Introduction Robotics

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Outline

- Practical information: 4L160 – Introduction Robotics
- Robotics history
- Rigid motions and homogenous transformations

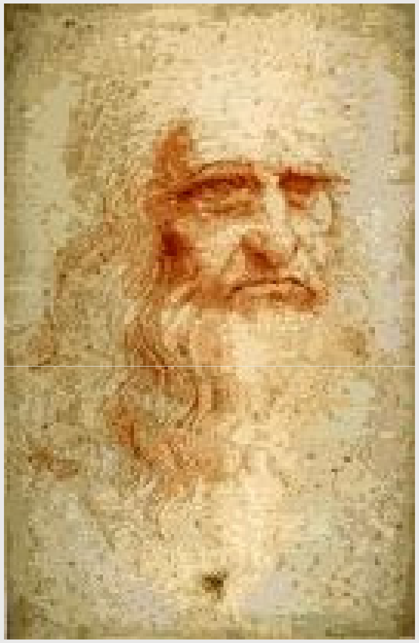
Robotics History

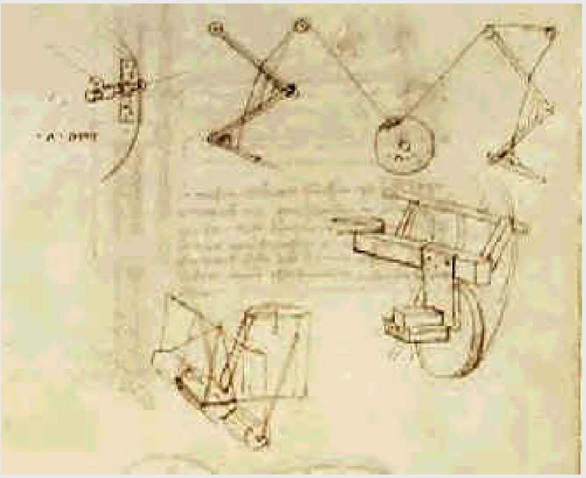
Vision

- Greeks: Aristotle writes

  “If every tool, when ordered, or even of its own accord, could do the work that befits it... then there would be no need either of apprentices for the master workers or of slaves for the lords.”
- Automata: water clocks, mechanical animals, mechanical orchestra etc.

Leonardo’s robot (1495)

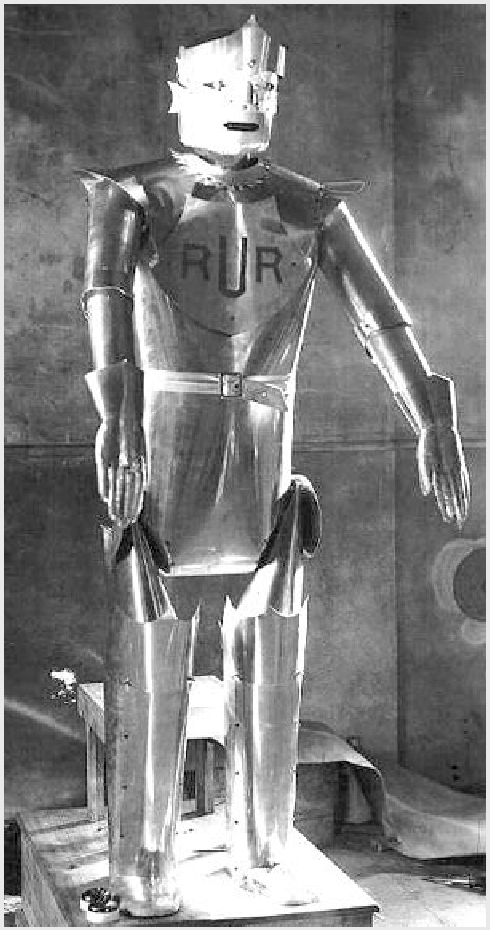




Jacques de Vaucanson (1738)

J. de Vaucanson and his ‘duck’

The notion of “Robot” (1921)



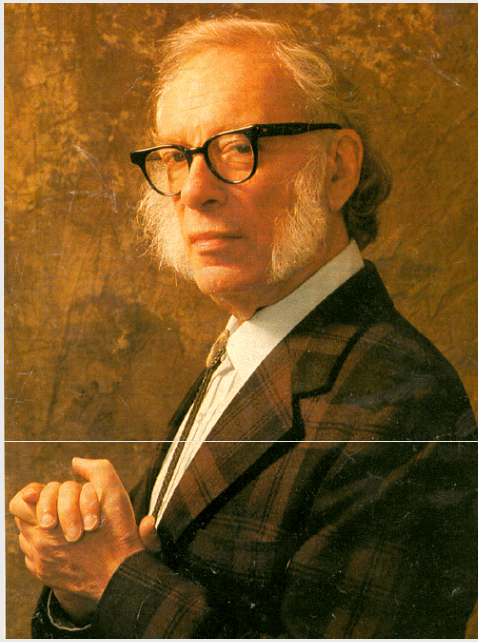
- Karel Capek’s play “Rossuum’s Universal Robots”
- “Robot” is coined from Czech word “robota”
- “Robota” means labor

Trademarks of science fiction

Robot Maria in classic SF movie "Metropolis" of Fritz Lang (1926)

Famous R2-D2 (left) and C3PO (right) in SF saga "Start Wars" of George Lucas (1977)

Three Laws of Robotics



Formulated by Russian/American writer Isaac Asimov:

1. *A robot may not harm a human being, or, through inaction, allow a human being to come to harm.*
2. *A robot must obey the orders given to it by human beings except where such orders would conflict with the First Law.*
3. *A robot must protect its own existence, as long as such protection does not conflict with the First or Second Law.*

A definition of “Robot”

Robot Institute of America:

Robot is a reprogrammable, multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks.

