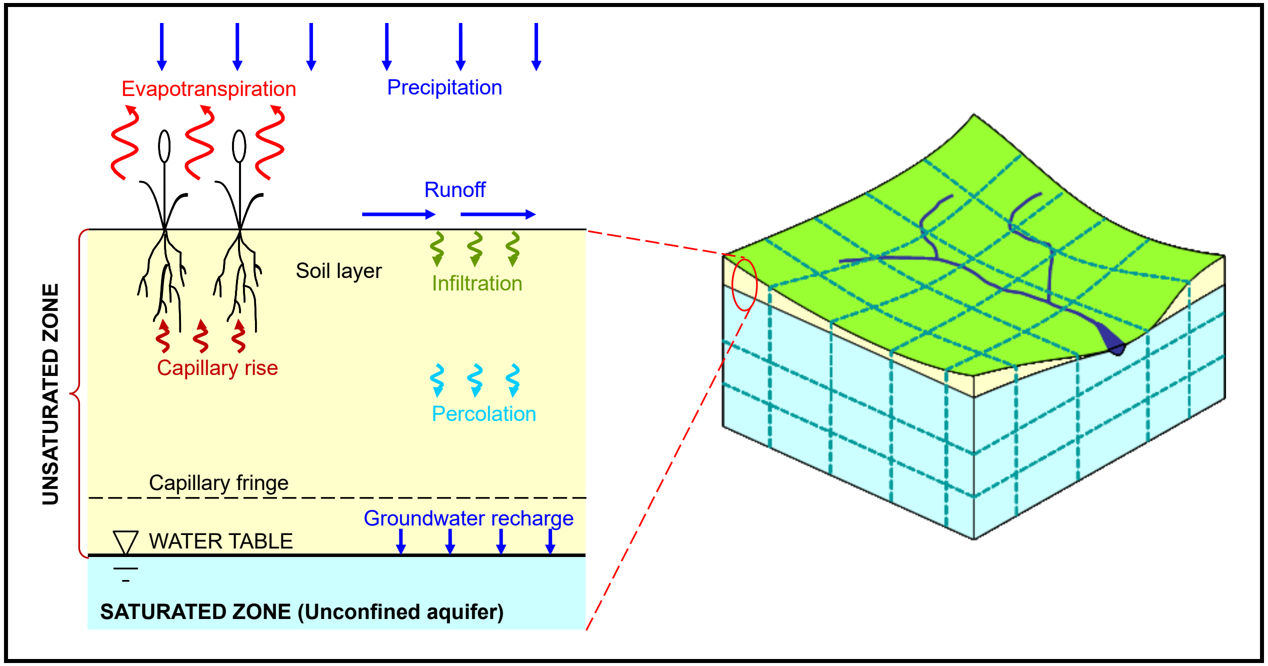
Cold regions hydrology

Andrew Ireson andrew.ireson@usask.ca
School for Environment and Sustainability/Global Institute for Water Security

4. Unfrozen soil processes



Learning Objectives

After this lecture you should be able to:

- Describe the physical nature of soils and soil mass/volume relations
- Understand water content and storage, how they are quantified and how these quantities are related
- Write the soil water balance equation, describing all the states and boundary fluxes
- Understand the concepts of field capacity, permanent wilting point, and root available water
- Understand water content and matric potential and their inter-relationship through the soil moisture characteristic curve
- Understand hydraulic conductivity in variably saturated soils
- Sketch profiles of water content and matric potential for hydrostatic and steady-state infiltration scenarios
- Understand the use of Richards' Equation to simulate soil moisture dynamics

INTRODUCTION TO SOILS AND THEIR HYDROLOGICAL SIGNIFICANCE

Soils: the critical zone

Soils *make all the important decisions* about what a watershed will do with the water it receives from precipitation:

- How much water infiltrates into the ground vs runs off into streams/lakes/wetlands?
- Which plants are supported and hence how much water is lost to evapotranspiration
- How much water will be stored through drier periods to support vegetation, including crops
- How much water will be passed on to the aquifers as groundwater recharge

Hence, soils are ***"the critical zone"*** of the watershed

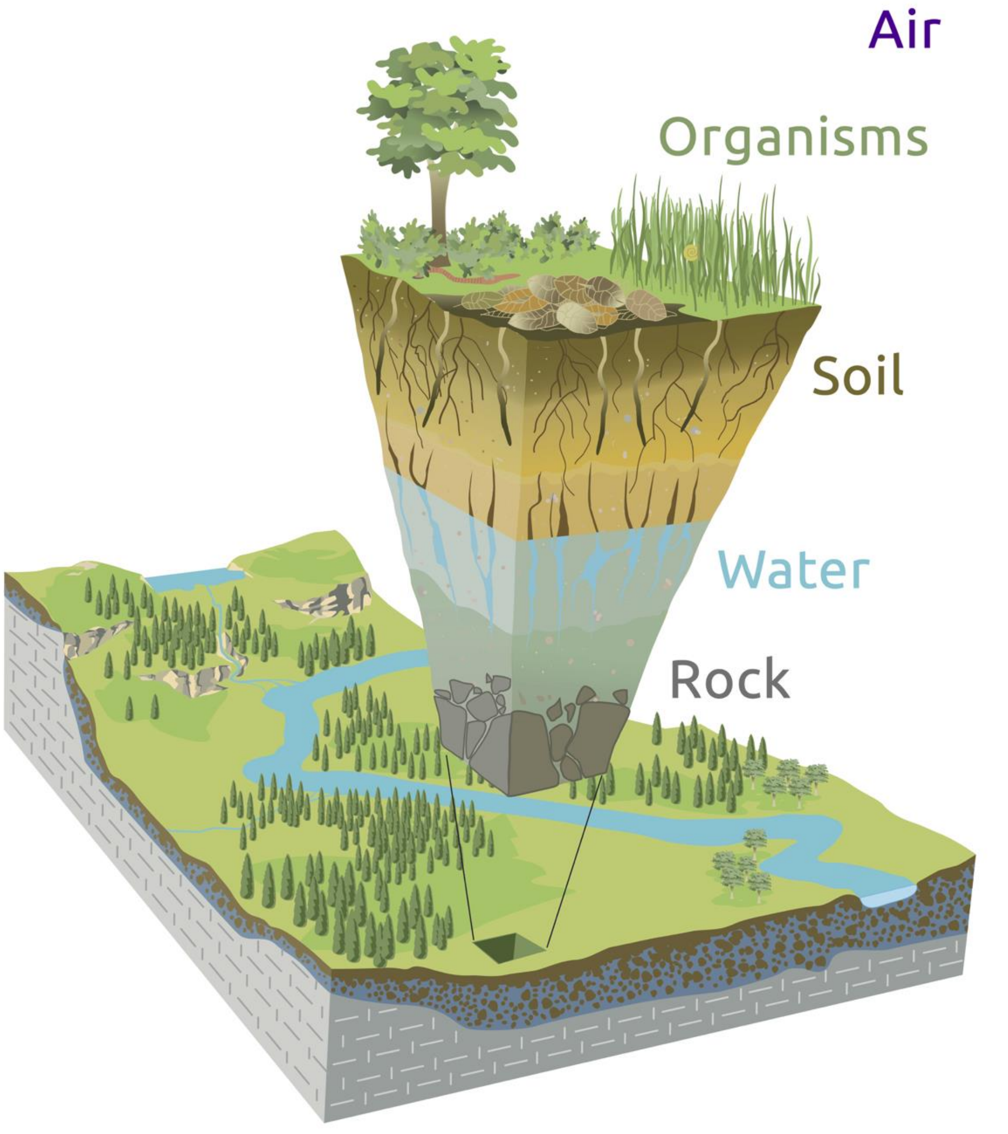
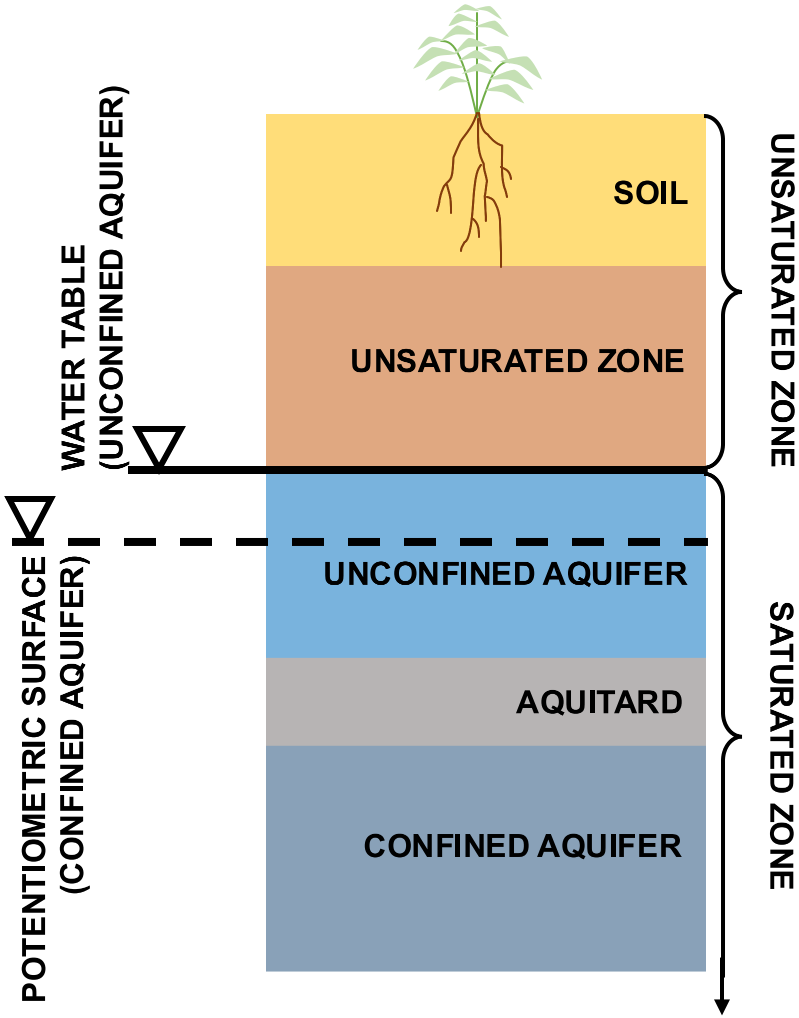


