Cold regions hydrology

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2. Thermodynamics of cold regions hydrology

Learning Objectives

You should be able to:

- Understand what energy, work and heat refer to and how these concepts are useful
- Understand what a system is, including an isolated, closed and open system
- Understand what a state is and what a flow is
- Write out the conservation equation for a given system
- Calculate sensible heat, latent heat, radiation flux, conduction flux and advection flux

The ideas in this lecture are foundational for the subsequent lectures and understanding snow and frozen ground hydrological processes.

Energy: work and heat

Thermodynamics is the study of energy. So, consider:

- What is energy?
- What is work?
- What is heat?
- Why are work and heat different kinds of the same thing?

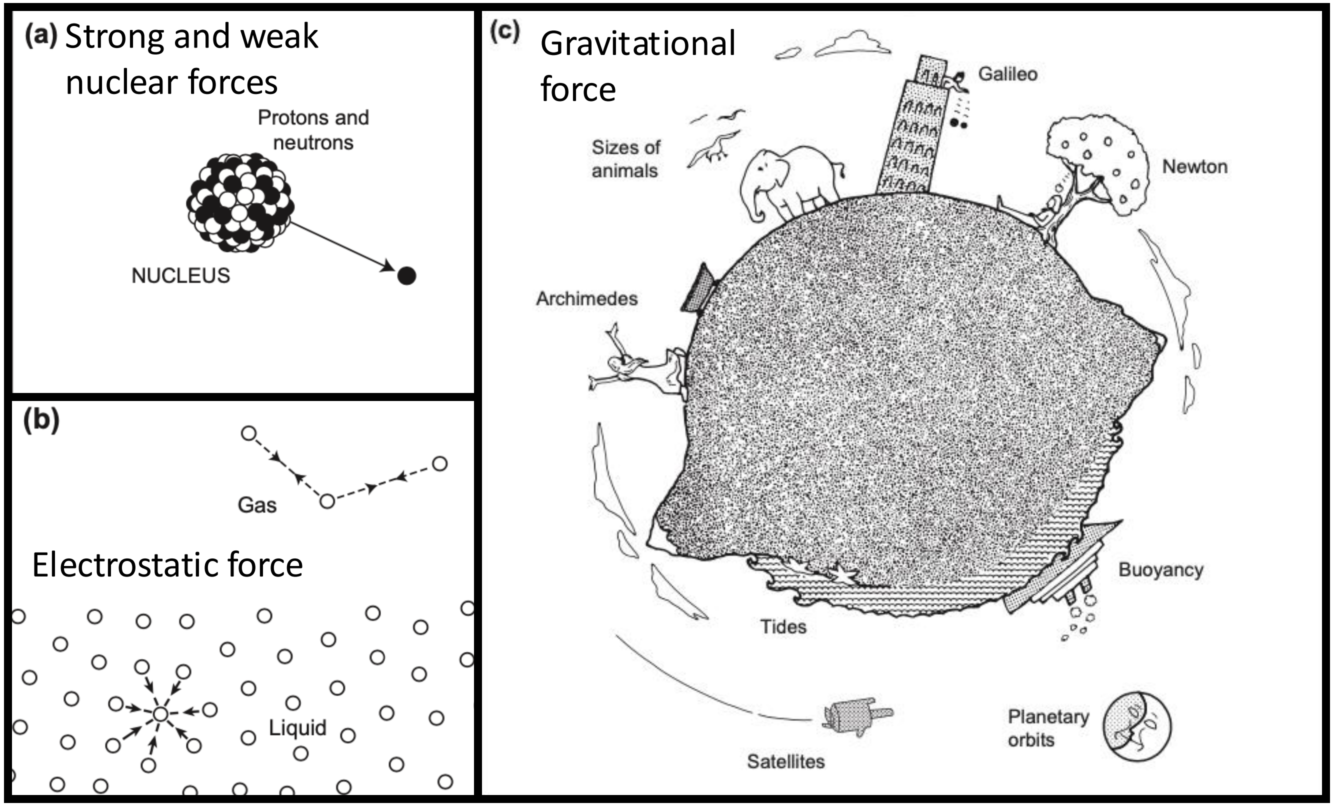
Energy is best thought of as a mysterious "**conserved quantity**".

The absolute value of energy is normally meaningless. We are only interested in **changes in energy** – in time or in space.

Tracking changes in energy allows us to **understand how systems work** and **make predictions** about how something will respond to change.

PART ONE: WORK

Forces of nature

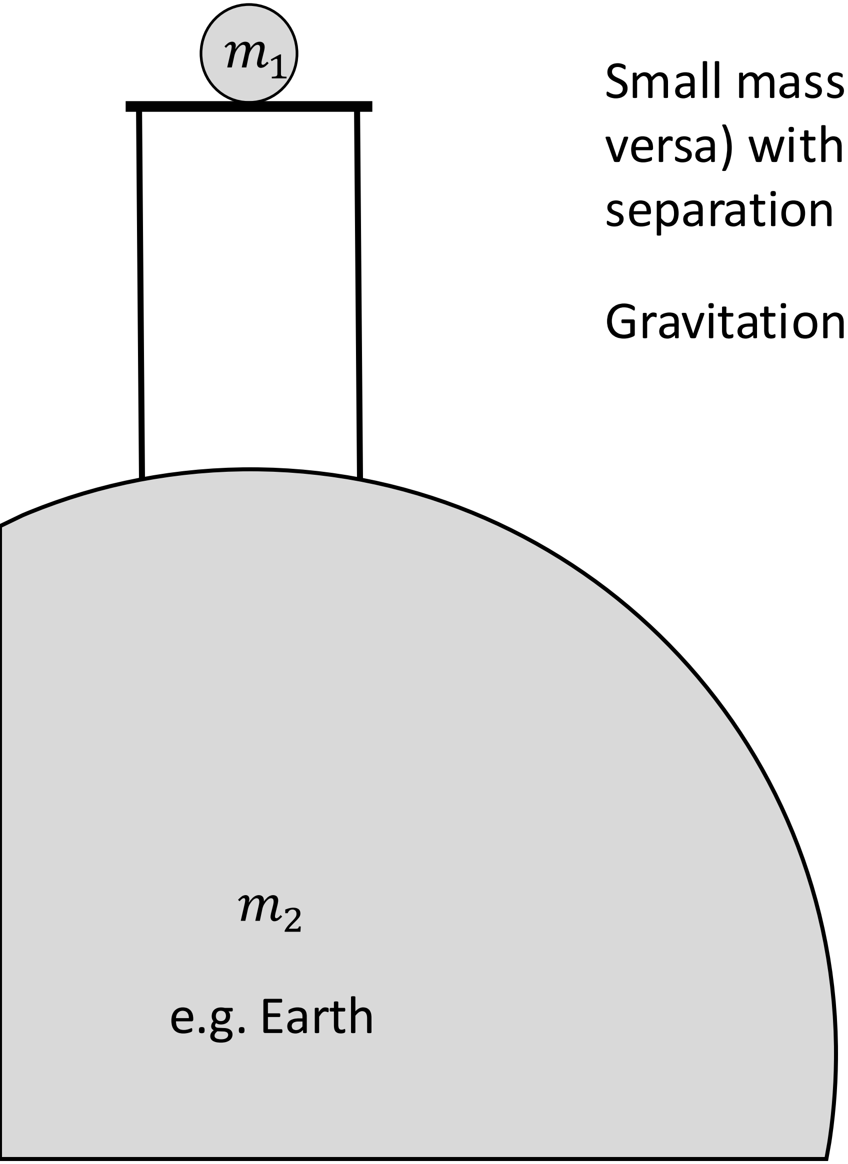


Two objects (e.g. atoms, planets, person, ball) subject to these forces either attract or repel one another.

"***Force***" is a quantity reflecting how strongly they attract together or repel apart.

From: Israelachivili, 2011. Intermolecular and surface forces. Academic Press.

Gravitational force



Small mass m_1 is attracted to large mass m_2 (and vice-versa) with a magnitude that is a function of the separation distance and the two masses:

Gravitational attraction force:

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

Near the surface of the earth we say

$$F \approx g m_1$$

Where

$$g = \frac{G m_2}{r^2}$$

Work: energy from force

When a force exists between two objects, there is the potential to induce movement, and "energy" is a quantity that accounts for this.

