edX Robo4 Mini MS – Locomotion Engineering

Block 1 – Week 3 – Unit 1

Raibert Vertical Hopper

Video 6.1

Segment 6.0.1
Motivation & Agenda

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with
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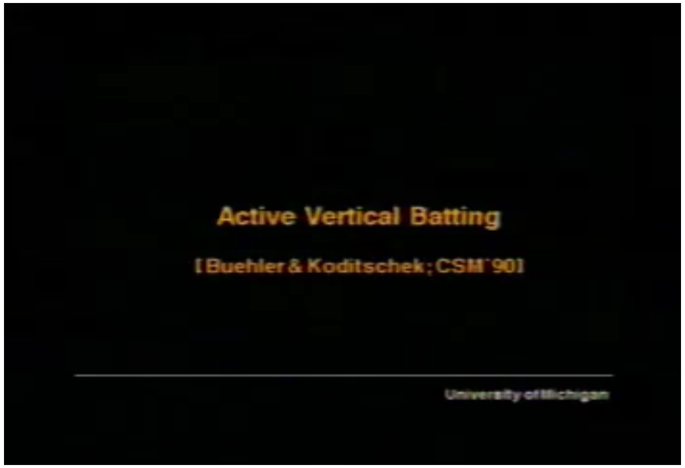
July, 2017

Agenda

- Historical
  - core component of Raibert's Hoppers
  - inspired much further work
- Conceptual
  - 1 DoF vertical "heartbeat"
  - organizing rhythm around which other DoF are coordinated
- Technical
  - first look at hybrid dynamics
  - central feature of locomotion

https://www.youtube.com/watch?v=XFXj81mvInc

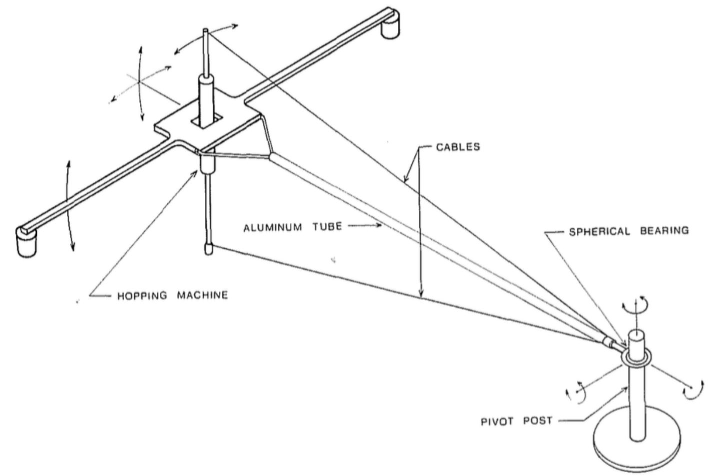
M. H. Raibert. Legged Robots That Balance. Cambridge: MIT Press, 1986

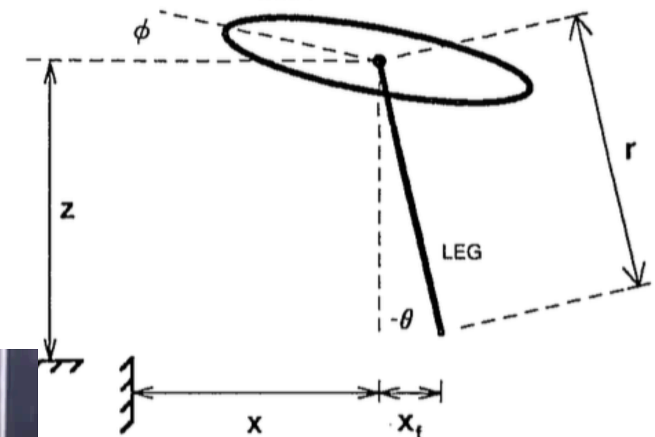


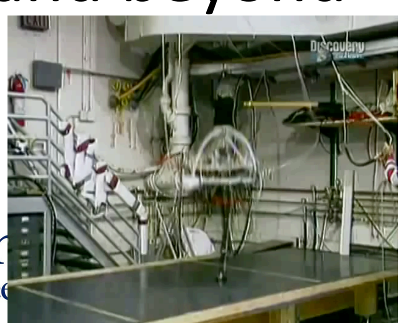
M. Buehler, D. E. Koditschek, and P. J. Kindlmann, "A family of robot control strategies for intermittent dynamical environments," *IEEE Control Systems Magazine*, vol. 10, pp. 16–22, 1990.