Image: Chorover, J., R. et al., 2007. Soil biogeochemical processes in the critical zone. Elements 3, 321-326. (artwork by R. Kindlimann). Accessed from http://criticalzone.org/

Soil: a definition

- Soil can be defined[1] as the weathered and fragmented outer layer of the earth's surface, formed by disintegration and decomposition of rocks by physical and chemical processes, and by the activity and accumulation of residues of numerous plants and animals.
- The soil is the upper part of the unsaturated zone, where the pore-space is normally filled with both air and water
- The soil is generally where the roots are located, while the unsaturated zone extends down to the water table.



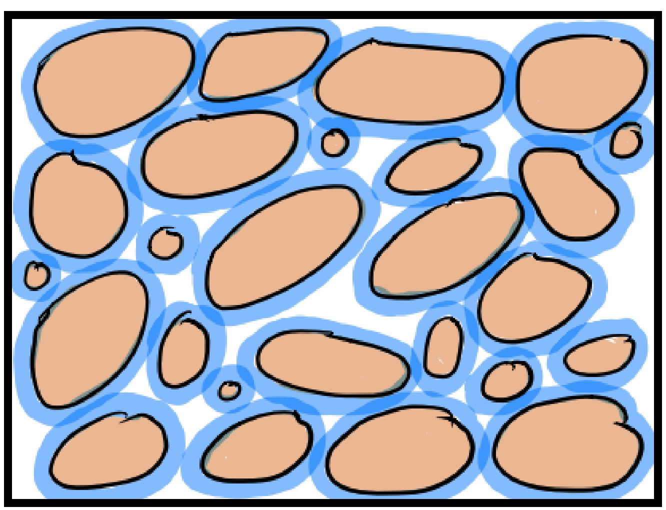
1. Hillel, D. 1998. Environmental Soil Physics, p. 3

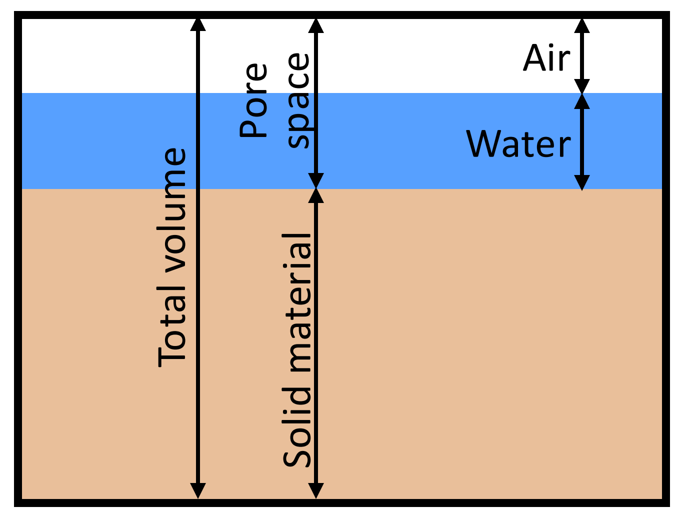
Soil composition

Soil comprises material in three phases:

Occupy the **pore space**: Liquid, Gas

		Volume (L^3)	Mass (M)
Solid	Soil matrix	V_S	M_s
Liquid	Water	V_L	M_L
Gas	Air	V_G	M_G **(negligible)**
	Total	$V_T = V_s + V_L + V_G$	$M_T = M_s + M_L$





Static mass/volume properties

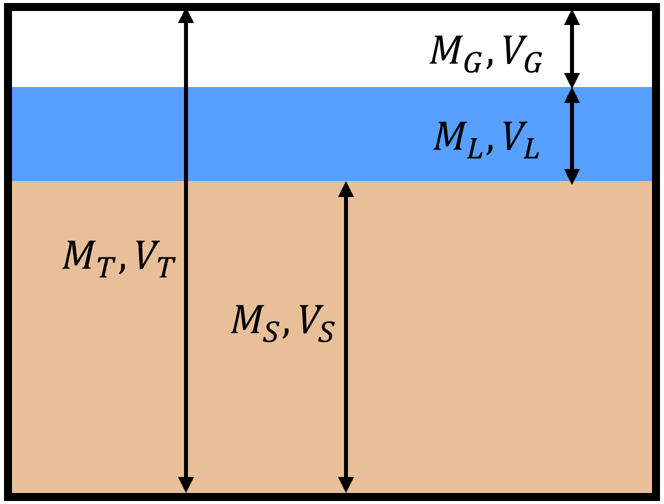
The following simple properties are typically assumed to be constants for a given soil:

Porosity (m^3/m^3) $$n = \frac{V_L + V_G}{V_T}$$

Void ratio (m^3/m^3) $$e = \frac{V_L + V_G}{V_S}$$

Dry bulk density (kg/m^3) $$\rho_b = \frac{M_S}{V_T}$$

Particle density (kg/m^3) $$\rho_s = \frac{M_S}{V_S}$$



$$V_T = V_L + V_G + V_S$$

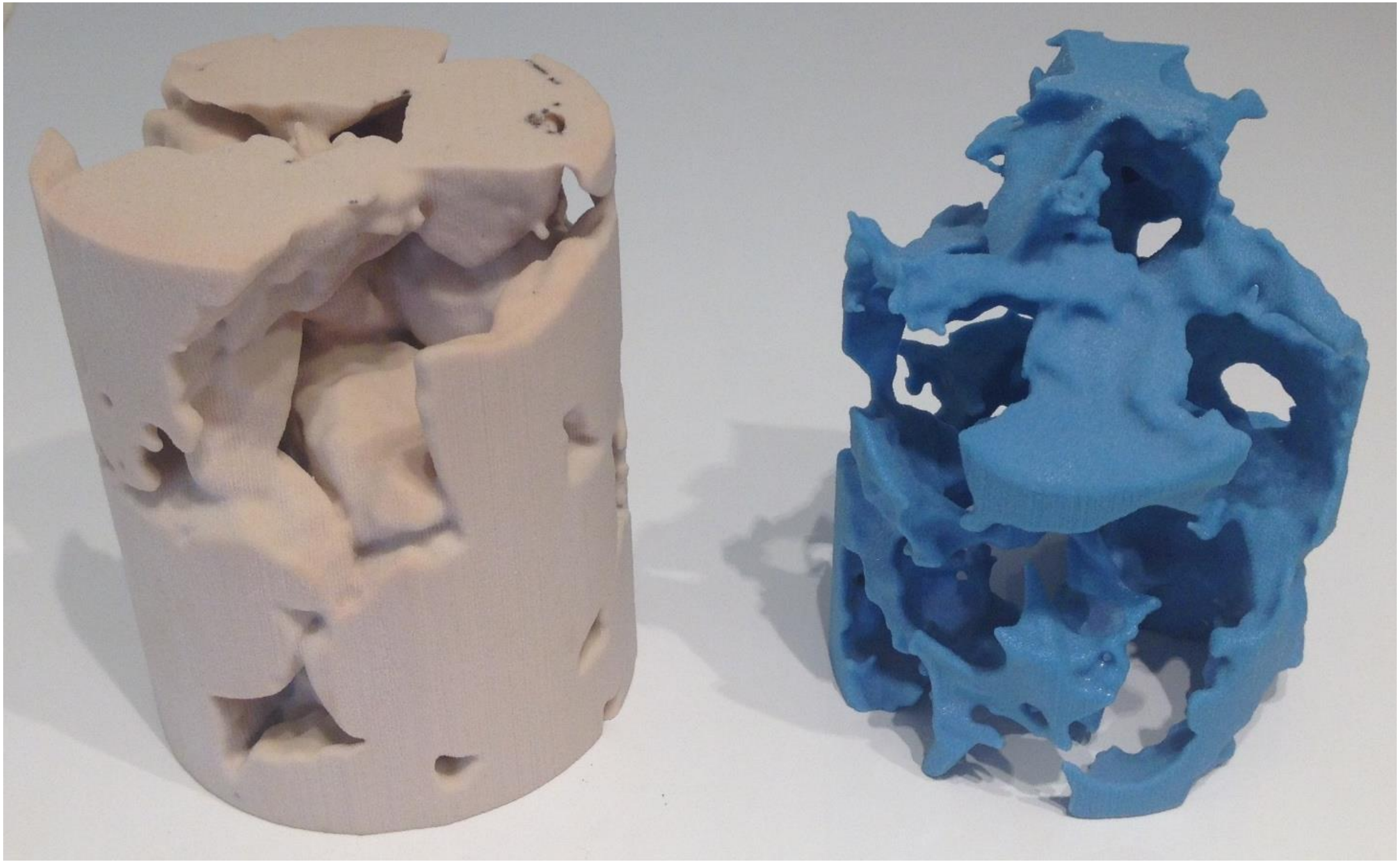
$$M_T = M_L + \cancel{M_G} + M_S$$

negligible

Note that
$$n = \frac{e}{1+e} = \frac{\rho_s - \rho_b}{\rho_s}$$

Recall generally that density (ρ) is defined as mass per volume (M/V), and that mass is always conserved, while volume is often not (i.e. things expand/contract)

Pore space



Magnified 3D printing of the pore space

Solid phase of the soil

The solid phase of the soil consists of particles of varying shape and size, packed in various ways. Soil can be classified on the basis of particle size.

Particles < 2 µm diameter are clay (chemically and physically reactive)

Particles > 2 µm diameter are non-clay fraction (inert, silt, sand or gravel)

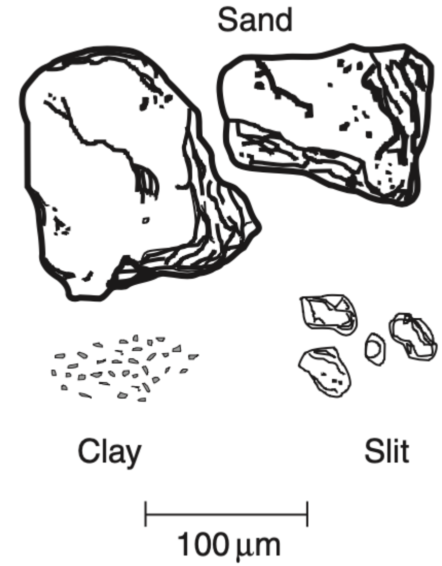
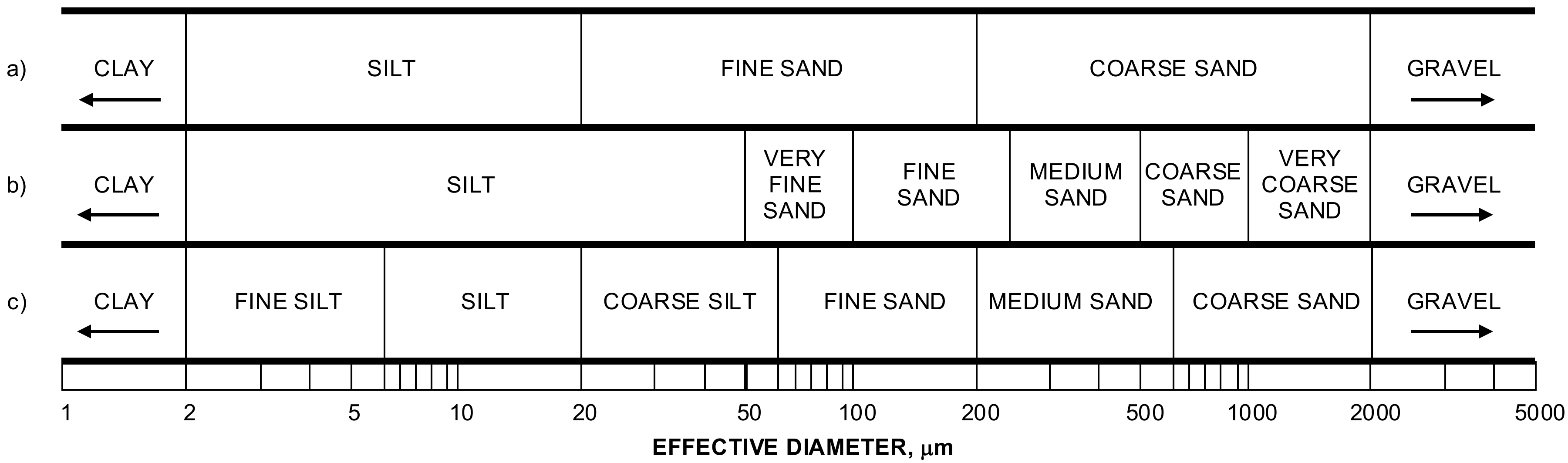


Image above from Hillel, D. 2004. Introduction to Environmental Soil Physics, p. 41

