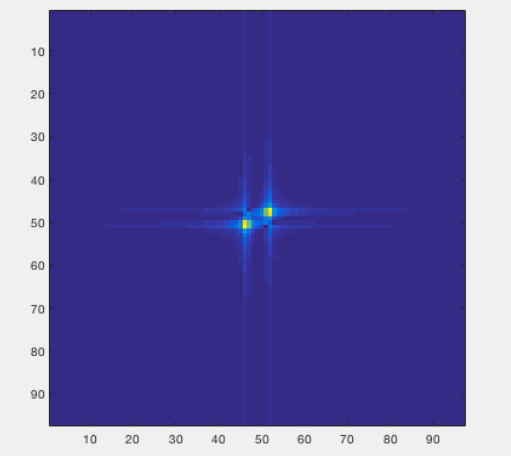
2D rotation

$$f * R\begin{pmatrix} x \\ y \end{pmatrix} \multimap\!\bullet\; \mathcal{F}\left(R\begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix}\right)$$





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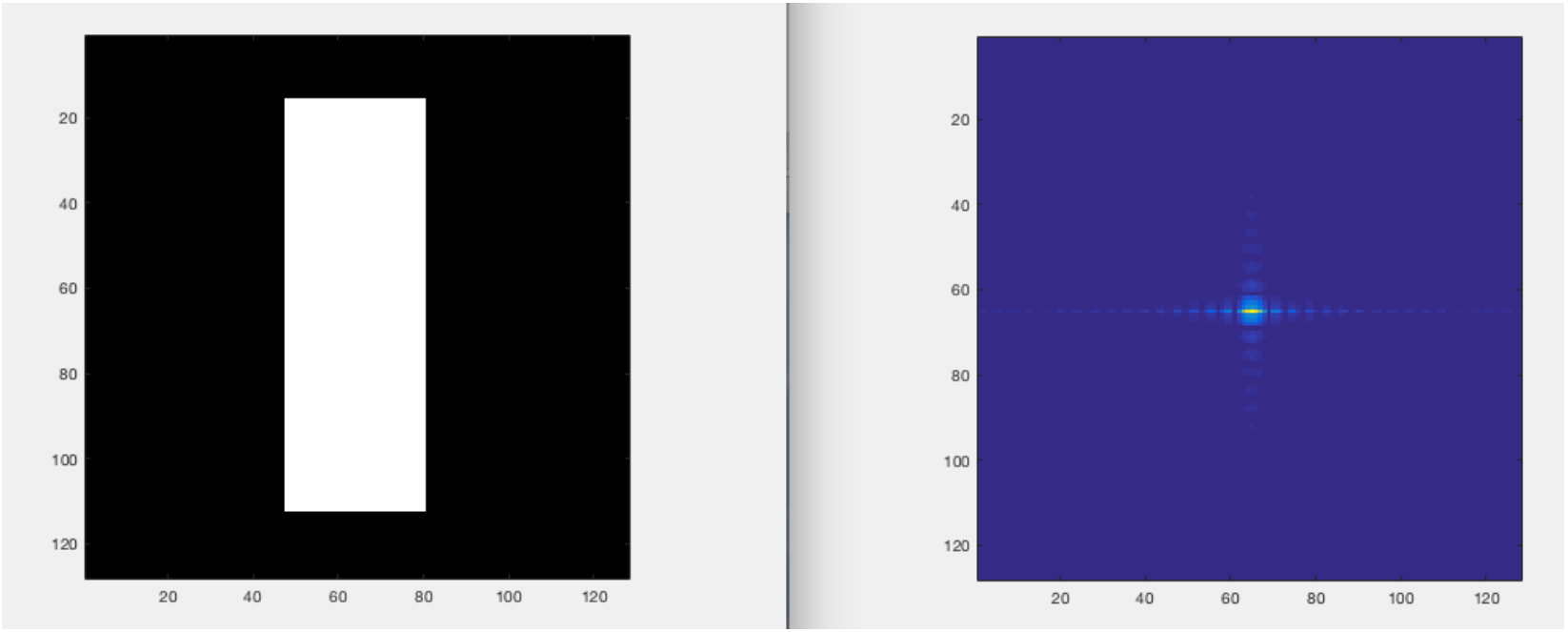
2D Fourier of a box



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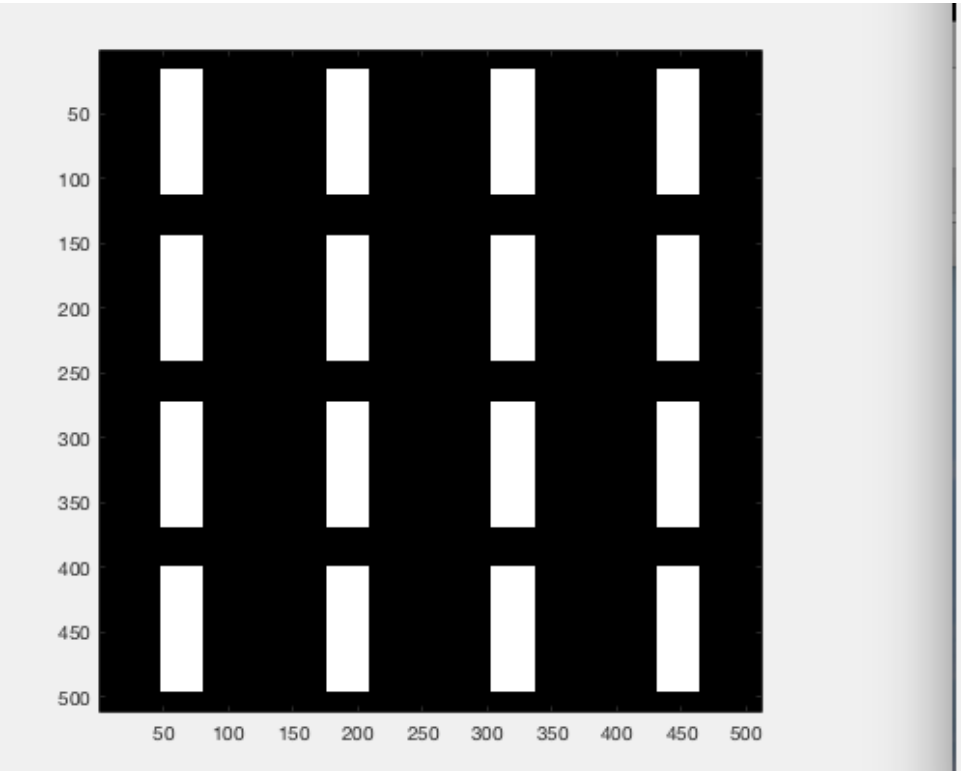
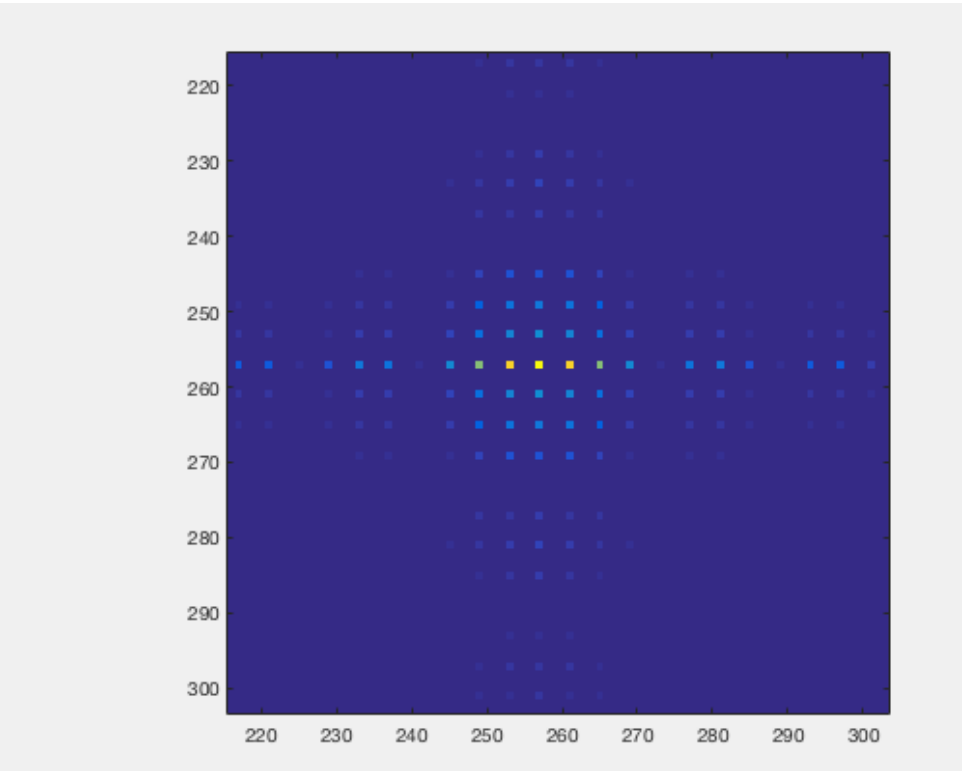
How do we model other periodic patterns?





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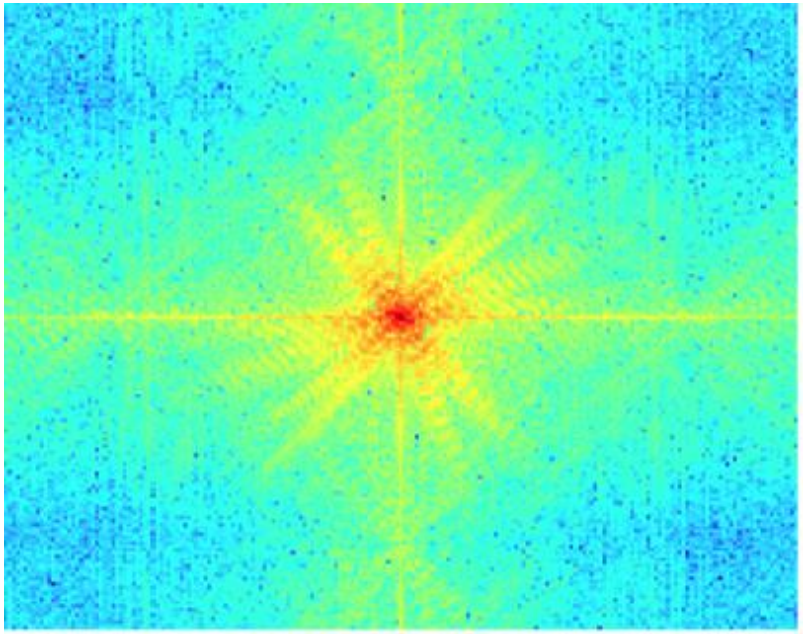
Clue about orientation of edges



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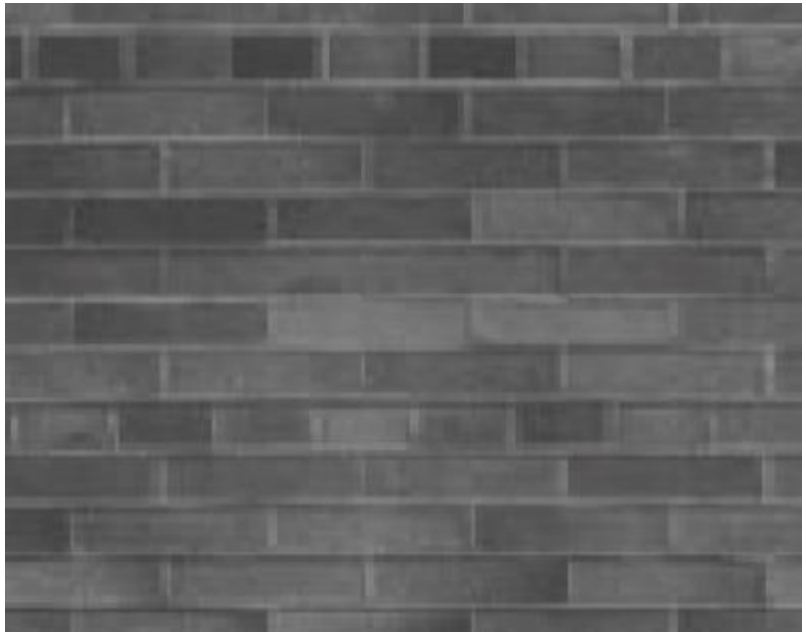
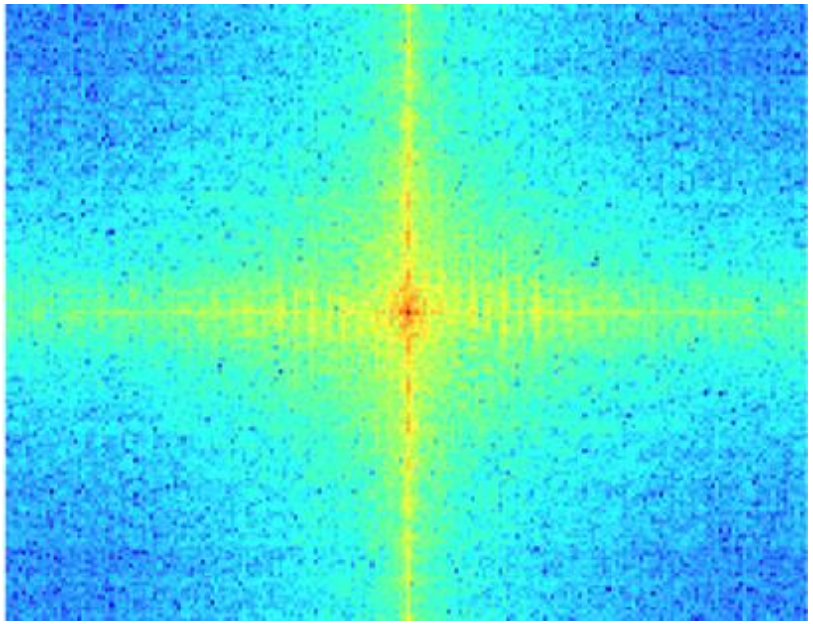
Clue about periodicity





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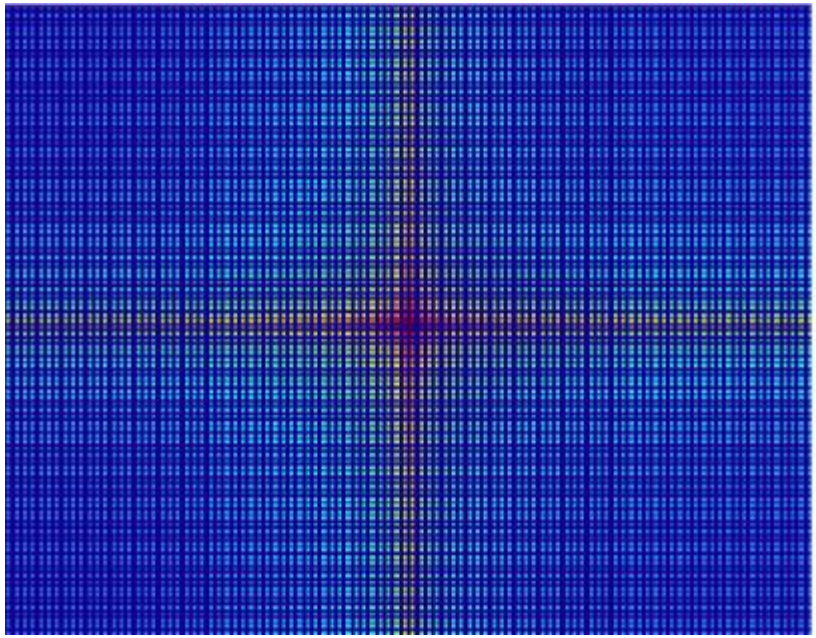
Clues about contrast



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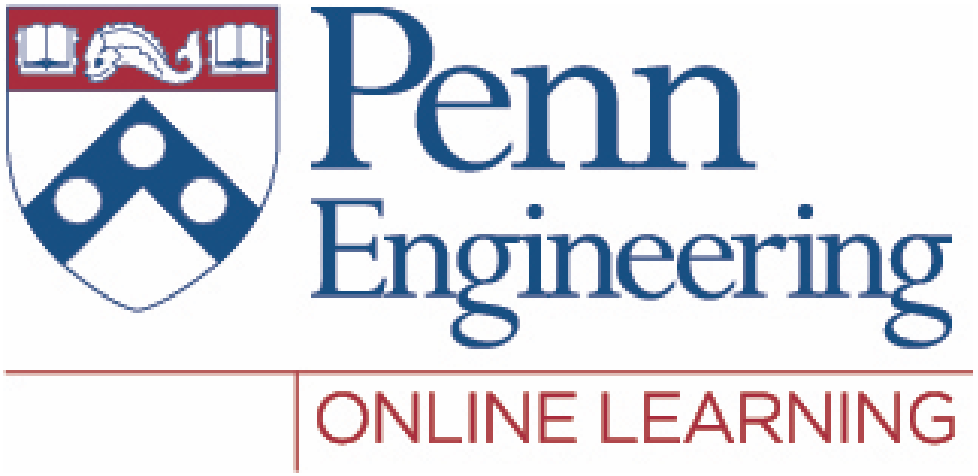
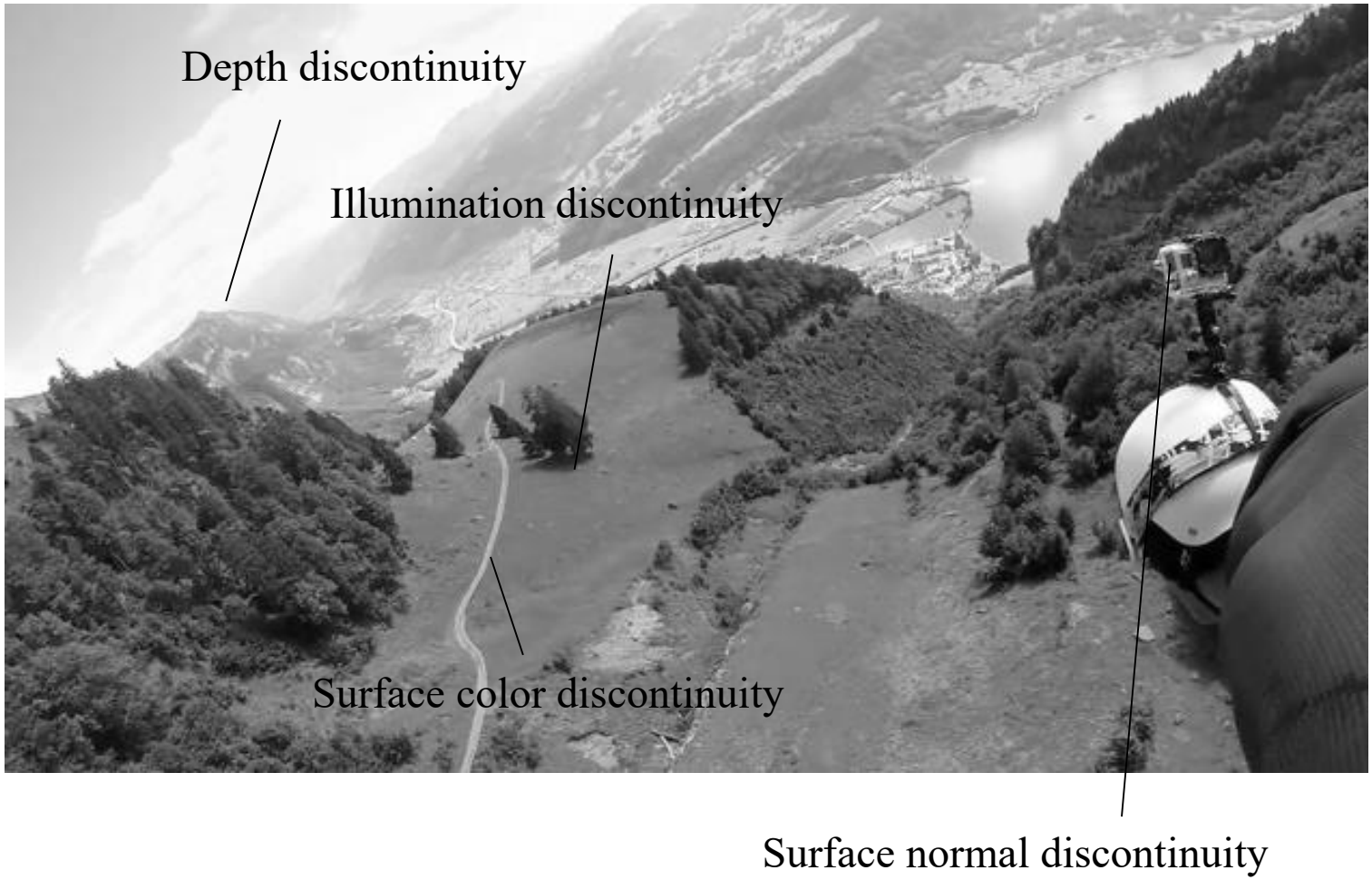


Video 2.9
Jianbo Shi

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Edge Formation Factors



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What's in front?

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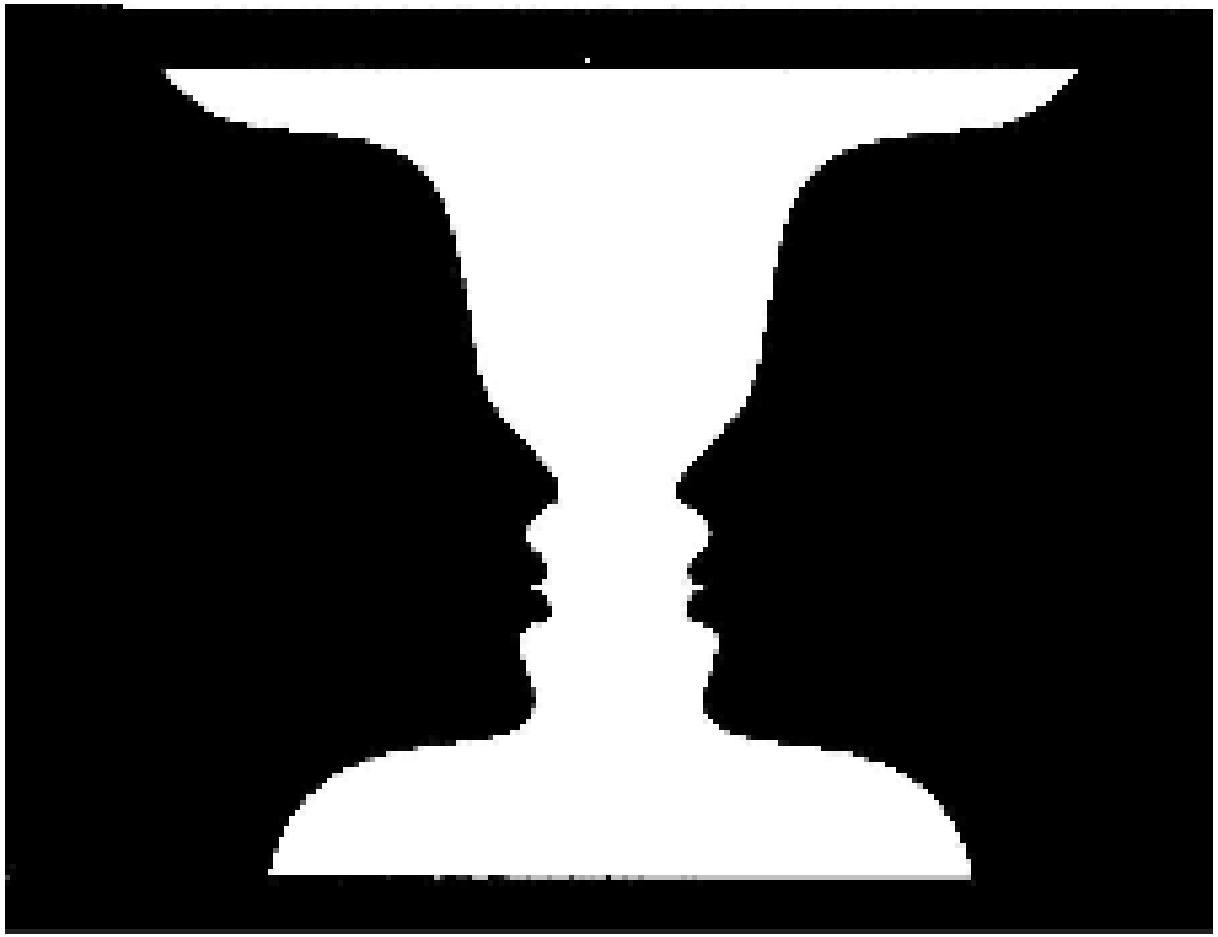
What it is?



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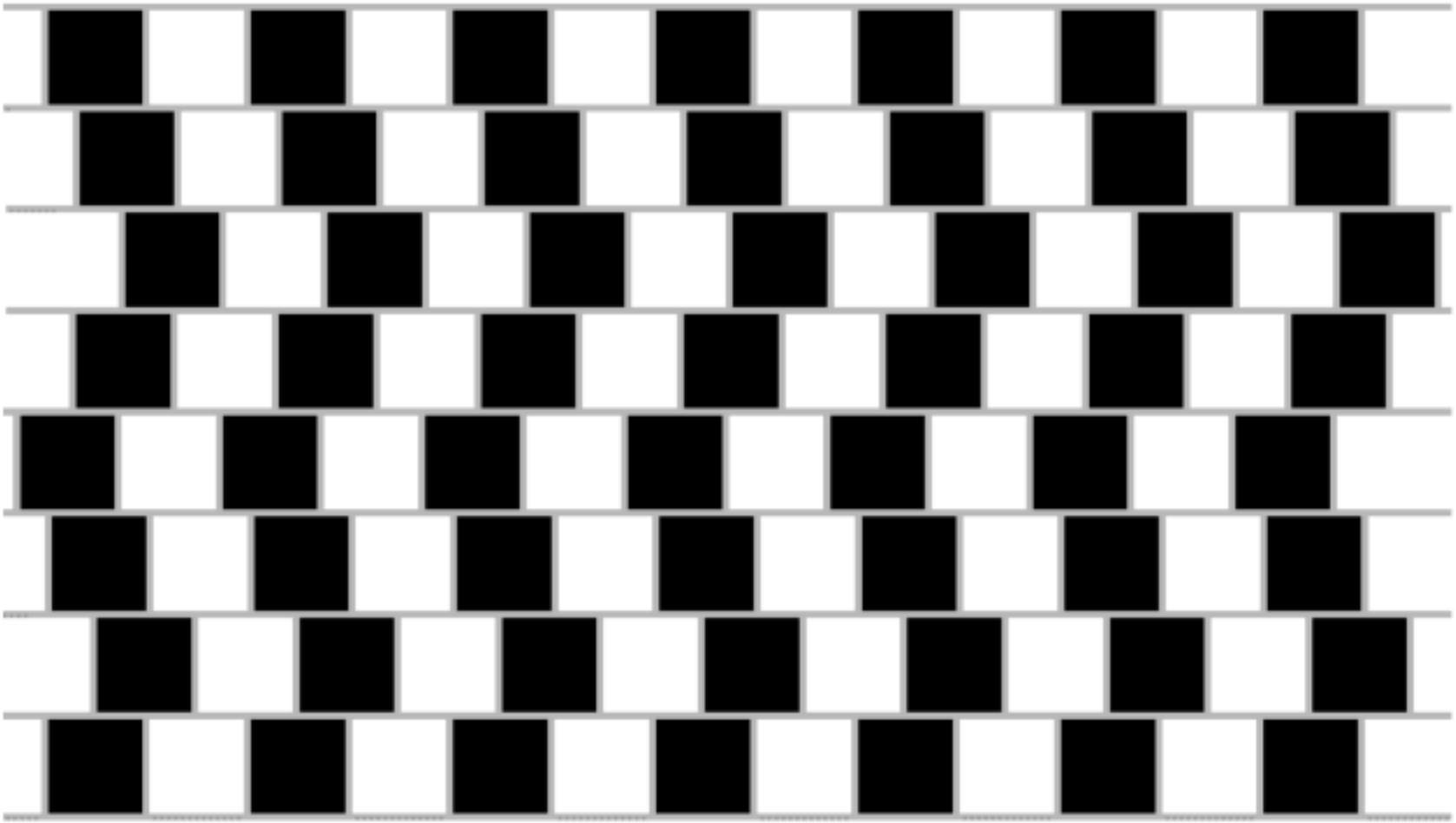
http://www.cfar.umd.edu/~fer/optical/index.html

The rows of black and white squares are all parallel.
The vertical zigzag patterns disrupt our horizontal perception.