Sagittal Plane Machine

- Raibert reduced complexity
  - initially, of control task
  - by mechanical restriction
  - to 2 dim. workspace
  - yields 3 DoF configuration space
- Control Ideas Generalize
  - to spatial machine in 3 dim
  - with 6 DoF
  - and beyond





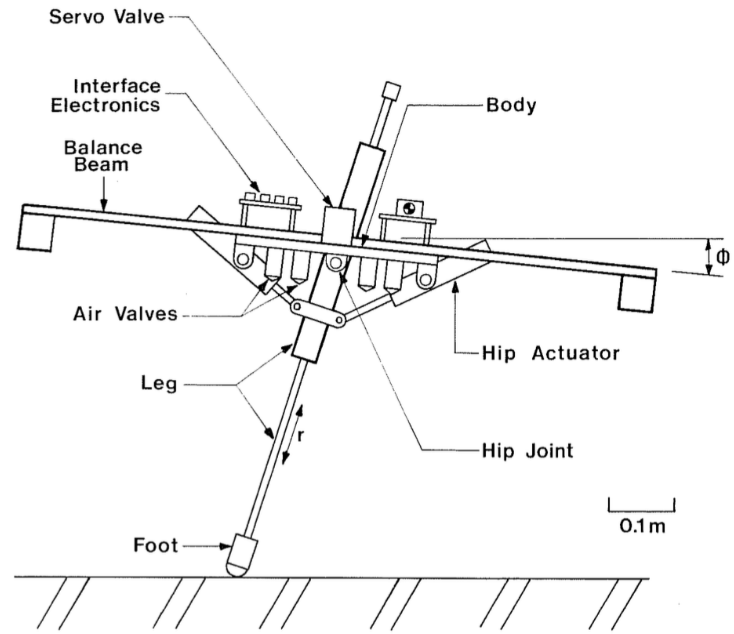


figures from:
M. H. Raibert. Legged Robots That Balance. Cambridge: MIT Press, 1986

video from:
Plastic Pals. "Robots from MIT's Leg Lab". YouTube video, 02:28. Posted [October 31, 2011].
https://www.youtube.com/watch?v=XFXj81mvInc

Robust Modular Robot Controller

- Problem setting
  - 3 DoF (RPR) kinematics
  - 6 dim dynamics
    - highly nonlinear (and “hybrid”)
    - tightly coupled
    - underactuated



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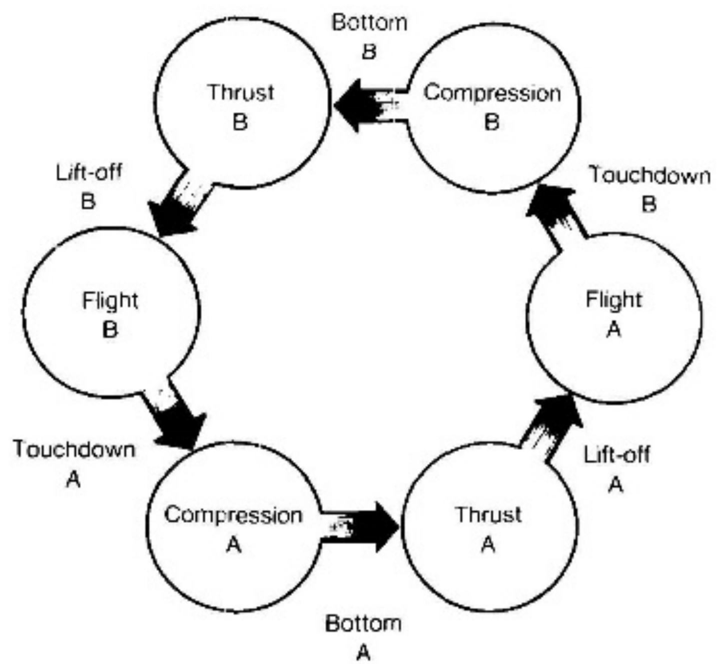
- Raibert’s control policies
  - use (mostly)“proportional-derivative” feedback loops
  - assume each DoF is decoupled from others
  - “time-share” the actuation resources
    - air valve adjustment of leg spring in stance for height control
    - hip actuator control of pitch in stance
    - hip actuator selection of leg angle in flight for fore-aft velocity