Chat with serious impact

George Devol and Joseph F. Engelberger at a cocktail party in 1956:

GD: 50 percent of the people who work in factories are really putting and taking.

JE: Why are machines made to produce only specific items?

GD: How about approaching manufacturing the other way around, by designing machines that could put and take anything?

Unimation, Inc., the world's first robot company, was formed as a result of this discussion.

And the impact was ...

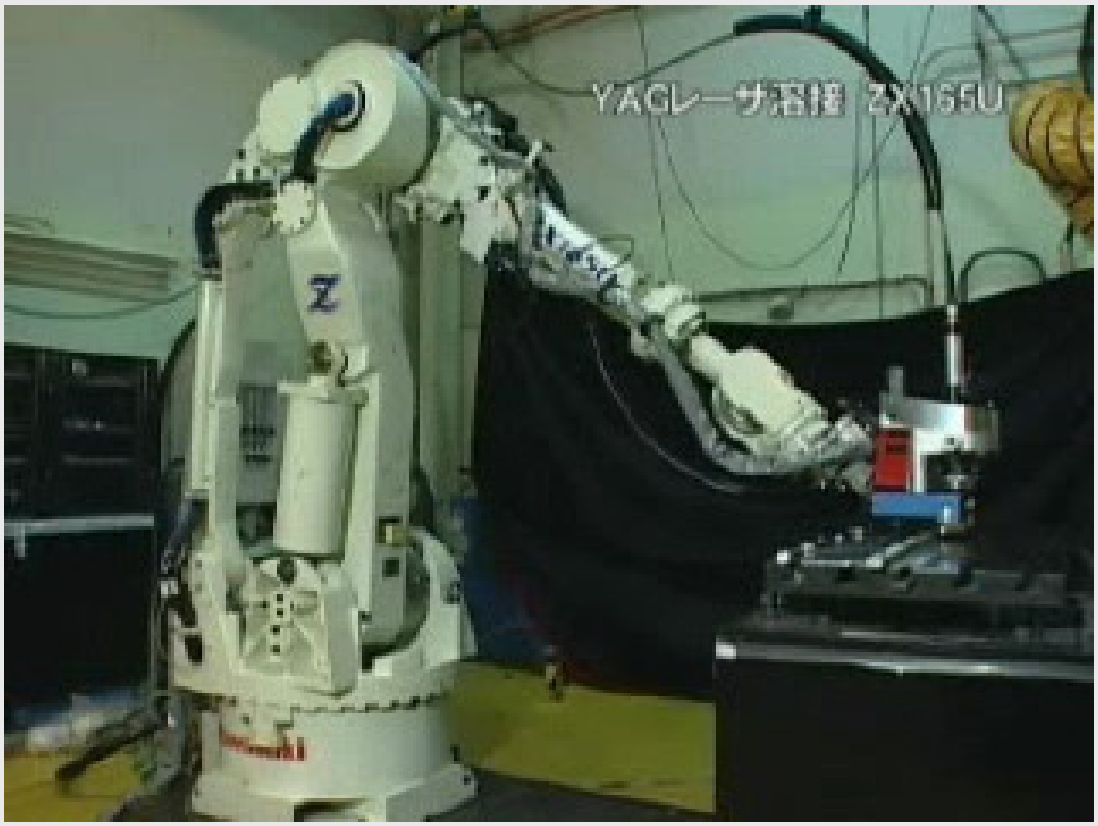
- 1961: The first Unimate robot was installed in a plant of General Motors in New Jersey.
- 1963: The first artificial robotic arm to be controlled by computer was designed at Rancho Los Amigos Hospital, California.

Number of industrial robots at the end of 2003:

Japan 350.000	Italy 50.000
Germany 112.700	France 26.000
North America 112.000	Spain 18,000
	UK 14.000

Nowadays robots ...

http://www.robotclips.com/



… and what we aim at

- household robots
- service robots
- robots in dangerous environments
- medical robotics
- human-robot collaboration

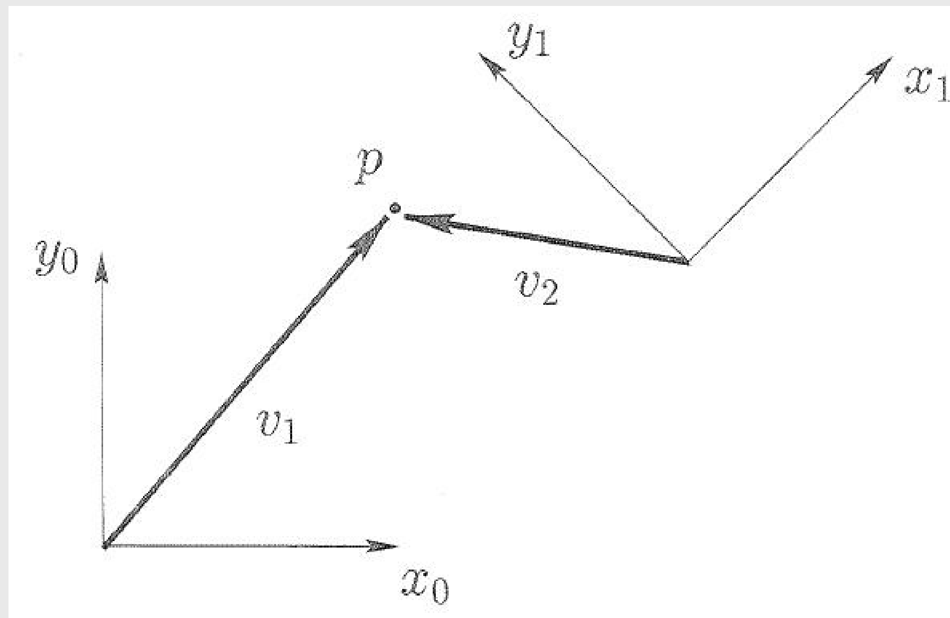
Rigid Motions and Homogenous Transformations

Scope

- Establish coordinate frames to represent positions and orientations of rigid objects.
- Determine transformations among various coordinate frames.
- Introduce concept of homogenous transformations to:
  - simultaneously describe the position and orientation of one coordinate frame relative to another,
  - perform coordinate transformations (represent various quantities in different coordinate frames).