Work, W (J), is the energy that is expended as a force, F, moves an object through some distance, x. Work is defined

$$W = \int F.dx$$

And for constant forces over straight distances we have

$$W = F.x$$

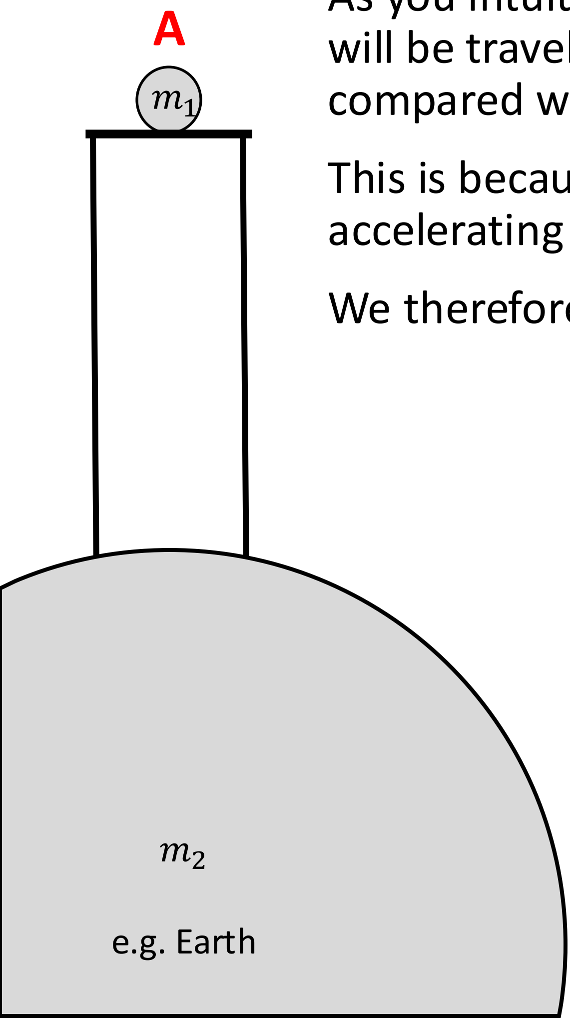
Potential energy, E (J), is the amount of work that can be done by a force in moving an object. It is a relative measure defined between some actual state and some reference state (datum).

Work done and expended potential always balance, as energy is conserved (first law of thermodynamics). Hence

Potential energy in current state, relative to some reference state **=** **Work done in moving object from current state to reference state**

$$E = W$$

Gravitational potential



Compare A and B.

As you intuitively know from living on Earth, if we release m_1 it will be travelling much faster when it hits the ground in case A compared with case B.

This is because the force has had longer to act on m_1 in case A, accelerating it for longer, giving it more velocity.

We therefore say that A has a greater ”potential” than B.

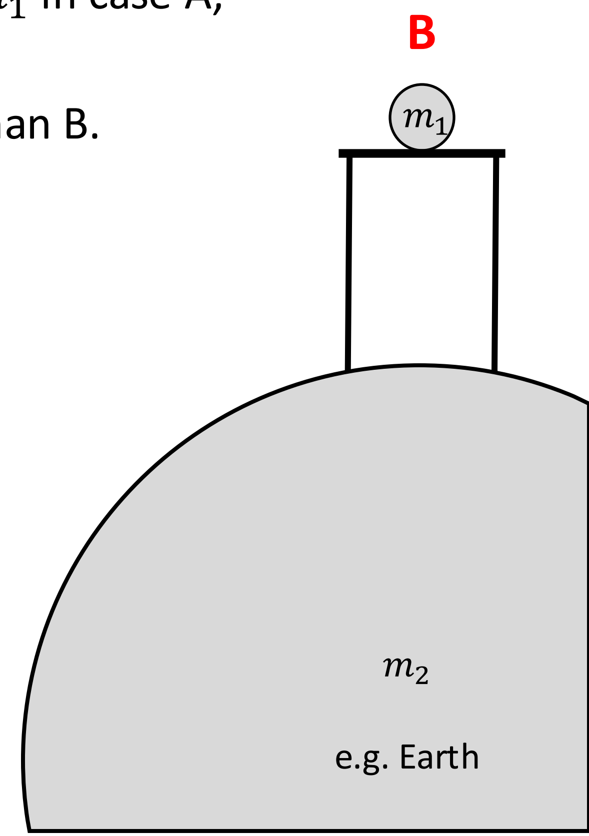
The potential energy before the drop, E_g is therefore equal to the work done during the drop, W

If we assume the ball travels in a straight line, with a constant force, F, then:

$$W = Fz = m_1 g z$$

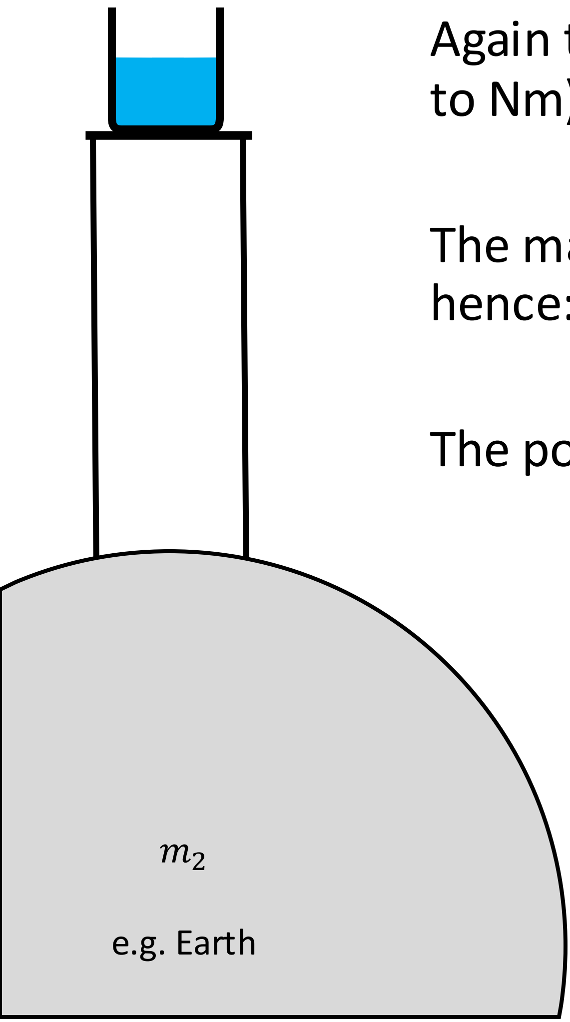
Hence

$$E_g = m_1 g z$$



Gravitational potential of fluids

The same gravitational laws hold for fluids. Consider the beaker of water shown:



Again the gravitational potential energy, E_g (Joules, J, equivalent to Nm) is

$$E_g = mgz$$

The mass of water is the density, ρ (kg/m^3) x volume, V (m^3), hence:

$$E_g = \rho V g z$$

The potential energy can be expressed per unit volume as

$$\frac{E_g}{V} = \rho g z$$

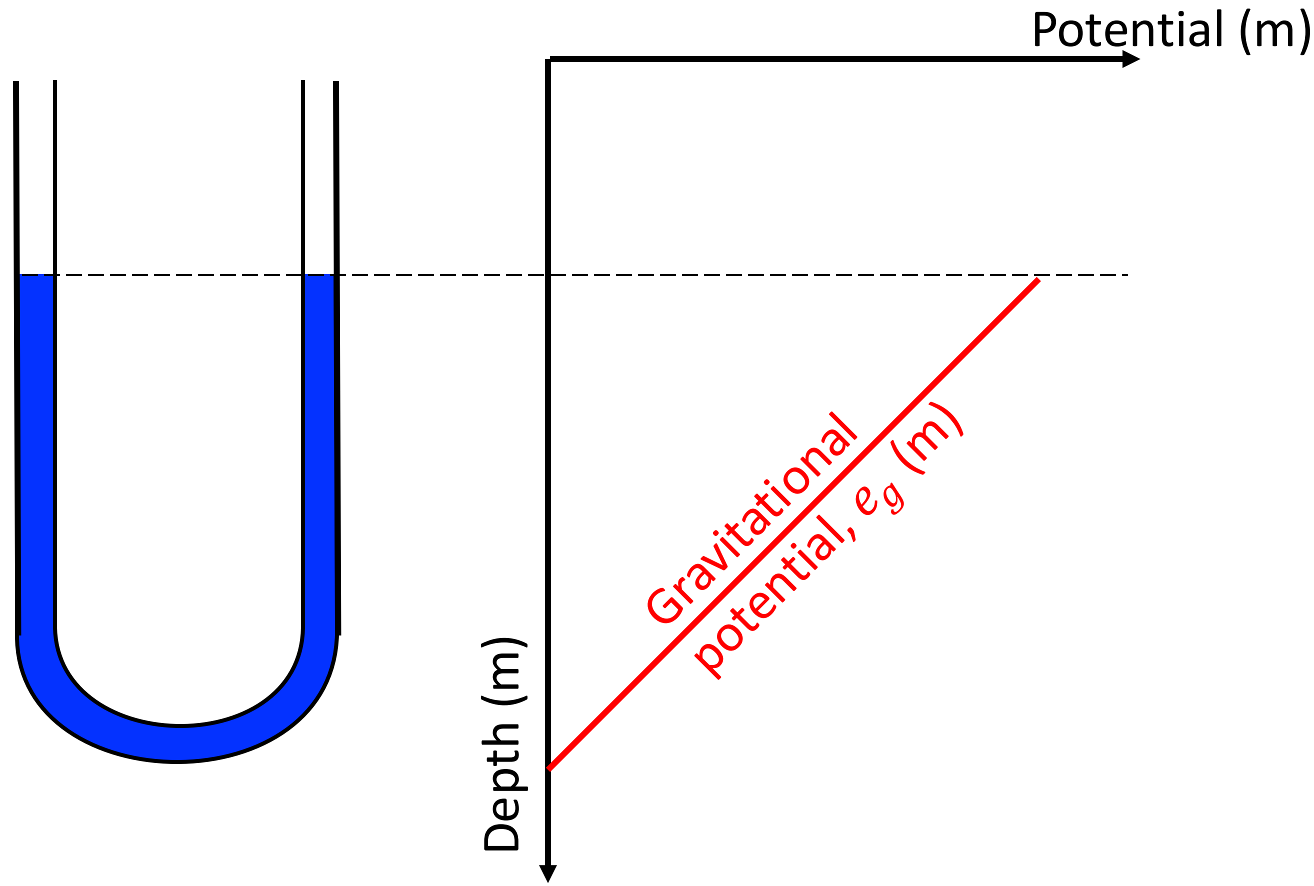
Or as potential energy per unit weight, e_g (J/kg), as

$$e_g = \frac{E_g}{\rho V g} = z$$

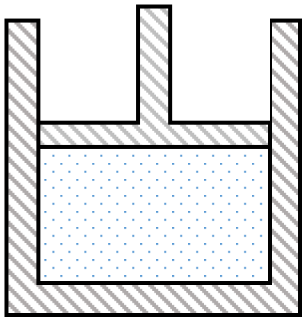
The gravitational potential energy per unit weight has units of length and is equivalent to the elevation of the water body in question. This will be a very convenient unit to work with