Control of Hybrid Dynamics

- legged locomotion results from
  - ground reaction forces (GRF)
  - imparted to the body mass center
  - through the limbs
- limbs make & break ground contact
  - introducing impulses
  - changing the GRF dynamics
  - yield hybrid dynamics
    - continuous-in-time force-driven
    - discrete-in-time event-driven
- control of hybrid dynamics
  - by actuation through limbs
  - must be coordinated in continuous time
  - and sensitive to the limb-contact events



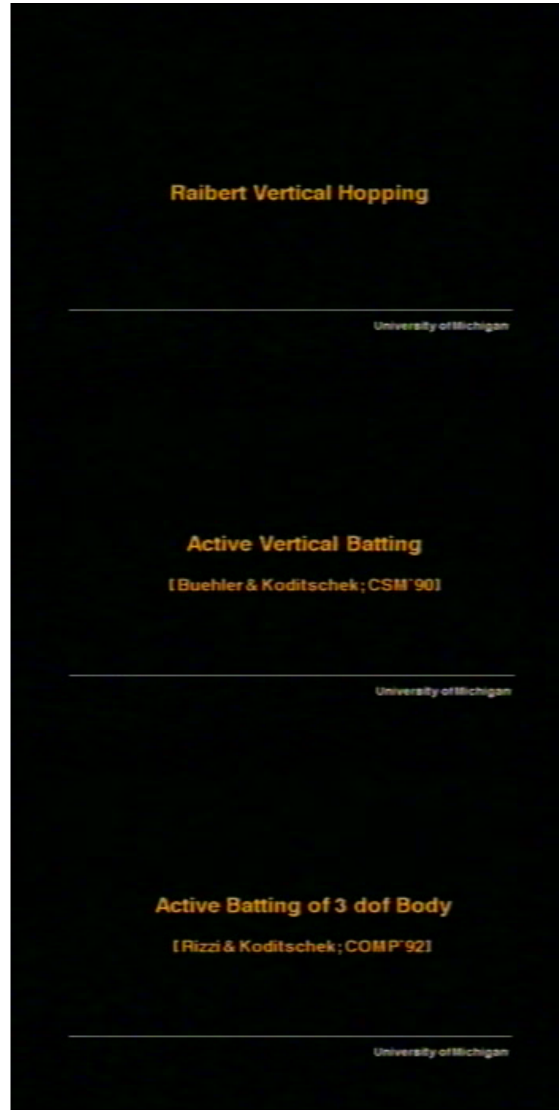
M. H. Raibert, "Legged Robots," *Commun. ACM*, vol. 29, no. 6, pp. 499–514, Jun. 1986.

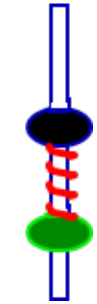
Other Machines "Anchor" Vertical Hopper

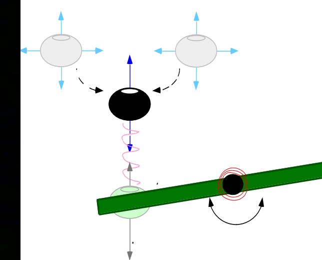
- Vertical hopper
  - Can be used as a "template"
  - to be "anchored"
  - as the "heartbeat"
- In more complex systems
  - 2 DoF
  - 3 DoF
  - … and beyond