Synthetic approach to represent locations

- Coordinates are not assigned.
- Direct reasoning about geometric entities (e.g. points or lines):
  - x_0 and y_0 are perpendicular to each other,
  - $v_1 \times v_2$ defines another vector,
  - $v_1 \cdot v_2$ is dot product and defines scalar.

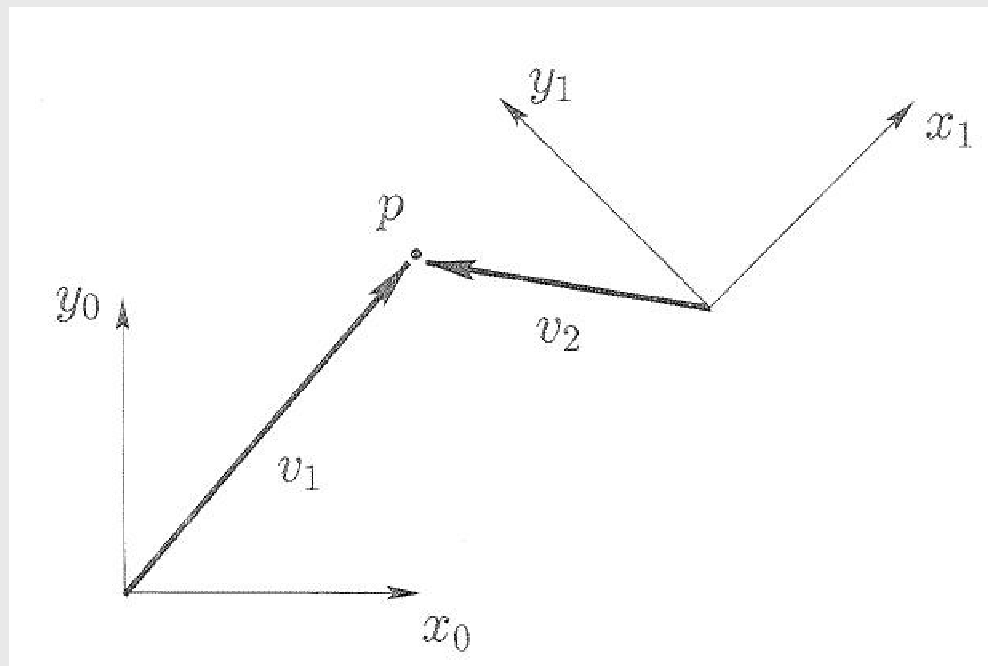


Analytic approach to represent locations

- Coordinates are assigned w.r.t. frames $o_0x_0y_0$ and $o_1x_1y_1$.
- Analytic reasoning via algebraic manipulations.

$$p^0 = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \; p^1 = \begin{bmatrix} -2.8 \\ 4.2 \end{bmatrix}$$

$$o_1^0 = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \; o_0^1 = \begin{bmatrix} -10.6 \\ 3.5 \end{bmatrix}$$

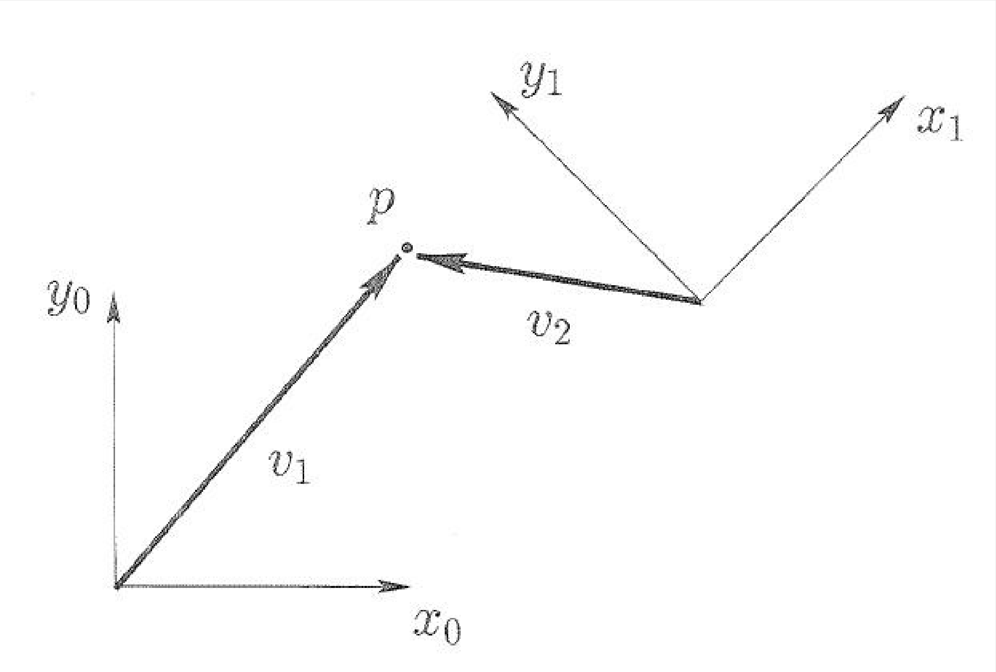


Representing vectors

- Free vectors are not constrained to be located at a particular point in space.

$$v_1^0 = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \qquad v_1^1 = \begin{bmatrix} 7.77 \\ 0.8 \end{bmatrix}$$

$$v_2^0 = \begin{bmatrix} -5.1 \\ 1 \end{bmatrix} \qquad v_2^1 = \begin{bmatrix} -2.89 \\ 4.2 \end{bmatrix}$$



- ${v_1}^0 + {v_2}^1$ does not make sense since $o_0x_0y_0$ and $o_1x_1y_1$ are not parallel.