Cornelia Fermüller

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

j

i

121	121	118	111	. . .	21
134	136	137	132	. . .	23
133	131	136	136	. . .	25
136	145	148	151	. . .	34
137	140	147	149	. . .	54
. . .	. . .	. . .	. . .	. . .	. . .
231	233	243	244	. . .	179

Any 2D matrix can be seen as an image

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Linear Filtering

Image I

200	130	20
255	100	10
200	100	30

Kernel f

0.5	0	0
0	1.5	0.5
0	-0.5	0

	205	

Image *filtering:* Replacing each pixel value by weighted average of its neighbors

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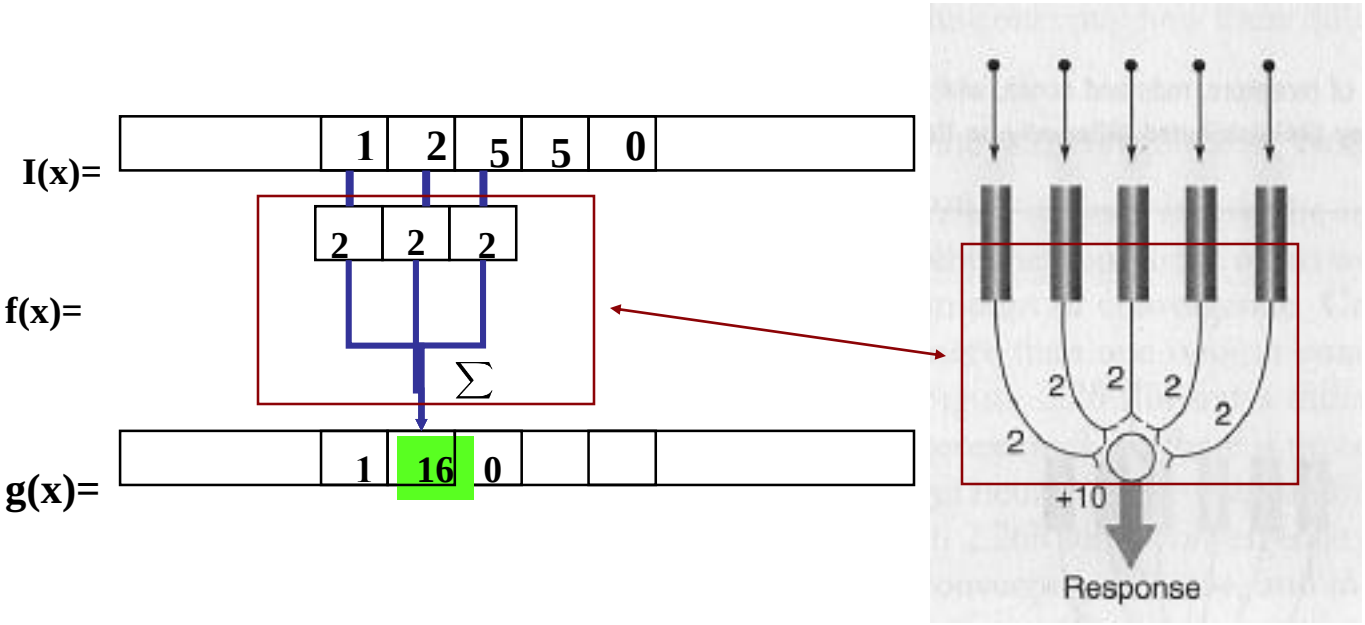
Simple Neural Network



This leads to parallel computation

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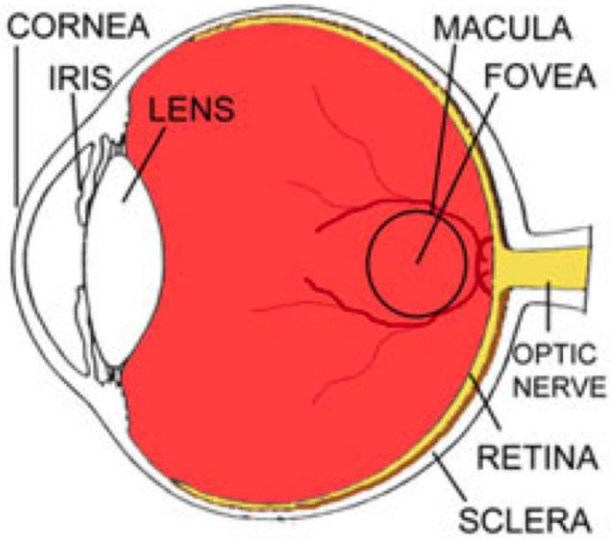
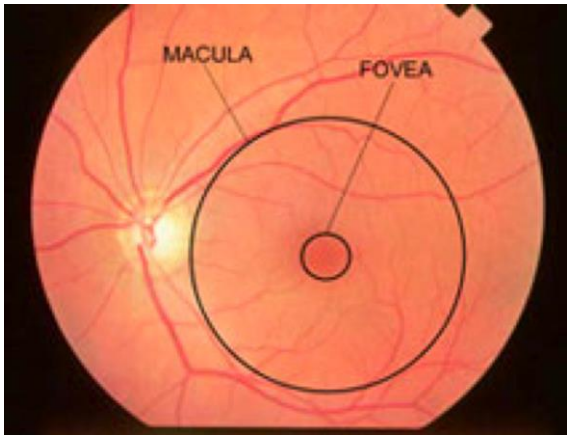
Anatomy of eye





The macula is the center of vision (and retina) and the fovea (FAZ) is the focal point approximately only 0.4mm in diameter. Reading, driving, etc. is all performed here.

http://www.retinatexas.com/Anatomy_eng.html

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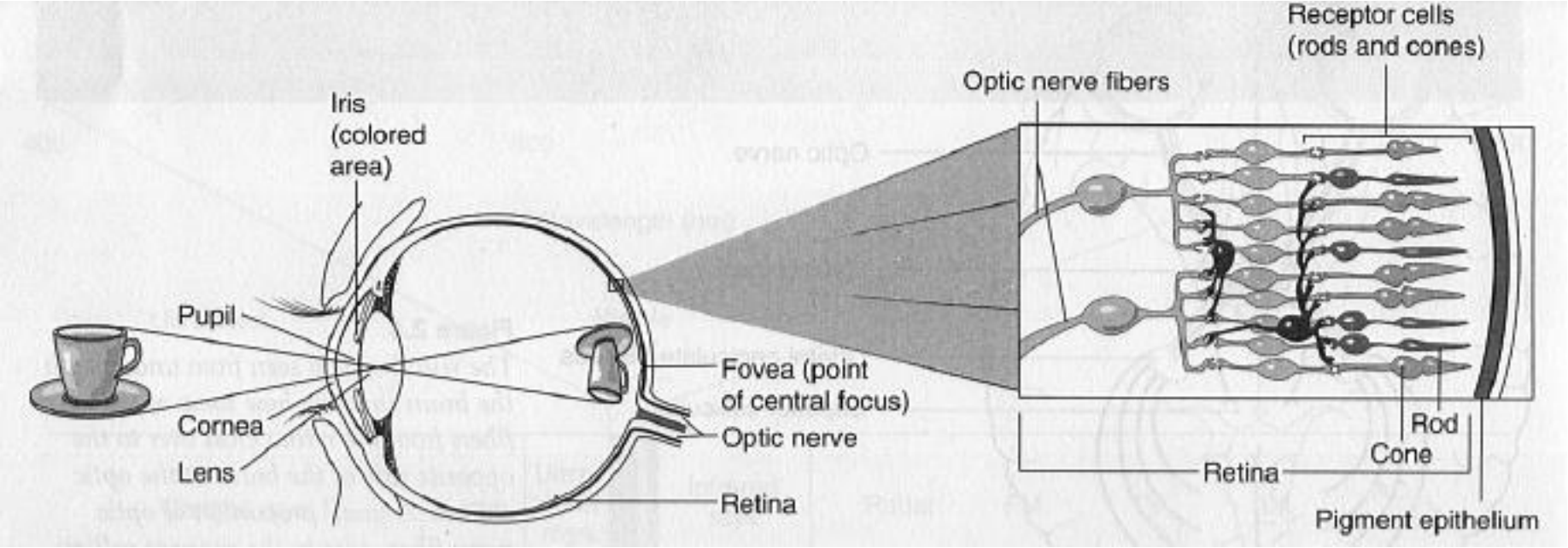
Photo-sensors



1) **Light pass through retina cells, excites Rod and Cone**
2) **Cone: color(spectral) sensitive, R,G,B, 6 Million**
3) **Rod: more photo sensitive, peak at 580nm(yellow), 120 M**
4) **What happens if you miss one type of Cone cells?**

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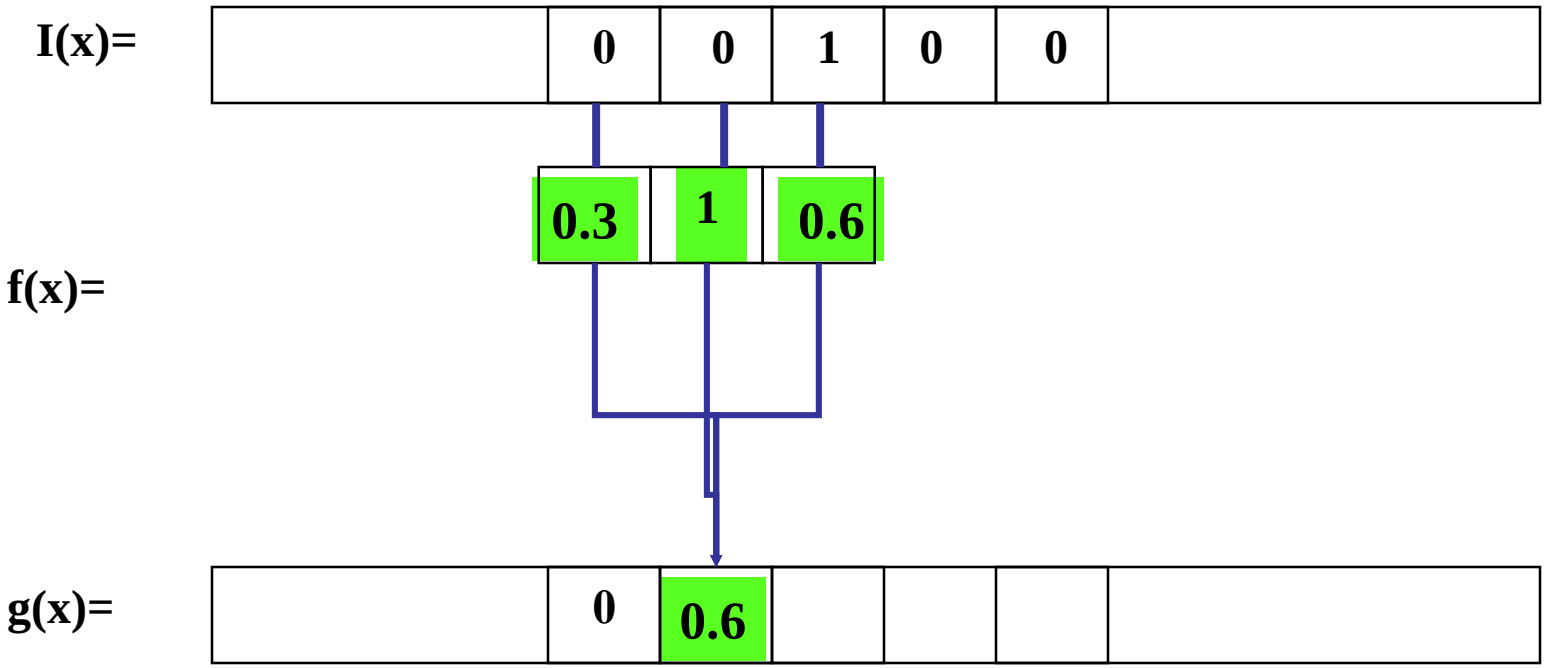
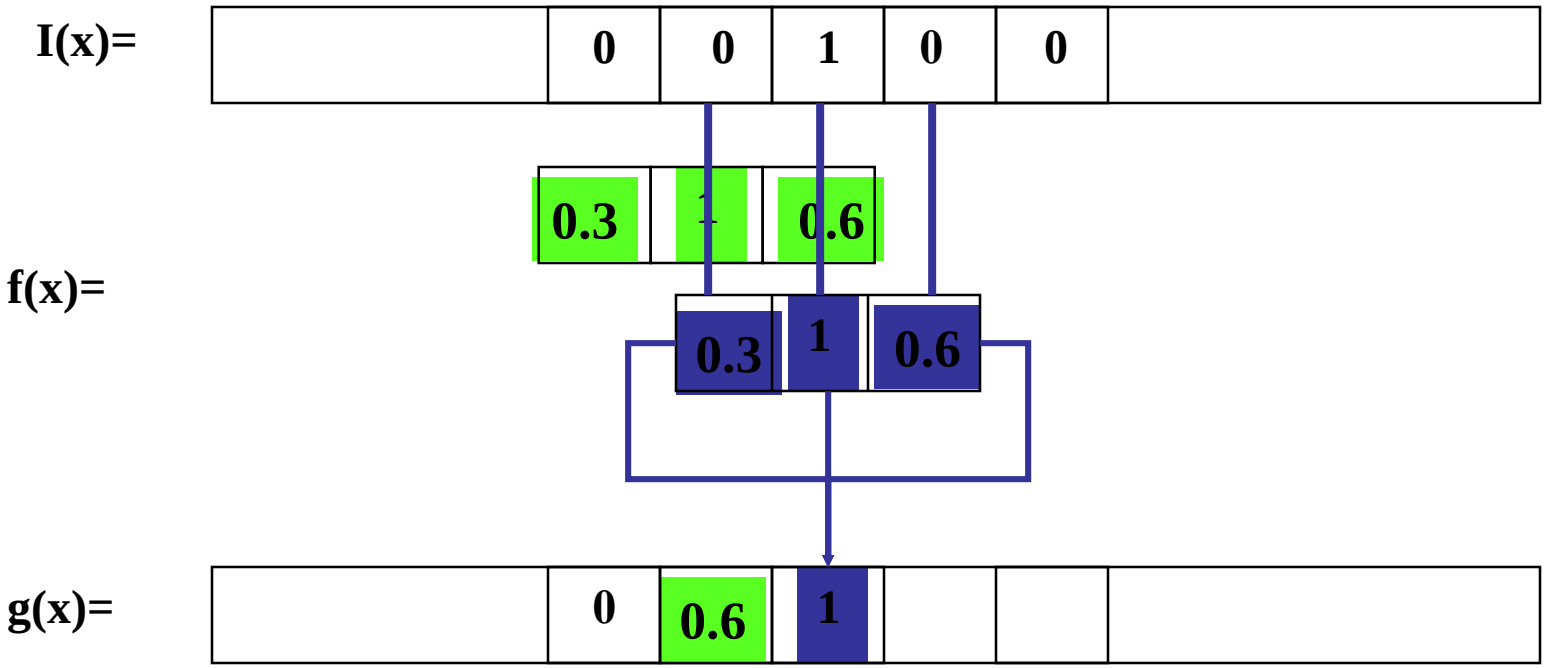
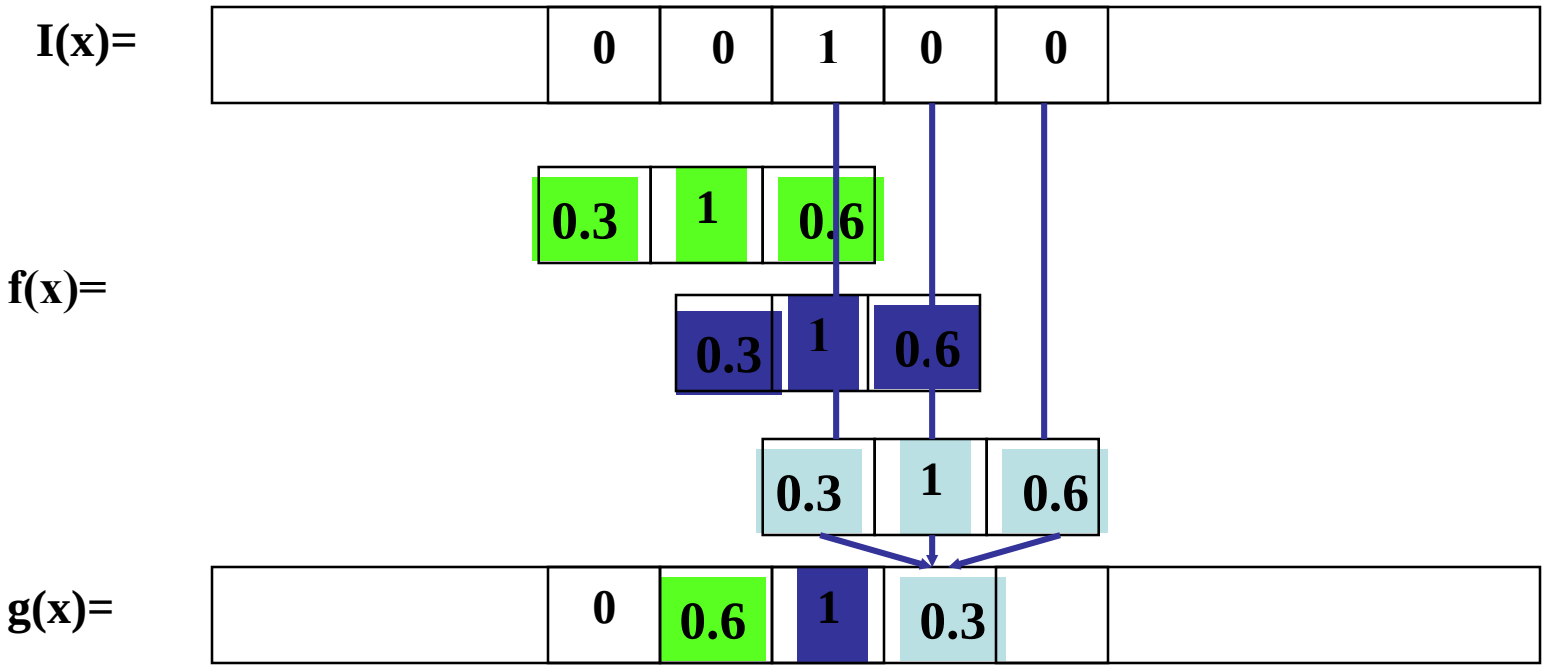


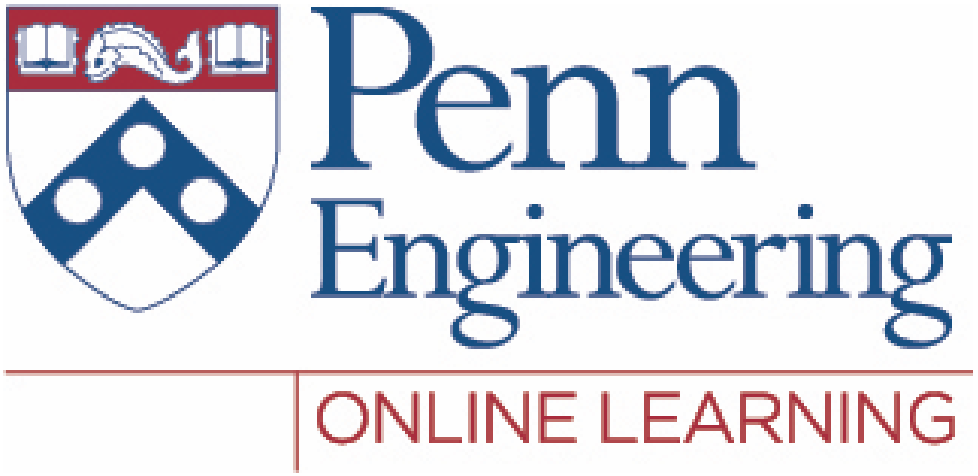
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi



Video 2.10
Jianbo Shi

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
0	1	0
0	0	0

Filter
Kernal

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
0	1	0
0	0	0

Kernal

Image no change

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
1	0	0
0	0	0

Kernal

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
1	0	0
0	0	0

Kernal

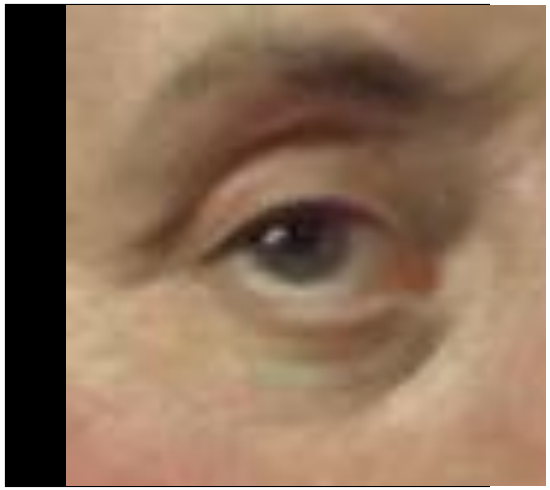


Image Shifted

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
0.3	0.3	0.3
0	0	0

Kernal

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
0.3	0.3	0.3
0	0	0

Kernal

Blurred
(horizontal)

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0.3	0
0	0.3	0
0	0.3	0

Kernal

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0.3	0
0	0.3	0
0	0.3	0

Kernal

Blurred (vertical)