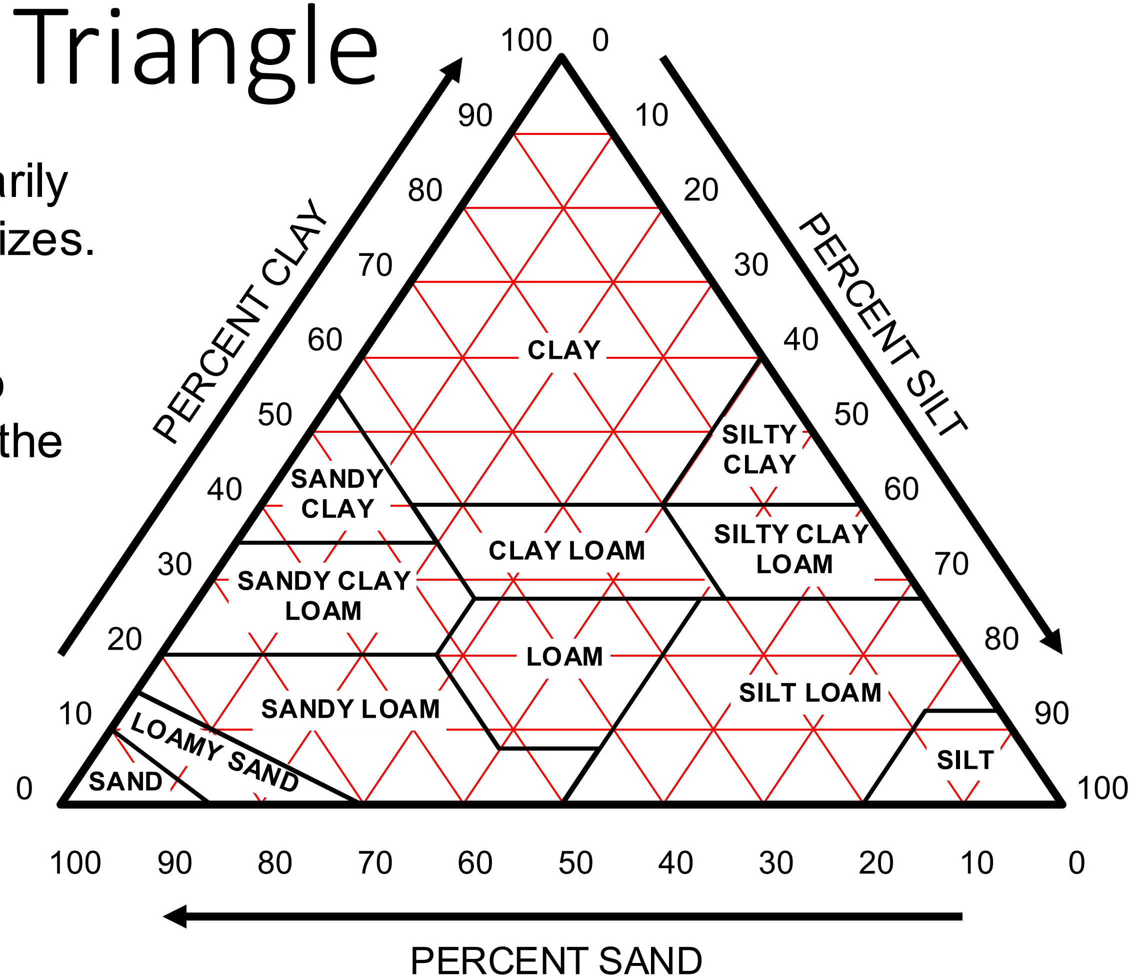


a) International Society of Soil Science (ISS).
b) United States Department of Agriculture (USDA).
c) British Standards Institution (BSI).

USDA Textural Triangle

Any given soil does not necessarily contain particles with **uniform** sizes.

Rather, there is a particle size distribution. A convenient way to describe such a soil is by using the textural triangle, on the basis of the silt, sand and clay content:



Particle diameters

Clay: below 2 µm

Silt: 2 to 50 µm

Sand: 50 to 2000 µm

Exercise:

Classify the following soils using the textural triangle

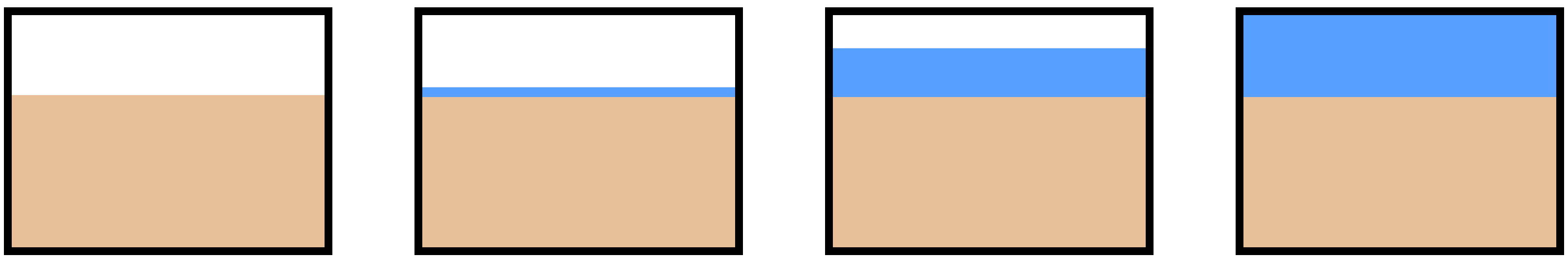
% Sand	% Clay	% Silt	Classification
82	5		
33		33	
	66	14	

Dynamic state variables

The volume/mass of water and air in the soil pores changes with time, and therefore, rather than "properties" we define "state-variables" that describe these.

The most commonly used state-variable is the volumetric water content:

$$\theta = V_L/V_T$$



Oven dry soil	Effectively dry soil (Residual water content)	Moist soil	Saturated soil
$\theta = 0$	$\theta = \theta_R$	$\theta_R < \theta < \theta_S$	$\theta = \theta_S = n$

Where θ_R is the residual water content and θ_S is the saturated water content. We also define the effective saturation, S_e as

$$S_e = \frac{\theta - \theta_R}{\theta_S - \theta_R} \qquad 0 \leq S_e \leq 1$$

MEASUREMENT OF SOIL WATER CONTENT

1. Gravimetric method

The most direct measurement of soil moisture is obtained by taking a core from the field using an auger, measuring its mass and volume, then drying in an oven and taking the new mass. We have

$M_{wet} =$ mass of sample immediately after it's taken (kg)

$M_{dry} =$ mass of sample after drying in an oven at 105°C for 24 hours (kg)

$V_{core} =$ volume of core sample (m^3)

From this we find the gravimetric water content, θ_G (kg/kg):

$$\theta_G = \frac{M_L}{M_S} = \frac{M_{wet} - M_{dry}}{M_{dry}}$$

And the volumetric water content is given by

$$\theta = \theta_G \frac{\rho_b}{\rho_L} \qquad \text{where } \rho_b = \frac{M_{dry}}{V_{core}}$$

1. Gravimetric method

Advantages:

- Only truly direct method – no calibration

Disadvantages:

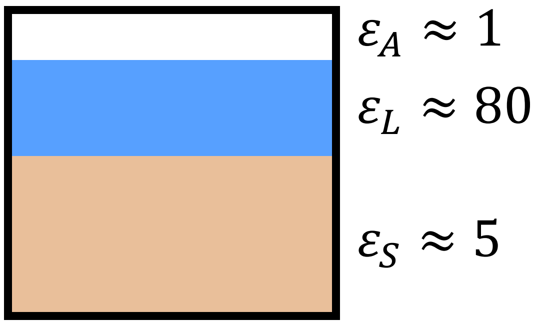
- Laborious and time consuming
- Soil corers tend to compress the soil – error in volumetric water content
- Invasive and destructive
- One-off measurement

Practical uses:

- Used as basis for calibration of indirect methods

2. Dielectric methods

The ”dielectric constant”, ε, is a measurable material property (enough said!). Water happens to have a highly distinct dielectric constant, compared with soil and air:



Hence, if the volume weighted bulk dielectric constant, ε_b, of a control volume of soil is measured, we can write out the mixing equation[1], as

$$\varepsilon_b^\alpha = \varepsilon_L^\alpha \theta + \varepsilon_S^\alpha (1-n) + \varepsilon_A^\alpha (n-\theta)$$

And this can be solved for θ

1. Kelleners, T. J., & Norto, J. B. (2012). Determining Water Retention in Seasonally Frozen Soils Using Hydra Impedance Sensors. *Soil Science Society of America Journal*, *76*(1), 36. https://doi.org/10.2136/sssaj2011.0222

2. Dielectric methods

Advantages:

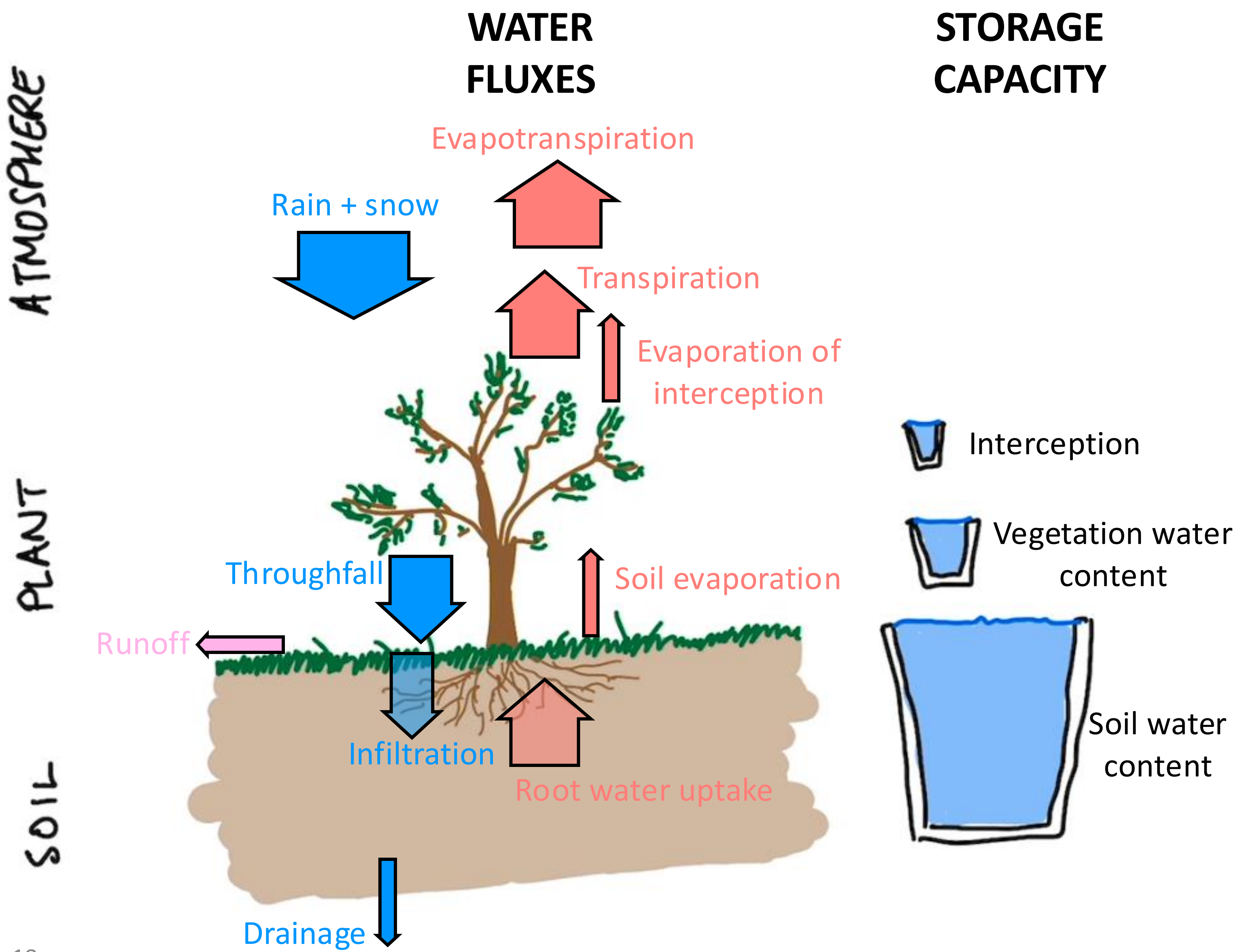
- Relatively cheap
- Can be continuously logged

Disadvantages:

- Accuracy and precision still subject of some debate
- Manufacturers standard calibrations can be inadequate

THE SOIL WATER BALANCE

Soil and plant water balance

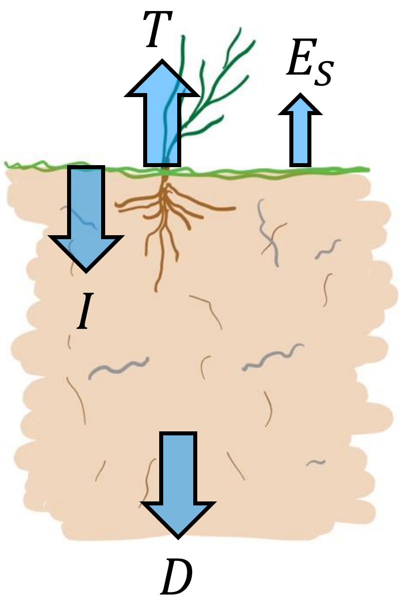


Soil water balance

Neglecting the vegetation canopy, the basic soil water balance equation is

$$\frac{dS}{dt} = I - E_s - T - D$$

Where I is infiltration (mm/d), E_S is soil evaporation (mm/d), T is transpiration (mm/d) and D is drainage (mm/d).



To solve the water balance, we need to quantify each of these fluxes.

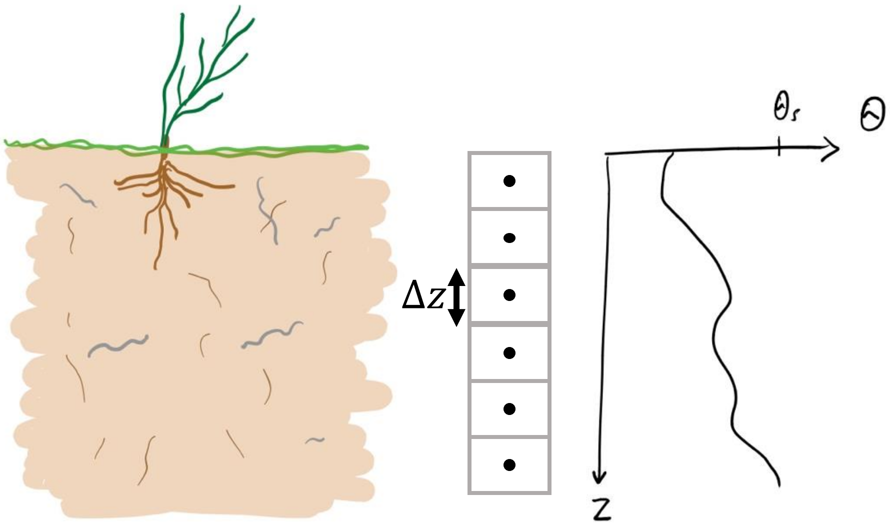
Infiltration: is the amount of rainfall or snowmelt that can enter the soil and not runoff. It depends on the properties of the soil and how wet the soil is (and whether the soil is frozen)

Evaporation and transpiration: depend on the meteorological conditions, soil properties and vegetation. Sometimes E_S is ignored for vegetated surfaces.

Drainage: is the gravity driven percolation of water down into the ground, forming recharge to aquifers. This depends on the soil properties and wetness of the soil.

Soil storage

Water content is given by $\theta = \frac{V_L}{V_T}$



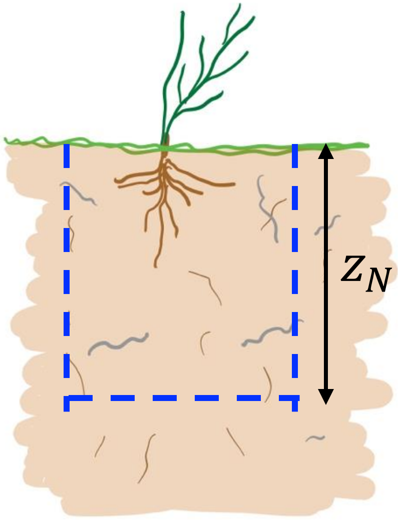
Water content is measurable and is useful when we want to look at the depth distribution of water in the soil/unsaturated zone. We discretize the soil into cells, and account for the water content in each cell.

When we are looking at **water balances**, storage, S, is a more useful metric.