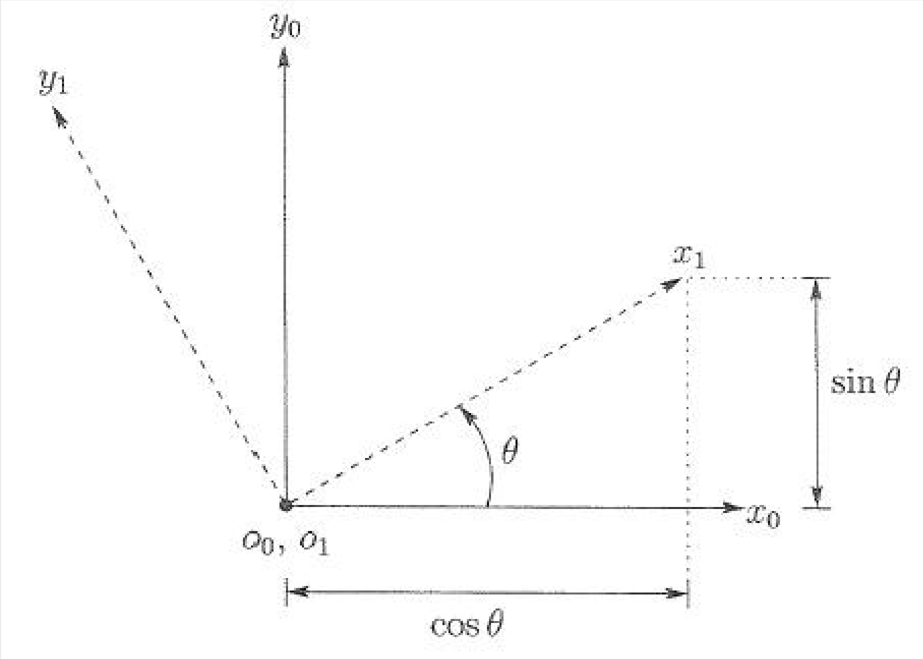
Representing rotations in coordinate frame 0

- Rotation matrix

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 \end{bmatrix}$$

- x_i and y_i are the unit vectors in $o_i x_i y_i$

$$R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

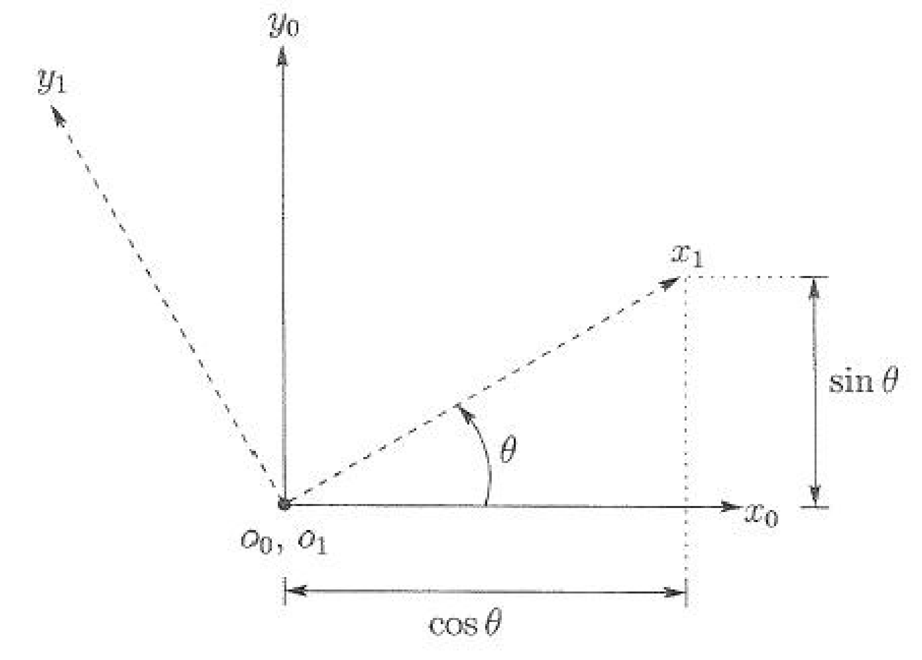


Representing rotations in coordinate frame 1

$$R_0^1 = \begin{bmatrix} x_0 \cdot x_1 & y_0 \cdot x_1 \\ x_0 \cdot y_1 & y_0 \cdot y_1 \end{bmatrix}$$

$$R_0^1 = (R_1^0)^T \qquad (R_1^0)^T = (R_1^0)^{-1}$$

$R^T = R^{-1} :\Leftrightarrow R$ is orthogonal



Special orthogonal group

Set of $n \times n$ orthogonal matrices is $SO(n)$: special orthogonal group of order n

For any $R \in SO(n)$:

- $R^T = R^{-1}$
- the columns (rows) of R are mutually orthogonal
- each column (row) of R is a unit vector
- $\det R = 1$

Representing rotations in  (1/4)