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

Image

0	0	0
0	2	0
0	0	0

−

0	0	0
0.3	0.3	0.3
0	0	0

Filter
Kernal

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear Filter

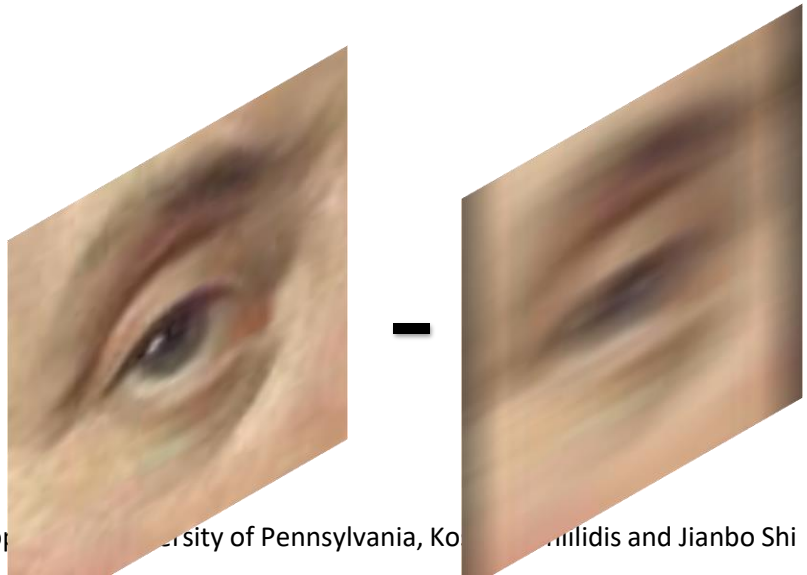
Image

0	0	0
0	2	0
0	0	0

−

0	0	0
0.3	0.3	0.3
0	0	0

Kernal Image



Linear Filter

Image

0	0	0
0	2	0
0	0	0

−

0	0	0
0.3	0.3	0.3
0	0	0

Kernal Image

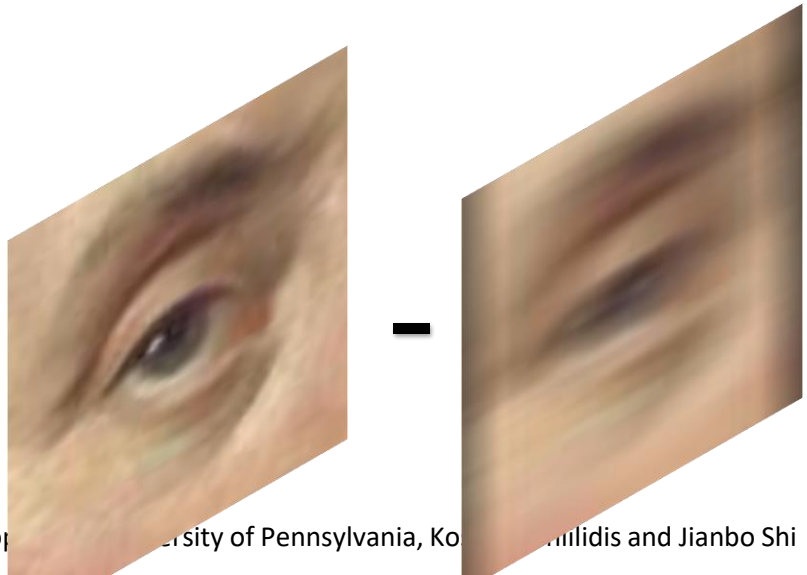


Image filtering

$$g[m,n] = \sum_{k,l} I(m+k, n+l) * f(k,l)$$

Output Image

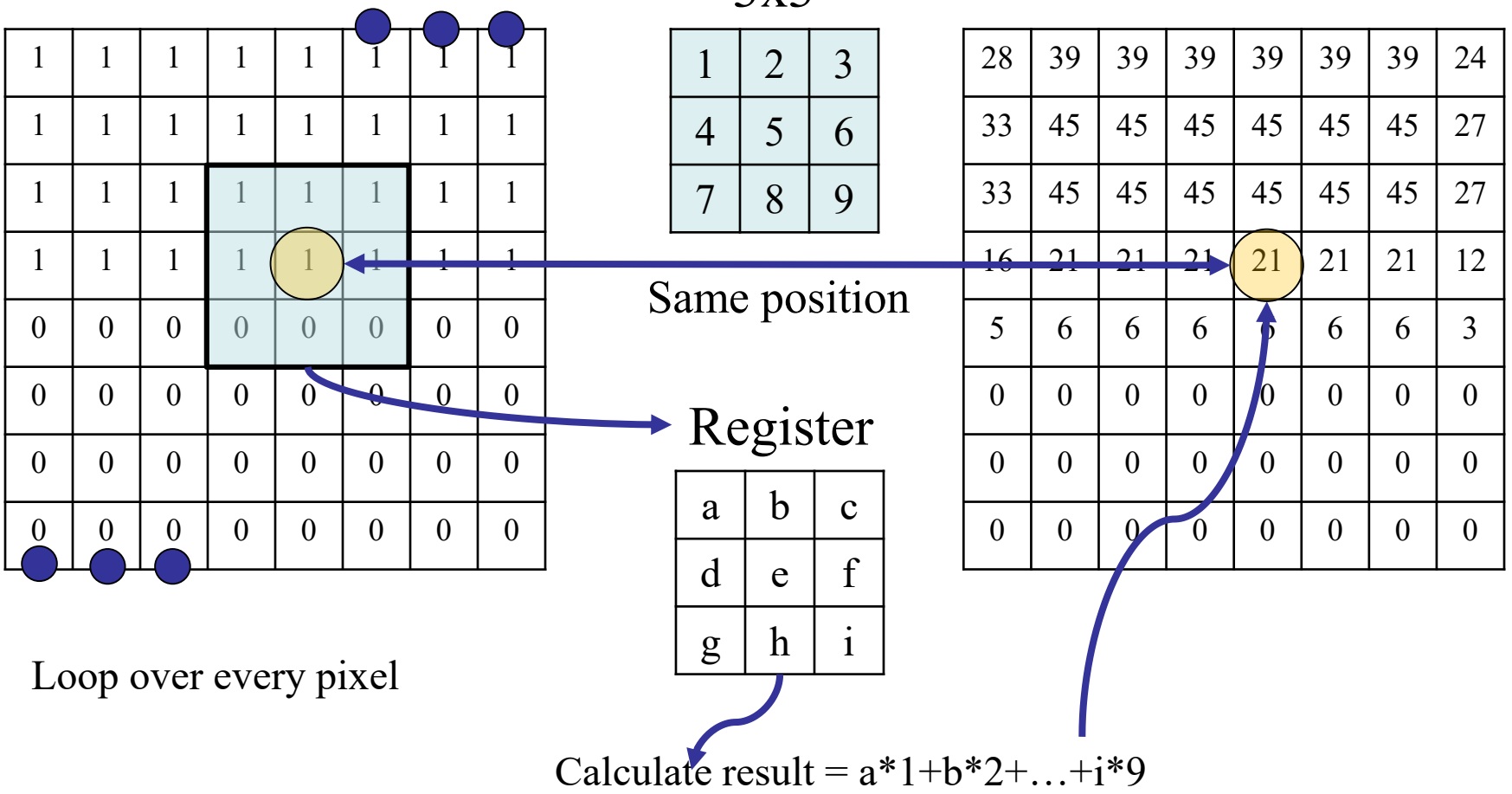
Input Image

Kernal Image

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Image filtering

$$g[m,n] = \sum_{k,l} I(m+k, n+l) * f(k,l)$$



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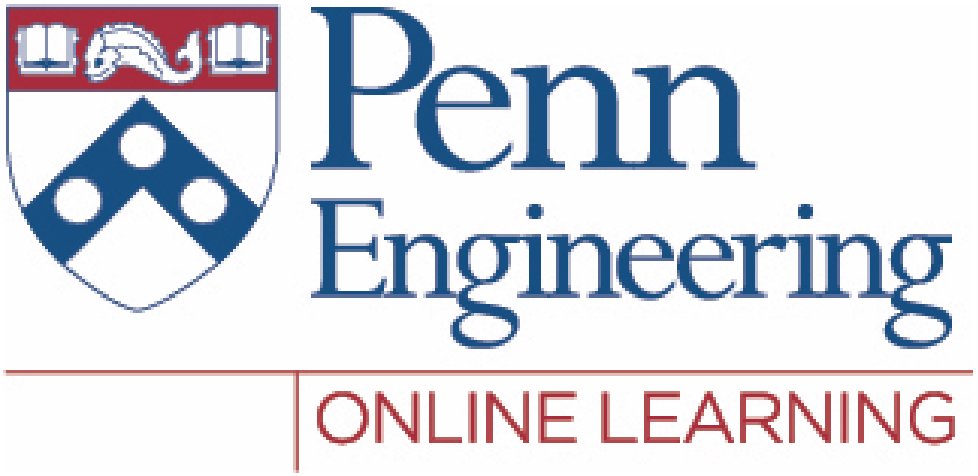
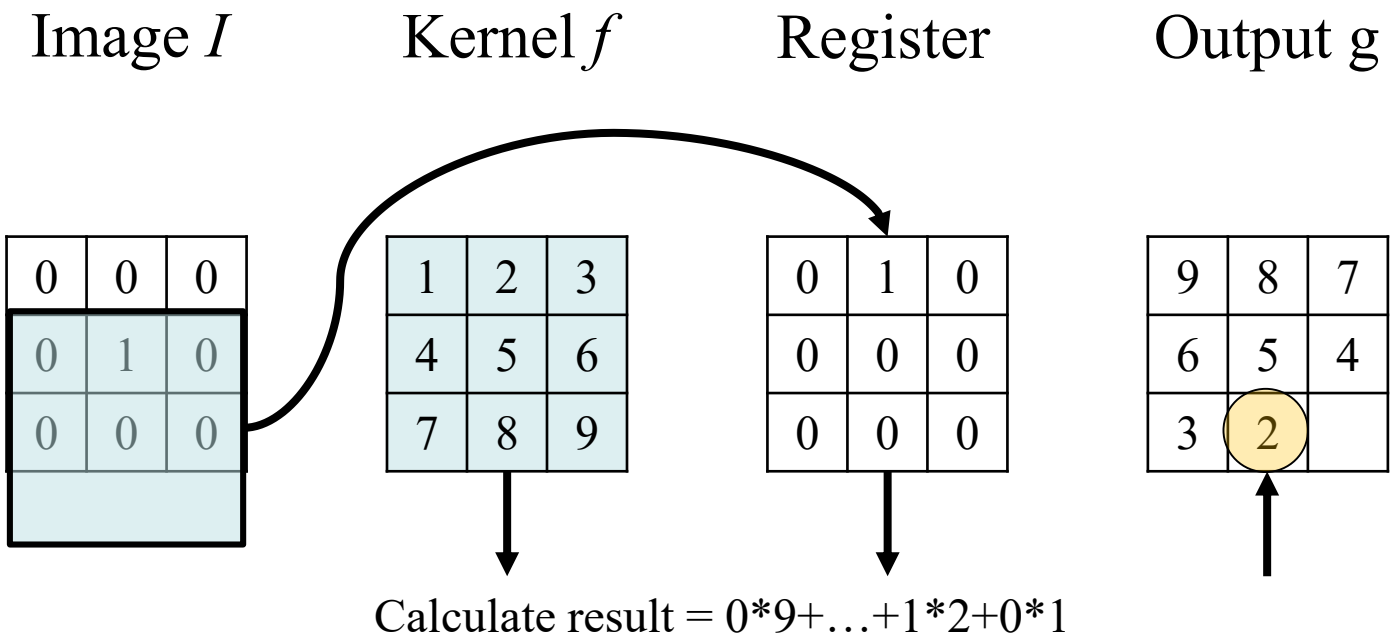


Video 2.11
Jianbo Shi

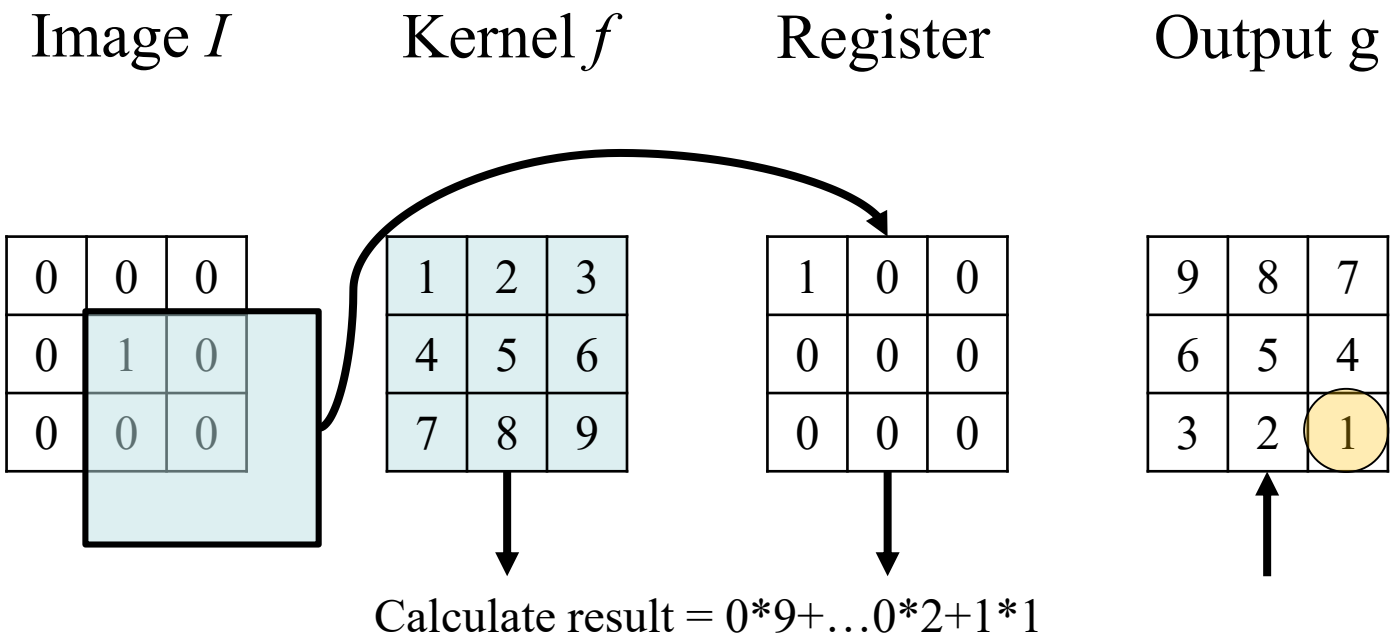
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Special case: impulse function



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Special case: impulse function



<Note> The output is the kernel flipped left-right, up-down!

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Convolution $I \otimes f$

- Let I be an Signal(image), Convolution kernel f,

$$g[m,n] = \sum_{k,l} I(m-k, n-l) * f(k,l)$$

Output Image

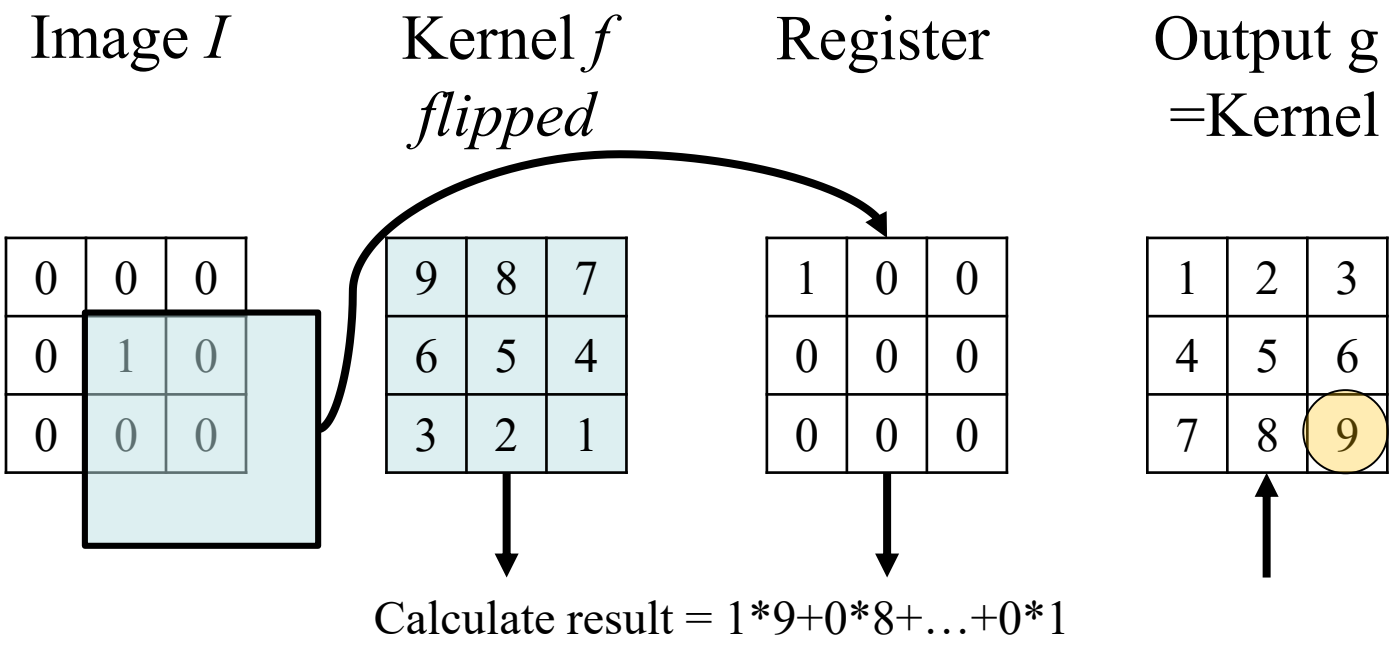
Input Image

Kernal Image

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Convolution

- Convolution is filtering with kernel flipped



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Impulse functions shift images

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
0	0	0
0	0	0

Kernel f'

0	0	0
0	0	0
0	0	1

Result $I \otimes f$

e	f	0
h	i	0
0	0	0

- In this case the resulting image shifted to the upper left

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Impulse functions shift images

Image I

1	0	0
0	0	0
0	0	0

Kernel f

a	b	c
d	e	f
g	h	i

Kernel f'

i	h	g
f	e	d
c	b	a

Result $I \otimes f$

e	f	0
h	i	0
0	0	0

- In this case the resulting image shifted to the upper left

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Convolution is associative

I

a	b	c
d	e	f
g	h	i

f

1	0	0
0	0	0
0	0	0

$I \otimes f$

e	f	0
h	i	0
0	0	0

f

1	0	0
0	0	0
0	0	0

I

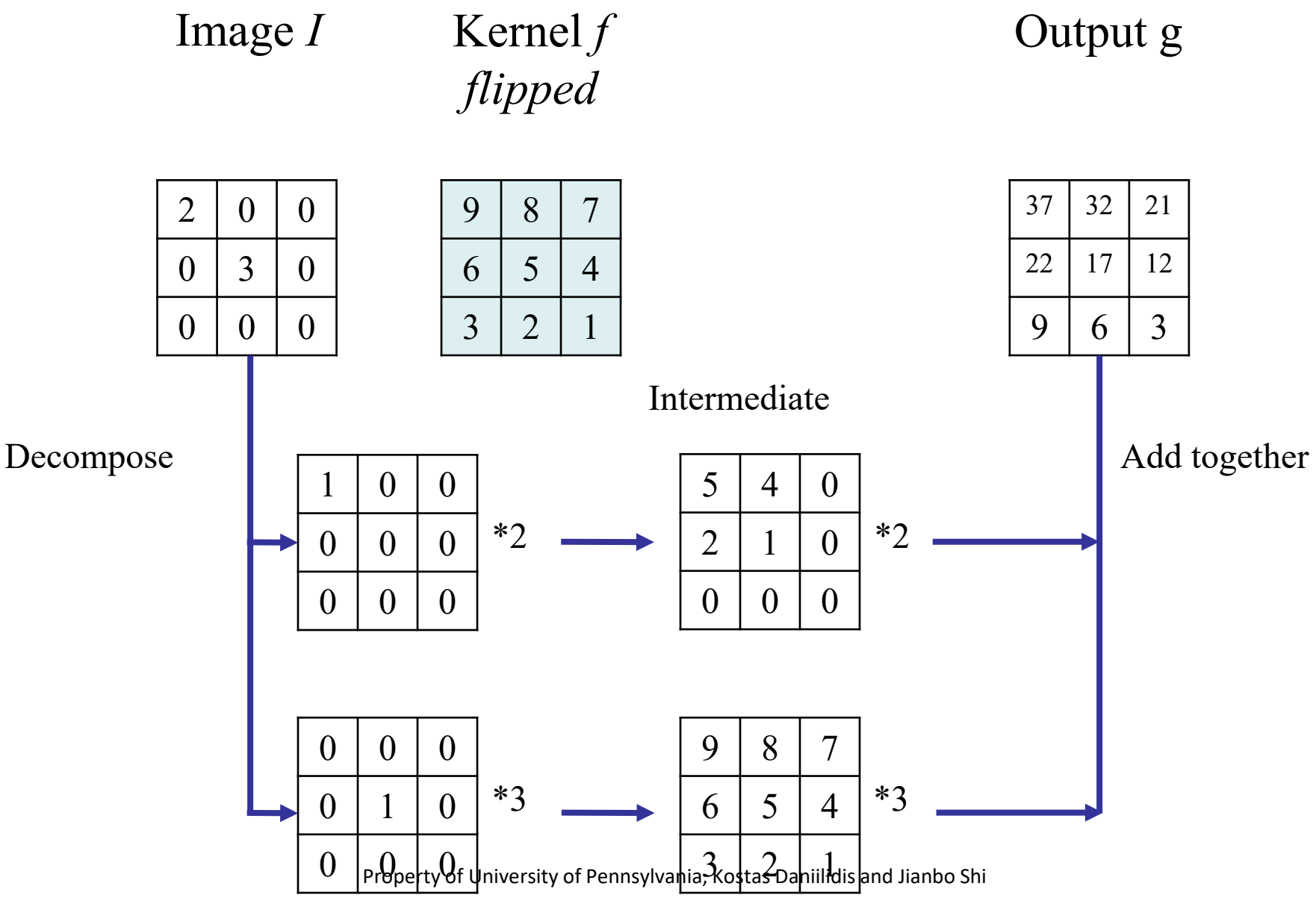
a	b	c
d	e	f
g	h	i

$f \otimes I$

e	f	0
h	i	0
0	0	0

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear independence



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Linear independence

Image I

0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0

5x5

Kernel f

1	0	0
0	1	0
0	0	0

Output $g=g_1+g_2$

1	1	1	1	0
1	1	1	1	0
1	1	1	1	0
1	1	1	1	0
0	1	0	1	0

Kernel f_1

1	0	0
0	0	0
0	0	0

Output g_1

1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
0	0	0	0	0

Kernel f_2

0	0	0
0	1	0
0	0	0

Output g_2

0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0
0	1	0	1	0

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

- Convolution is commutative

$$I \otimes f$$

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
1	0	0
1	1	0

Decompose

Image I

a	b	c
d	e	f
g	h	i

$\otimes$

1	0	0
0	0	0
0	0	0

e	f	0
h	i	0
0	0	0

0	0	0
1	0	0
0	0	0

b	c	0
e	f	0
h	i	0

0	0	0
0	0	0
1	0	0

0	0	0
b	c	0
e	f	0

0	0	0
0	0	0
0	1	0

0	0	0
a	b	c
d	e	f

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

- Convolution is commutative

$$f \otimes I$$

Image I

a	b	c
d	e	f
g	h	i

Kernel f

1	0	0
1	0	0
1	1	0

Decompose

1	0	0
0	0	0
0	0	0

e	f	0
h	i	0
0	0	0

0	0	0
1	0	0
0	0	0

$\otimes$

a	b	c
d	e	f
g	h	i

b	c	0
e	f	0
h	i	0

0	0	0
0	0	0
1	0	0

0	0	0
b	c	0
e	f	0

0	0	0
0	0	0
0	1	0

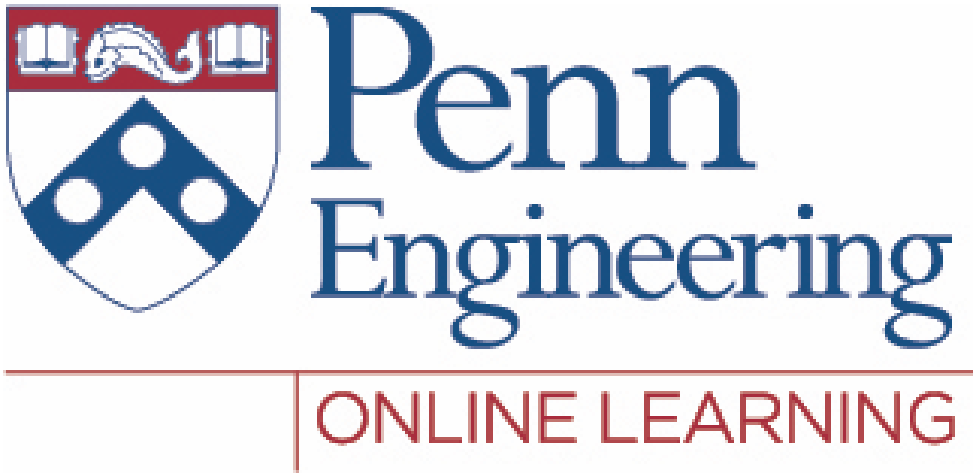
0	0	0
a	b	c
d	e	f

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Proof of Commutative property

- $g[m,n] = I \otimes f = f \otimes I$
- $g[m,n] = I \otimes f = \sum_{k,l} I(m-k, n-l) * f(k,l)$
- Let $k' = m - k, l' = n - l,$
  then $k = m - k', l = n - l'$
- $g[m,n] = \sum_{k',l'} I(k',l') * f(m-k', m-l') = f \otimes I$

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

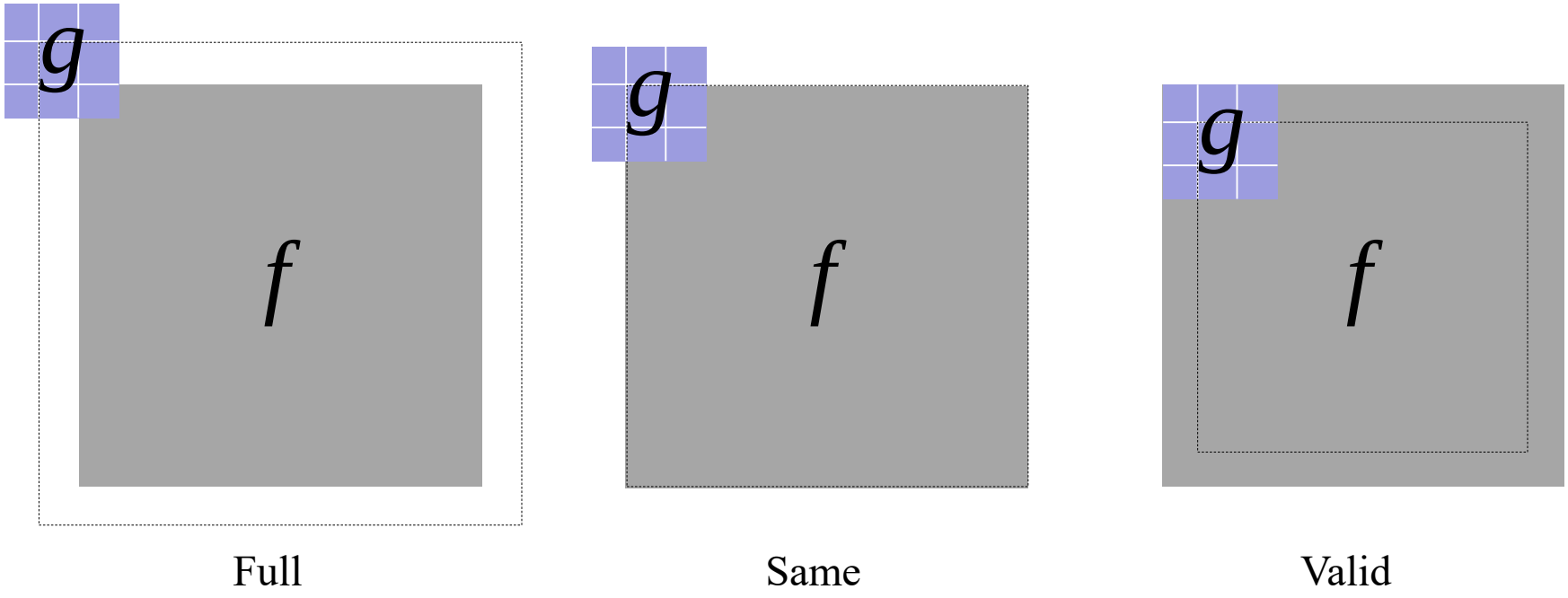


Video 2.12
Jianbo Shi

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Output Size of Image Convolution

$f \otimes g$



Full Same Valid

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

Same: output_size = f_size

Valid: output_size = f_size – (g_size – 1)

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

2D visualization of convolution (full)

Image I

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel f

1	2	3
4	5	6
7	8	9

3x3

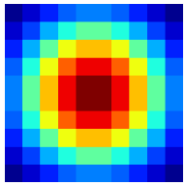
Output g

1	3	6	6	6	6	6	6	5	3
5	12	21	21	21	21	21	21	16	9
12	27	45	45	45	45	45	45	33	18
12	27	45	45	45	45	45	45	33	18
11	24	39	39	39	39	39	39	28	15
7	15	24	24	24	24	24	24	17	9
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

10x10

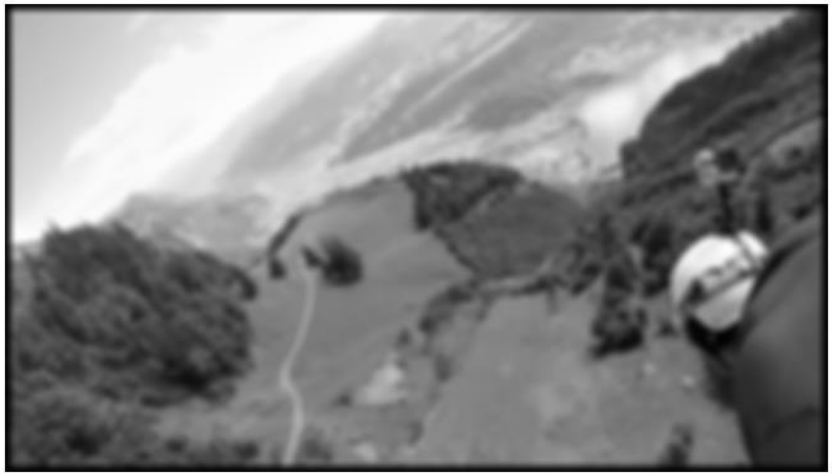
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Full

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

```
>> full = filter2(g, im, 'full');
>> size(full)

ans =

   369   649
```


Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

2D visualization of convolution (same)

Image I

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel f

1	2	3
4	5	6
7	8	9

3x3

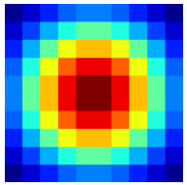
Output g

12	21	21	21	21	21	21	16
27	45	45	45	45	45	45	33
27	45	45	45	45	45	45	33
24	39	39	39	39	39	39	28
15	24	24	24	24	24	24	17
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

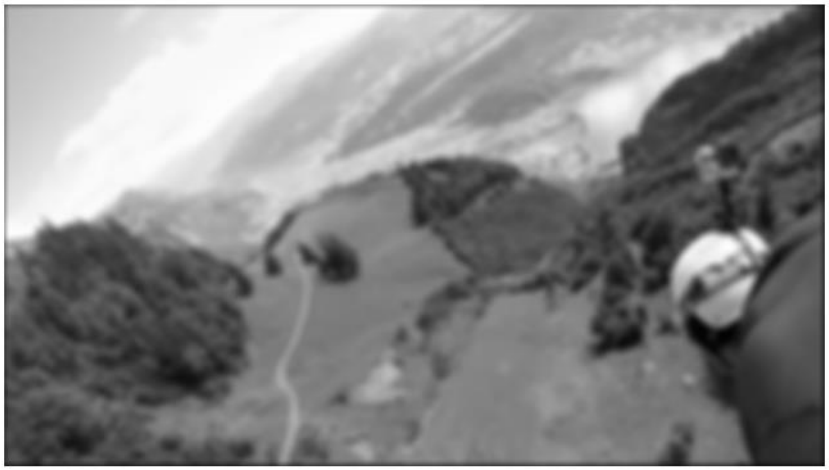
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Same

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

Same: output_size = f_size

```
>>same = filter2(g, im, 'same');
>> size(same)

ans =

   360   640
```


Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

2D visualization of convolution (valid)

Image I

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8x8

Kernel f

1	2	3
4	5	6
7	8	9

3x3

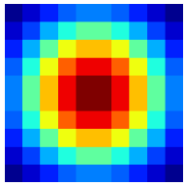
Output g

45	45	45	45	45	45
45	45	45	45	45	45
39	39	39	39	39	39
24	24	24	24	24	24
0	0	0	0	0	0
0	0	0	0	0	0

6x6

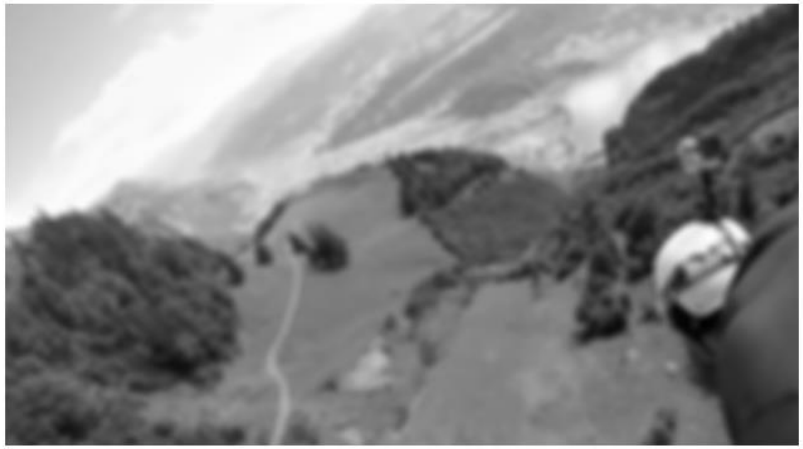
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Output Size of Image Convolution



g : 10 x 10 Gaussian kernel

f : 640 x 360 resolution



Valid

filter2(g, f, shape) in MATLAB

Full: output_size = f_size + g_size – 1

Same: output_size = f_size

Valid: output_size = f_size – (g_size – 1)

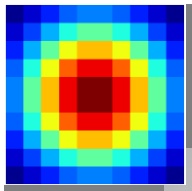
```
>> valid = filter2(g, im, 'valid');
>> size(valid)

ans =

   351   631
```

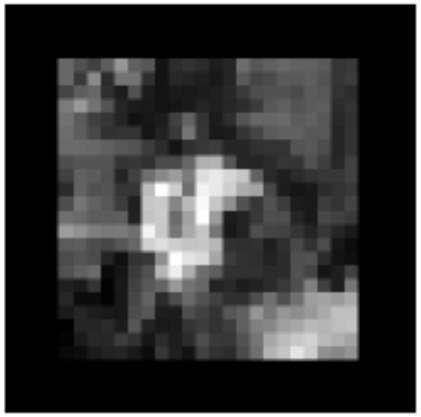

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Image Boundary Effect

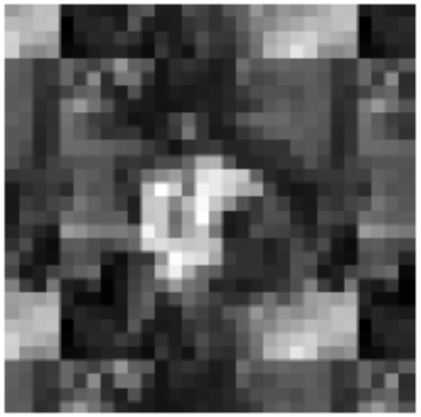


The filter window falls off at the edge of image.