S is the amount of water in the complete soil profile, expressed as a depth. In other words, if you removed all the soil moisture and ponded on the surface, this is how deep it would be. Storage can be found from

$S = \sum_{i=1}^{n} \theta_i \Delta z$ or in continuous notation $S = \int_{z=0}^{L} \theta dz$

Soil storage thresholds



Consider a soil profile of some depth, z_N

The soil would be fully saturated when all the pore-space is water filled, i.e.

$$S_{sat} = \int_{z=0}^{z_N} n.\,dz = n.\,z_N$$

In practice a soil will rarely remain saturated for long, because of gravity drainage. If an initially saturated soil is left for a period of days with no rainfall, it will eventually stop draining (or drainage will become negligibly small). This defines the soil's *field capacity:*

$S_{FC} =$ amount of storage after gravity drainage stops

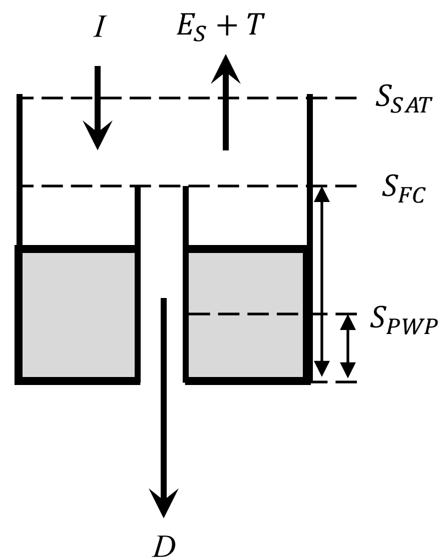
However, it is still possible to reduce the storage further by evapotranspiration. Transpiration will dry the soil until there is insufficient water for the plants to access, and they die. This storage threshold is know as *permanent wilting point:*

$S_{PWP} =$ minimum amount of storage possible in the field, at which point further transpiration losses are impossible

(Note: soils can be oven dried below S_{PWP}, but typically not in the field)

Soil storage thresholds

We can depict these storage thresholds on a bucket-type conceptual model for the soil:



Note, all of these parameters have units of length, and represent an equivalent depth of ponded water.

As is clear from the diagram, drainage can only occur when

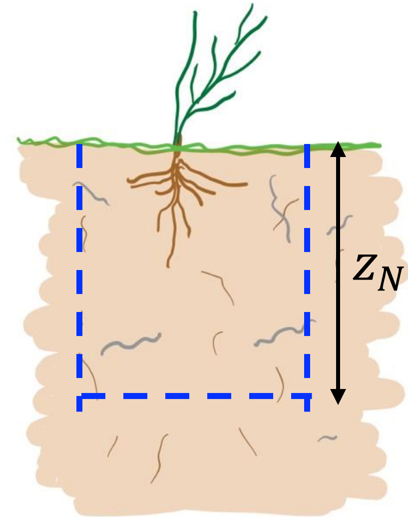
$$S_{FC} < S < S_{SAT}$$

And evapotranspiration can only occur when

$$S_{PWP} < S < S_{SAT}$$

Soil physics approach

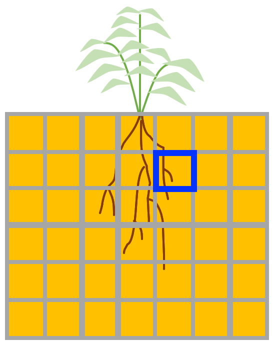
The concept of field capacity is based on treating the entire soil profile as a lumped entity. We know that within this volume the distribution of water (and temperature and pressure and solutes) is non-uniform and that the soil properties are heterogeneous. Therefore the quantity S_{FC} combines a number of sources of information (soil texture, depth, landscape position, vegetation).



With soil-physics, we attempt to deal with smaller volumes of soil where we can reasonably assume that the states and properties are uniform and homogeneous.

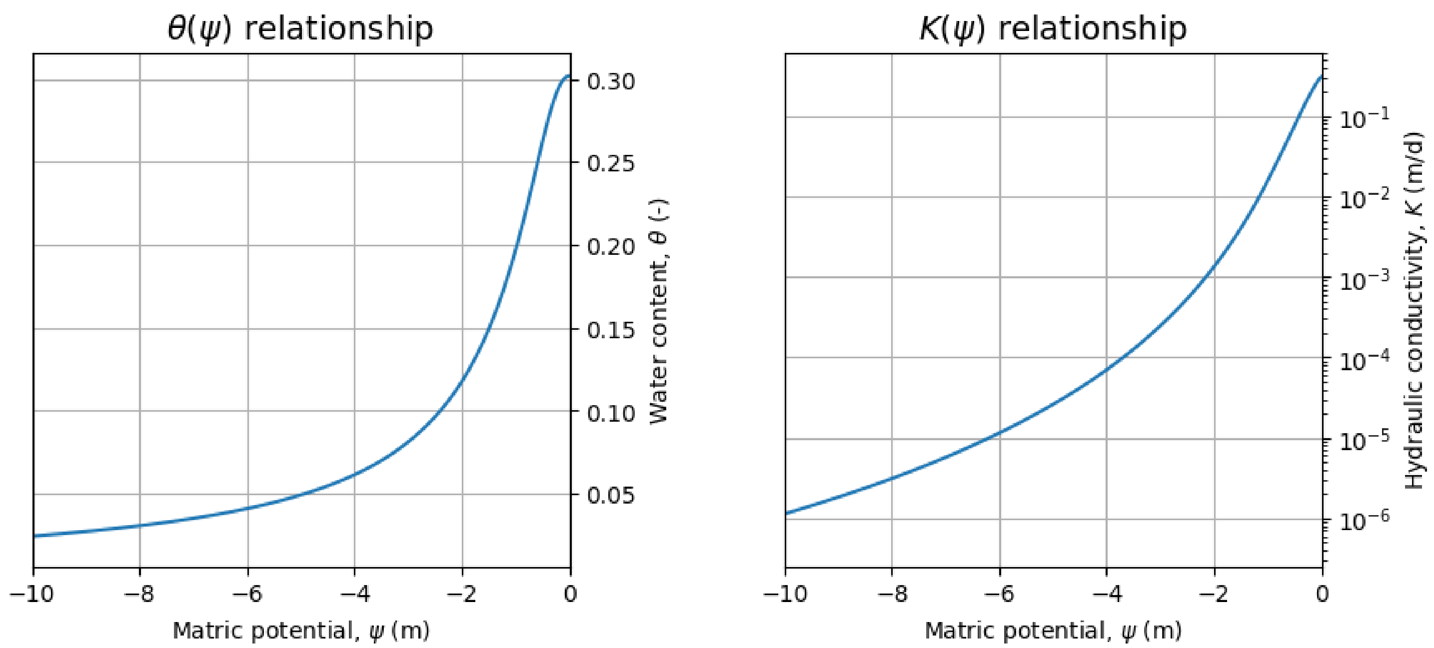
We call such a volume a "*representative-elementary-volume*".

We attempt to define more physically fundamental states and properties, and we attempt to use physical laws to explain the storage and movement of water.



Soil physics: what do we need to know

Soil physics approaches to modelling infiltration and drainage fluxes and water content dynamics invariably require estimates of the "***soil hydraulic properties***", which are provided in two graphs, e.g.:



Within these graphs we have three inter-related variables: **water content** θ, **matric potential** ψ (labelled above as soil water tension) and **hydraulic conductivity**, K.