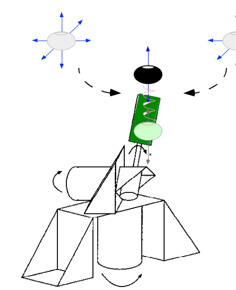
Raibert, MIT Press, 1986, *op. cit.*

Buehler, et al. *IEEE Control Systems Magazine*, 1990, *op. cit.*





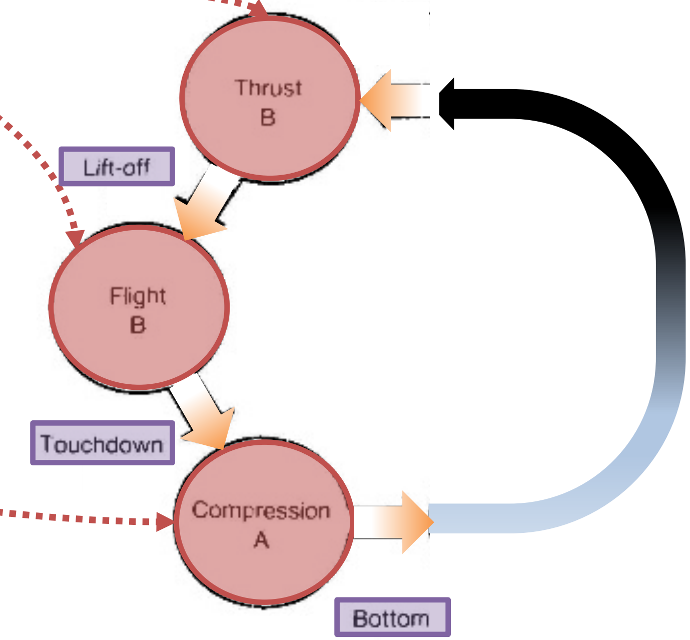




A. Rizzi, L. Whitcomb, and D. Koditschek, "Real-Time Control of a Spatial Robot Juggler," *IEEE Computer*, vol. 25, no. 5, pp. 12–24, 1992.

Initial Approach to Vertical Hopper

- Model Continuous time flows
  - each mode of contact
  - governed by different VF
- Model natural guard conditions
  - physical event interrupts mode
  - locomotion: typically LO/TD
- Study/Express mode map
- Model reset map
- Compose
  - mode map ∘ reset map
  - further compose each composition in turn
- End up with return map



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Week 6 – Unit 1

Raibert Vertical Hopper

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Continuous Time Models

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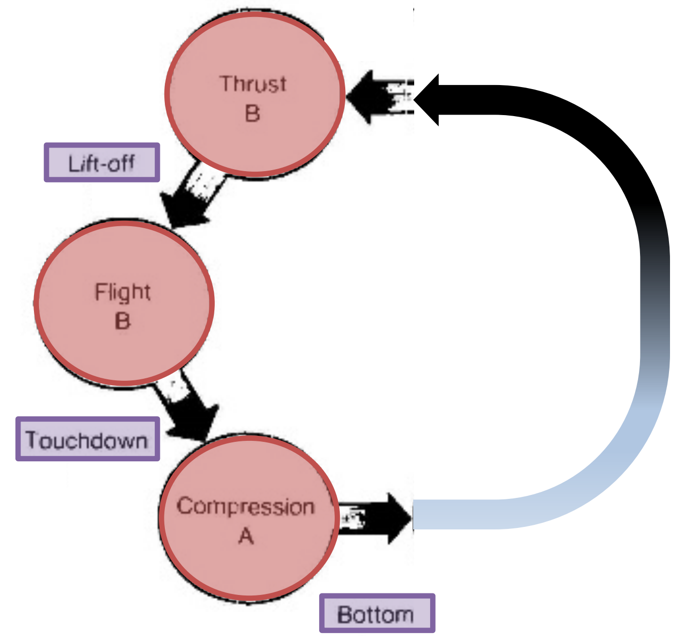

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Gravity Loaded Hooke's Law Spring

- Rework Seg.1.2.1.1-eqn (4)

$$
\begin{aligned}
m\ddot{\chi} &= \Phi_{\mathrm{D}}(\dot{\chi}) + \Phi_{\mathrm{HS}}(\chi - \chi_r) + \Phi_{\mathrm{G}}(\chi) \\
&= -b\dot{\chi} - k\,(\chi - \chi_r) - mg \qquad (1) \\
&=: \Phi_{\mathrm{GLH}}(\mathbf{x}); \quad \mathbf{x} := \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix}
\end{aligned}
$$

- Note differences from Φ_{DHO}
  - using variable χ to denote vertical extension
    - rather than b_y as in pendulum or $x := \chi - \chi_r$ as in DHO
    - to focus on role of relaxed spring position $\chi_r > 0$
  - addition of gravitational force, $\Phi_{\mathrm{G}} := -mg$

Additional Modeling Assumptions

- Simplify physics
  - unit mass, m:=1; gravity-matched spring, $m\ \chi_r = g$
  - touchdown/liftoff
    - spring is disengaged (braked) below rest length: $\chi_{td} = \chi_{lo} = 0 < \chi_r$
    - to assure some spring potential in each stance
    - and (with gravity-matched spring) zero potential force at $\chi = 0$
- Yields familiar 1st order VF (f_{DHO} from Seg.1.2.1.2)

$$f_{\mathrm{GLH}}(\mathbf{x}) := A_{GLH}\mathbf{x}$$

$$A_{GLH} := \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2(1+\beta^2) & -2\omega\beta \end{bmatrix} \tag{2}$$

  - with new parameters, ω,β >0 replacing physical ones in f_{DHO}
  - introduced to assure under-damped system
- But now working with hybrid dynamics
  - this VF only models "passive" stance mode $\chi < 0$
  - actuation, introduced next, yields new "active" mode
  - introduce formal guard and reset maps in next segment