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi



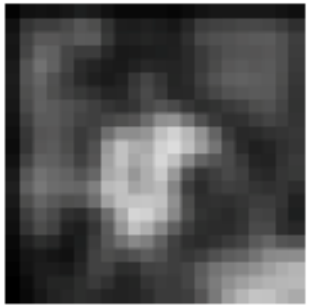
zero



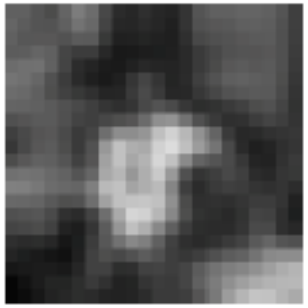
wrap

clamp

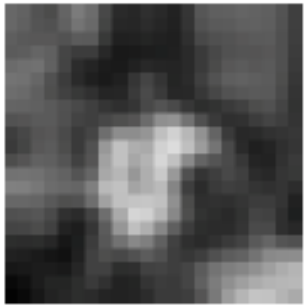
mirror



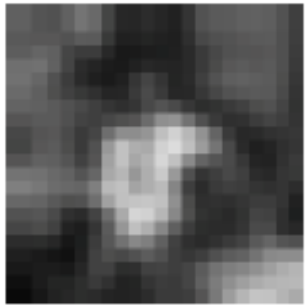
blurred zero



normalized zero



blurred clamp



blurred mirror

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Image Extrapolation (Mirroring)

Code

```
J = imread('image.bmp');
figure; imshow(J);
```

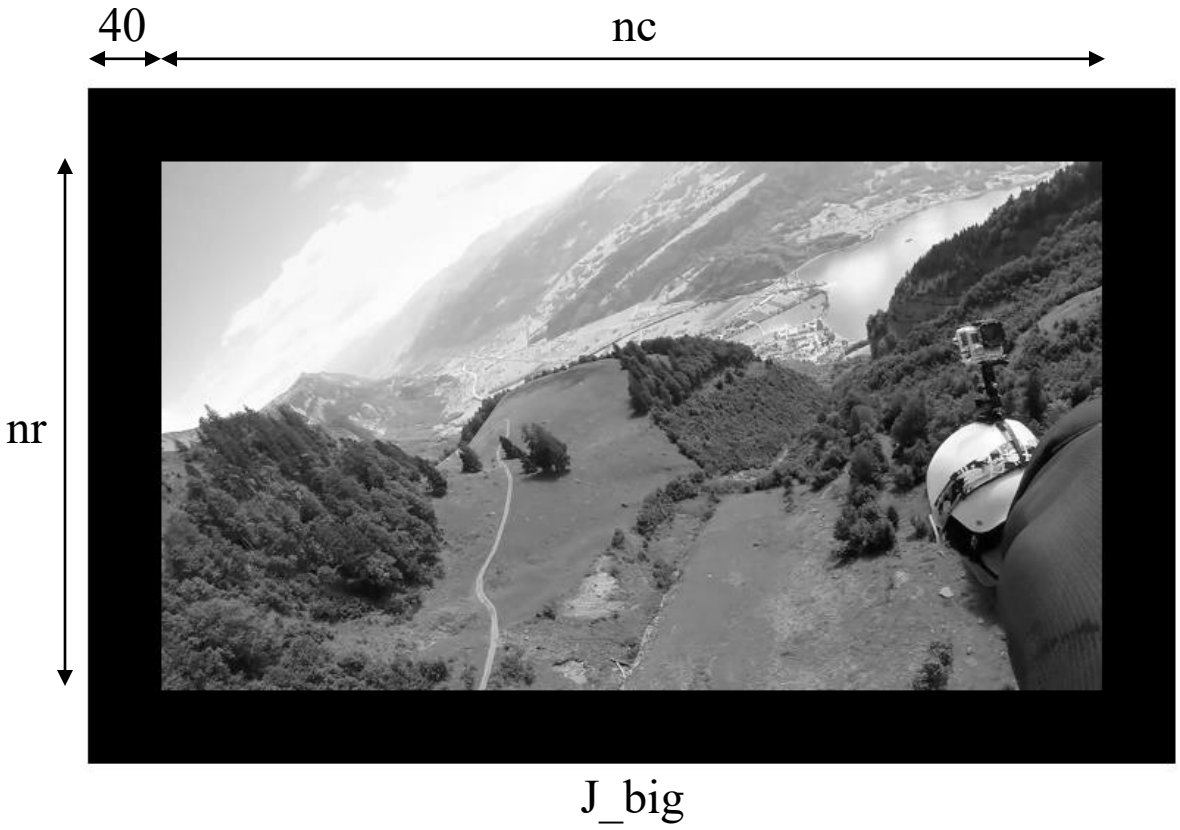
J

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Image Extrapolation (Mirroring)

Code

```
boarder = 40;
[nr,nc,nb] = size(J);
J_big = zeros(nr+2*boarder, nc + 2*boarder,nb);
J_big(boarder+1:boarder+nr,boarder+1:boarder+nc,:) = J;
```

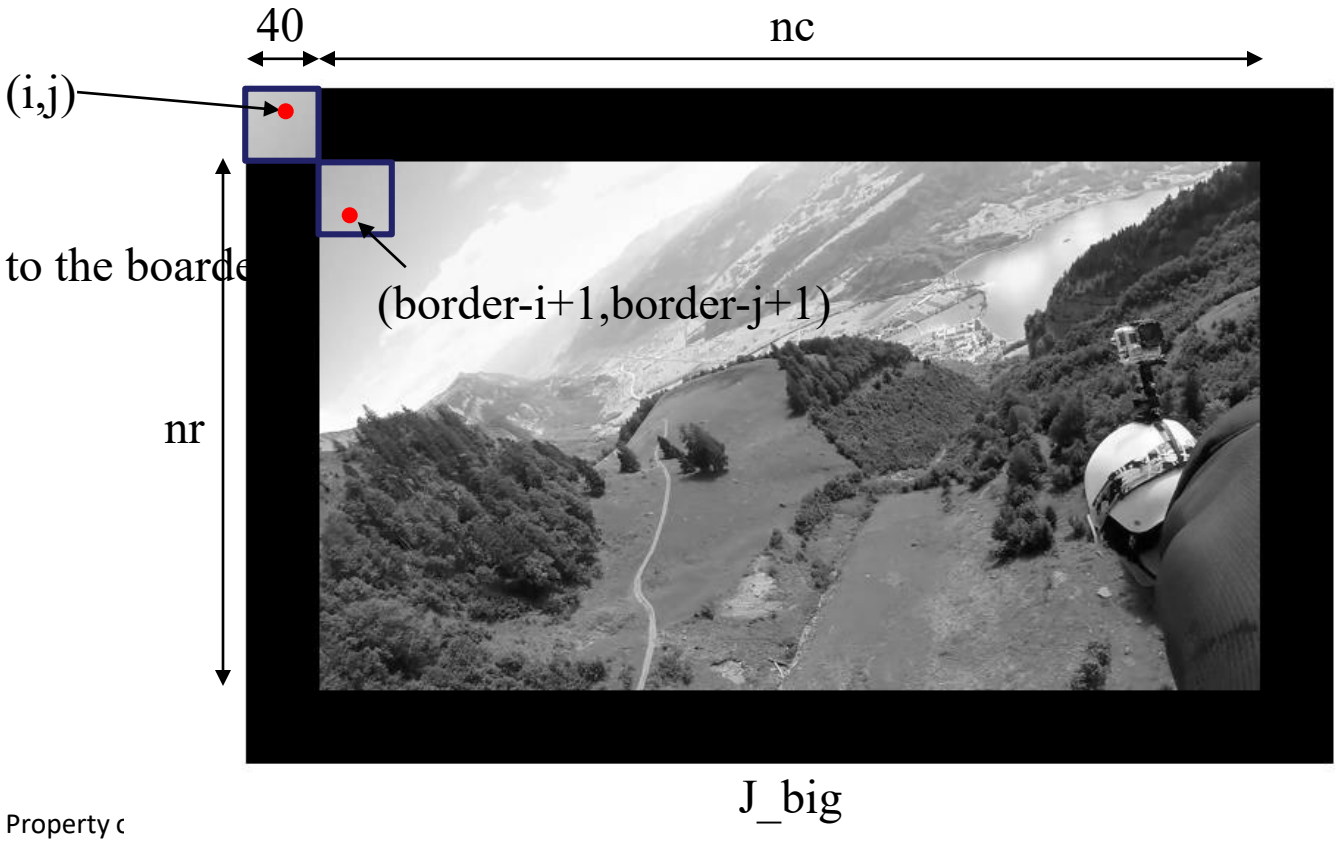


Property c

Image Extrapolation (Mirroring)

Code

```
for i=1:border,
    for j=1:border,
        J_big(i,j,:) = J(border-i+1,border-j+1,:);
    end
end
```



Mirroring with respect to the boarde

Property c

Image Extrapolation (Mirroring)

Code

```
for i=1:boarder,
   for j=border+1:border+nc,
         J_big(i,j,:) = J(border-i+1,j-border,:);
    end
end
```

Mirroring with respect to the boarde

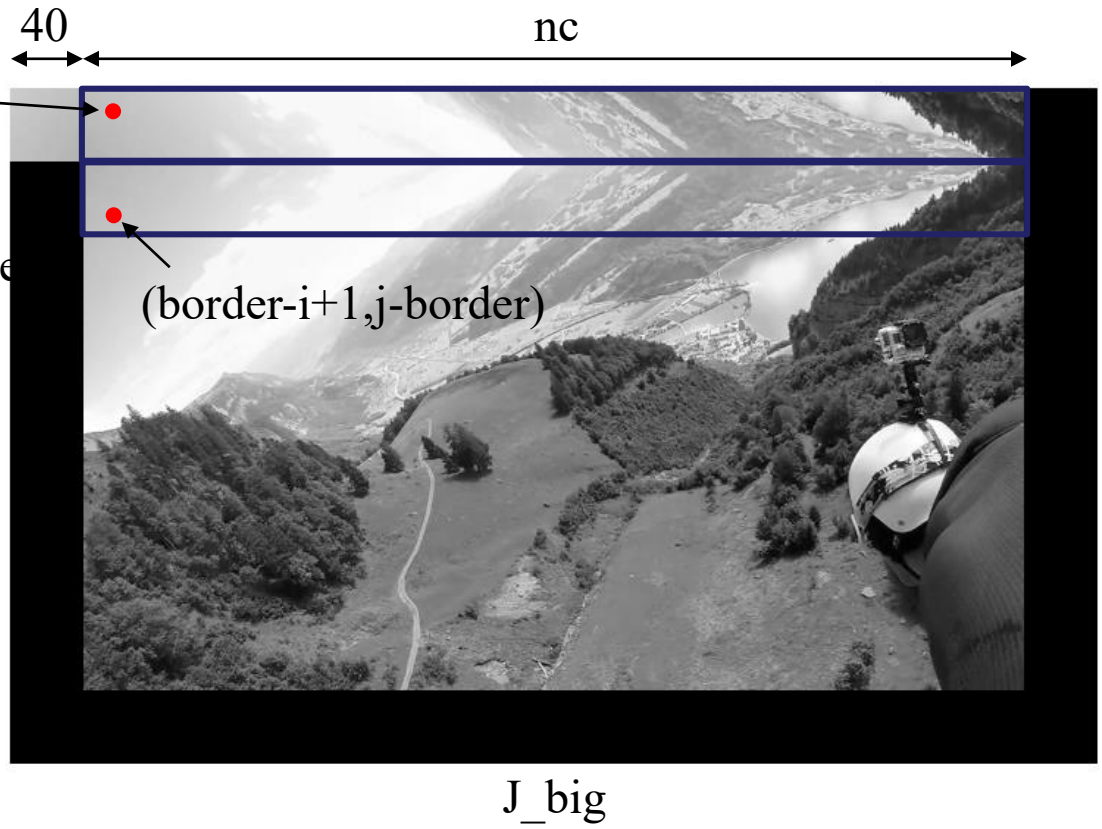


Image Extrapolation (Mirroring)

Code

```
for i=nr+border+1:border*2+nr,
    for j=border+1:border+nc,
        J_big(i,j,:) = J(2*nr-i+border+1,j-border,:);
    end
end
```

Mirroring with respect to the boarde

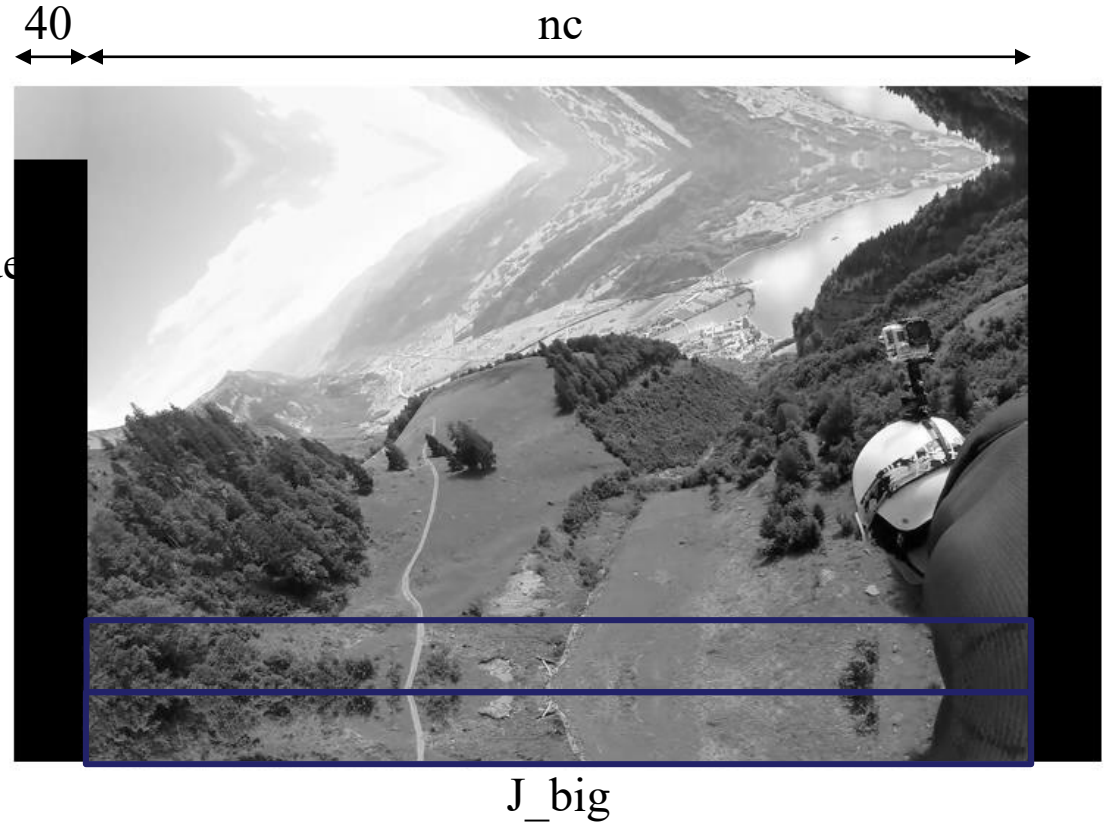
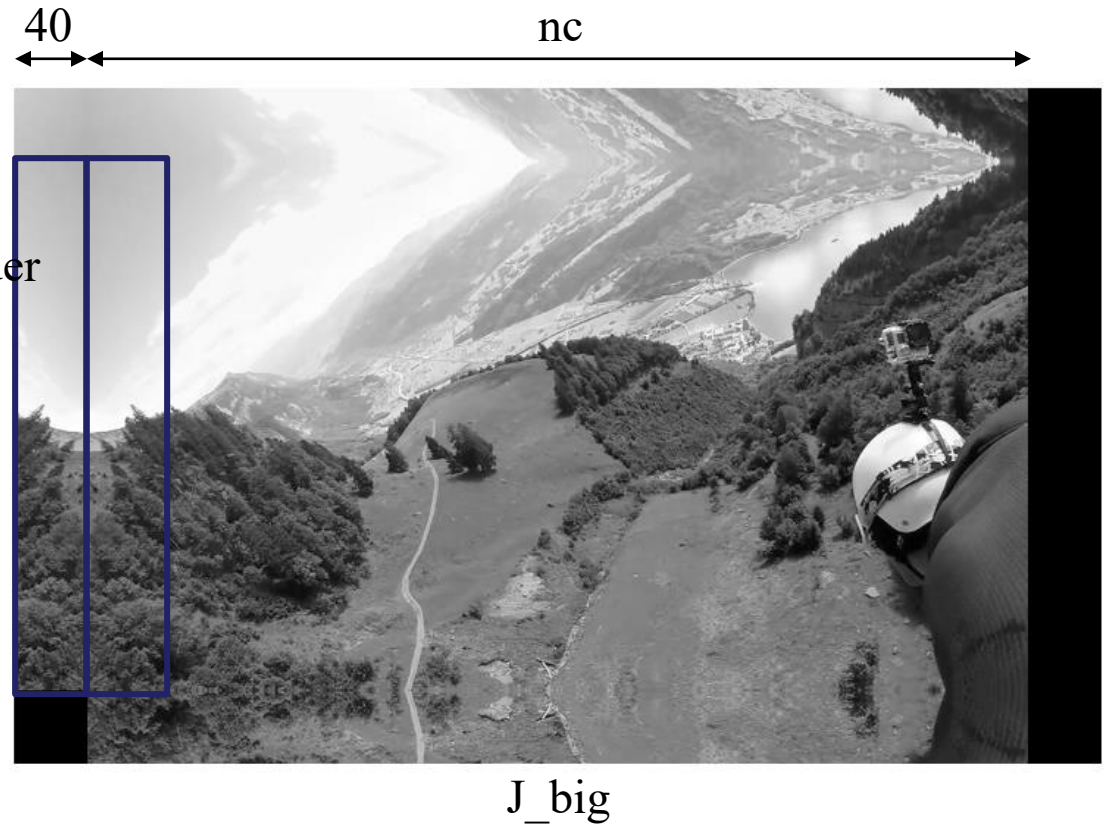


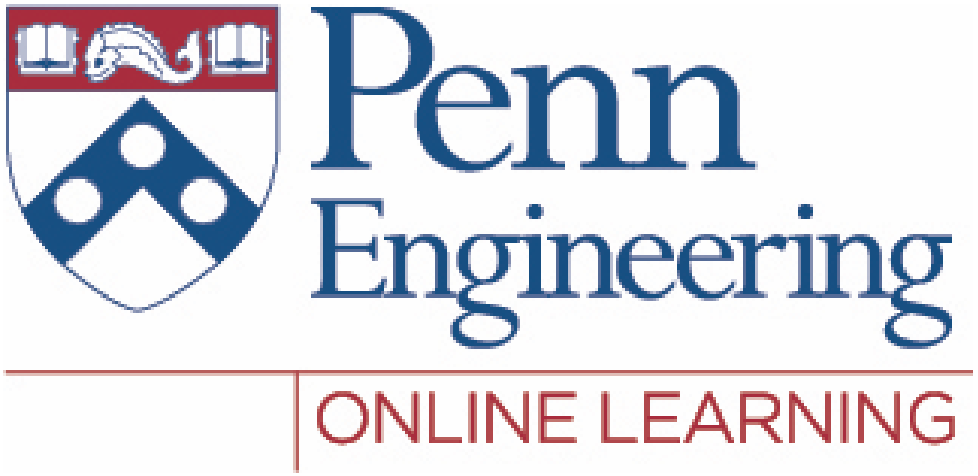
Image Extrapolation (Mirroring)

Code

```
for i=border+1:border+nr;
    for j=1:border,
        J_big(i,j,:) = J(i-border,border-j+1,:);
    end
end
```

Mirroring with respect to the boarder





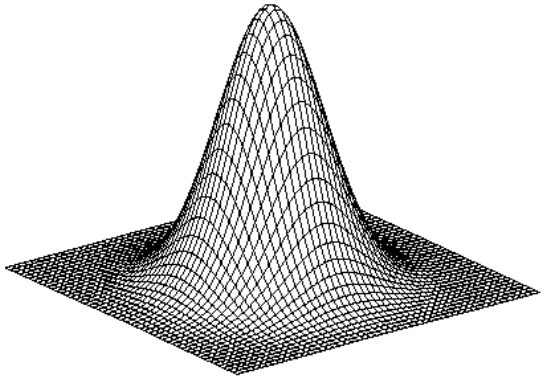
Video 2.13
Jianbo Shi

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Examples of image operation as convolution

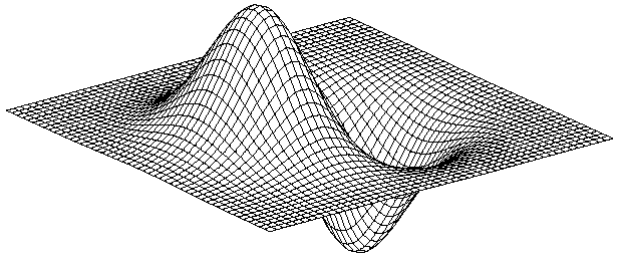
Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

2D filters, more on this later…



Gaussian

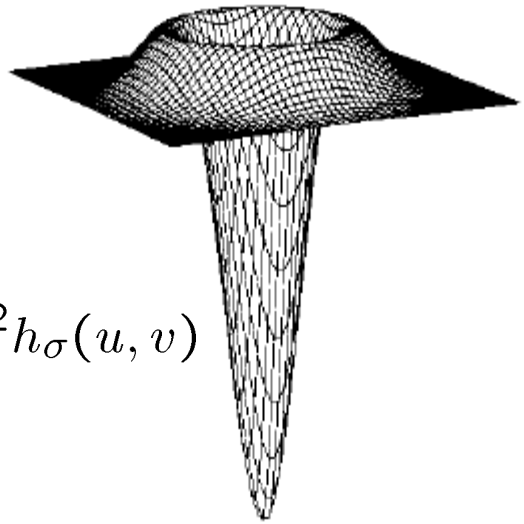
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

- is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

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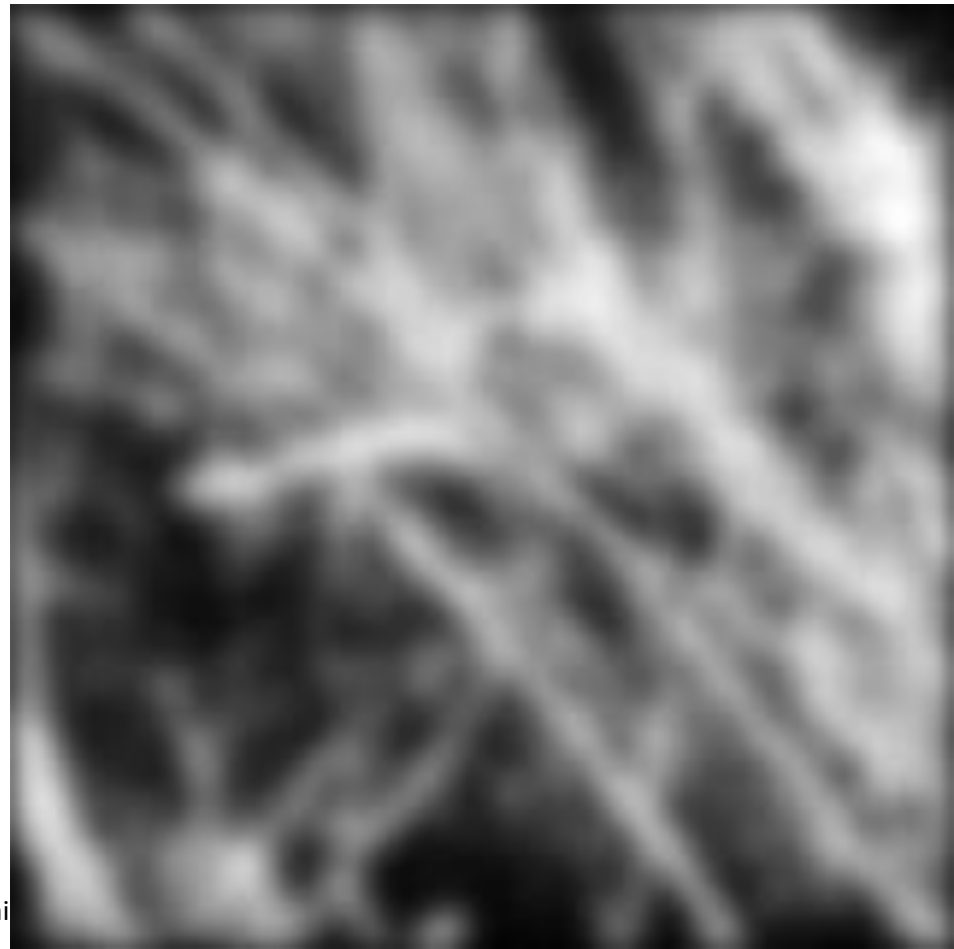
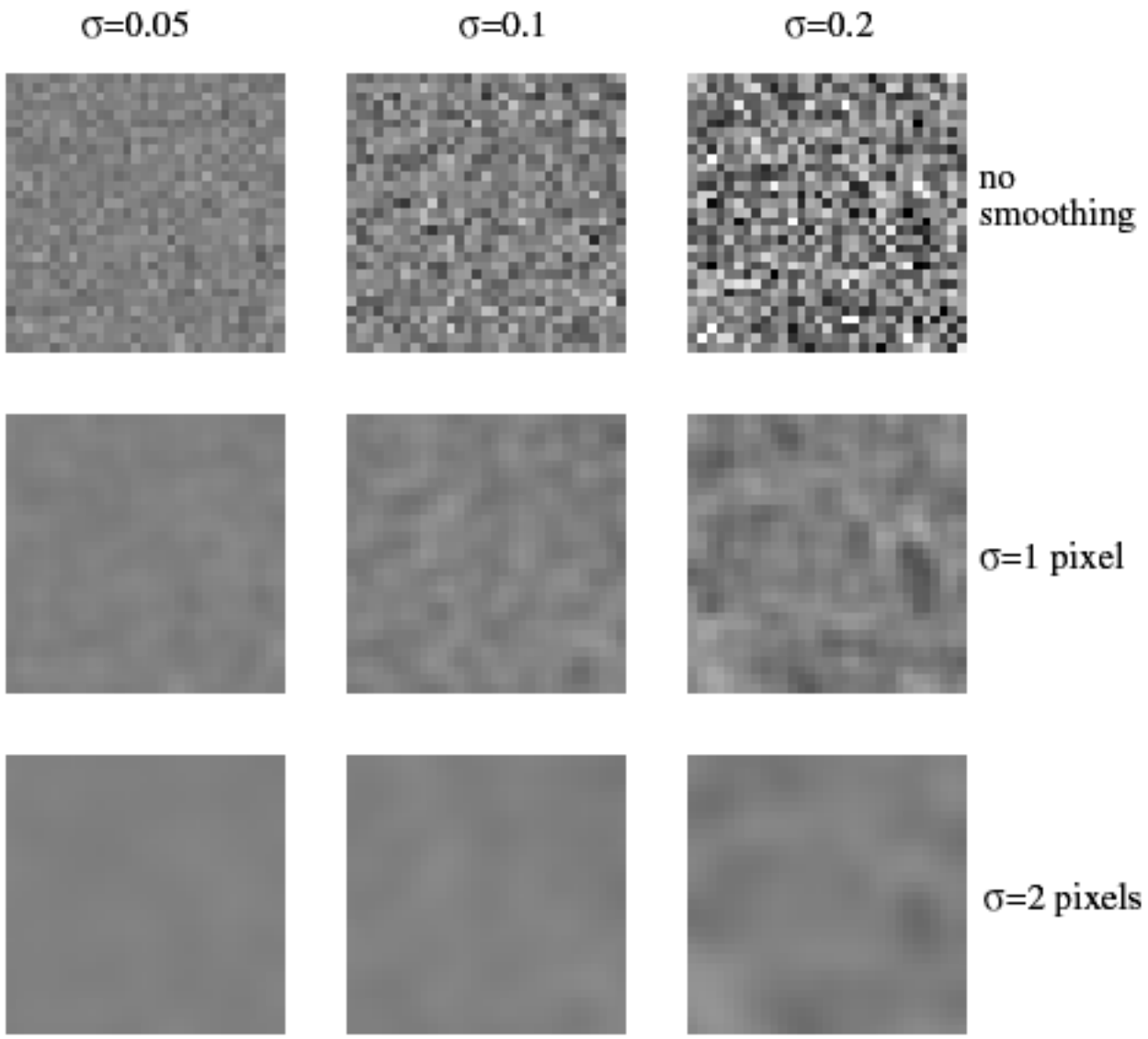
Smoothing with a Gaussian





The effects of smoothing
Each row shows smoothing with gaussians of different width; each column shows different realizations of an image of gaussian noise.