Gravitational potential in a water column

Consider a u-tube filled with water. As in any water column, there is a distribution of gravitational potential within the column, as shown



Pressure potential



Consider a small vertical displacement of the piston shown, downward by Δz

The change in potential energy of the fluid is again

$$\Delta E = \int F.dz$$

Since the $F = PA$ (pressure x area) we have

$$\Delta E = \int P.A.dz$$

For constant P and A we have

$$\Delta E = P\Delta V$$

So in any instant the pressure potential energy is

$$E_p = PV$$

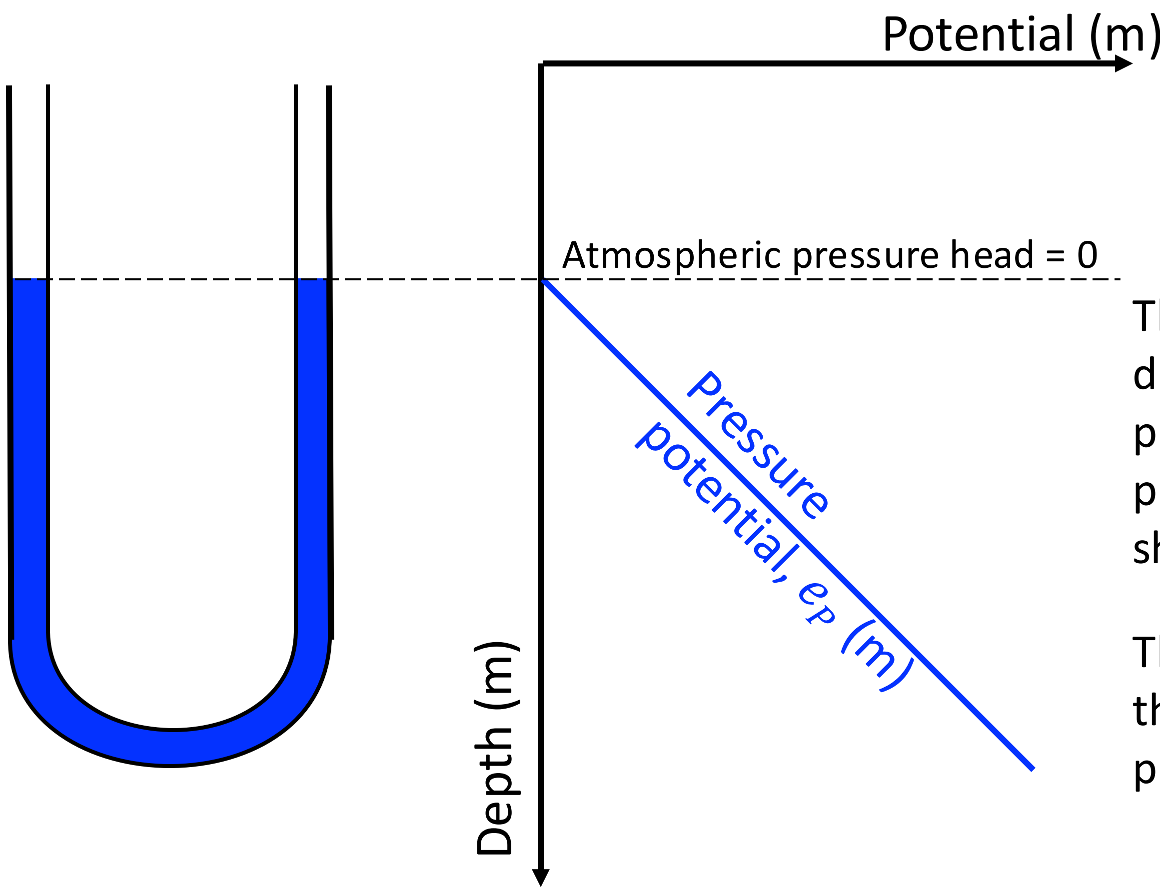
Expressed again per unit weight (i.e. overall units of length), this is

$$e_p = \frac{E_p}{\rho V g} = \frac{P}{\rho g}$$

Pressure potential in a water column

Consider again the u-tube. The pressure at the free-water surface must equal to atmospheric pressure.

At any point below the surface, the pressure increases, due to the weight of the water column above the point.



Thus, the distribution of pressure potential is as shown.

This is known as the hydrostatic pressure.

Total potential: hydraulic head

Considering the two forces of gravity and pressure, we have a total potential per unit weight of water (units of L) of

$$e_T = e_g + e_P$$

Which we can express as

$$e_T = z + \frac{P}{\rho g}$$

Here, e_T is exactly equivalent to **hydraulic head**, h (L), as used in groundwater. We hence define pressure head, p (L) as

$$p = \frac{P}{\rho g}$$

And we can write

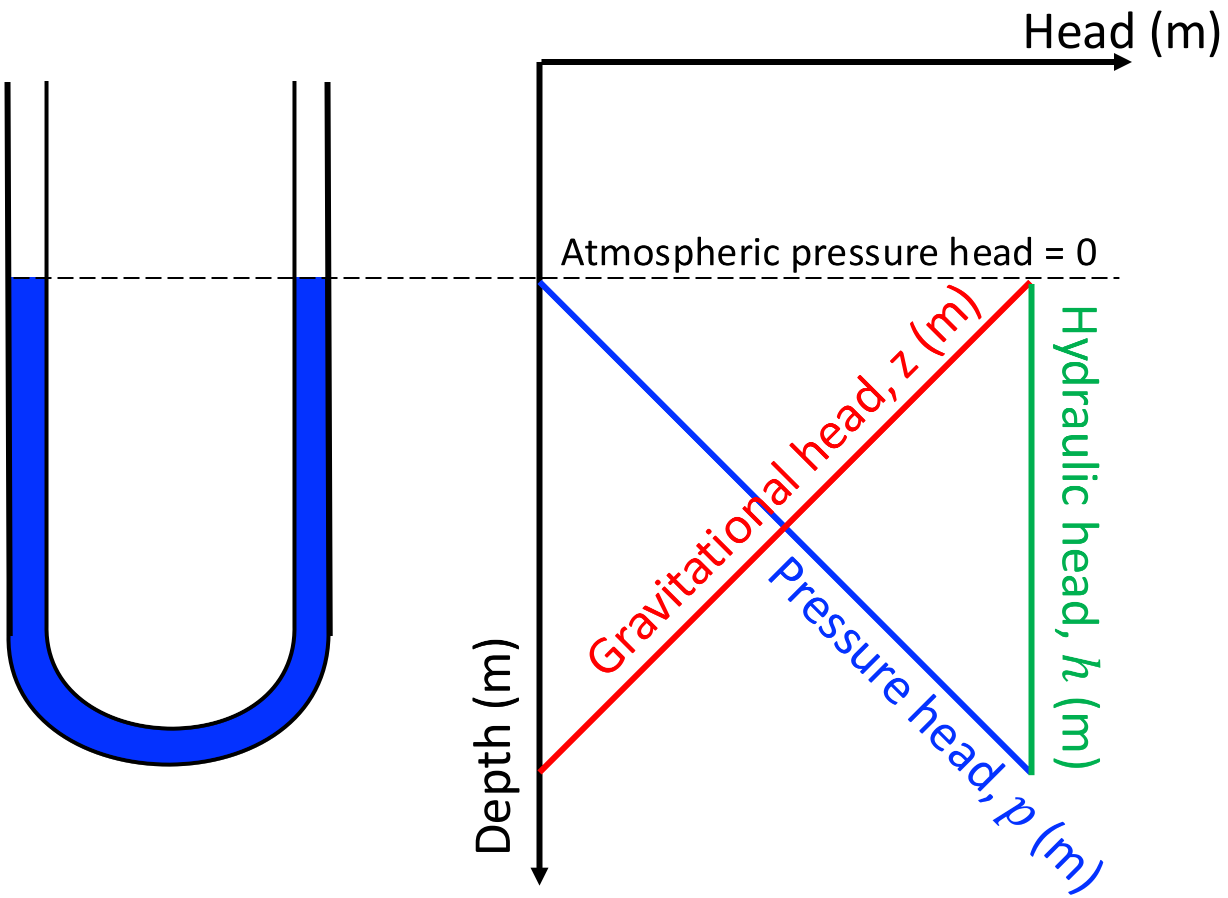
$$h = z + p$$

Flow in any fluid is driven by gradients in $\boldsymbol{h}$

Note, there are other potential energy terms, such as the kinematic energy and the osmotic potential, but for our purposes we can ignore these.