Active Stance Mode: Energy Injection

- Raibert's original hopper had an "air-spring"
  - pneumatic pump adjusted pressure in air-chamber
  - controller added fixed energy (via pressure)
  - at "bottom": creates new "active" mode with own VF
- Present model uses "series-elastic" actuator (SEA)
  - force control via spring rest-length adjustment
  - popularized by Pratt; still actively developed, e.g.
    - https://www.youtube.com/watch?v=wJwKwLUUTjc

    C. S. Knabe, V. Orekhov, M. A. Hopkins, B. Y. Lattimer, and D. W. Hong, "Two configurations of series elastic actuators for linearly actuated humanoid robots with large range of motion," in *Humanoid Robots (Humanoids), 2014 14th IEEE-RAS International Conference on*, 2014, pp. 1096–1096
  - present use: introduce constant upward thrust force
    - initiated at "bottom" as before (controlled to perform positive work)
    - restores energy lost to damping each successive stance

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Active Stance Mode Control

- SEA force control: an "inverse statics" control policy
  - assume actuator can move spring rest length, χ_r
    - exact precision in position and timing
    - known inverse statics for gravity-loaded Hooke's spring

$$\Phi_{\mathrm{GHS}}(\chi) := \Phi_{\mathrm{HS}}(\chi) + \Phi_{\mathrm{G}}(\chi) = -k(\chi - \chi_r) - mg$$

$$\Rightarrow \Phi_{\mathrm{GHS}}^{-1}(\Phi) = -\left(\Phi + mg - \chi\right)/k$$

  - given a desired reference force profile, $\Phi_{\mathrm{ref}}(t)$
    - actuator rest length command

$$\chi_r^*(t) := \Phi_{\mathrm{GHS}}^{-1} \circ \Phi_{\mathrm{ref}}(t) = -(\Phi_{\mathrm{ref}}(t) + mg - \chi)/k$$

    - injects exactly the right force at the right time
- Get new dynamics

$$\begin{aligned} m\ddot{\chi} &= \Phi_{\mathrm{D}}(\dot{\chi}) + \Phi_{\mathrm{GHS}}^{-1} \circ \Phi_{\mathrm{ref}}(t) \\ &= -b\dot{\chi} + \Phi_{\mathrm{ref}}(t) \end{aligned}$$

Constant Thrust Control Policy

- Stance control strategy
  - hold rest length fixed at nominal $\chi_r^*(t) \equiv \chi_r$
    - until compression ends at "bottom"
    - defined by vanishing of guard function $\gamma_{\mathrm{comp}}(\mathbf{x}) := \dot{\chi}$
  - at which point inject constant upward thrust force

$$\Phi_{\mathrm{ref}}(t) \equiv \tau$$

  - for brief fixed time interval after bottom

$$t \in [t_b, t_b + \Delta_\tau]$$

- Yields damped constant thrust VF

$$f_{\mathrm{DCT}}(\mathbf{x}) = A_{DCT}\mathbf{x} + a_{DCT}$$

$$A_{DCT} := \begin{bmatrix} 0 & 1 \\ 0 & -b/m \end{bmatrix} \qquad (3)$$

$$a_{DCT} := \begin{bmatrix} 0 \\ \tau/m \end{bmatrix}$$

Ballistic Flight Mode

- Decompression continues after end of thrust
  - rest length again fixed at nominal $\chi_r^*(t) \equiv \chi_r$
    - until decompression ends at "liftoff"
    - defined by vanishing of guard function $\gamma_{\text{decomp}}(\mathbf{x}) := \chi$
  - at which point ground reaction forces cease
    - only force is due to gravity
    - yields flight mode VF