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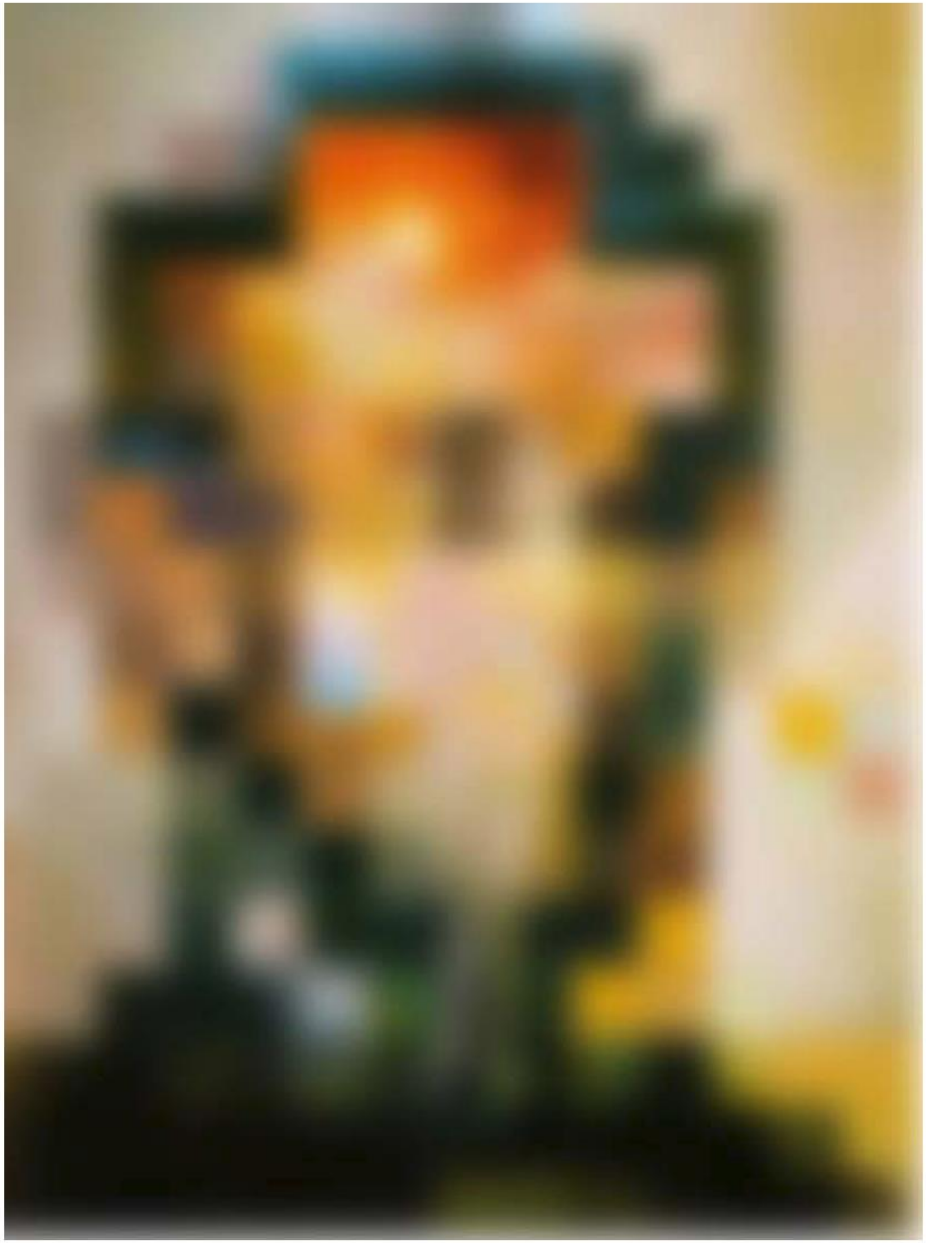
Salvador Dali, *"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln"*, 1976



Salvador Dali, *"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln"*, 1976

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Image smoothing can remove noise, and also …

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20
40
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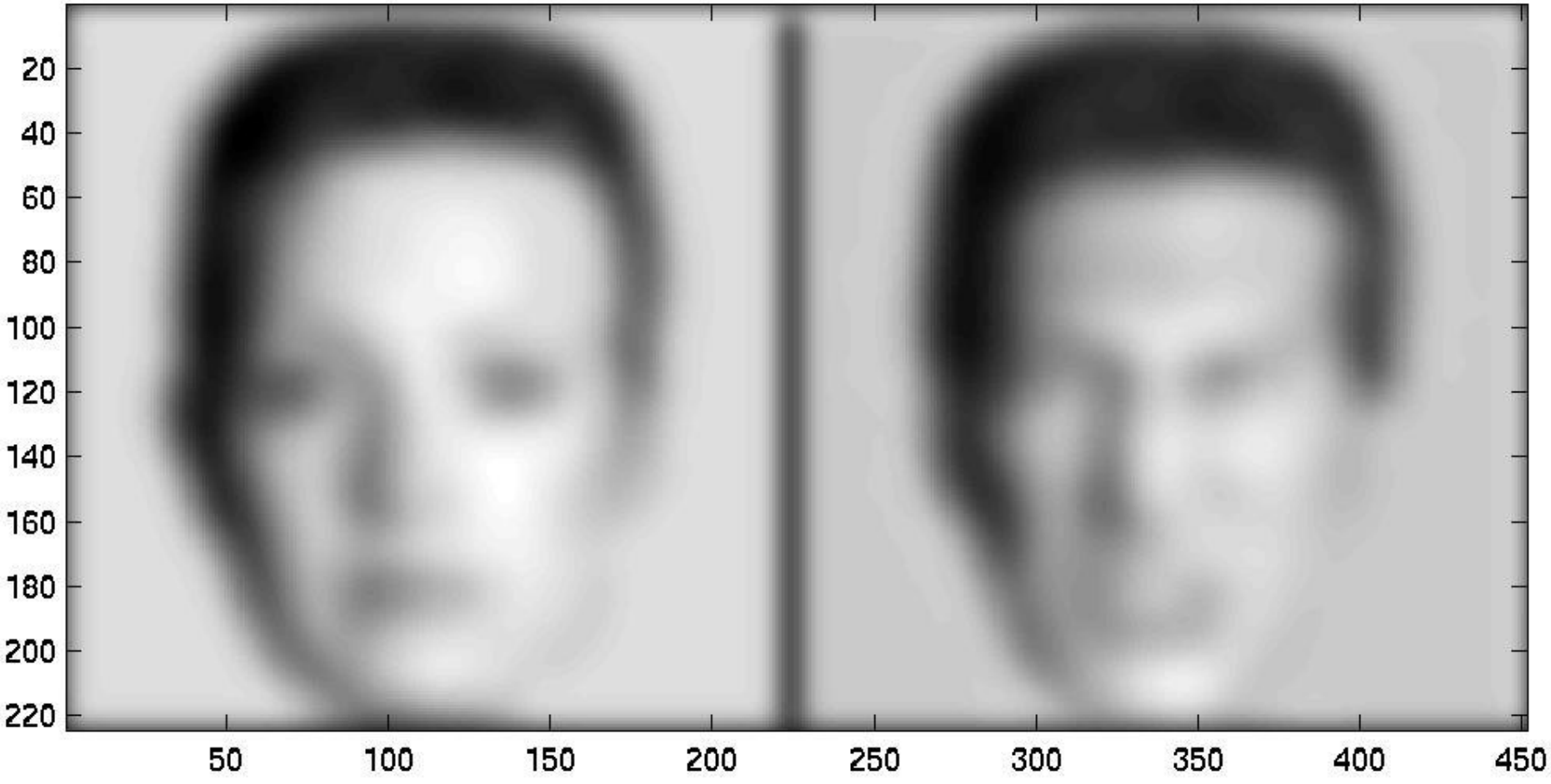
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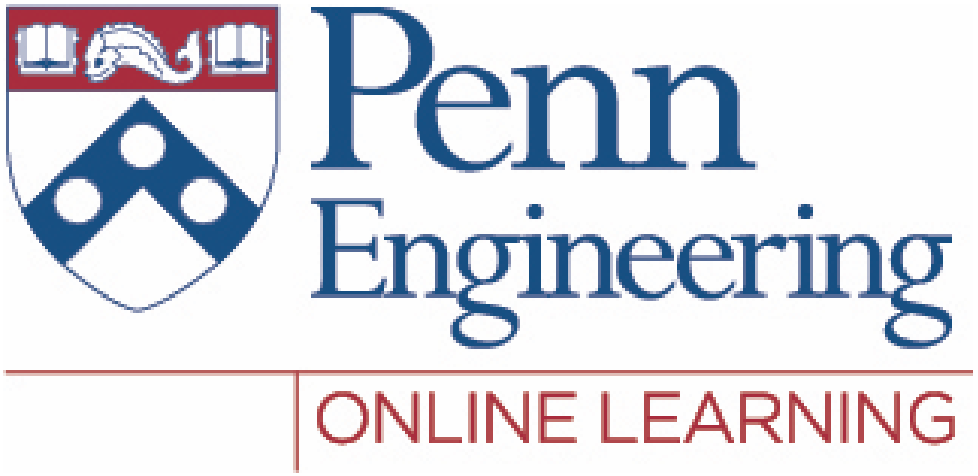

Video 2.14
Jianbo Shi

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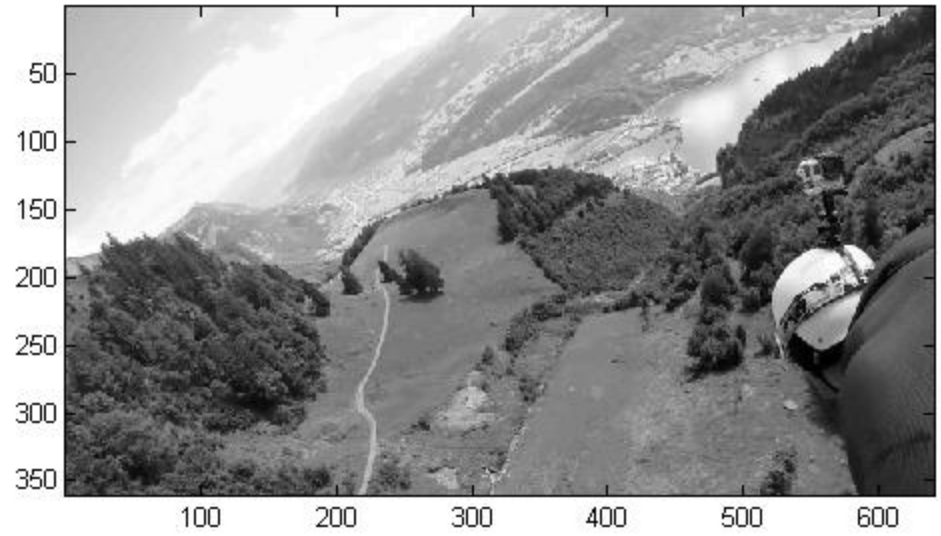
Edge Detection

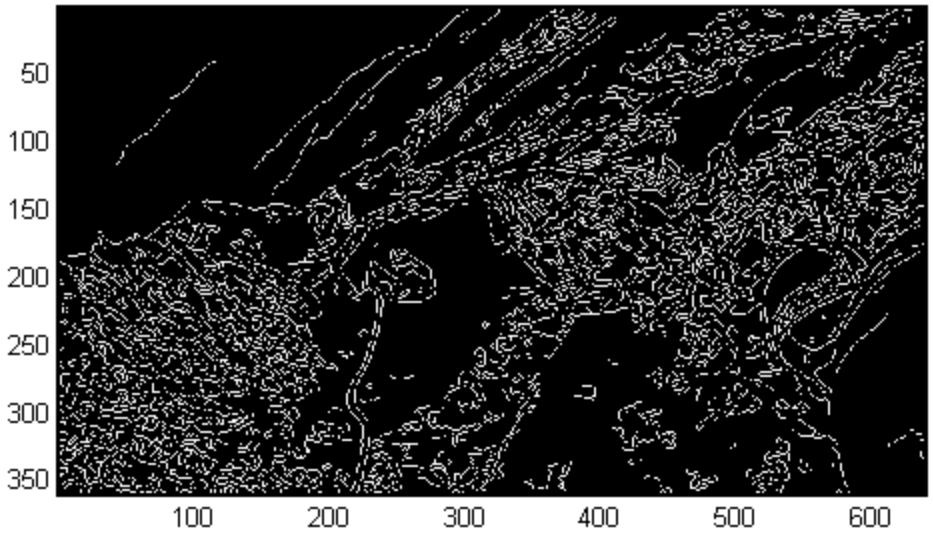
Code

```
Jb = rgb2gray(J);

imagesc(Jb);axis image; colormap(gray);

bw = edge(Jb,'canny');
```





Canny Edge Detection

$$B(i,j) = \begin{cases} 1 & \text{if } I(i,j) \text{ is edge} \\ 0 & \text{if } I(i,j) \text{ is not edge} \end{cases}$$

Objective: to localize edges given an image.

Original image, I

Binary image indicating edge pixels

Edge map image, B

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Canny Edge Detection

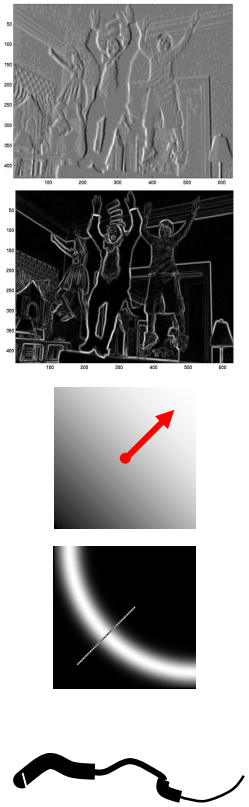
1. Filter image by derivatives of Gaussian

2. Compute magnitude of gradient

3. Compute edge orientation

4. Detect local maximum

5. Edge linking



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1) Compute Image Gradient

the first order derivative of Image I in x, and in y direction

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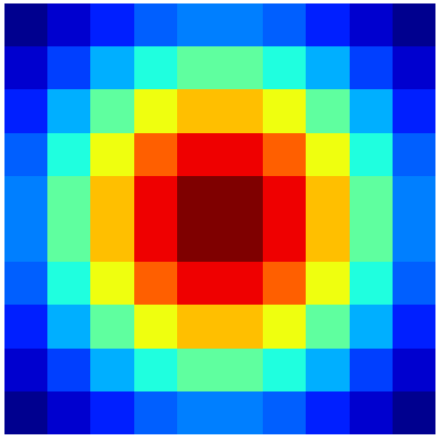
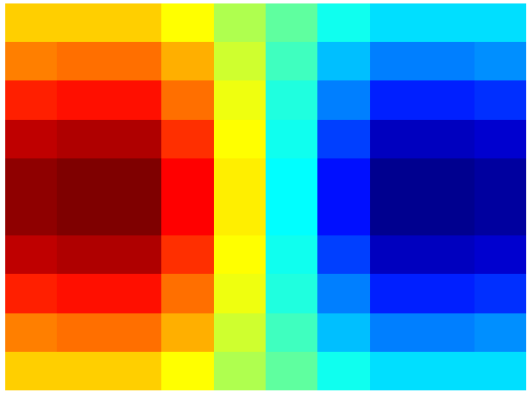
Edge Detection, Step 1, Filter out noise and compute derivative:

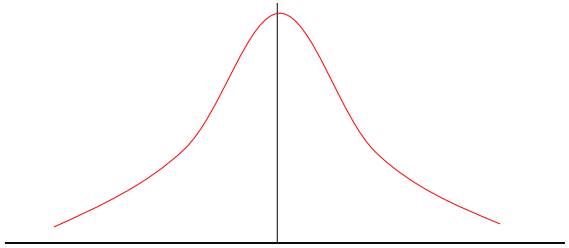
$$(\frac{\delta}{\delta x} \otimes G)$$

Gradient of Gaussian

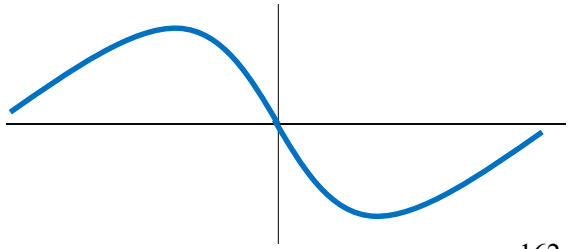


$$\frac{\delta}{\delta x}$$





$$\frac{\delta}{\delta x}$$



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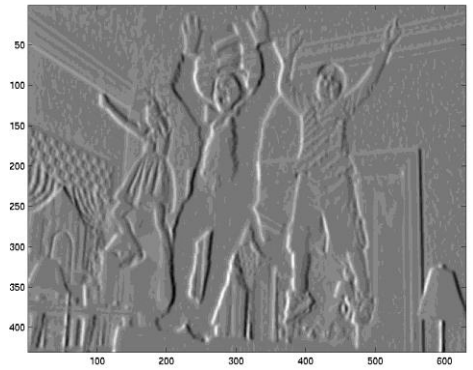
Edge Detection, Step 1, Filter out noise and compute derivative:

Image

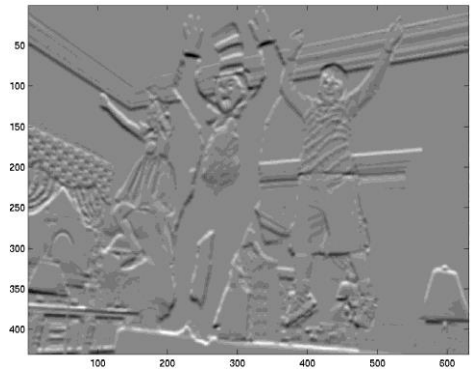
$$\otimes\left(\frac{\delta}{\delta x} \otimes G\right)$$

Smoothed Derivative

I_x



I_y



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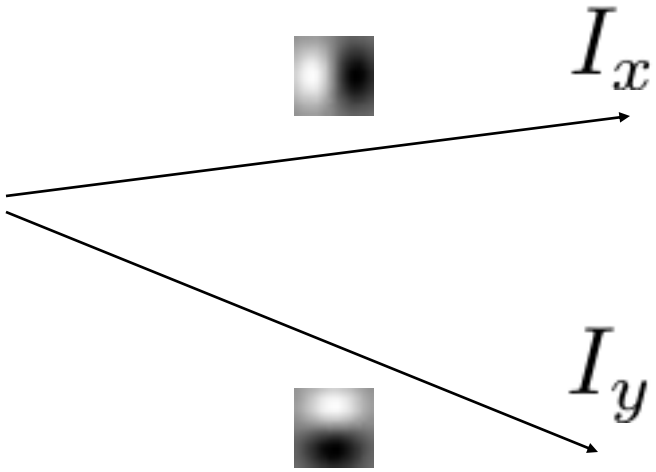
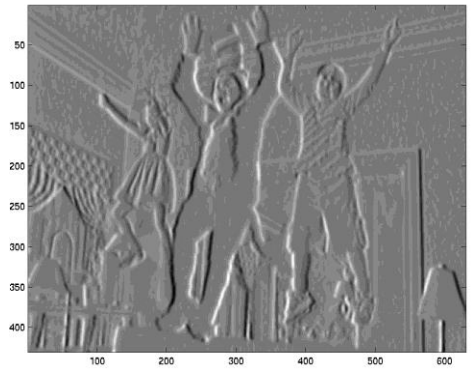
Edge Detection, Step 1, Filter out noise and compute derivative:

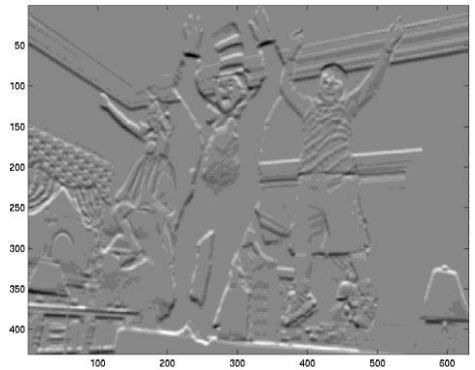
In matlab:

>> [dx,dy] = gradient(G); % G is a 2D gaussain

>> Ix = conv2(I,dx,'same'); Iy = conv2(I,dy,'same');







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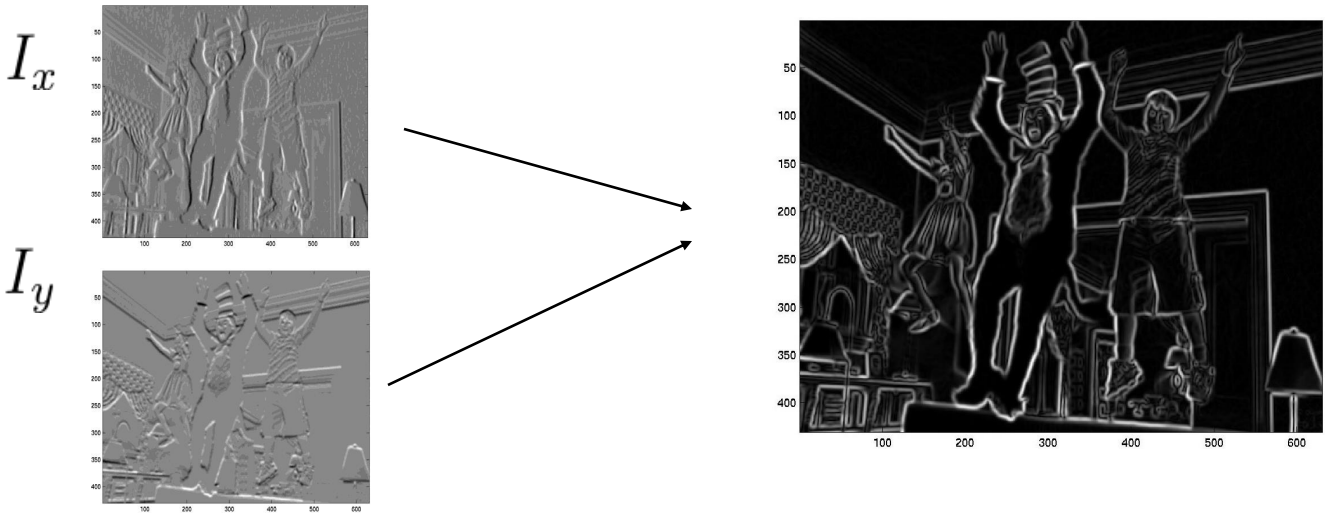
Edge Detection: Step 2
Compute the magnitude of the gradient

In Matlab:
>> Im = sqrt(Ix.*Ix + Iy.*Iy);



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We know roughly where are the edges, but we need their precise location.