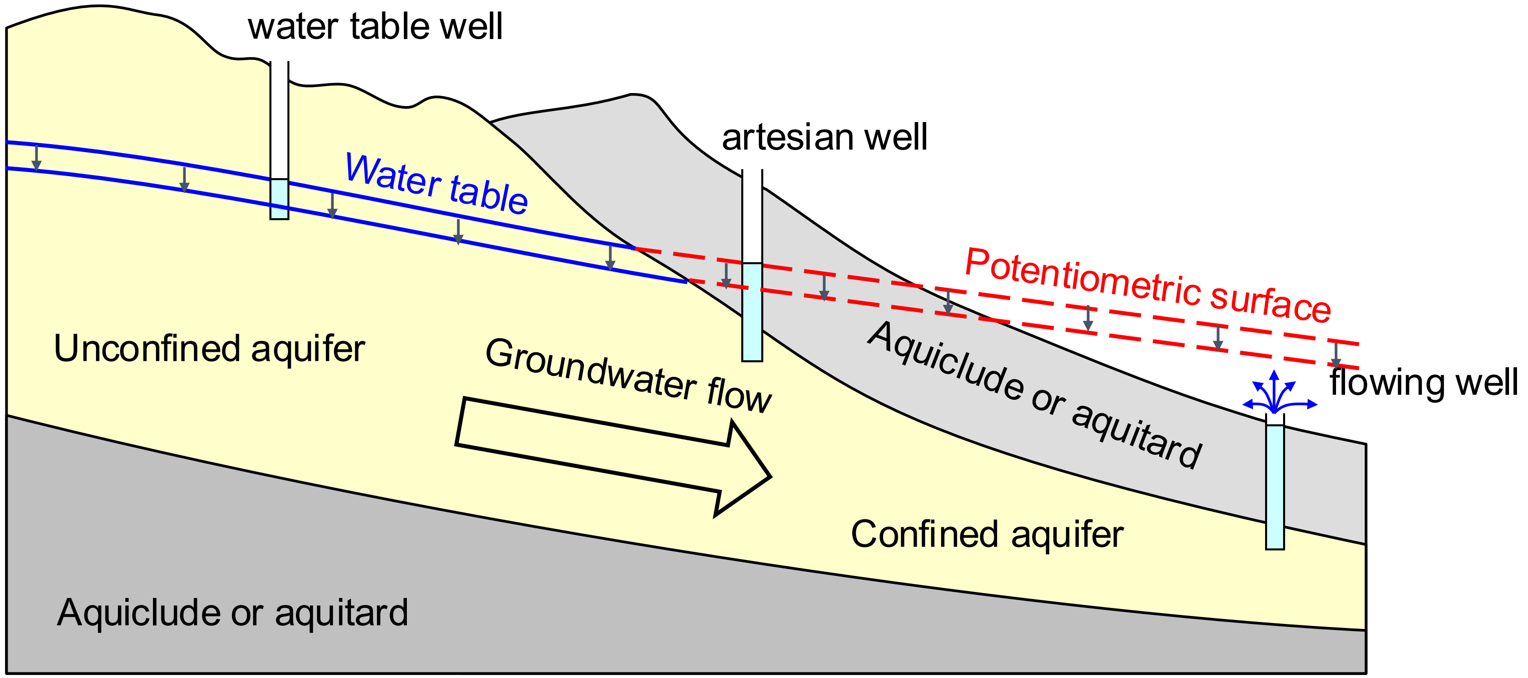
Hydraulic head

The hydraulic head in the u-tube is obtained by summing the gravitational and pressure heads, as shown:



In this case the hydraulic head is constant, indicating no gradient in h is present and hence there is no flow, and the system is hydrostatic.

Hydraulic head – groundwater flow



The groundwater system is in equilibrium (no flow) when the hydraulic head is equal everywhere

When hydraulic head is non-uniform, the system is not in equilibrium and there is groundwater flow **from high to low head.**

PART TWO: HEAT

Systems

What is a system?

We will define a system as *a group of one or more elements which interact with one another and with their surrounding environment.*

Examples:

The contents of a beaker

The solar system

The atmosphere

A watershed

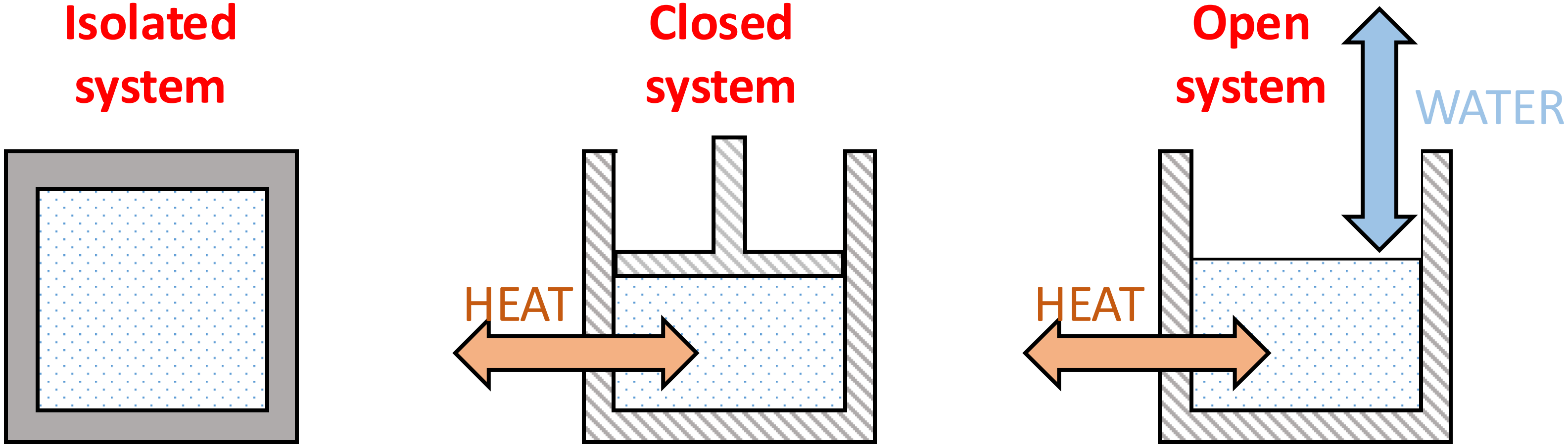
A human body

Systems

A system must have a clearly defined border or boundary.

We are typically concerned with the mass of a substance (here water) within the system, as well as the exchanges of that substance across the boundaries.

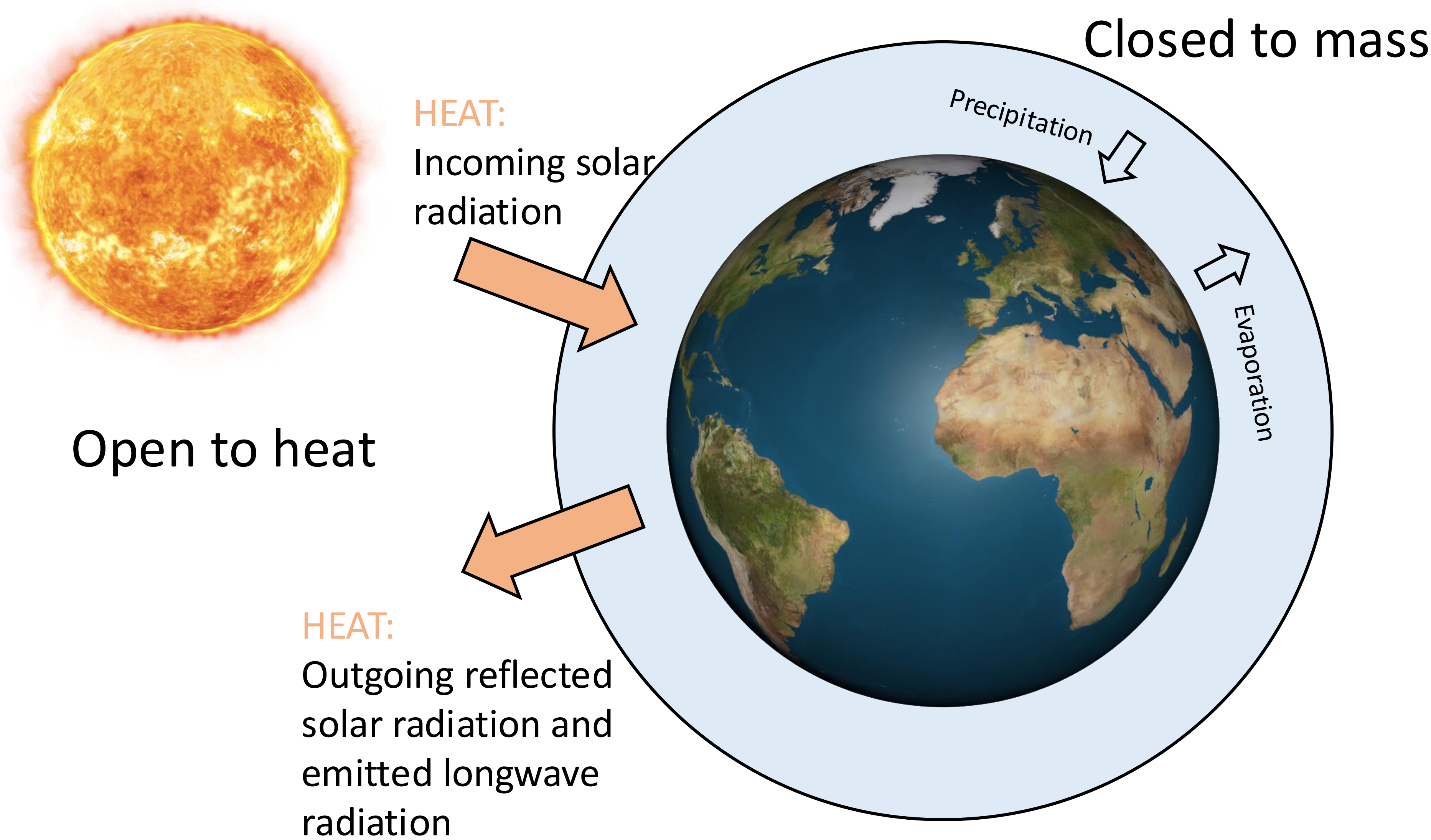
Isolated system	**Closed system**	**Open system**