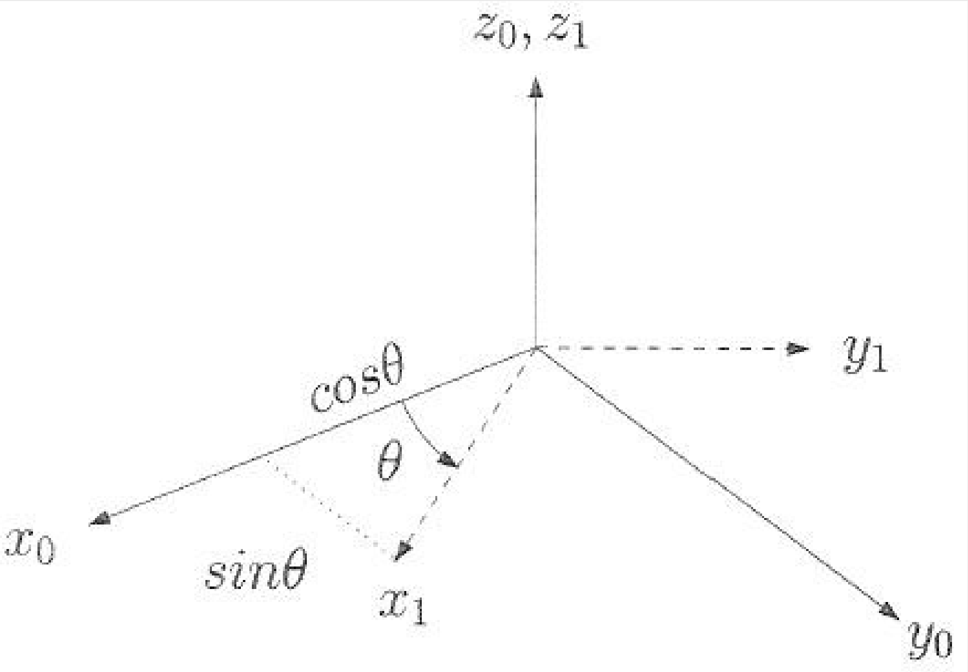
Each axis of the frame $o_1x_1y_1z_1$ is projected onto $o_0x_0y_0z_0$:

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

$$R_1{}^0 \in \mathrm{SO}(3)$$

Representing rotations in 3-D (2/4)

Example: Frame $o_1x_1y_1z_1$ is obtained from frame $o_0x_0y_0z_0$ by rotation through an angle θ about z_0 axis.



$$x_1 \cdot x_0 = \cos\theta \qquad y_1 \cdot x_0 = -\sin\theta$$
$$x_1 \cdot y_0 = \sin\theta \qquad y_1 \cdot y_0 = \cos\theta$$
$$z_1 \cdot z_0 = 1$$

all other dot products are zero

$$R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Representing rotations in 3-D (3/4)

Basic rotation matrix about z-axis

$$R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} =: R_{z,\theta}$$

$$R_{z,0} = I$$

$$R_{z,\theta} R_{z,\phi} = R_{z,\theta+\phi}$$

$$(R_{z,\theta})^{-1} = R_{z,-\theta}$$

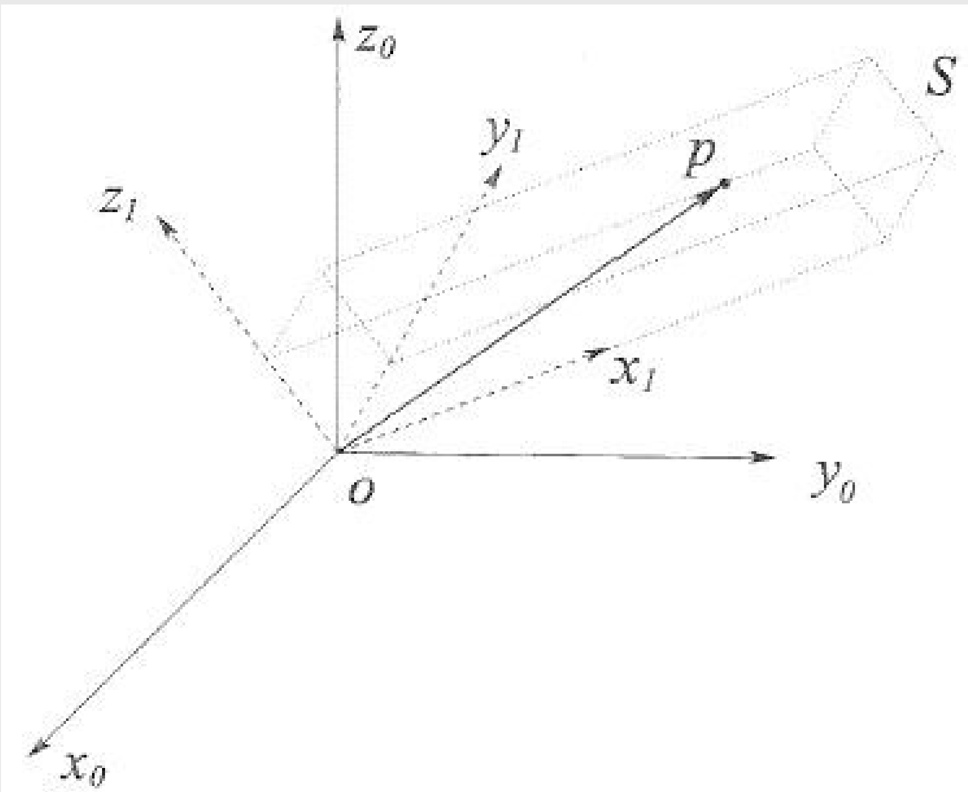
Representing rotations in 3-D (4/4)

Similarly, basic rotation matrices about x- and y-axes:

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

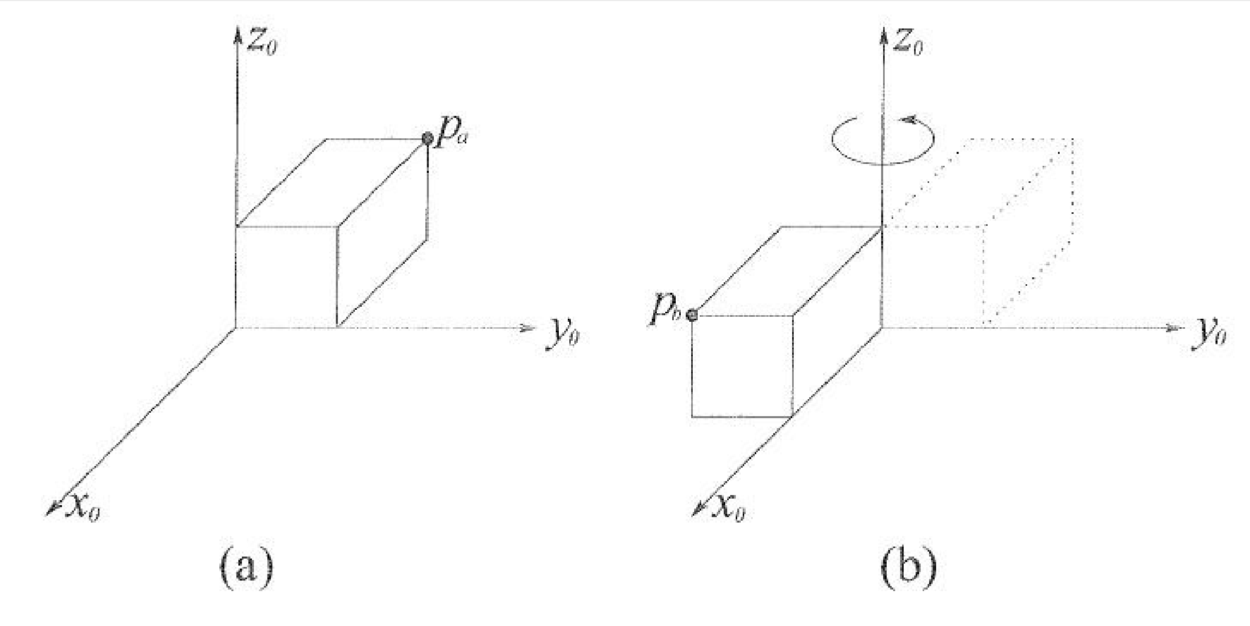
Rotational transformations (1/2)



p^i: coordinates of p in $o_i x_i y_i z_i$

$$p^0 = R_1^0 p^1$$

Rotational transformations (2/2)



- Frame $o_1x_1y_1z_1$ is attached to the block.
- In figure (a) $o_1x_1y_1z_1$ coincides with $o_0x_0y_0z_0$.

$$p_b^0 = R_1^0 p_b^1 = R_{z,\pi} p_b^1$$

$$p_b^1 = p_a^0$$

$$p_b^0 = R_{z,\pi} p_a^0.$$

Similarity transformations (1/2)

- Matrix representation of a general linear transformation is mapped from one frame to another using similarity transformation.

- If A is matrix representation of a linear transformation in $o_0x_0y_0z_0$ and B is the representation of the same linear transformation in $o_1x_1y_1z_1$, then

$$B = (R_1^0)^{-1} A R_1^0$$

where R_1^0 is the coordinate transformation between $o_1x_1y_1z_1$ and $o_0x_0y_0z_0$.

Similarity transformations (2/2)

Example: Frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by rotation:

$$R_1^0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0. \end{bmatrix}$$

If $A = R_{z,\theta}$ relative to frame $o_0x_0y_0z_0$, then relative to frame $o_1x_1y_1z_1$ we have

$$B = (R_1^0)^{-1} A R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & s_\theta \\ 0 & -s_\theta & c_\theta. \end{bmatrix}$$

Composition of rotations w.r.t. the Current Frame

- Frame relative to which a rotation occurs is the current frame.
- Rotations with respect to the current frames:

$$o_0x_0y_0z_0 \xrightarrow[R_1^0]{\text{rotation r.t. } 0} o_1x_1y_1z_1 \xrightarrow[R_2^1]{\text{rotation r.t. } 1} o_2x_2y_2z_2$$

- Composition:

$$R_2^0 = R_1^0 R_2^1.$$

Composition of rotations w.r.t. the Fixed Frame

- We can select one fixed frame as the reference one.
- Let $o_0x_0y_0z_0$ be the fixed frame.
- Rotations with respect to $o_0x_0y_0z_0$:

$$o_0x_0y_0z_0 \xrightarrow[R_1^0]{\text{rotation r.t. } 0} o_1x_1y_1z_1 \xrightarrow[R]{\text{rotation r.t. } 0} o_2x_2y_2z_2$$

- Rotation R can be represented in the current frame $o_1x_1y_1z_1$:

$$R_2^1 = (R_1^0)^{-1}RR_1^0. \qquad \text{[see similarity transformations]}$$

- Composition: $R_2^0 = R_1^0R_2^1 = R_1^0(R_1^0)^{-1}RR_1^0 = RR_1^0$

Parameterization of rotations (1/4)

- A rigid body has at most 3 rotational degrees of freedom.
- Rotational transformation $R \in SO(3)$ has 9 elements r_{ij} that are not mutually independent since