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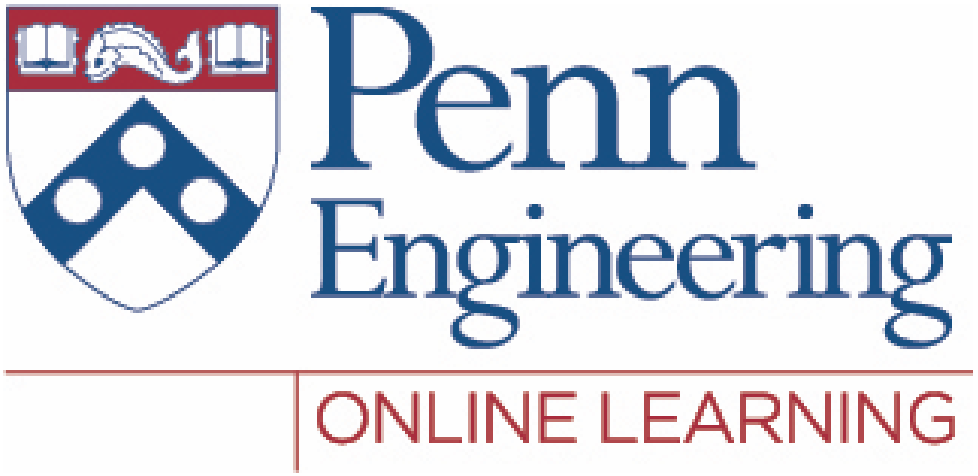

Video 2.15
Jianbo Shi

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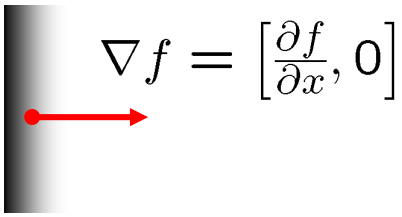
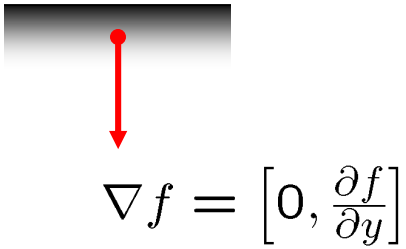
Finding the orientation of the edge

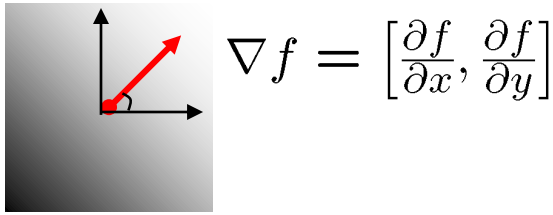
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient points in the direction of most rapid change in intensity







- The image gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

  - how does this relate to the direction of the edge?

$$\theta_{edge} = tan^{-1}(-\frac{\delta f}{\delta x} / \frac{\delta f}{\delta y})$$

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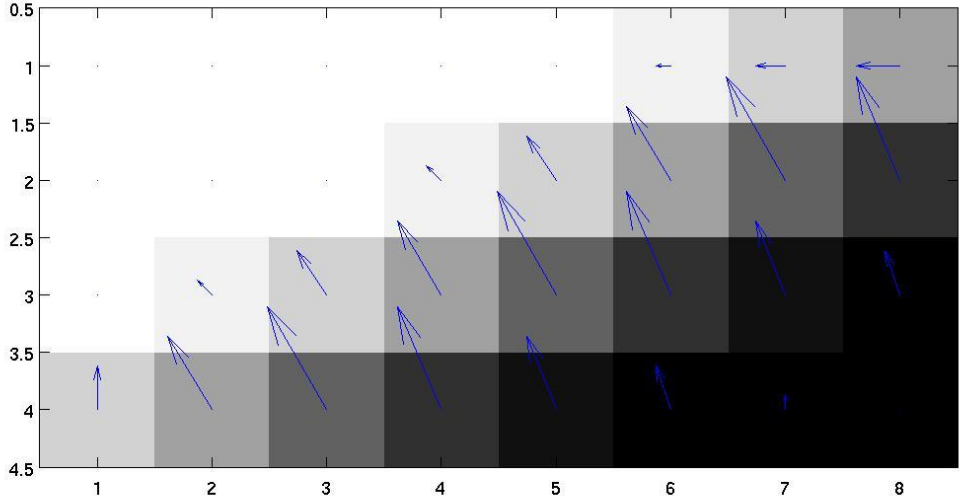
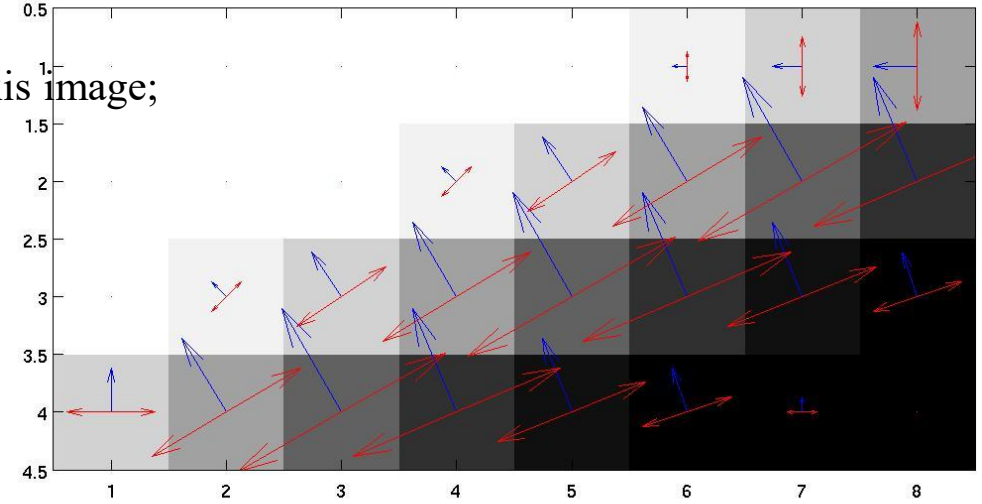
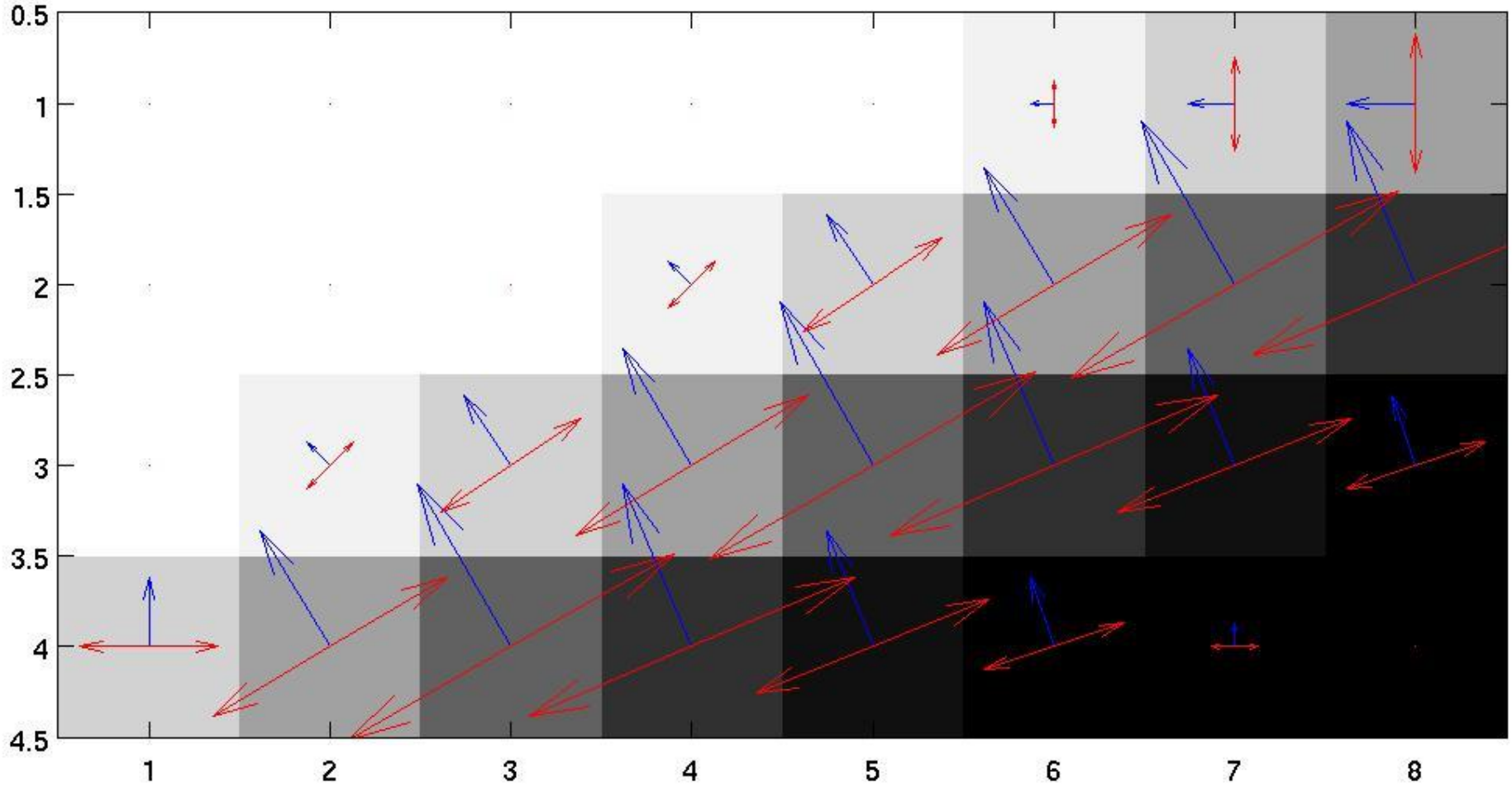
```
%% define image gradient operator
dy = [1;-1];
dx = [1,-1];

%% compute image gradient in x and y
AA_y = conv2(AA,dy,'same');
AA_x = conv2(AA,dx,'same');

Jy = AA_y(1:end-2,2:end-1);
Jy(1,:) = 0;

Jx = AA_x(2:end-1,1:end-2);
Jx(:,1) = 0;

%% display the image gradient flow
figure(3);clf;imagesc(J);colormap(gray);axis image;
hold on;
quiver(Jx,Jy);
quiver(-Jy,Jx,'r');
quiver(Jy,-Jx,'r');
```





Property of Univ


0.5
1
1.5
2
2.5
3
3.5
4
4.5
1
2
3
4
5
6
7
8

[gx,gy] = gradient(J);
mag = sqrt(gx.*gx+gy.*gy); imagesc(mag);colorbar

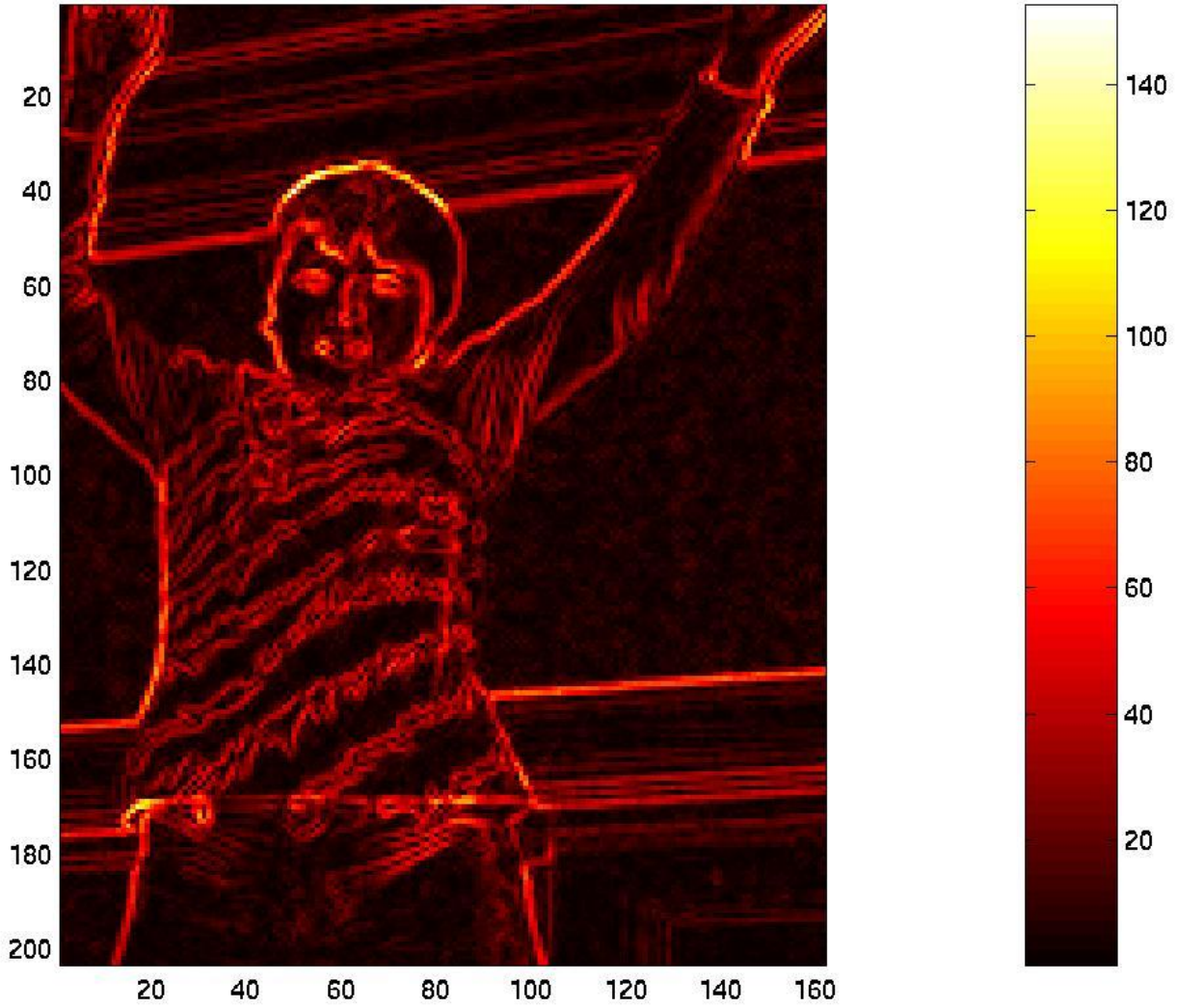


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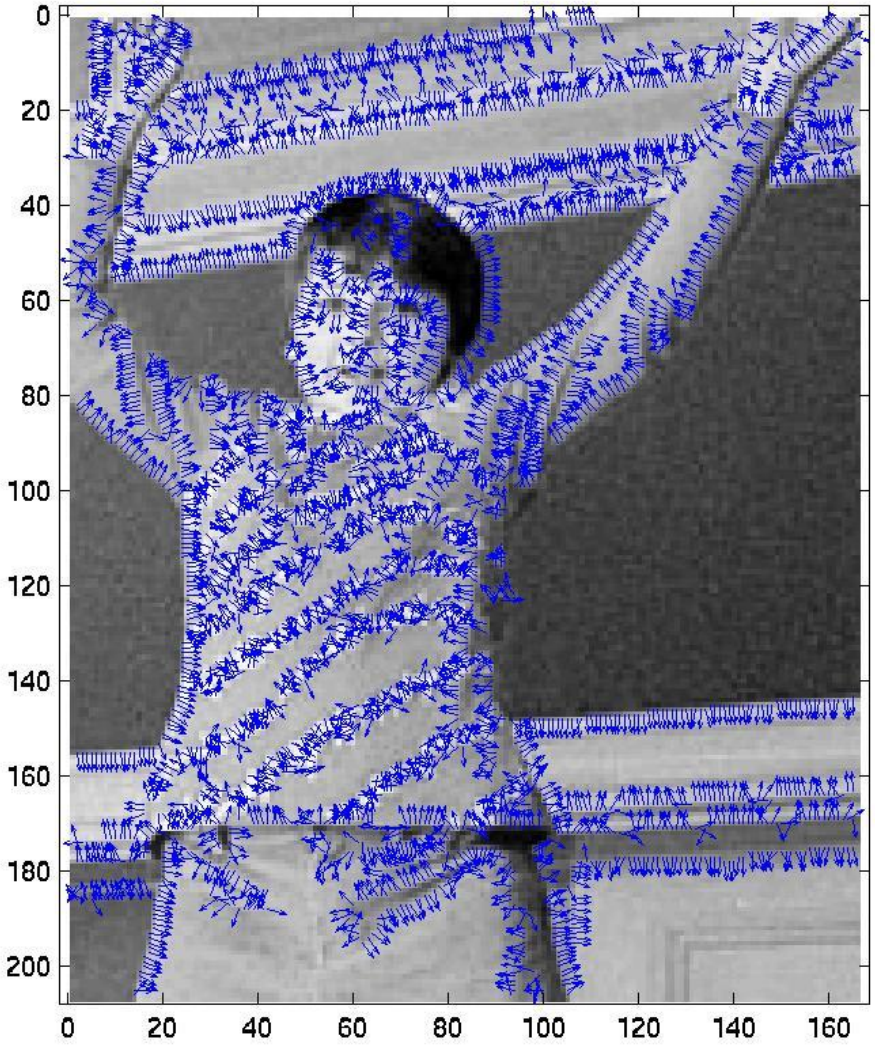
image gradient direction:



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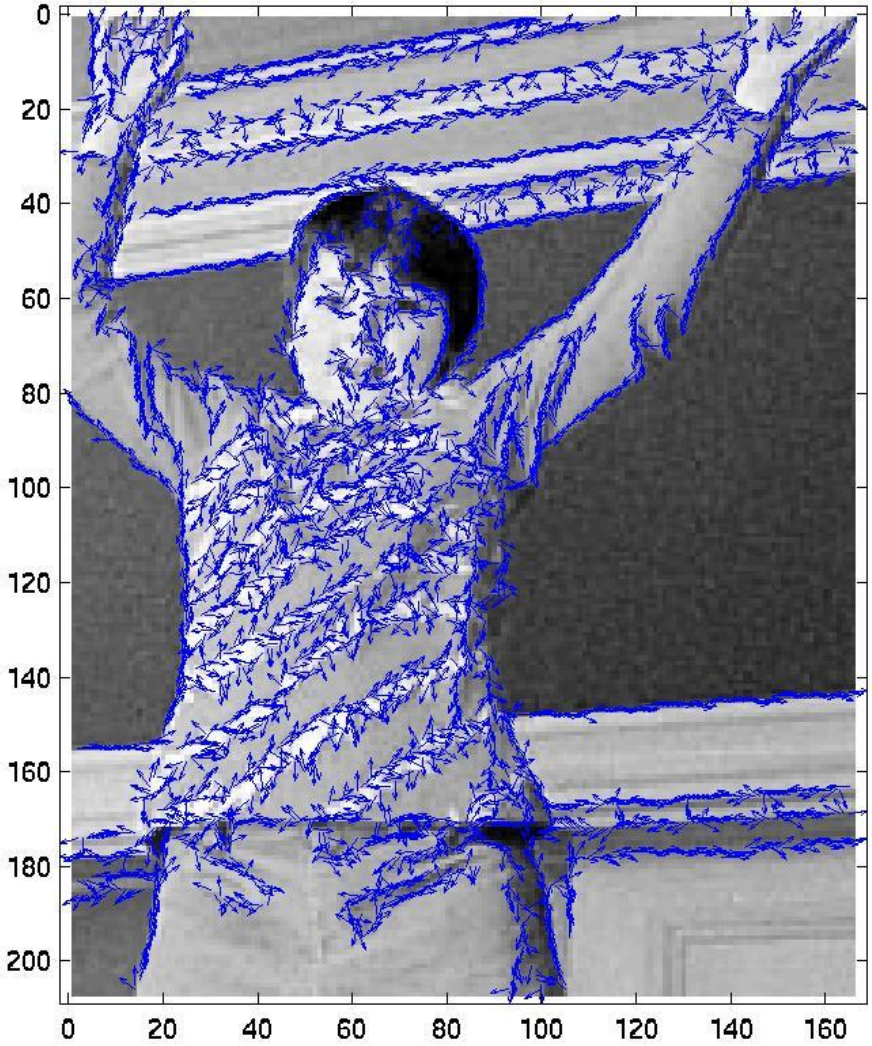
Edge orientation direction:



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[gx,gy] = gradient(J);
th = atan2(gy,gx); % or you can use:[th,mag] = cart2pol(gx,gy);
imagesc(th.*(mag>20));colormap(hsv); colorbar

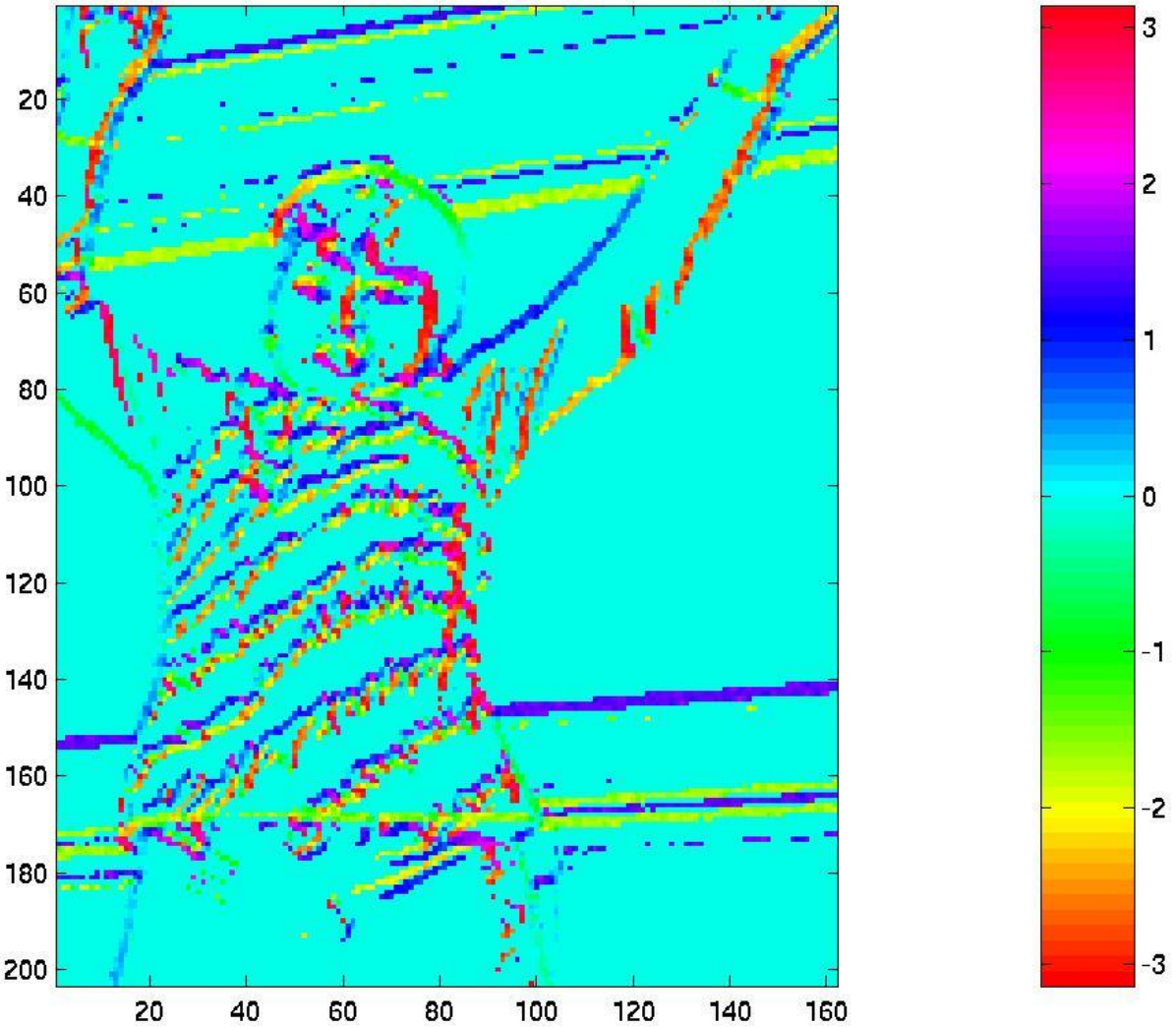

Image Scale

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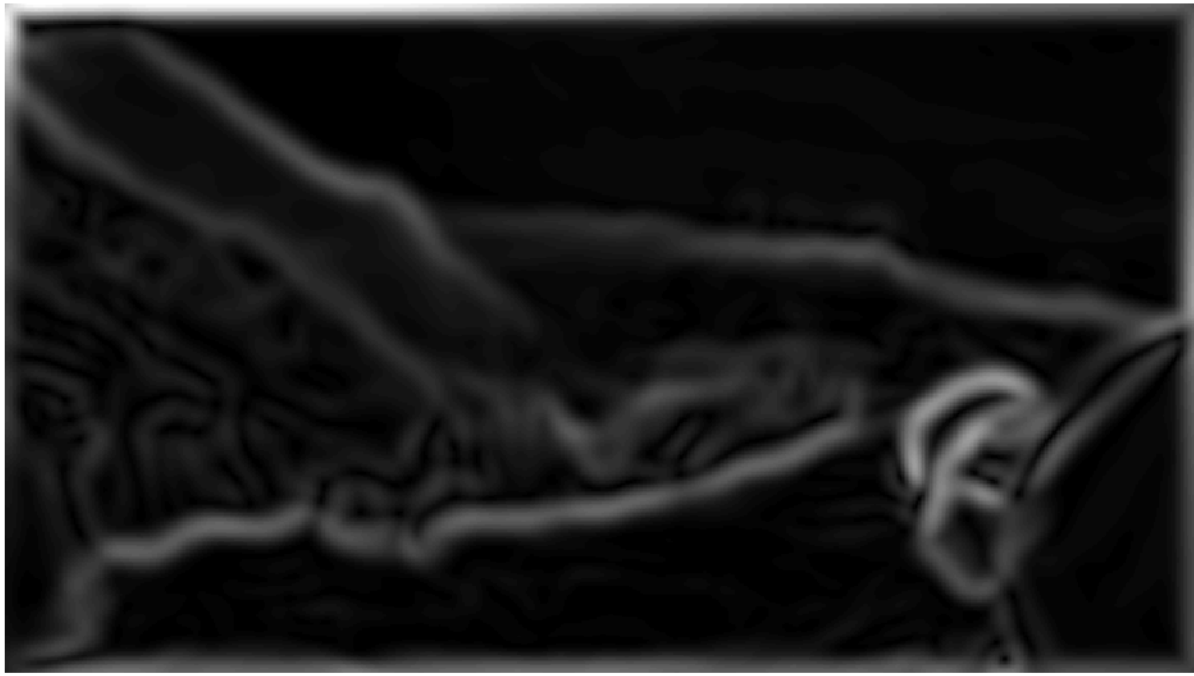
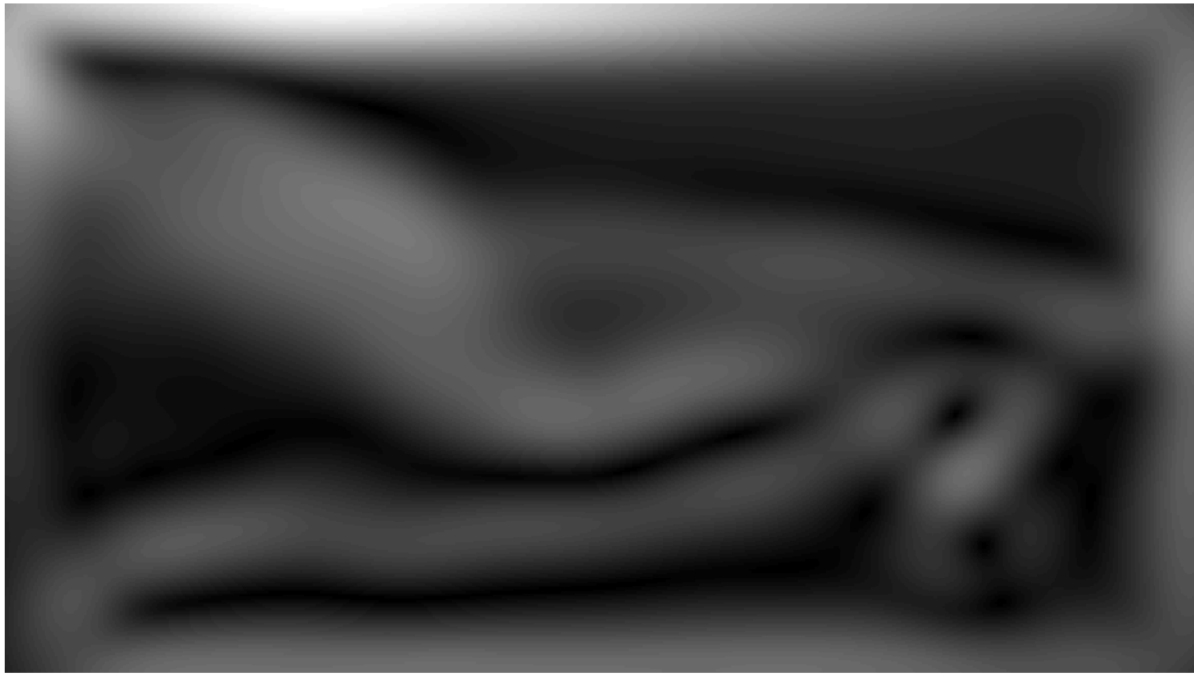
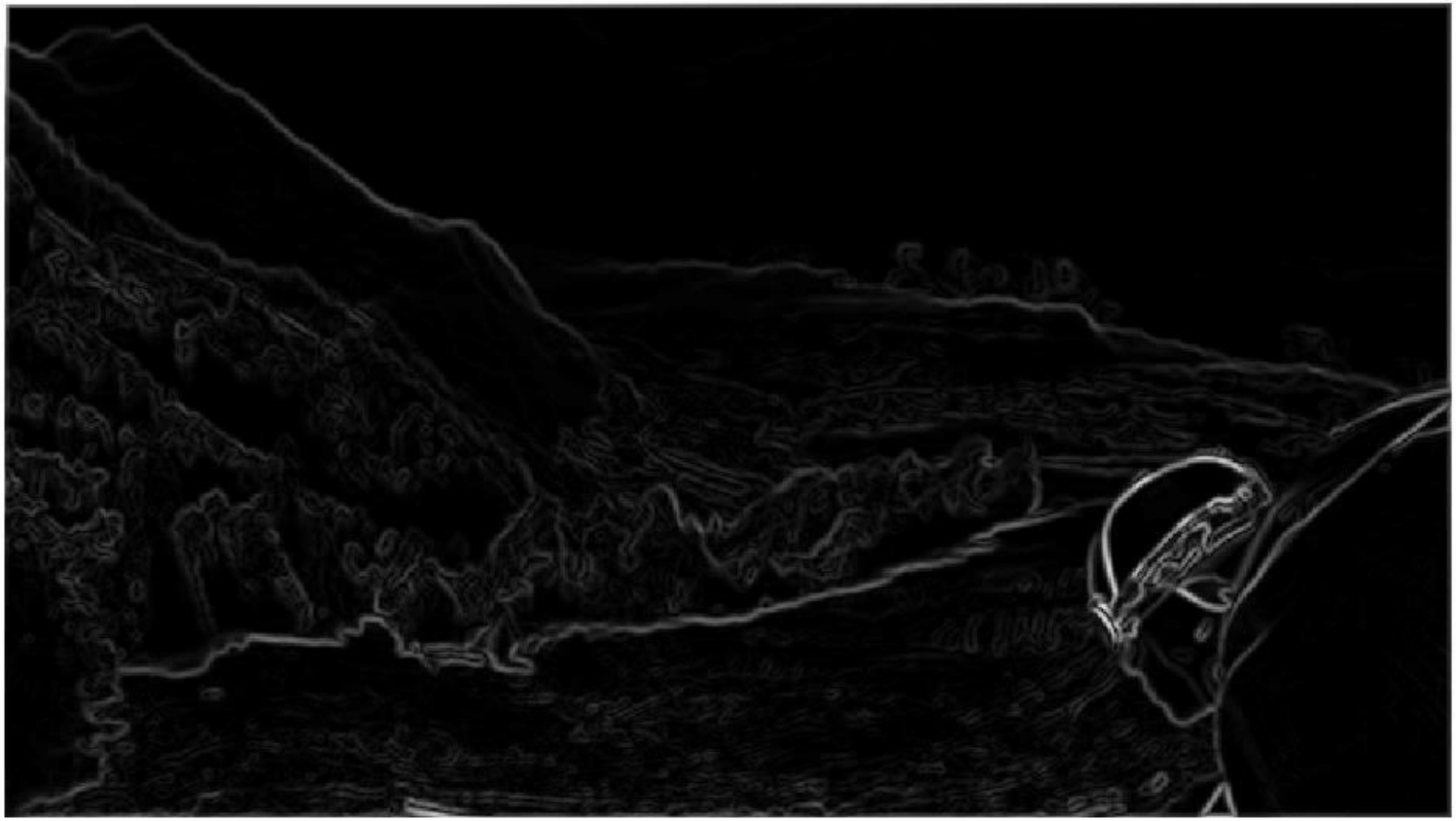
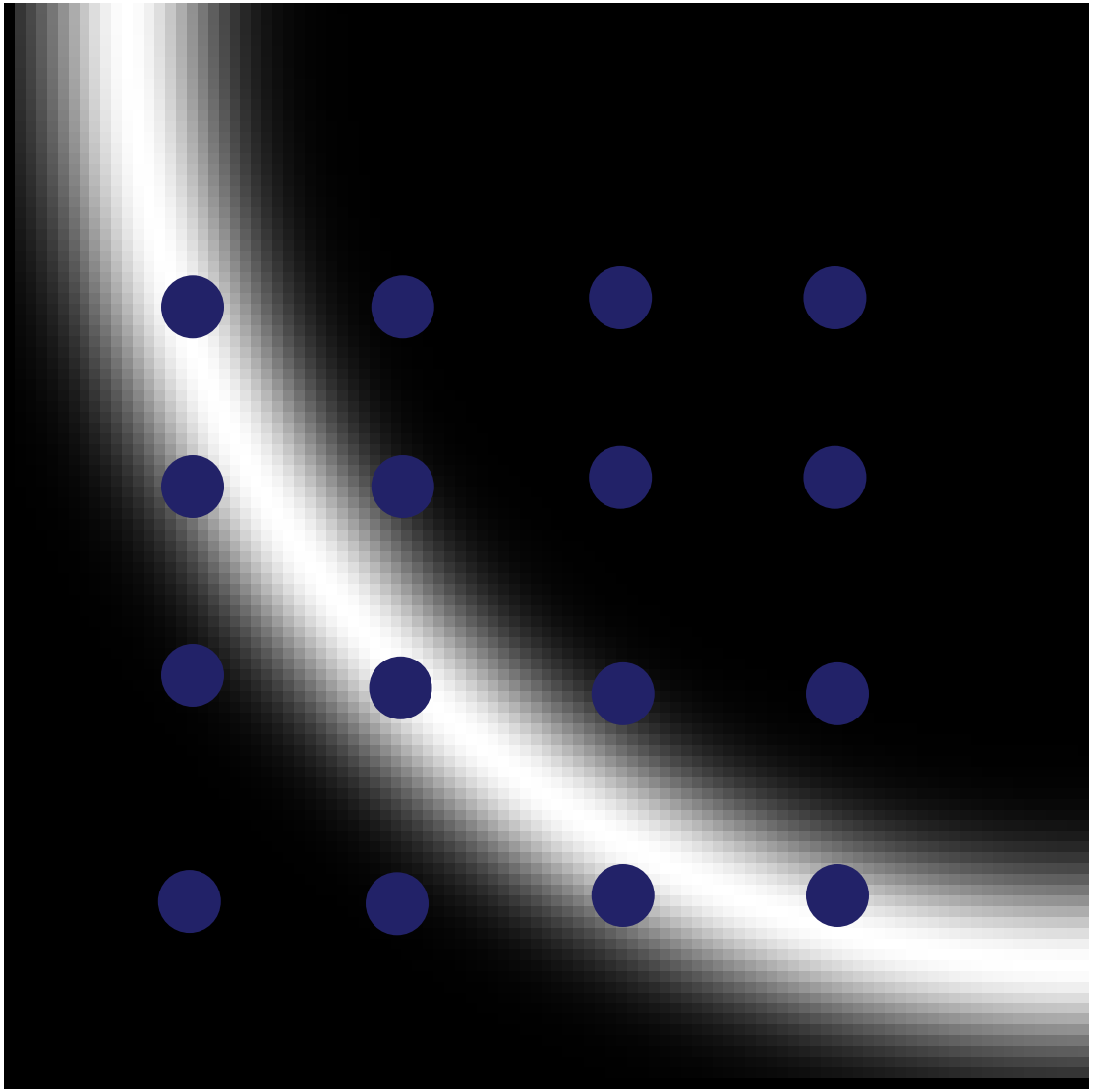