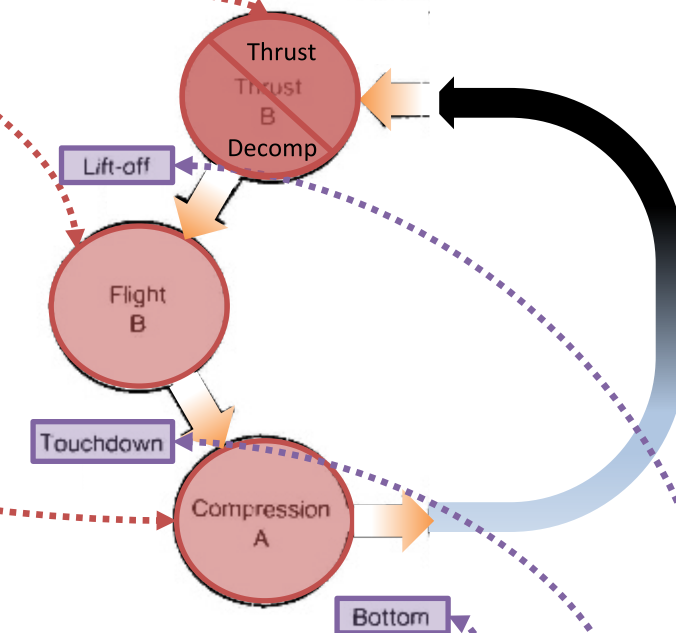
$$f_{\text{BF}}(\mathbf{x}) = A_{BF}\mathbf{x} + a_{BF}$$

$$A_{BF} := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad (4)$$

$$a_{BF} := \begin{bmatrix} 0 \\ -\mathrm{g}/m \end{bmatrix}$$

Moving Ahead

- Model Continuous time flows
  - each mode of contact
  - governed by different VF
- Model natural guard conditions
  - physical event interrupts mode
  - locomotion: typically LO/TD
  - had to introduce extra mode
    - decompression
    - model as timed event
    - to simplify analysis
- Next up:
  - study/express mode maps
  - model the reset maps



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Continuous Time Mode Maps

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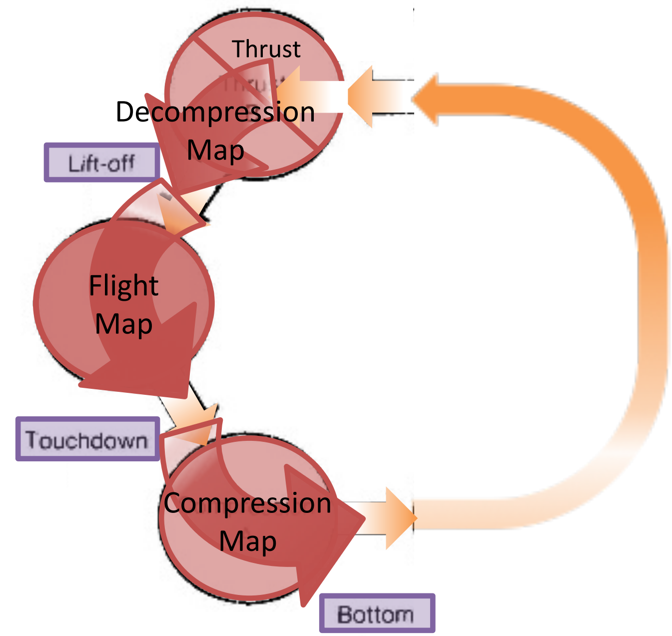

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Ballistic Flight Mode Flow Map Definition

- Exercises for this segment show

$$f_{\mathrm{BF}}(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ -\mathrm{g} \end{bmatrix}$$
$$\Rightarrow f_{\mathrm{BF}}^{t}(\mathbf{x}_0) = \begin{bmatrix} \chi_0 + \dot{\chi}_0 t - \frac{1}{2}\mathrm{g}t^2 \\ \dot{\chi}_0 - \mathrm{g}t \end{bmatrix} \qquad (5)$$

- Flight ends at Touchdown
  - which, recall, occurs when $\chi = \chi_{td} := 0$
  - represented by the vanishing of guard $\gamma_{\mathrm{BF}}(\mathbf{x}) := \chi$
- Now solve the vanishing condition for t

$$0 = \gamma_{\mathrm{BF}} \circ f_{\mathrm{BF}}^{t}(\mathbf{x}_0) = \chi_0 + \dot{\chi}_0 t - \frac{1}{2}\mathrm{g}t^2$$

- to get implicit function,

$$T_{td}(\mathbf{x}_0) := \frac{1}{\mathrm{g}}\left(\dot{\chi}_0 + \sqrt{\dot{\chi}_0^2 - 2\mathrm{g}\chi_0}\right)$$

- yielding mode flow map

$$\mathbf{x}_{td} = \boldsymbol{\ell}_{\mathrm{BF}}^{\mathrm{td}}(\mathbf{x}_0) := f_{\mathrm{BF}}^{T_{td}(\mathbf{x}_0)}(\mathbf{x}_0) = \begin{bmatrix} 0 \\ -\sqrt{\dot{\chi}_0^2 + 2\mathrm{g}\chi_0} \end{bmatrix} \qquad (6)$$