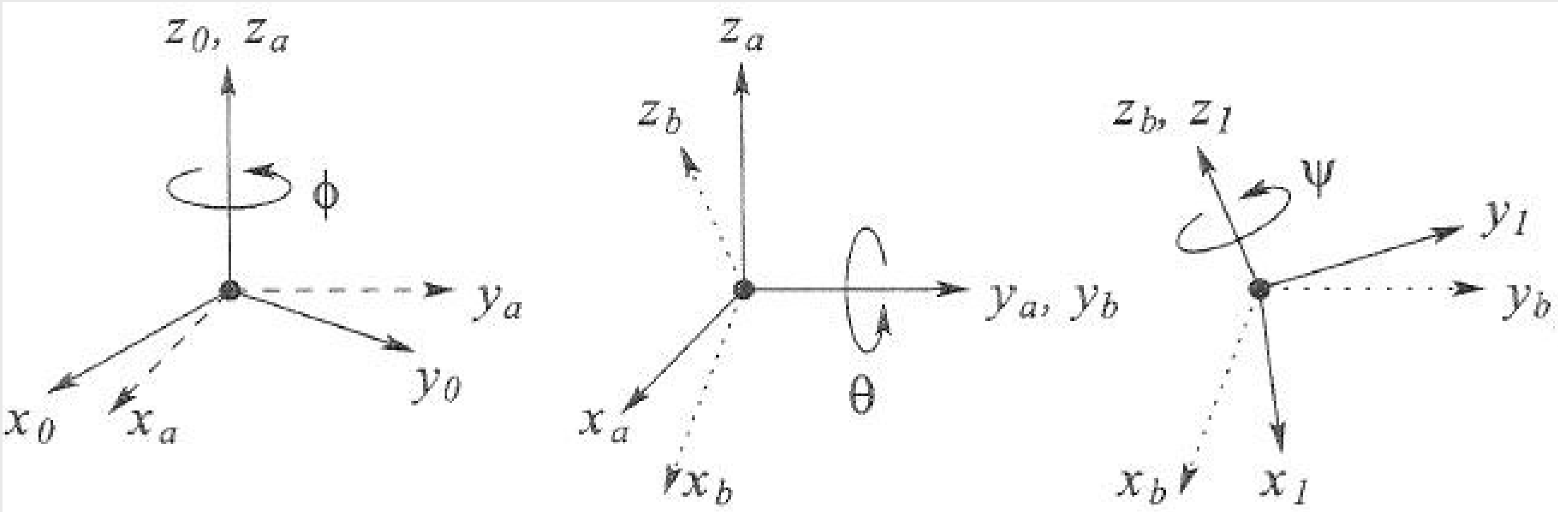
$$\sum_i r_{ij}^2 = 1 \qquad j = 1,2,3 \qquad \text{[columns are unit vectors]}$$

$$r_{1i}r_{1j} + r_{2i}r_{2j} + r_{3i}r_{3j} = 0 \qquad i \neq j \qquad \text{[columns are unit mutually orthogonal]}$$

- Given constraints define 6 independent equations with 9 unknowns; consequently, there are only 3 free variables.

Parameterization of rotations (2/4)

Euler angles

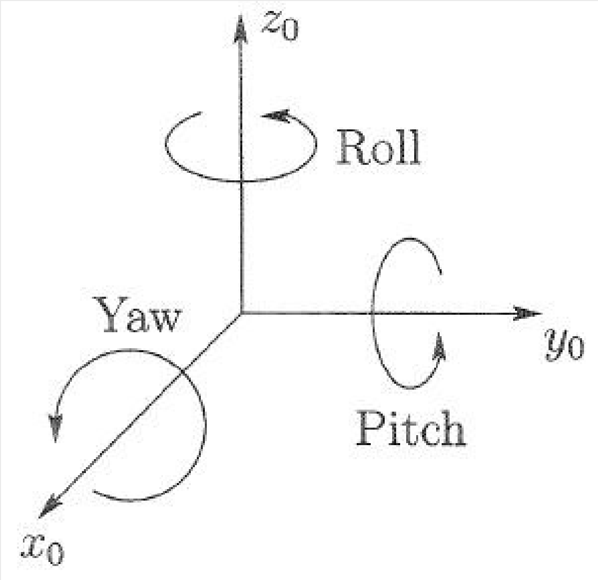


ZYZ—Euler angle transformation:

$$R_{ZYZ} = R_{z,\phi} R_{y,\theta} R_{z,\psi} = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Parameterization of rotations (3/4)

Roll, pitch, yaw angles

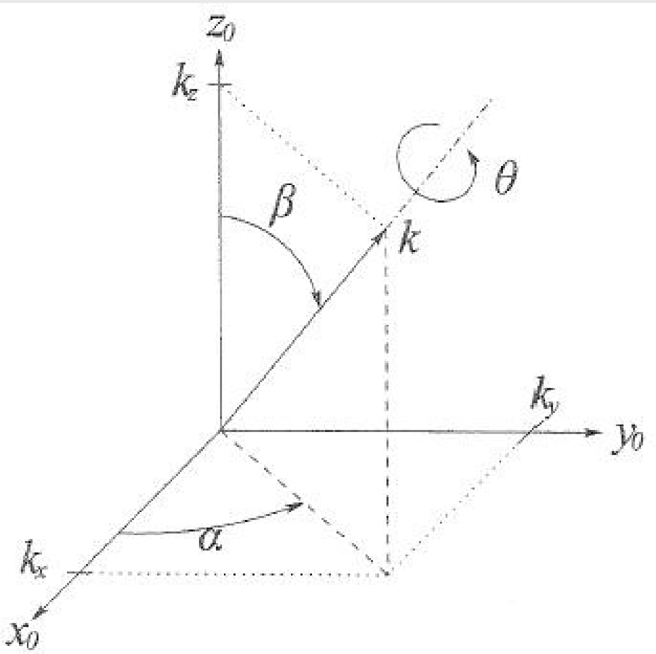


XYZ—yaw-pitch-roll angle transformation:

$$R = R_{z,\phi} R_{y,\theta} R_{x,\psi} = \begin{bmatrix} c_\phi c_\theta & -s_\phi c_\psi + c_\phi s_\theta s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ s_\phi c_\theta & c_\phi c_\psi + s_\phi s_\theta s_\psi & -c_\phi s_\psi + s_\phi s_\theta c_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{bmatrix}$$