No exchange of heat or mass (e.g. water) with surroundings	No exchange of mass with surroundings, but can gain/lose heat	Can exchange heat and mass with surroundings

Question: Is planet Earth an isolated, closed or open system?

Earth



Earth is therefore a closed system

States and fluxes

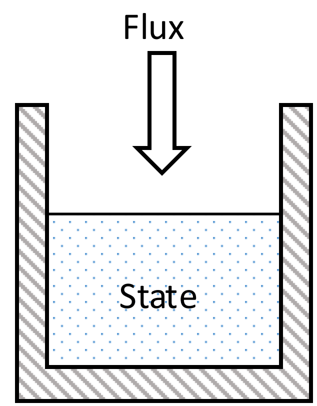
Mass cannot be created or destroyed: mass is therefore conserved

Energy cannot be created or destroyed: energy is therefore conserved (1st Law of Thermodynamics)

These two laws are of fundamental importance in hydrology (and all sciences!)

A **State Variable** is a quantity that tracks the mass or energy within a system

A **flow** or **flux** is a quantity that tracks the movement of mass or energy into/out of the system



Conservation equation:

Change in **state** over some time interval	=	Sum of all **fluxes** over all boundaries in that time interval
	Or	
Change in storage	=	Inflows – outflows
	Or	
$\Delta\mathrm{m}$	=	$m_{in} - m_{out}$

1st Law of thermodynamics

Energy is conserved. It cannot be created or destroyed. I ma

Change in internal energy

=

Sum of all of the energy that enters

-

Sum of all the energy that leaves.

Or

$$\Delta U = j_{IN} - j_{OUT}$$

Heat fluxes

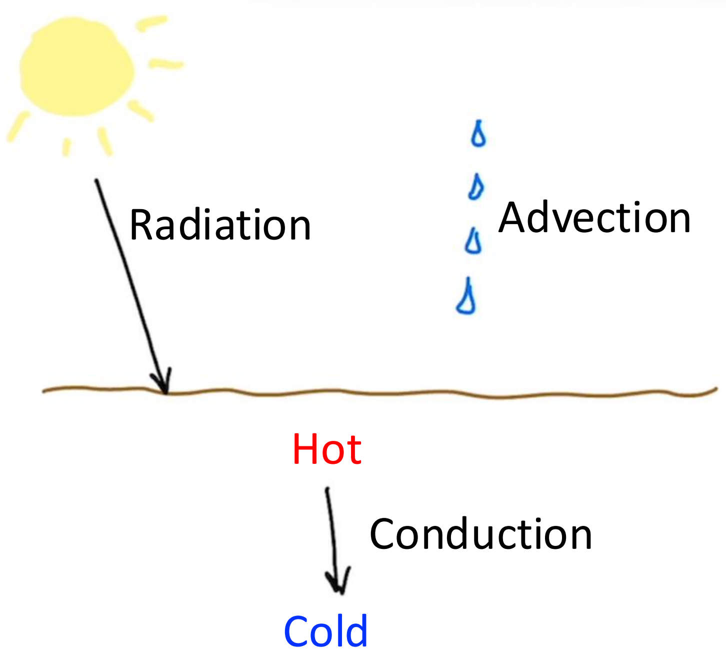
There are three basic types of heat flux:

Radiation $j_r = \varepsilon\sigma T^4$ (J/m²/s) Stephan Boltzmann Law

Conduction $j_c = -\kappa\frac{dT}{dx}$ (J/m²/s) Fourier's law

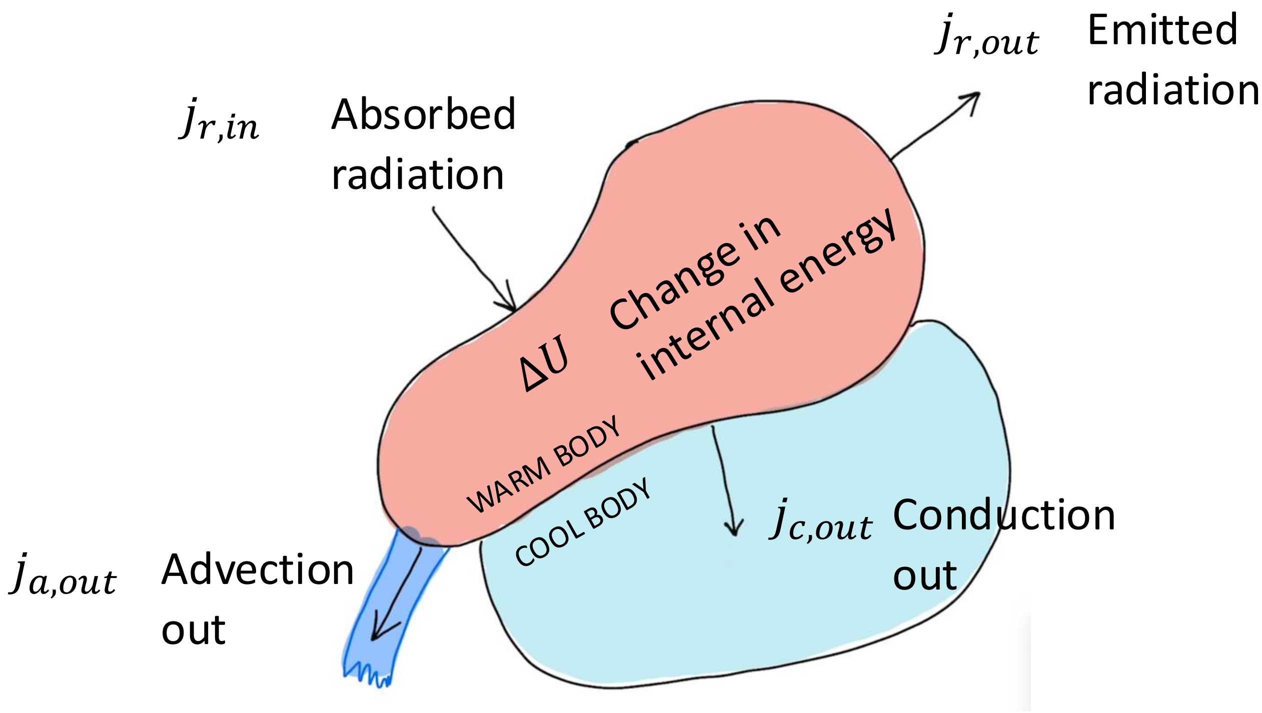
Advection $j_a = c_p\rho qT$ (J/m²/s)

or **Convection**



In all cases the heat flux is given in Joules per unit area per time.