Gravity Loaded Hooke's Law Spring Flow

- Recall from Seg. 1.2.1.2 & exercises RC coordinates

$$\begin{bmatrix}\psi\\ \dot\psi\end{bmatrix} = \mathbf{y} = h_{\mathrm{RC}}(\mathbf{x}) = S\mathbf{x}; \quad S := \begin{bmatrix}\omega\sqrt{1+\beta^2}\chi + \beta\dot\chi/\sqrt{1+\beta^2}\\ \dot\chi/\sqrt{1+\beta^2}\end{bmatrix}$$

- Reveals "oscillatory" (underdamped) GLH flow

$$\exp\left(tSA_{GLH}S^{-1}\right) = \mathrm{e}^{-\beta t}R(\omega t); \quad R(\omega t) := \begin{bmatrix}\cos\omega t & -\sin\omega t\\ \sin\omega t & \cos\omega t\end{bmatrix}$$

- Simplify still further via nonlinear ("polar") $\mathrm{CC_D}$

$$\begin{bmatrix}\rho\\ \phi\end{bmatrix} := \mathbf{p} = h_{\mathrm{P}}(\mathbf{y})\mathbf{p} = h_{\mathrm{P}}(\mathbf{y}) := \begin{bmatrix}\psi^2 + \dot\psi^2\\ \arctan\dot\psi/\psi\end{bmatrix} \qquad (6)$$

(don't confuse with Seg. 1.2.2.1 vector $\boldsymbol{q} = \begin{bmatrix}\theta\\ \dot\theta\end{bmatrix}$!)

- Exercises: decoupled conjugate VF with simple flow

$$\dot{\mathbf{p}} = \begin{bmatrix}\dot\rho\\ \dot\phi\end{bmatrix} = \begin{bmatrix}-2\omega\beta\rho\\ -\omega\end{bmatrix} =: f_{\mathrm{PRC}}(\mathbf{p}) \Rightarrow f^t_{\mathrm{PRC}}(\mathbf{p}_0) = \begin{bmatrix}\mathrm{e}^{-2\omega\beta t}\rho_0\\ \phi_0 - \omega t\end{bmatrix}$$

Compression Flow Map Definition

- Compression starts at touchdown, $\mathbf{x}_{td} = \begin{bmatrix} \chi_{td} \\ \dot{\chi}_{td} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\chi}_{td} \end{bmatrix}$
- Compression ends at "bottom"
  - which, recall, occurs when $\dot{\chi} = 0$
  - represented by the vanishing of guard $\gamma_{\text{comp}}(\mathbf{x}) := \dot{\chi}$

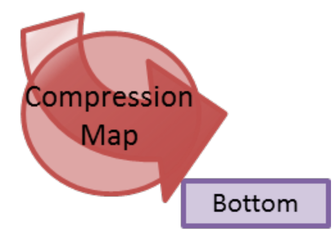
preferably expressed in polar RC coordinates

$$\tilde{\gamma}_{\text{comp}}(\mathbf{p}) := \gamma_{\text{comp}} \circ h_{\text{PRC}}^{-1}(\mathbf{p}) = \gamma_{\text{comp}} \circ h_{\text{RC}}^{-1} \circ h_{\text{P}}^{-1}(\mathbf{p})$$

$$= \sqrt{1+\beta^2}\dot{\psi}(\mathbf{p}) = \sqrt{1+\beta^2}\rho \sin\phi$$

- Solve vanishing condition for t in polar RC coords

$$n\pi = \phi_{td} - \omega t \Leftrightarrow t = T_{bot}(\mathbf{p}_{td}) := \frac{\phi_{td} - n\pi}{\omega}$$



- Yields mode flow map

$$\mathbf{p}_{bot} = \boldsymbol{\ell}_{\text{PRC}}^{\text{bot}}(\mathbf{p}_{td}) := f_{\text{PRC}}^{T_{bot}(\mathbf{p}_{td})}(\mathbf{x}_{td}) = \begin{bmatrix} \mathrm{e}^{-2\beta(\phi_{td} - n\pi)}\rho_{td} \\ n\pi \end{bmatrix} \qquad (7)$$

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Damped Constant Thrust Mode Flow