Different scale of image encodes different edge response.

Discretized pixel locations



(Forsyth & Ponce)

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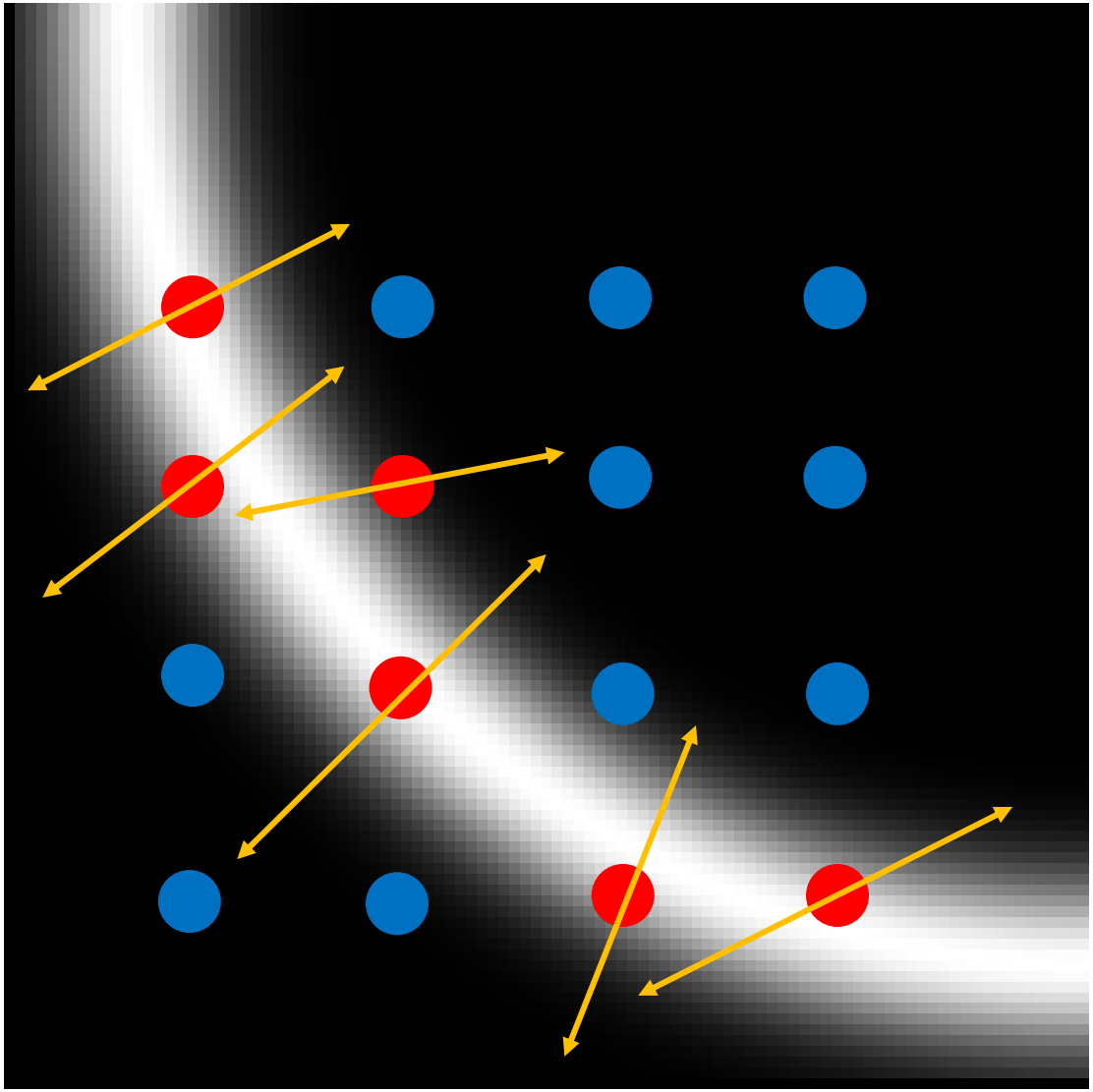
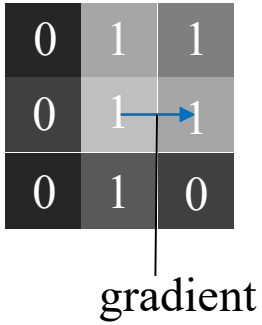
Thesholding





(Forsyth & Ponce)

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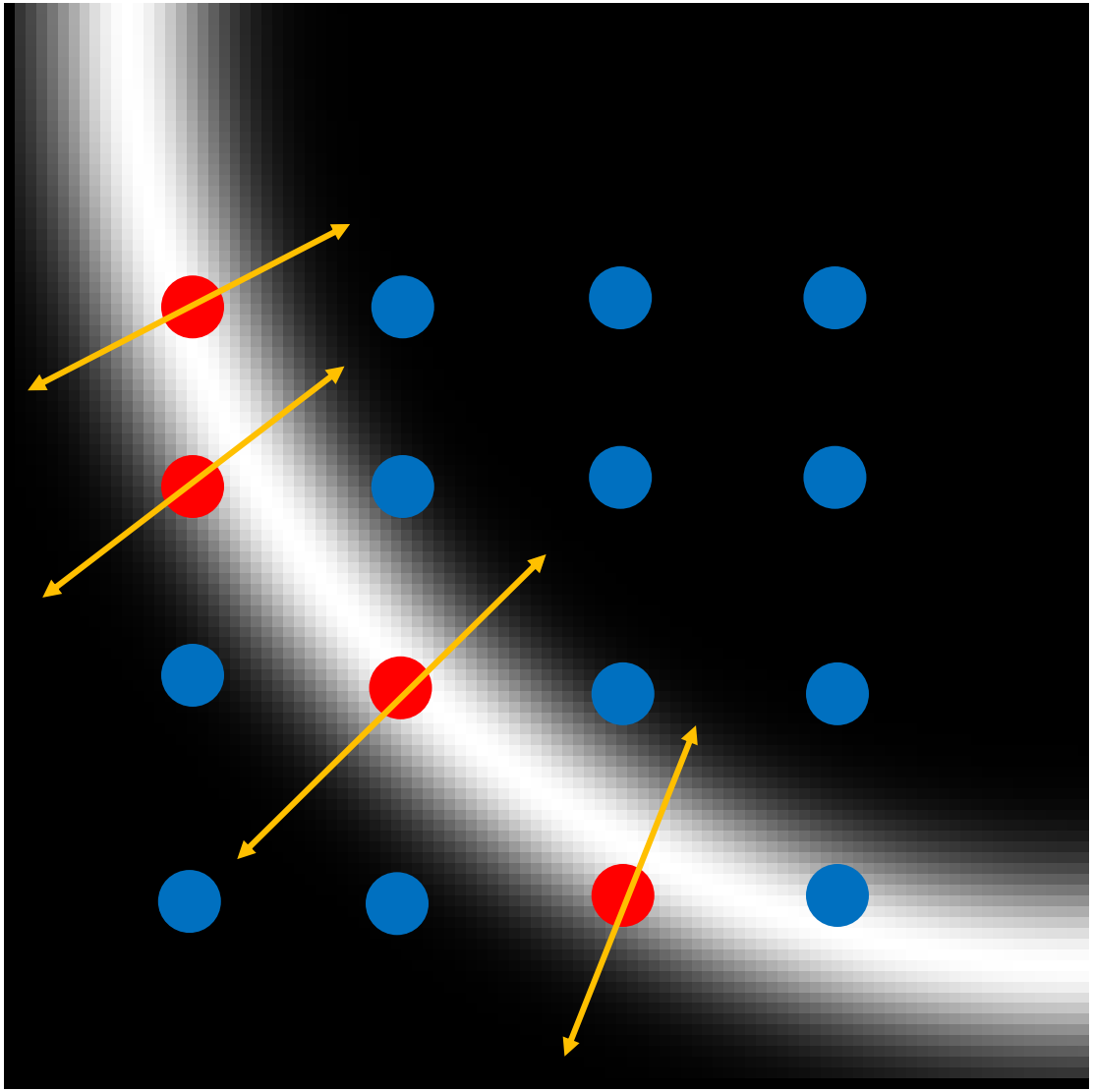
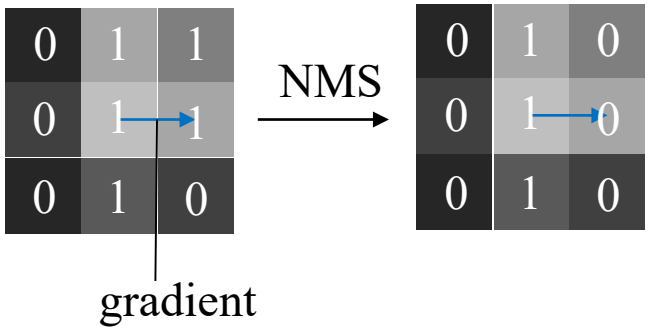
Non-maximum suppression along the line of the gradient





(Forsyth & Ponce)

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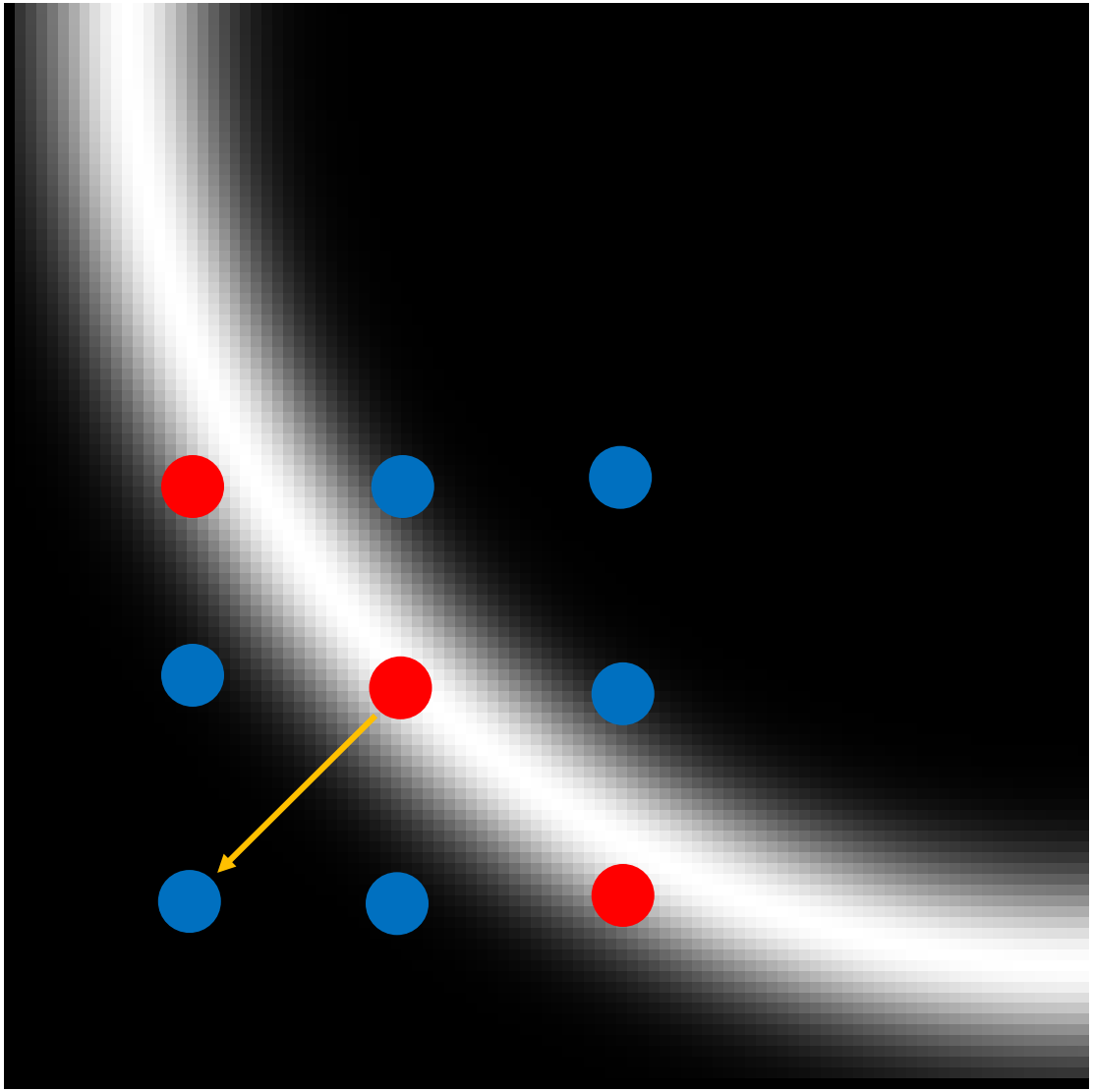
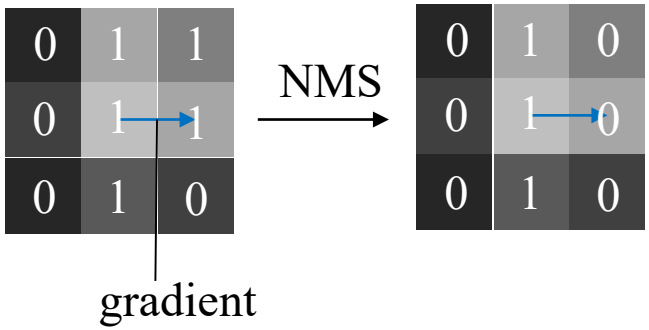
Gradient direction





(Forsyth & Ponce)

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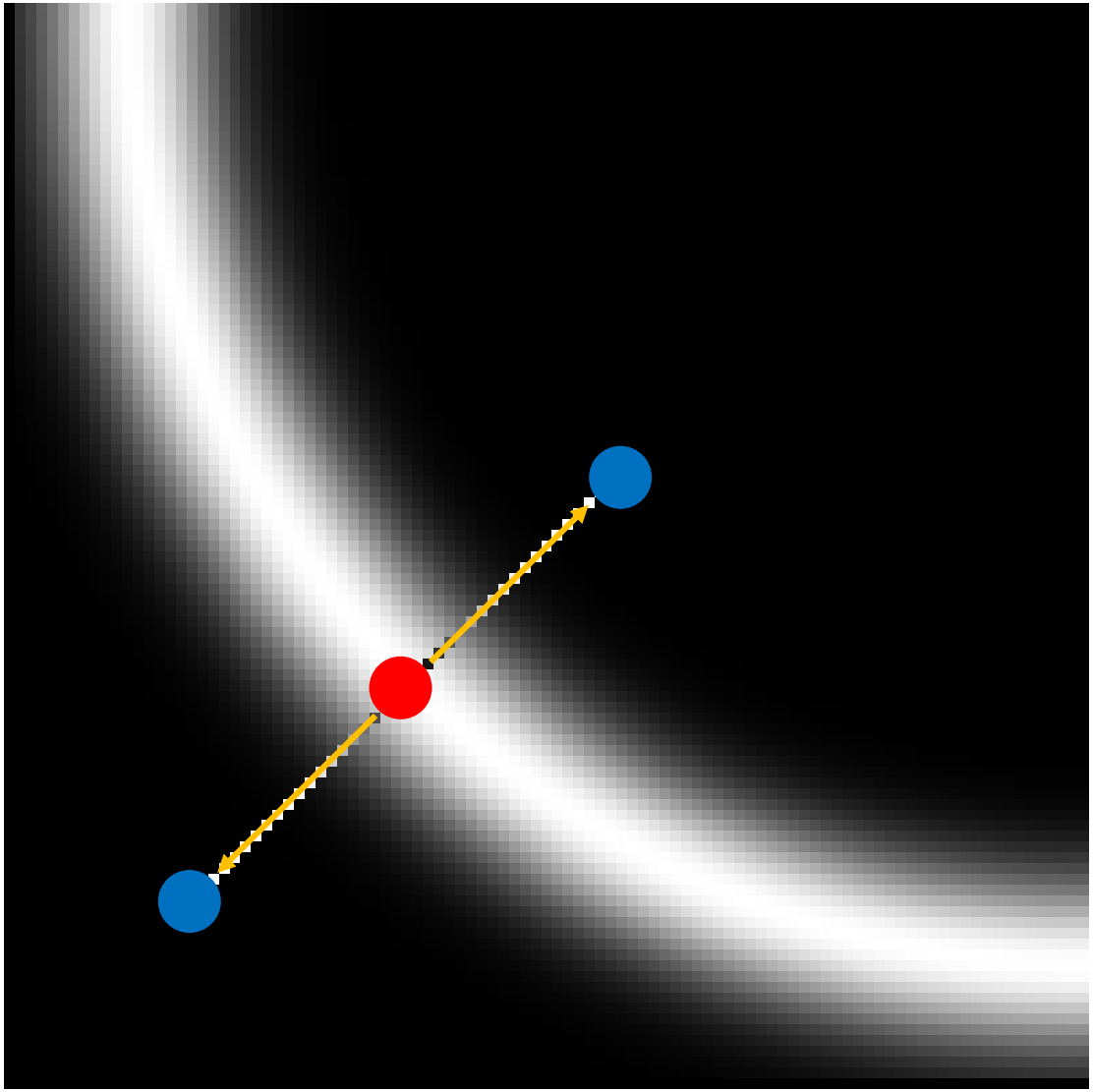
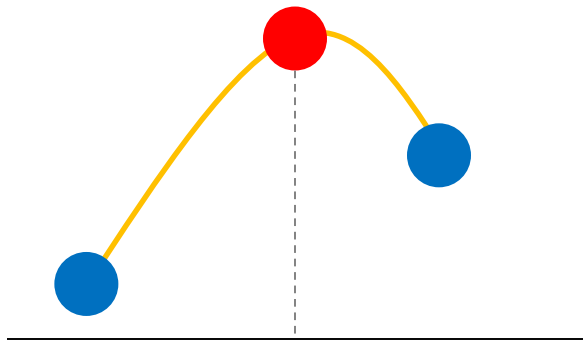
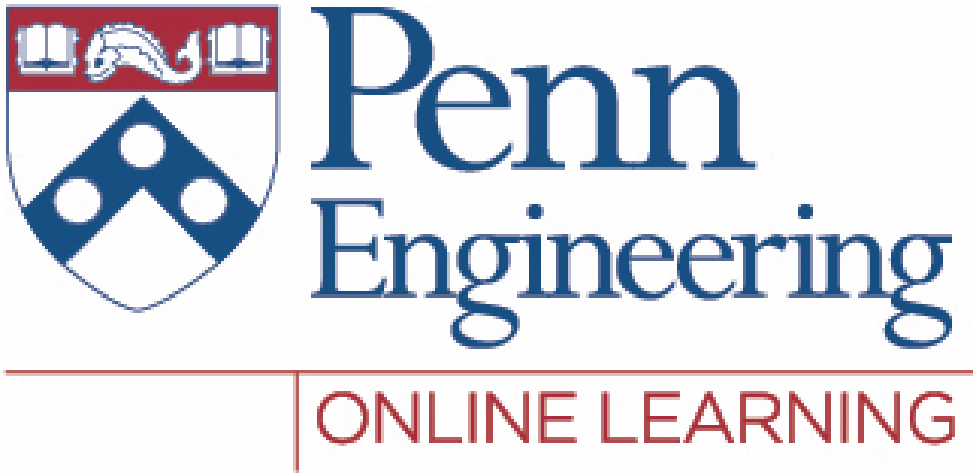




Local maximum

(Forsyth & Ponce)

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi



Video 2.16
Jianbo Shi

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Edge Detection

Code

```
Jb = rgb2gray(J);

imagesc(Jb);axis image; colormap(gray);

bw = edge(Jb,'canny');
```

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

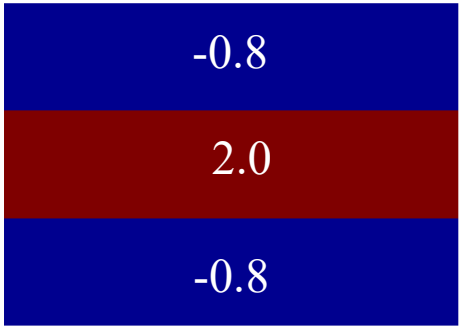
Numerical Image Filtering

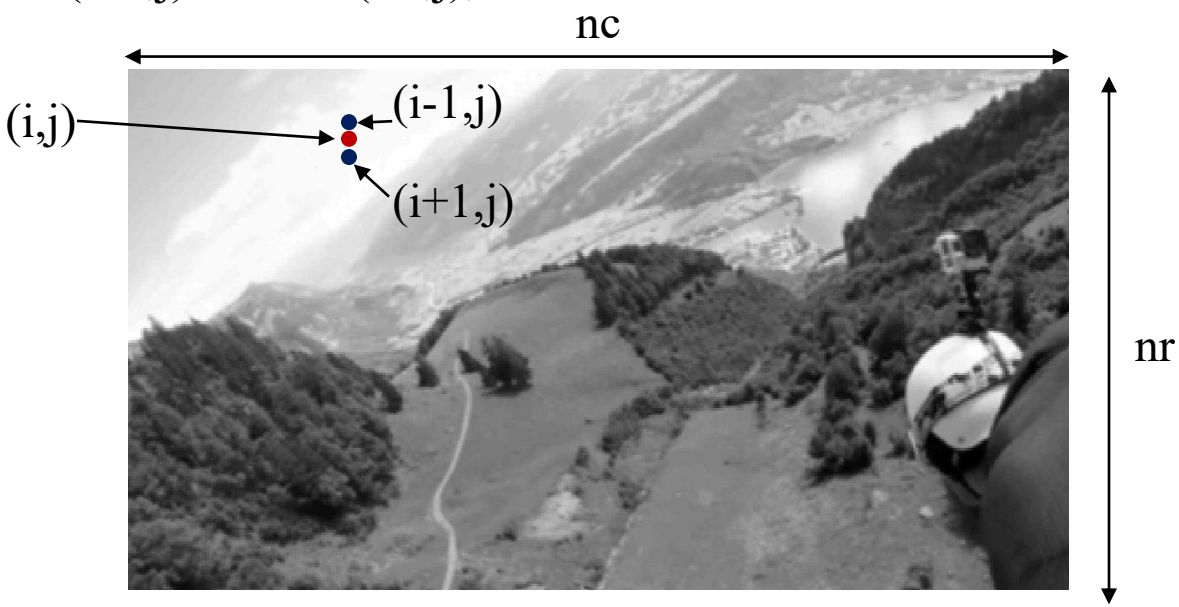
Looping through all pixels

```
[nr,nc] = size(Jb);
J_out = zeros(nr,nc);
for i=1:nr,
   for j=1:nc;
      if (i<nr) && (i>1),
         J_out(i,j) = 2*Jb(i,j) - 0.8*Jb(i+1,j) - 0.8*Jb(i-1,j);
      else
         J_out(i,j) = Jb(i,j);
      end
   end
end
figure; imagesc(J_out);
colormap(gray)
```