Heat states



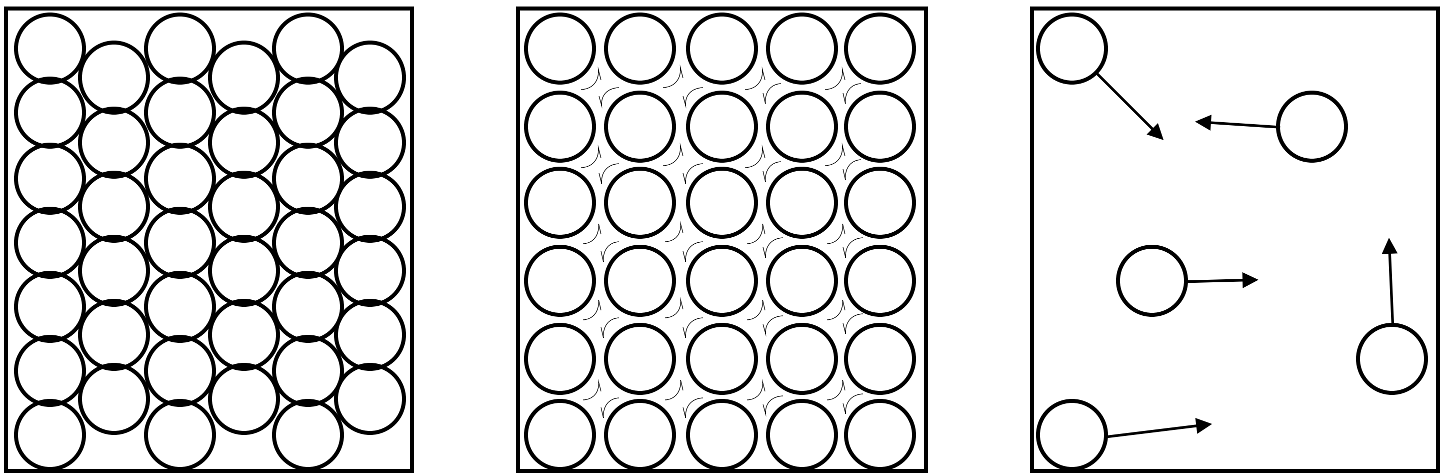
A state variable for the amount of energy within a system (depicted here as the red "warm body" is the **Internal Energy**

Here

$$\Delta U = j_{r,in} - j_{r,out} - j_{c,out} - j_{a,out}$$

States of water

A complication is that water (like all substances) exists in three physical states:



Solid Liquid Gas

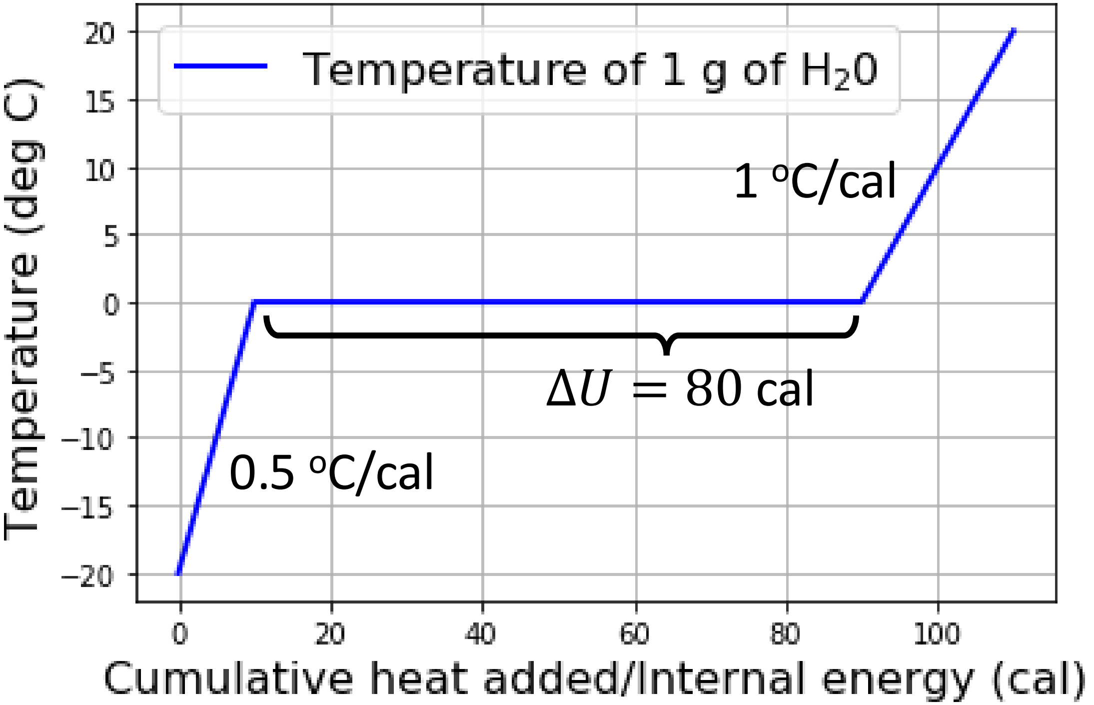
Freezing free water

It was a profound insight of thermodynamics to show that heat and mechanical work are equivalent – both types of energy, both can be measured in Joules.

Before this insight, however, heat energy was thought to be something separate, and was measured in Calories.

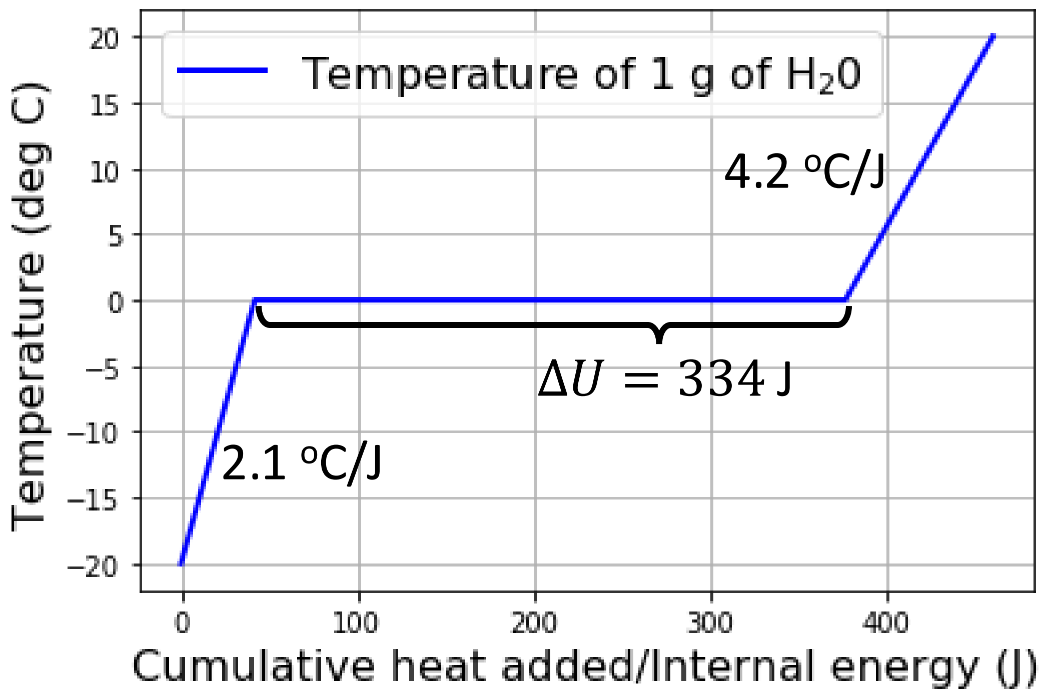
The following facts were known:

- It takes 1 calorie to raise 1 g of water 1 deg C
- It takes 0.5 calories to raise 1 g of ice 1 deg C
- It takes 80 calories to melt 1 g to ice
- It takes 540 calories to evaporate 1 g of liquid water to vapor

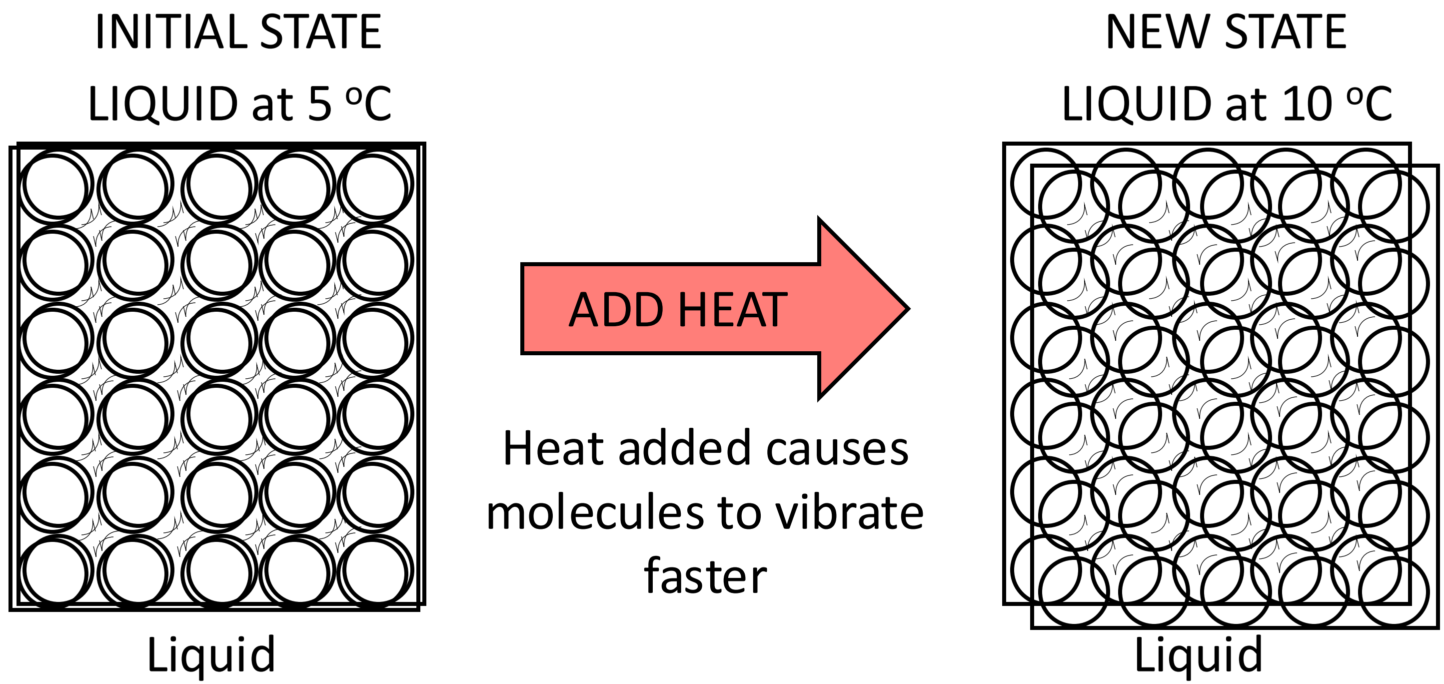


Heat and work

Since we now recognize that **heat** and **work** are equivalent, we use Joules, J, to represent both (though for frozen soil hydrology this doesn't actually have any implications as we don't need to convert heat to work or vice versa). 1 Cal = 4.186 Joules. So the plot of internal energy vs Temperature is:



Sensible heat



When heat is added to an object, the first thing that will happen is the molecules will absorb the heat and start to vibrate faster.