- Raibert's strategy of constant thrust from bottom
- Results in familiar 2nd order ODE $m\ddot{\chi} + \beta\dot{\chi} - \tau = 0$
- Yields 1st order VF $f_{\mathrm{DCT}}(\mathbf{x}) = A_{DCT}\mathbf{x} + a_{DCT}$

$$A_{DCT} := \begin{bmatrix} 0 & 1 \\ 0 & -\beta/m \end{bmatrix} \qquad (8)$$

$$a_{DCT} := \begin{bmatrix} 0 \\ \tau/m \end{bmatrix}$$

- With familiar flow derived from f_{D} in Seg.1.2.1.1
- Resulting in $\mathbf{x}_{et} = \mathbf{x}_{bot} + \mathbf{x}_t$

$$\mathbf{x}_t := \frac{\tau}{2\omega^2\beta^2}\begin{bmatrix} 2\omega\beta\Delta_\tau - \left(1 - \mathrm{e}^{-2\beta\Delta_\tau}\right) \\ \omega\beta\,(1 - \mathrm{e} - 2\beta\Delta_\tau) \end{bmatrix} \qquad (9)$$

D. E. Koditschek and M. Bühler. Analysis of a simplified hopping robot. The International Journal of Robotics Research, 10(6):587–605, Dec 1991

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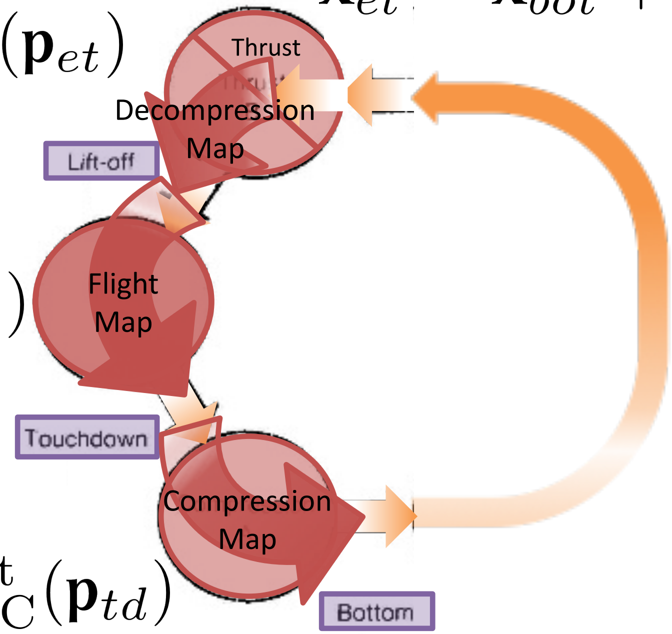
$$\mathbf{x}_{et} = \mathbf{x}_{bot} + \mathbf{x}_t$$

$$\mathbf{p}_{lo} = \boldsymbol{\ell}^{\mathrm{lo}}_{\mathrm{PRC}}(\mathbf{p}_{et})$$

$$\mathbf{x}_{td} = \boldsymbol{\ell}^{\mathrm{td}}_{\mathrm{BF}}(\mathbf{x}_0)$$

$$\mathbf{p}_{bot} = \boldsymbol{\ell}^{\mathrm{bot}}_{\mathrm{PRC}}(\mathbf{p}_{td})$$



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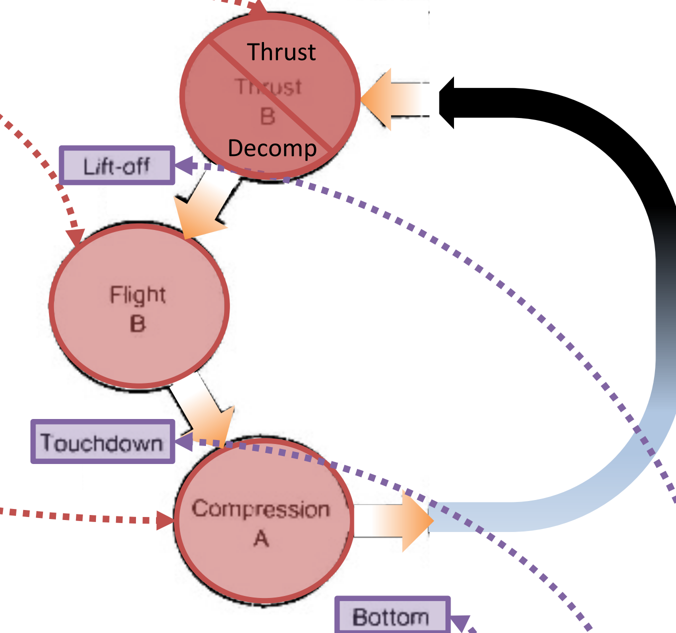

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