In the following lecture, we will try to understand what each of these variables represents…. we will need to revise some basic physics…

Matric potential – a conceptual definition

- We can define matric potential as the suction that the soil exerts on the water within the soil pores, due to capillary forces and adhesion.
- It represents how tightly the water is held by the soil, and hence how much energy is needed to extract that water.
- In wet soils, it is very easy to extract the water, so the matric potential is high – i.e. a negative value close to zero (e.g. -1 m)
- In dry soils, it is harder to extract the remaining water, so the matric potential is very low, i.e. a large negative value (e.g. -50 m)

Total potential: hydraulic head

Considering the two forces of gravity and pressure, we have a total potential per unit weight of water (units of L) of

$$e_T = e_g + e_P$$

Which we can express as

$$e_T = z + \frac{P}{\rho g}$$

Here, e_T is exactly equivalent to **hydraulic head**, h (L), as used in groundwater. We hence define pressure head, p (L) as

$$p = \frac{P}{\rho g}$$

And we can write

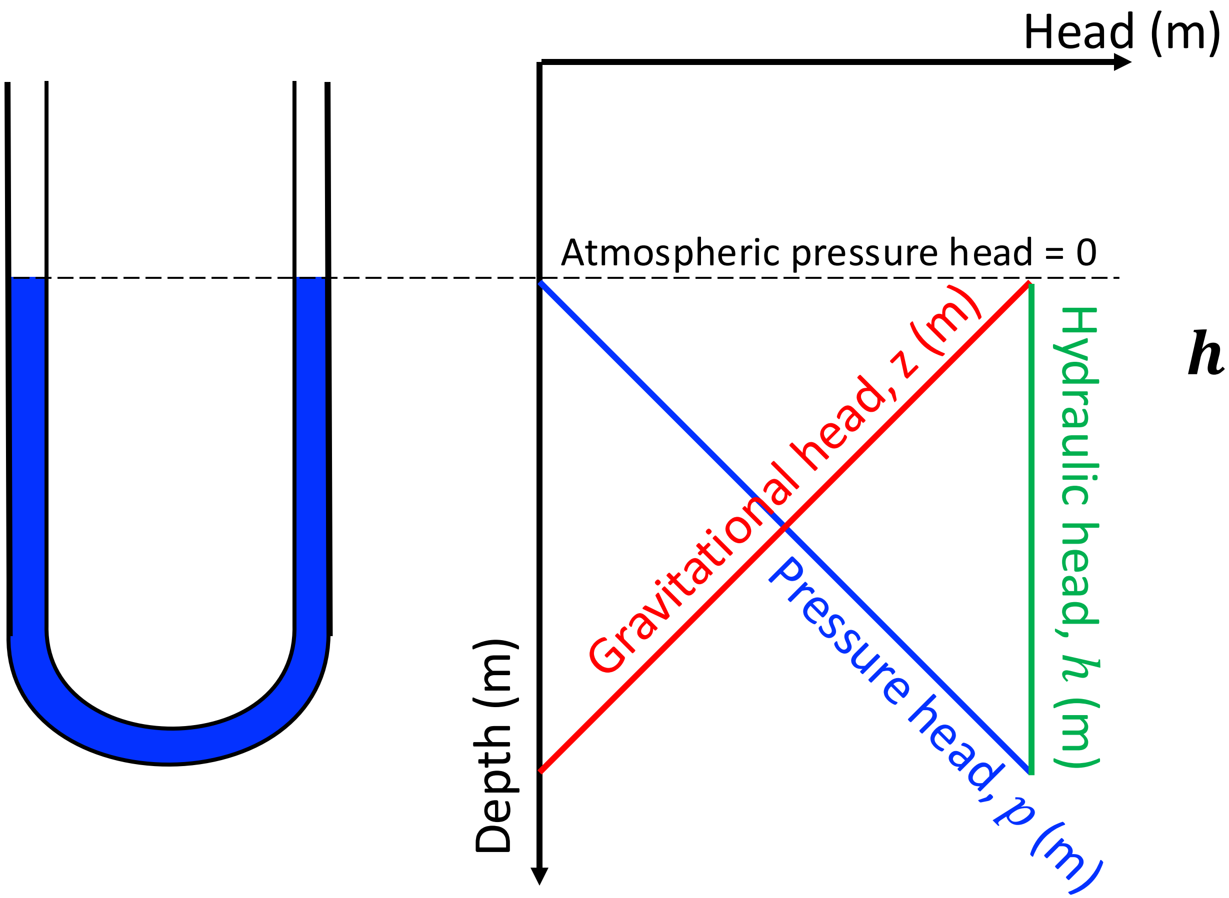
$$h = z + p$$

Flow in any fluid is driven by gradients in $\boldsymbol{h}$

Note, there are other potential energy terms, such as the kinematic energy and the osmotic potential, but for our purposes we can ignore these.

Hydraulic head

The hydraulic head in the u-tube is obtained by summing the gravitational and pressure heads, as shown:

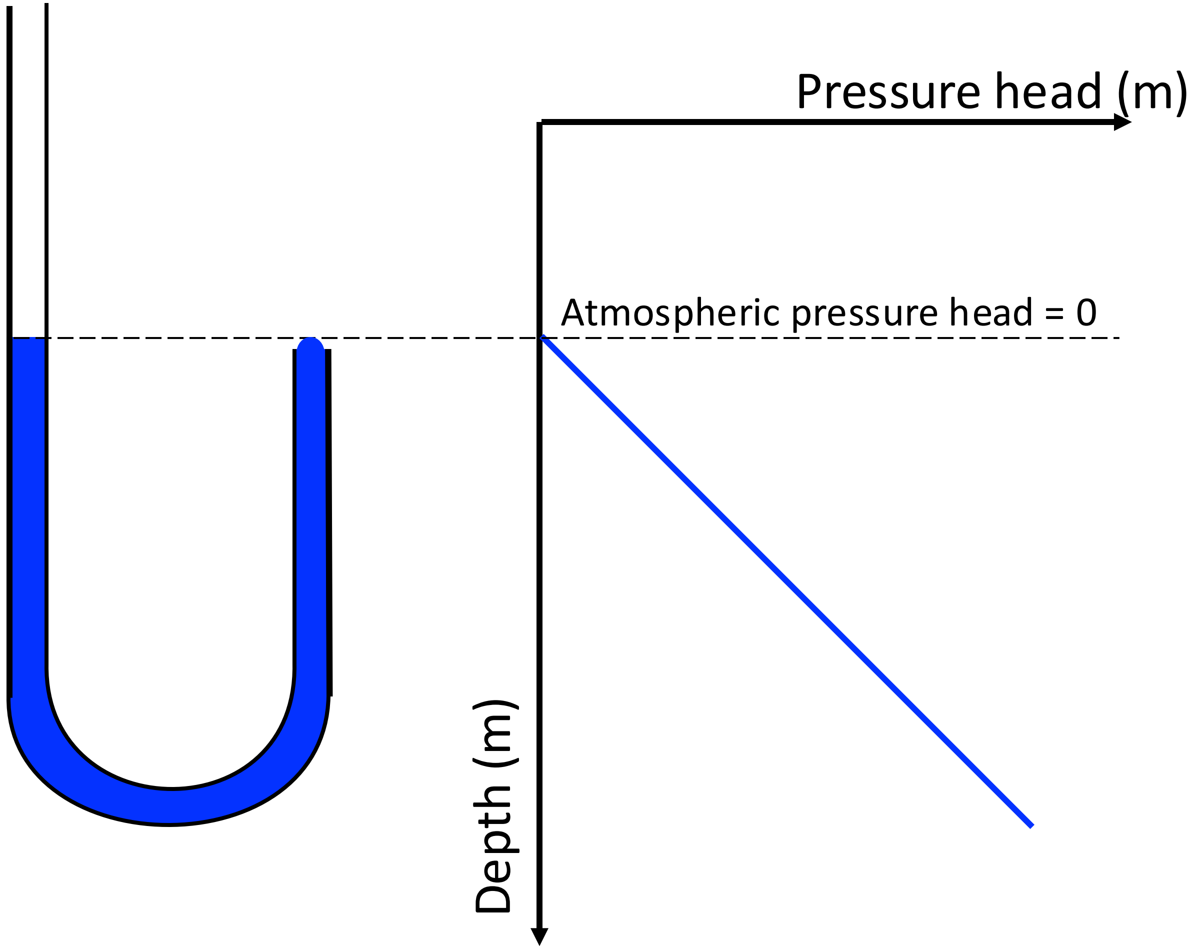


$$\boldsymbol{h = z + p}$$

In this case the hydraulic head is constant, indicating no gradient in h is present and hence there is no flow, and the system is hydrostatic.