We experience faster vibrations as hotter temperatures

This is called sensible heat.

$$\Delta U_S = c_p \Delta(mT)$$

Temperature

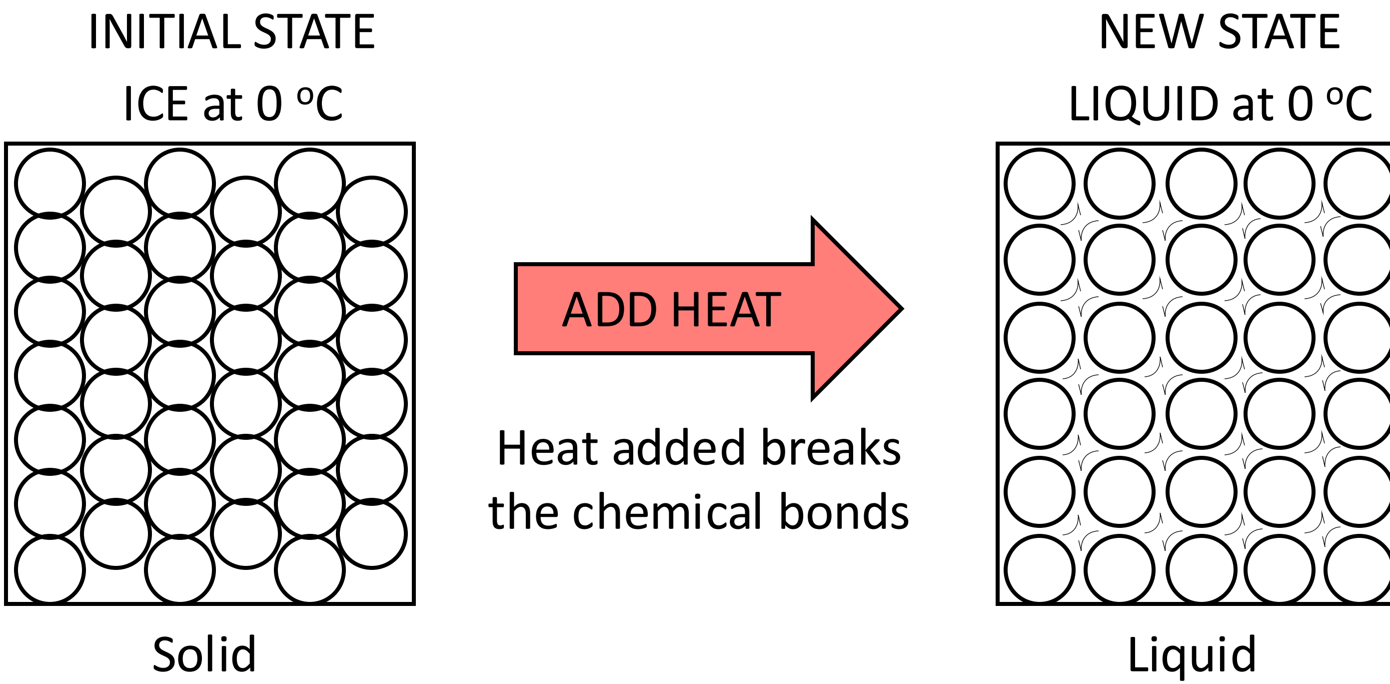
Temperature is normally measured in ^{o}C (degrees Celsius or centigrade) or in degrees Kelvin

	^{o}C	K
Liquid water freezes (at 1 atm)	0	273.15
Liquid water boils (at 1 atm)	100	373.15
Triple point (liquid, ice and vapor are stable)	0.01	273.16

Note that ΔT (^{o}C) $= \Delta T$ (K)

Temperature is associated with sensible heat. Temperature is sensed as hot or cold, but in reality it corresponds to hot fast the molecules are all vibrating.

Latent heat



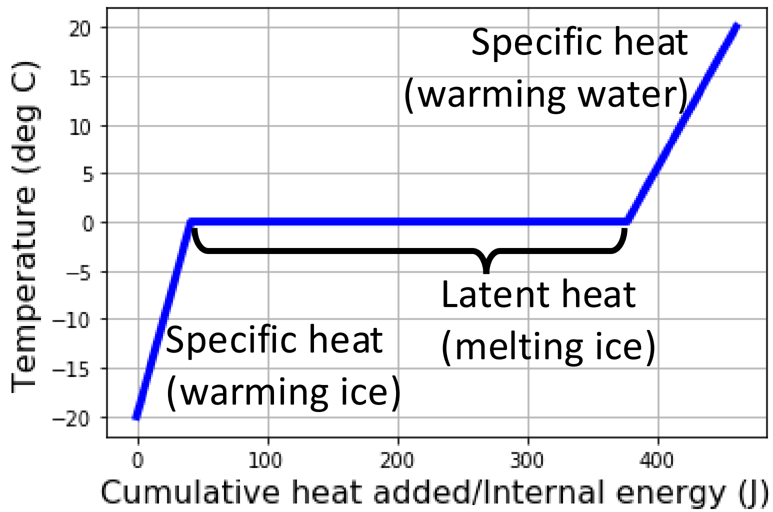
When the molecules in a solid are vibrating really fast, any additional heat absorbed will cause the chemical bonds to break.

This results in a change in phase – solid to liquid (or later liquid to gas)

This is called latent heat.

$$\Delta U_L = \lambda \Delta m$$

Sensible and latent heat of free water



Properties of water:

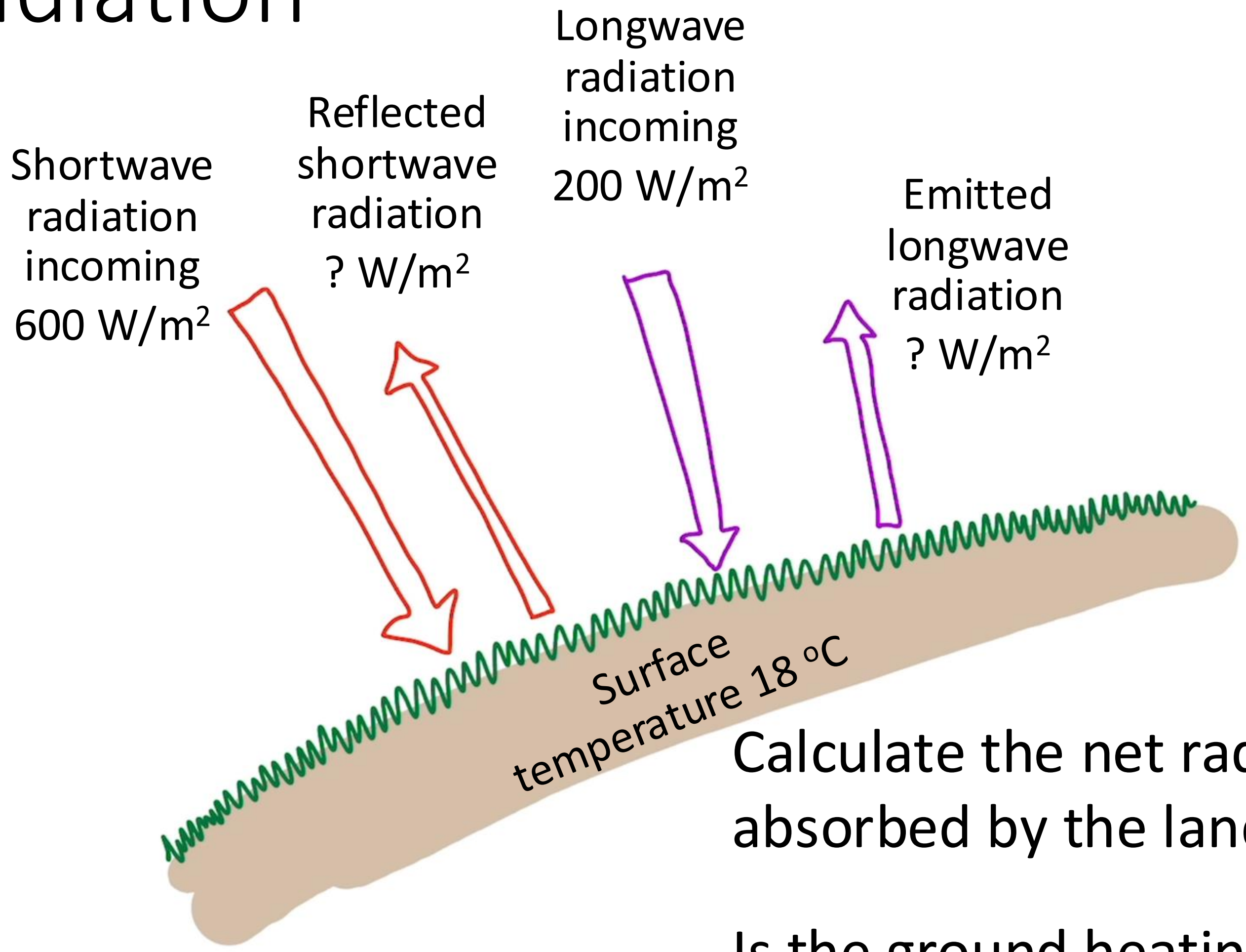
Specific heat capacity of water, $c_{p,liq} \approx 4180$ J/K/kg