Parameterization of rotations (4/4)

Axis / angle representation



$k = [k_x \; k_y \; k_z]^T$ (unit) axis vector in $o_0x_0y_0z_0$

$R_{k,\theta}$ represents rotation of θ about k

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta + k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix}$$

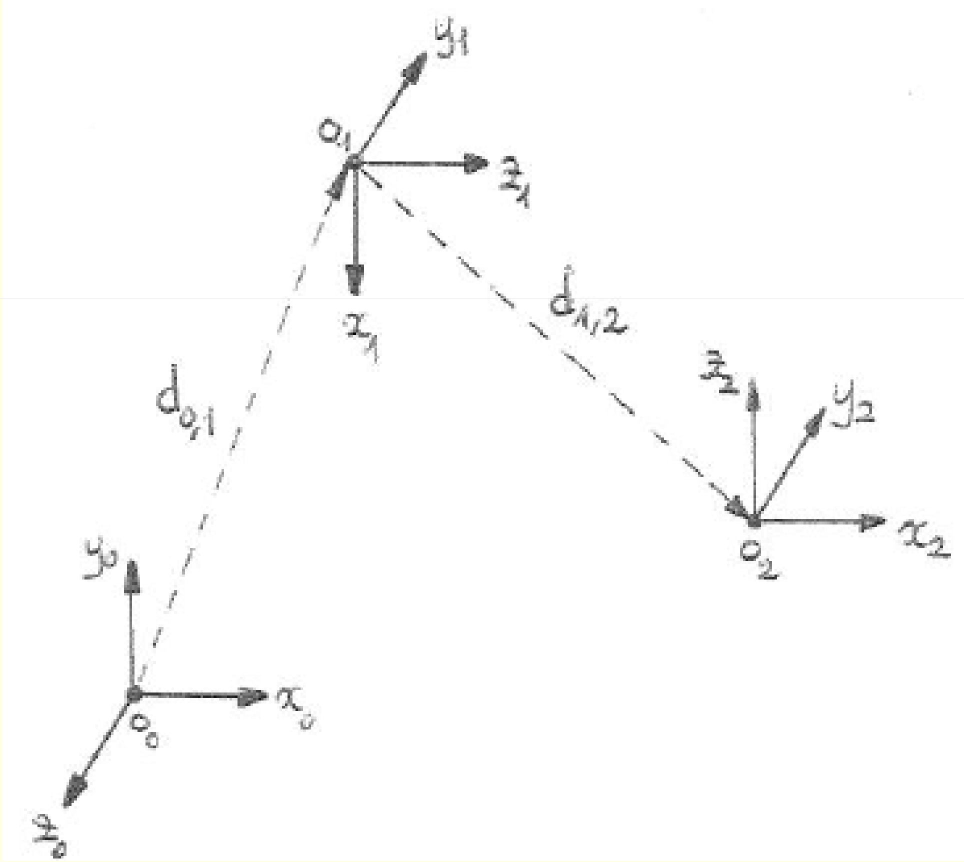
where $v_\theta = 1 - c_\theta$

Rigid motions (1/2)

- Rigid motion: a pure translation + a pure rotation.
- A rigid motion is an ordered pair (d, R) where $d \in \mathbb{R}^3$ and $R \in SO(3)$.
- The group of all rigid motions: special Euclidean group $SE(3)$

$$SE(3) = \mathbb{R}^3 \times SO(3)$$

Rigid motions (2/2)



$$p^1 = R_2^1 p^2 + d_{1,2}^1$$

$$p^0 = R_1^0 p^1 + d_{0,1}^0$$

$$p^0 = \underbrace{R_1^0 R_2^1}_{R_2^0} p^2 + \underbrace{R_1^0 d_{1,2}^1 + d_{0,1}^0}_{d_{0,2}^0}$$

$$p^0 = R_2^0 p^2 + d_{0,2}^0$$

Homogenous transformations (1/2)

- We have

$$R_2^0 = R_1^0 R_2^1 \qquad\qquad d_{0,2}^0 = R_1^0 d_{1,2}^1 + d_{0,1}^0.$$

- Note that

$$\begin{bmatrix} R_1^0 & d_{0,1}^0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^1 & d_{1,2}^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1^0 R_2^1 & R_1^0 d_{1,2}^1 + d_{0,1}^0 \\ 0 & 1 \end{bmatrix}.$$

- Consequently, rigid motion (d, R) can be described by matrix representing homogenous transformation:

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}.$$

Homogenous transformations (2/2)

- Since R is orthogonal, we have

$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}.$$

- We augment vectors p^0 and p^1 to get their homogenous representations

$$P^0 = \begin{bmatrix} p^0 \\ 1 \end{bmatrix} \qquad P^1 = \begin{bmatrix} p^1 \\ 1 \end{bmatrix}$$

and achieve matrix representation of coordinate transformation

$$P^0 = H_1^0 P^1.$$

Basic homogenous transformations

$$\mathrm{Trans}_{x,a} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha & 0 \\ 0 & s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{y,\beta} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Trans}_{z,c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{Rot}_{z,\gamma} = \begin{bmatrix} c_\gamma & -s_\gamma & 0 & 0 \\ s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$