Computation time: 0.050154 sec

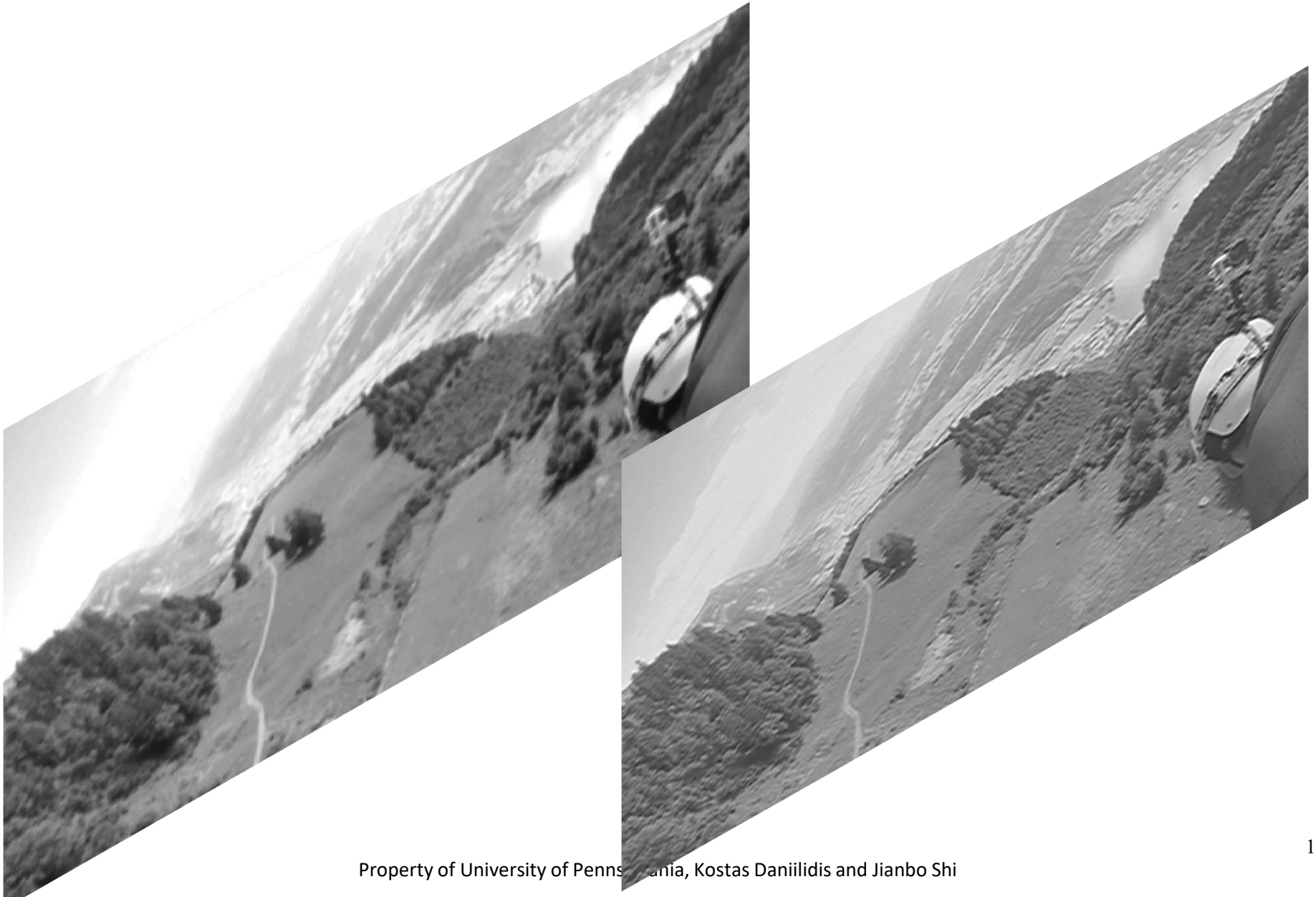
Filter





Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

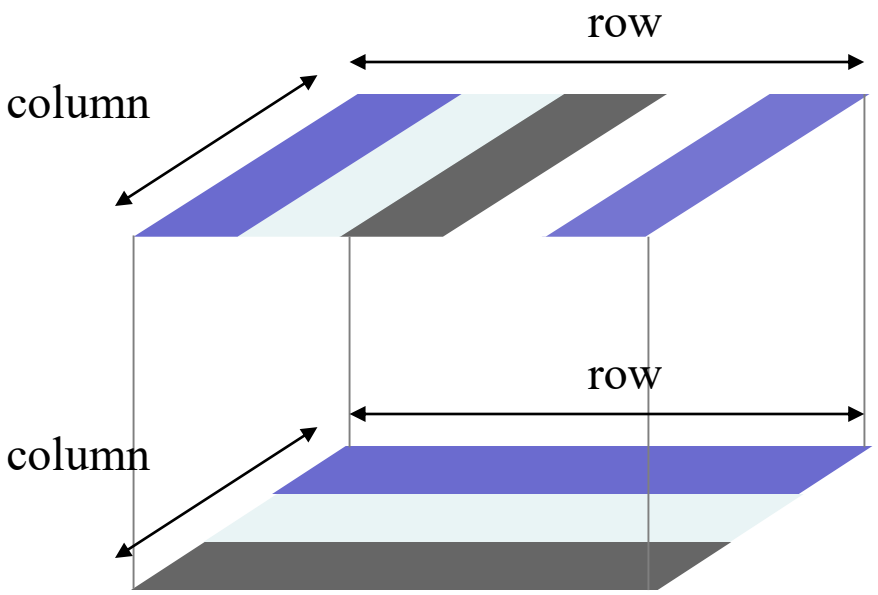
Numerical Image Filtering



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

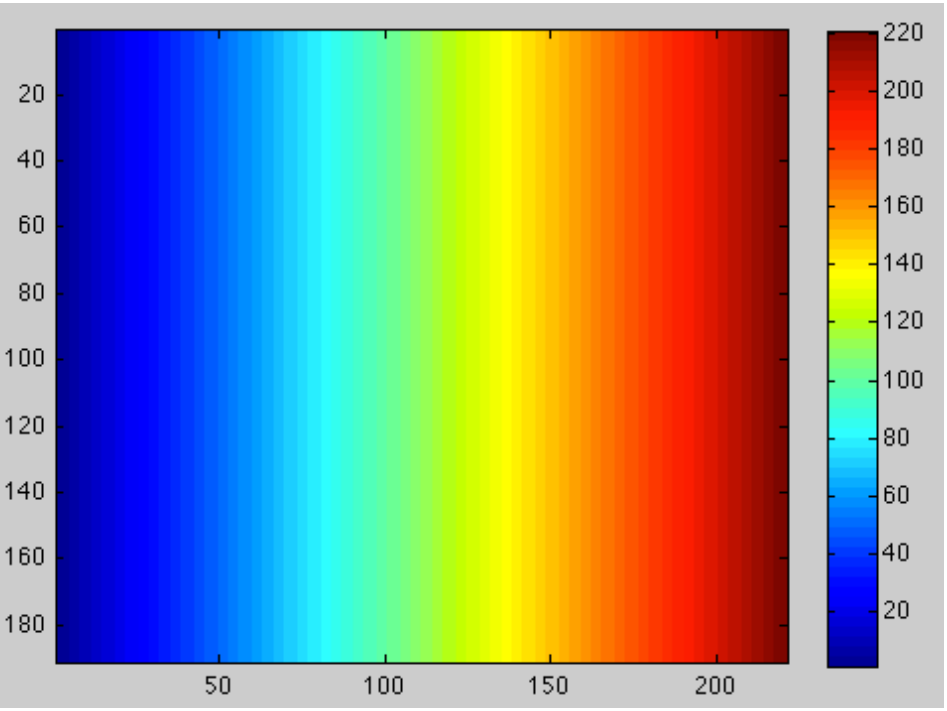
Convolution without Looping using meshgrid

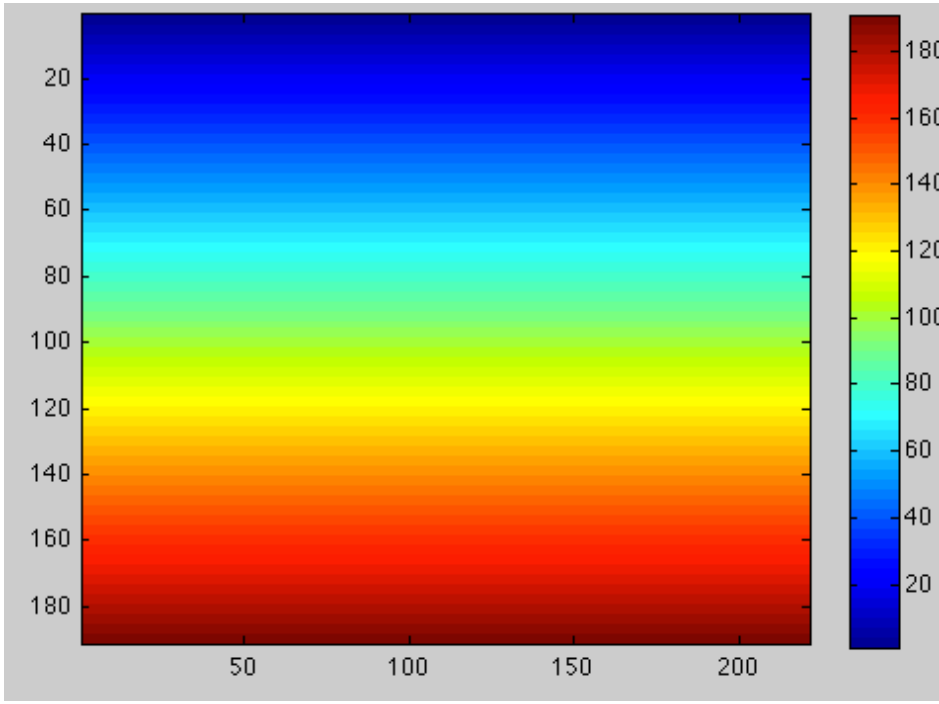
```
>> [x,y] = meshgrid(1:5,1:3)

x =

     1     2     3     4     5
     1     2     3     4     5
     1     2     3     4     5

y =

     1     1     1     1     1
     2     2     2     2     2
     3     3     3     3     3
```



Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

```
[x,y] = meshgrid(1:nc,1:nr);
figure(1); imagesc(x); axis image; colorbar; colormap(jet);
figure(2); imagesc(y); axis image; colorbar; colormap(jet);
```



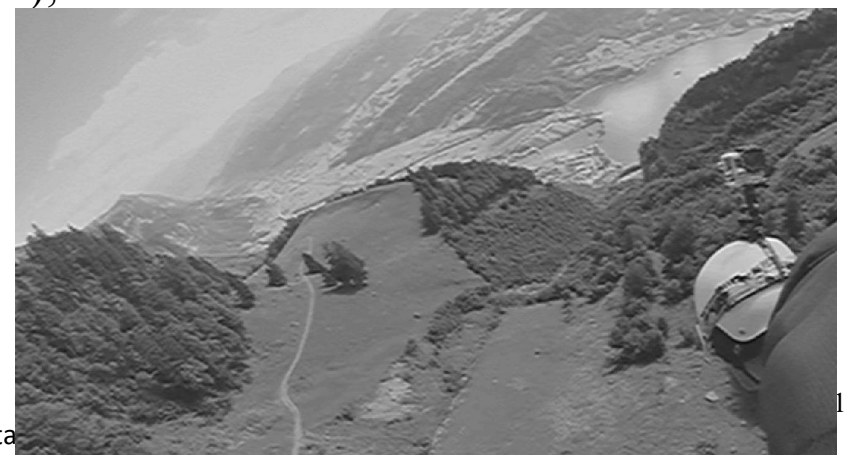


Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

Convolution without Looping using meshgrid

```
[x,y] = meshgrid(1:nc,1:nr);

y_up = y-1;
y_down = y+1;

y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));

J_out = 2*Jb(:) - 0.8*Jb(ind_up) - 0.8*Jb(ind_down);
J_out = reshape(J_out, nr, nc);

figure; imagesc(J_out);colormap(gray)
```

Computation time: 0.024047 sec



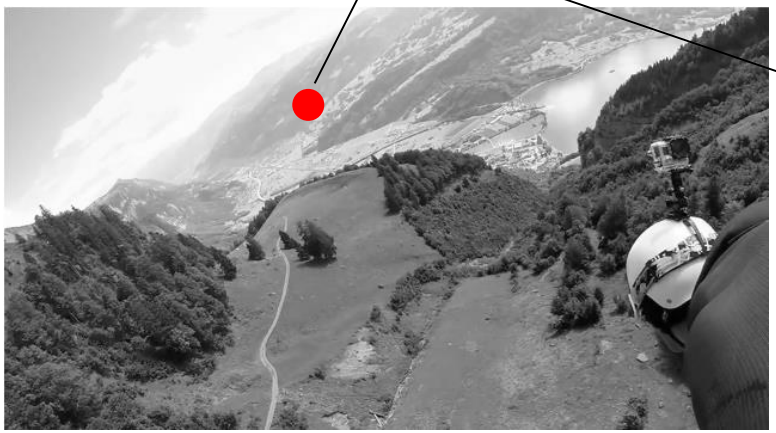
Property of University of Pennsylvania, Kosta

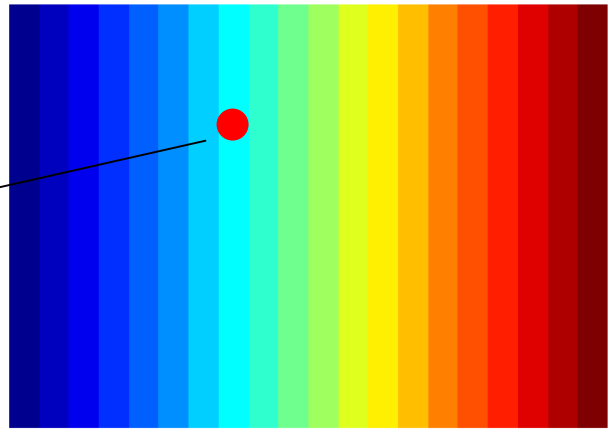
Convolution without Looping using meshgrid

[x,y] = meshgrid(1:nc,1:nr);

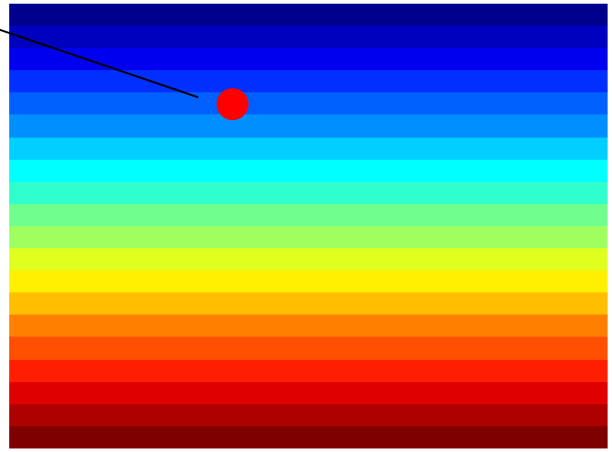
x and y are subscript indice.

Jb_{xy}





x

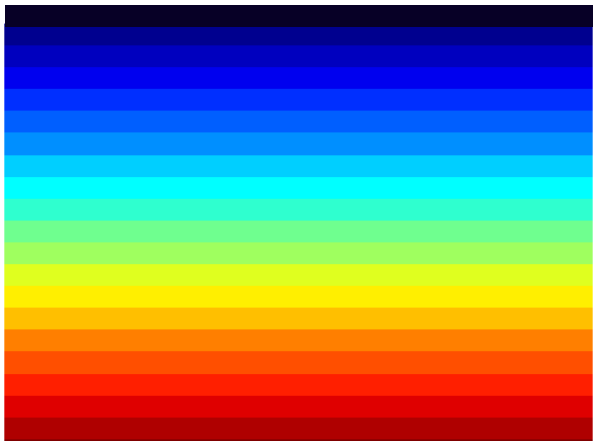


y

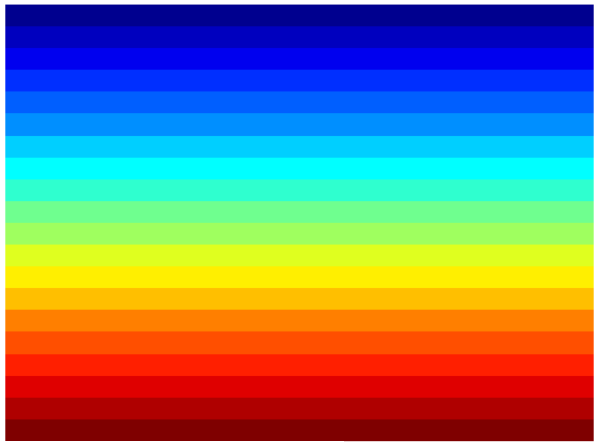
Convolution without Looping using meshgrid

```
[x,y] = meshgrid(1:nc,1:nr);

y_up = y-1;
y_down = y+1;
```

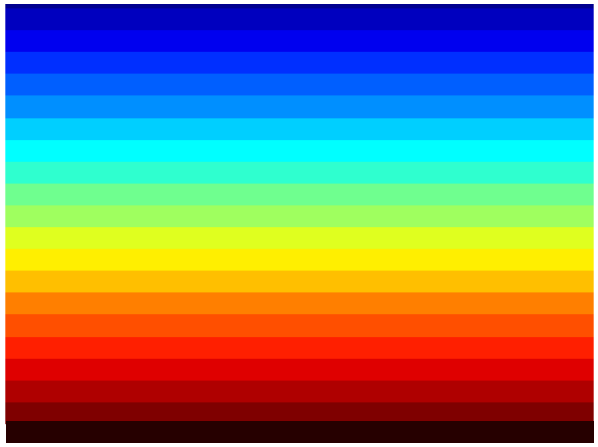
y_up



y



y_down



y_up =

0	0	0	0	0
1	1	1	1	1
2	2	2	2	2

y =

1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

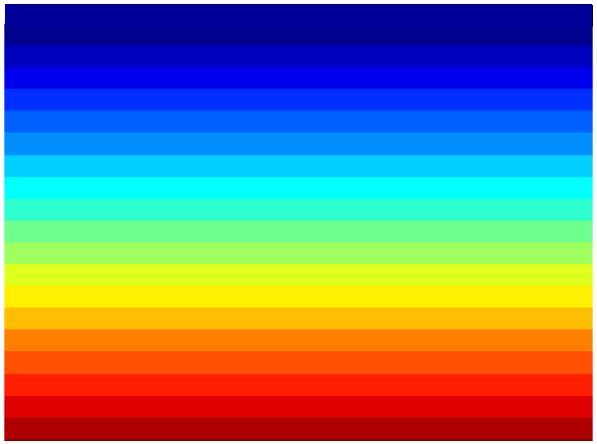
y_down =

2	2	2	2	2
3	3	3	3	3
4	4	4	4	4

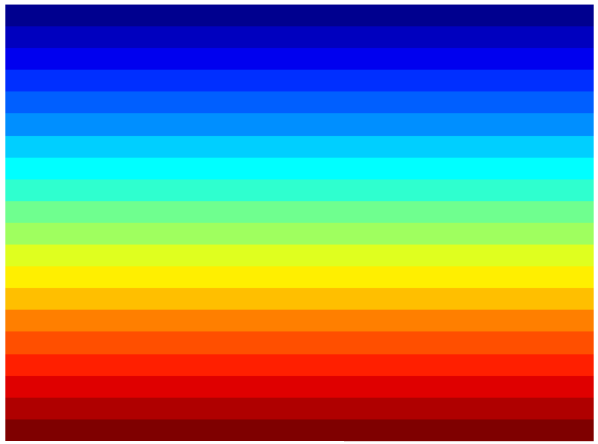
Convolution without Looping using meshgrid

```
y_up = y-1;
y_down = y+1;

y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));
```

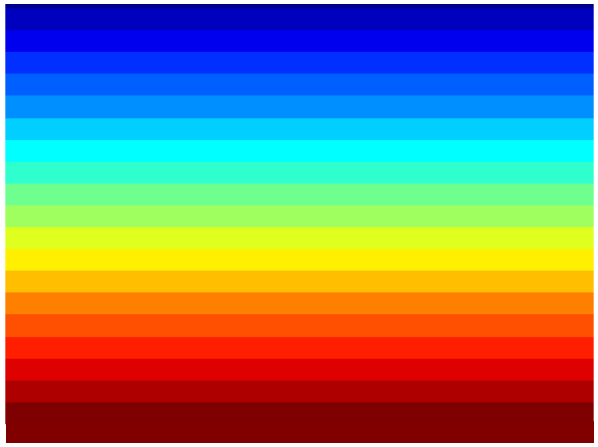
y_up



y



y_down



y_up =

1	1	1	1	1
1	1	1	1	1
2	2	2	2	2

y =

1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

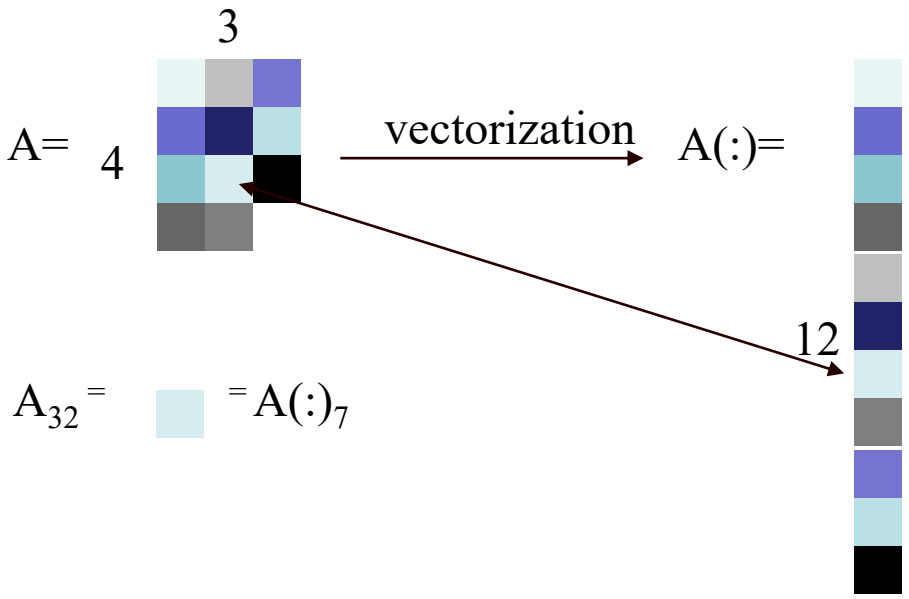
y_down =

2	2	2	2	2
3	3	3	3	3
3	3	3	3	3

Convolution without Looping using meshgrid

```
y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));
```

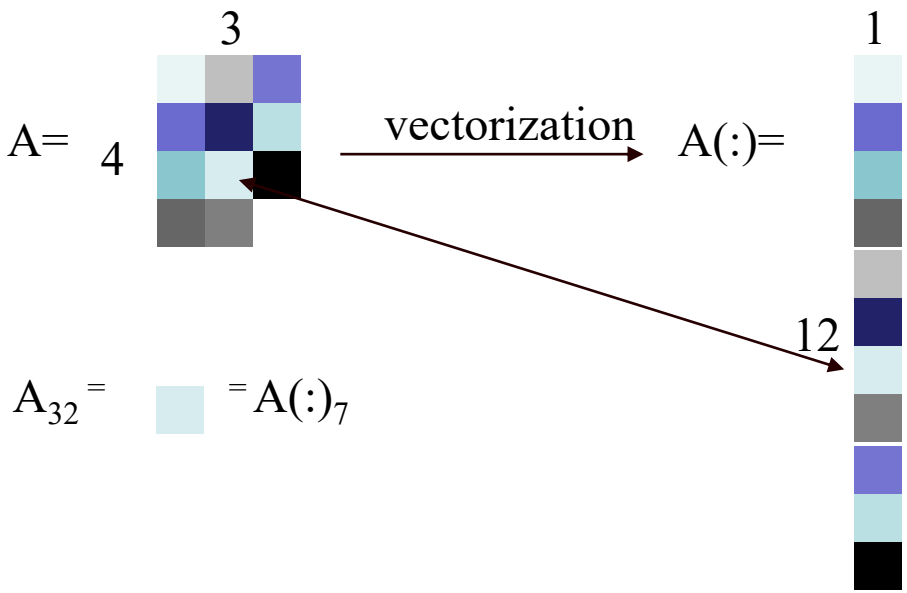
linear_index = sub2ind([n_row, n_col], row_subscript, col_subscript)



7 = sub2ind([4 3], 3,2)

Convolution without Looping using meshgrid

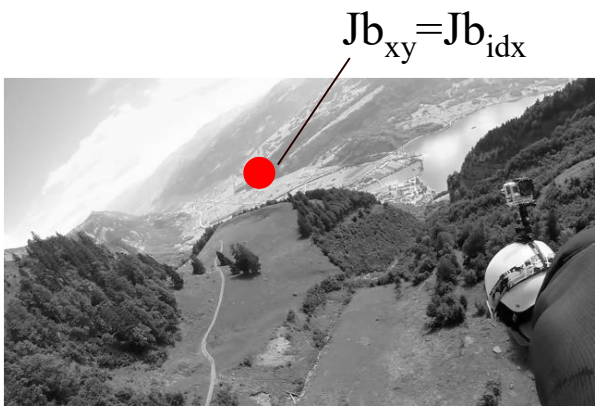
```
y_up = min(nr,max(1,y_up));  % keep y_up index within legal range of [1,nr]
y_down = min(nr,max(1,y_down));

ind_up = sub2ind([nr,nc],y_up(:),x(:));  % create linear index
ind_down = sub2ind([nr,nc],y_down(:),x(:));

linear_index = sub2ind([n_row, n_col], row_subscript, col_subscript)
```



7 = sub2ind([4 3], 3,2)

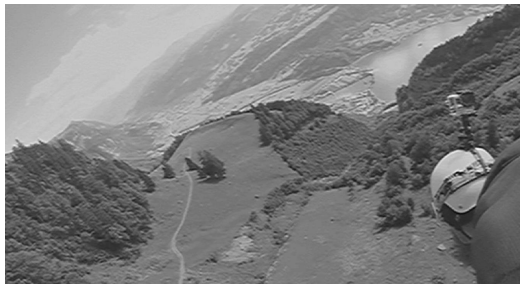
ind_up = sub2ind([nr,nc],y_up(:),x(:));

Operation on vectors



Convolution without Looping using meshgrid

```
J_out = 2*Jb(:) - 0.8*Jb(ind_up) - 0.8*Jb(ind_down);
J_out = reshape(J_out, nr, nc);

figure; imagesc(J_out);colormap(gray)
```



=

J_out

2

-0.8

-0.8

Jb

Jb(ind_up)

Jb(ind_down)

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi

With loop

Computation time: 0.050154 sec

Without loop

Computation time: 0.024047 sec

Property of University of Pennsylvania, Kostas Daniilidis and Jianbo Shi