Specific heat capacity of ice, $c_{p,ice} \approx 2100$ J/K/kg

Latent heat of fusion of water, $\lambda_f = 0.334 \times 10^6$ J/kg

Latent heat of vaporization of water, $\lambda_v = 2.26 \times 10^6$ J/kg

Worked examples
1) Radiation



Calculate the net radiation absorbed by the land surface.

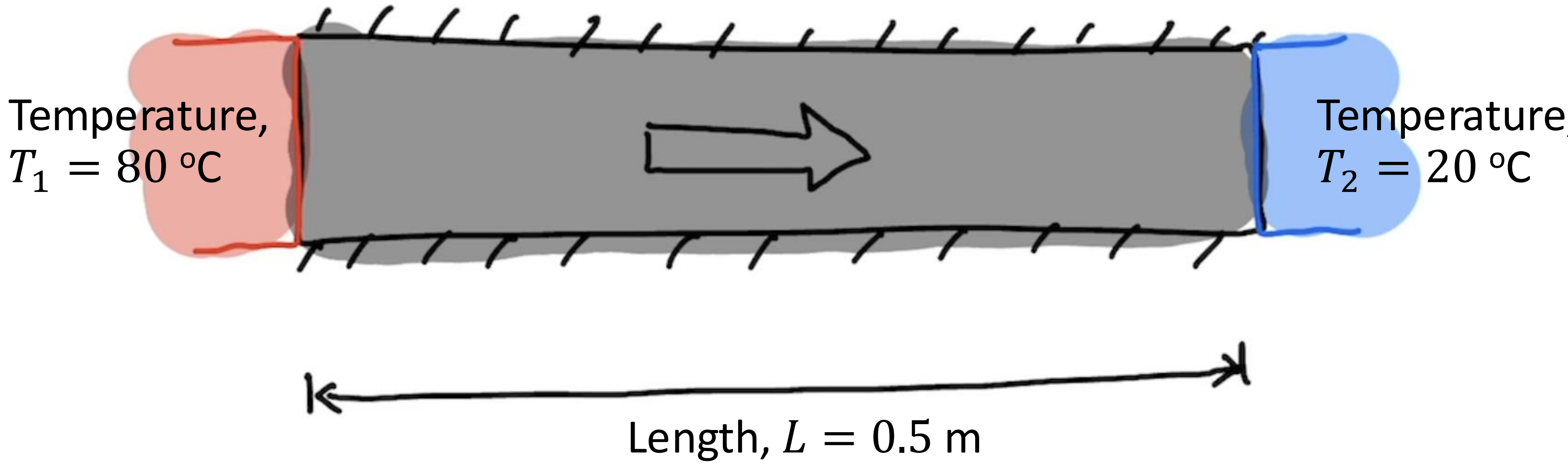
Is the ground heating or cooling?

Albedo, $\alpha = 0.2$

Emissivity, $\varepsilon = 0.9$

Stephan Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m²/K

Worked examples
2) Conduction



Calculate the heat flux through the bar above at steady-state conditions, assuming

1) $\kappa = 0.6$ W/m/K (equivalent to liquid water)

2) $\kappa = 2.2$ W/m/K (equivalent to ice)

3) $\kappa = 0.025$ W/m/K (equivalent to air)

Which material (liquid water, ice or air) is the best insulator?

Worked examples
3) Advection

Flow:

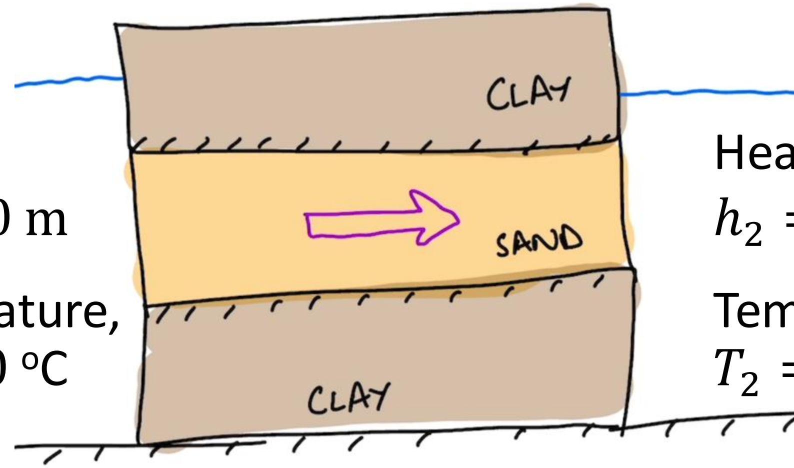
$$q = -K\frac{dh}{dx} \quad \text{m/d}$$

Relative advected heat flux:

$$j = c_p\rho q(T_2 - T_1)$$

Head,
$h_1 = 20$ m

Temperature,
$T_1 = 20$ °C



Head,
$h_2 = 15$ m

Temperature
$T_2 = 19$ °C

Length, $L = 100$ m

Aquifer hydraulic conductivity, $K = 0.1$ m/d

Calculate the relative advected heat flux through the confined aquifer shown.

Worked examples
4) Sensible heat

Consider that internal energy from sensible heat is given by

$$\Delta U = c_p \Delta(mT)$$

Assume we initially have 1 kg of liquid water at 5 ^{o}C. What is the internal energy?

We add to this 0.5 kg of liquid water at 5 ^{o}C. What is the new temperature and internal energy?

Next we add another 0.5 kg of liquid water at 20 ^{o}C. What is the new temperature and internal energy?

For liquid water ($c_p = 4180$ J/kg/K)

Worked examples
5) Sensible heat

Consider that internal energy from sensible heat is given by

$$\Delta U = c_p \Delta(mT)$$

Assume we initially have 1 kg of liquid water at 5 ^oC.

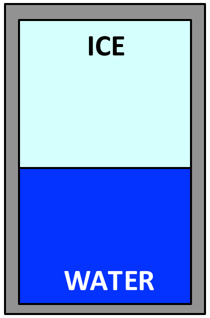
We add to this 0.5 kg of liquid water at 0 ^oC. What is the new temperature and internal energy?

Describe qualitatively what happened to the energy and temperature of the water. How can you explain this?

For liquid water ($c_p = 4180$ J/kg/K)

Worked examples
6) Sensible and latent heat

Imagine that in a perfectly insulated container we combine 0.5 kg of ice at 0^{o}C with 0.5 kg of liquid water at 0^{o}C. What happens?



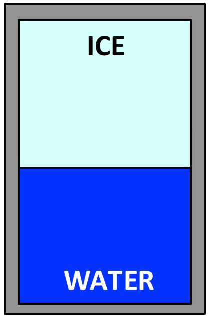
Specific heat capacity of water, $c_w \approx 4180$ J/K/kg

Specific heat capacity of ice, $c_i \approx 2100$ J/K/kg

Latent heat of fusion of water, $L_f = 0.334 \times 10^6$ J/kg

Worked examples
7) Sensible and latent heat

Now, imagine that we do the same experiment with 0.5 kg of ice at -4°C with 0.5 kg of liquid water at +1°C. What happens?



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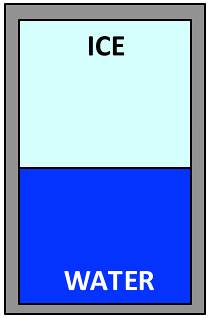
Latent heat of fusion of water, $L_f = 0.334 \times 10^6$ J/kg

Worked examples
8) Sensible and latent heat

Now, imagine that we do the same experiment with 10 kg of ice at -10ºC with 0.5 kg of liquid water at +0ºC. What happens?



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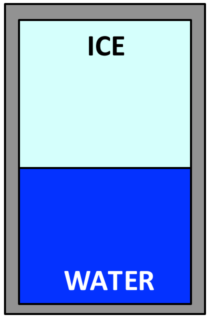
Worked examples
9) Sensible and latent heat

Now, imagine that we do the same experiment with 0.5 kg of ice at -1^{o}C with 10 kg of liquid water at +20^{o}C. What happens?



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