

Assignment 2

Wednesday, November 6, 2019 10:47 PM



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Lecture: Robot Mapping
Winter term 2013

Sheet 2

Topic: Bayes Filter

Submission deadline: Nov. 4, 2013

Submit to: robotmappingtutors@informatik.uni-freiburg.de

Exercise: Bayes Filter

A robot is equipped with a manipulator to paint an object. Furthermore, the robot has a sensor to detect whether the object is colored or blank. Neither the manipulation unit nor the sensor are perfect.

From previous experience you know that the robot succeeds in painting a blank object with a probability of

$$p(x_{t+1} = \text{colored} \mid x_t = \text{blank}, u_{t+1} = \text{paint}) = 0.9,$$

where x_{t+1} is the state of the object after executing a painting action, u_{t+1} is the control command, and x_t is the state of the object before performing the action.

The probability that the sensor indicates that the object is colored although it is blank is given by $p(z = \text{colored} \mid x = \text{blank}) = 0.2$, and the probability that the sensor correctly detects a colored object is given by $p(z = \text{colored} \mid x = \text{colored}) = 0.7$.

Unfortunately, you have no knowledge about the current state of the object. However, after the robot performed a painting action the sensor of the robot indicates that the object is colored.

Compute the probability that the object is still blank after the robot has performed an action to paint it. Use an appropriate prior distribution and justify your choice.

This part!

Handwritten solution for the Bayes Filter exercise. The page shows a coordinate system with points (x_1, y_1, θ_1) and (x_2, y_2, θ_2) . It includes matrices for rotation and translation, and a detailed calculation of the probability $p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored})$. The final answer is 0.027.

Handwritten notes on the right side of the page:

- $p(x_{t+1} = \text{colored} \mid x_t = \text{blank}, u_{t+1} = \text{paint}) = 0.9$
- $p(z = \text{colored} \mid x = \text{blank}) = 0.2$
- $p(z = \text{colored} \mid x = \text{colored}) = 0.7$
- $x_t = ?$
- $z_{t+1} = \text{colored}$
- What is the $p(x_{t+1} = \text{blank})$?
- $p(x_{t+1} = \text{colored} \mid z_{t+1} = \text{colored})$
- $p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored})$
- $p(x_{t+1} = \text{colored}) = p(z_{t+1} = \text{colored} \mid x_{t+1} = \text{colored}) \cdot p(x_{t+1} = \text{colored})$
- $p(x_{t+1} = \text{colored}) = \frac{0.7 \cdot p(x_{t+1} = \text{colored})}{0.7 \cdot p(x_{t+1} = \text{colored}) + 0.2 \cdot p(x_{t+1} = \text{blank})}$
- $p(x_{t+1} = \text{blank}) = \frac{0.2 \cdot p(x_{t+1} = \text{blank})}{0.7 \cdot p(x_{t+1} = \text{colored}) + 0.2 \cdot p(x_{t+1} = \text{blank})}$
- Now to compute $p(x_{t+1} = \text{blank})$
- given $u_{t+1} = \text{paint}$
- $\rightarrow p(x_{t+1} = \text{blank} \mid x_t = \text{blank})$
- $= \frac{p(x_{t+1} = \text{blank} \mid x_t = \text{blank}, u_{t+1} = \text{paint})}{p(x_t = \text{blank}) + p(x_{t+1} = \text{blank} \mid x_t = \text{colored}, u_{t+1} = \text{paint})}$
- $= \frac{0.9 \cdot p(x_t = \text{blank})}{0.9 \cdot p(x_t = \text{blank}) + 0.1 \cdot p(x_t = \text{colored})}$
- $= 0.9 \cdot p(x_t = \text{blank})$
- $\Rightarrow p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored})$
- $= 0.2 \times \frac{0.9 \cdot p(x_t = \text{blank})}{0.9 \cdot p(x_t = \text{blank}) + 0.1 \cdot p(x_t = \text{colored})} = 0.027$

Final Ans :

$$p(x_{t+1} = \text{blank} \mid z_{t+1} = \text{colored}) = 0.2 \times \frac{0.9 \cdot p(x_t = \text{blank})}{0.9 \cdot p(x_t = \text{blank}) + 0.1 \cdot p(x_t = \text{colored})} = 0.027$$

- $\frac{0.9}{0.9 - 0.057}$