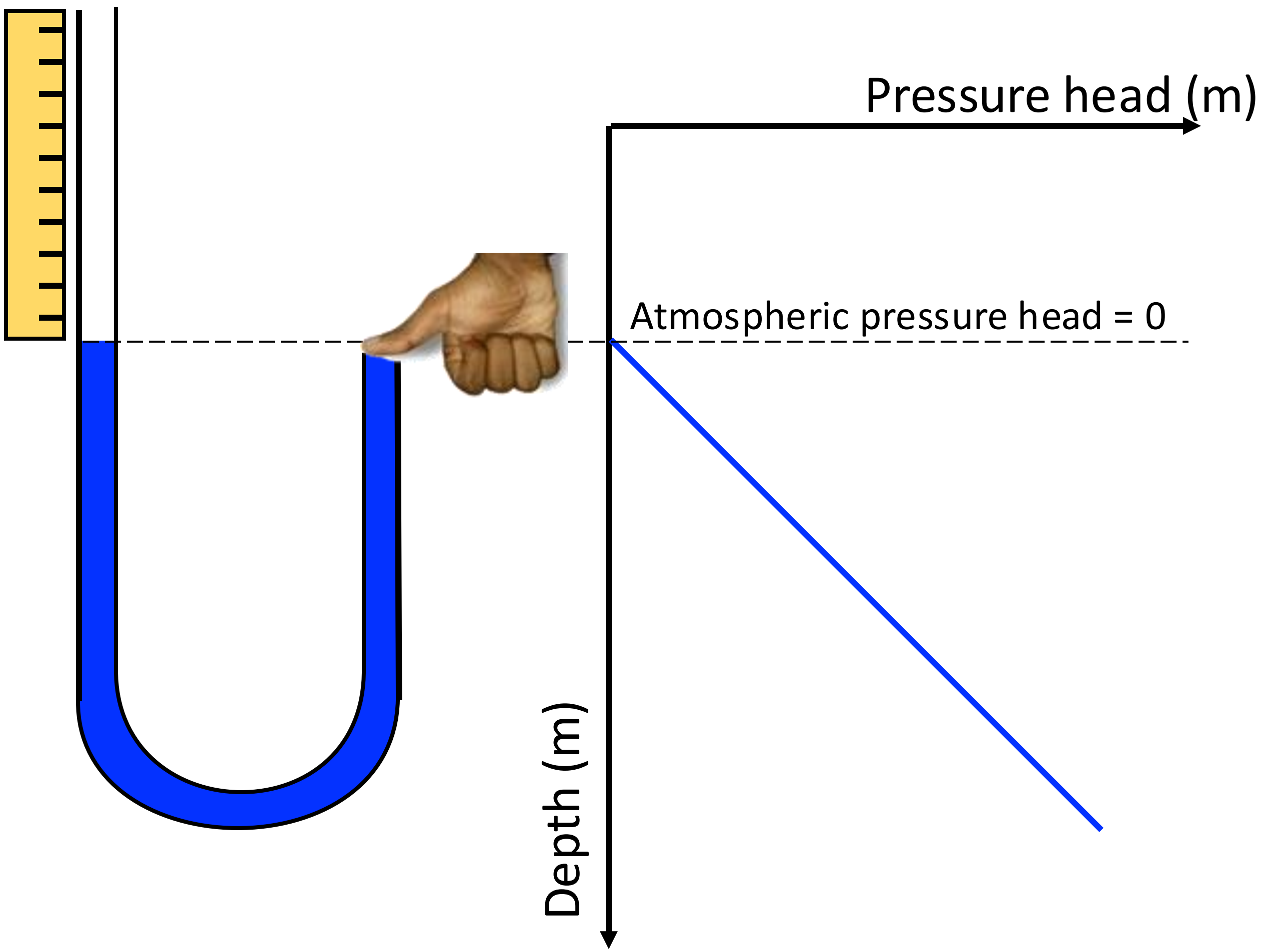
Negative pressures

Drop the right-hand side… nothing changes with the pressure head:



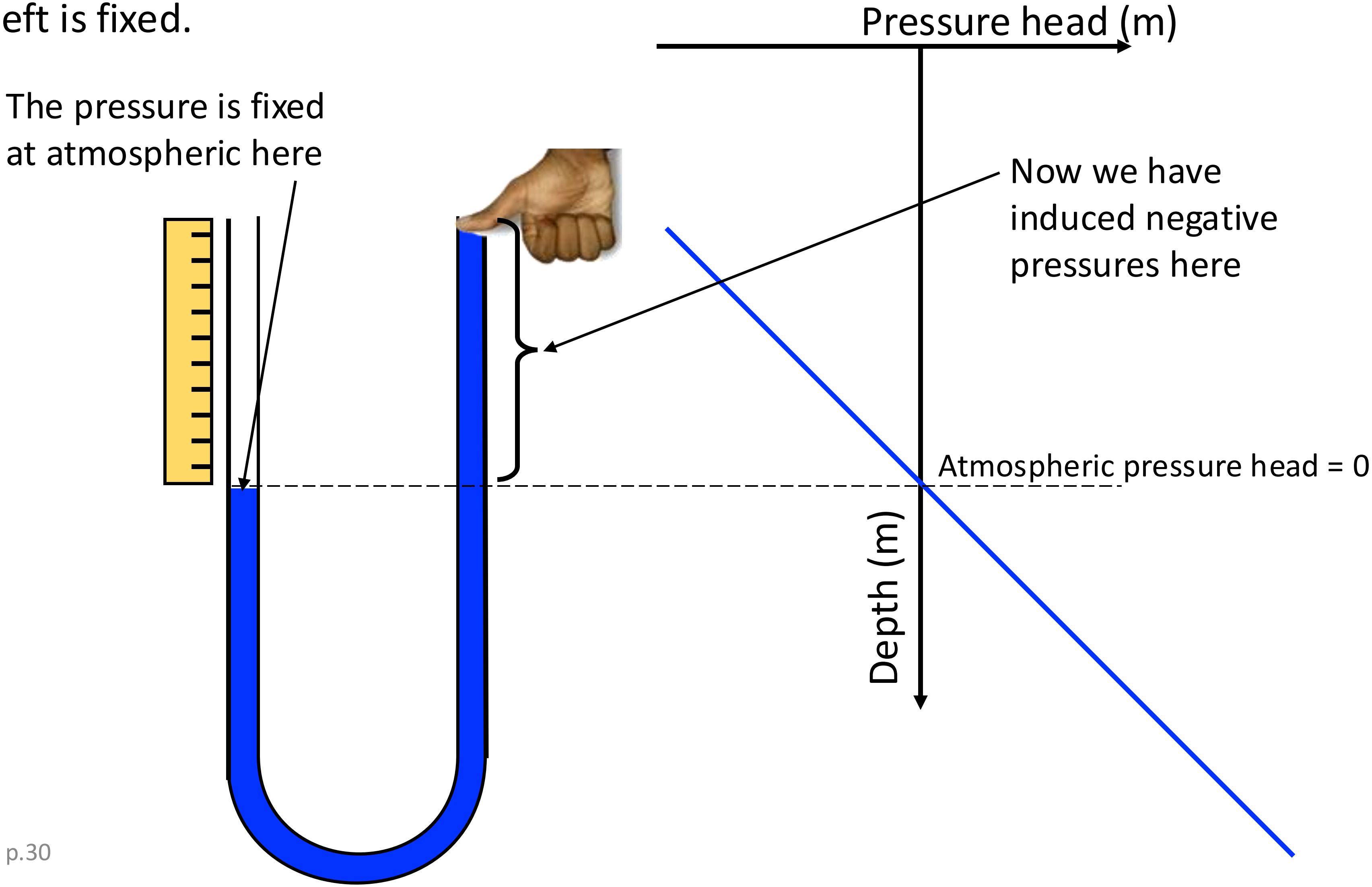
Negative pressures

Plug the right-hand side with your thumb – still nothing changes:



Negative pressures

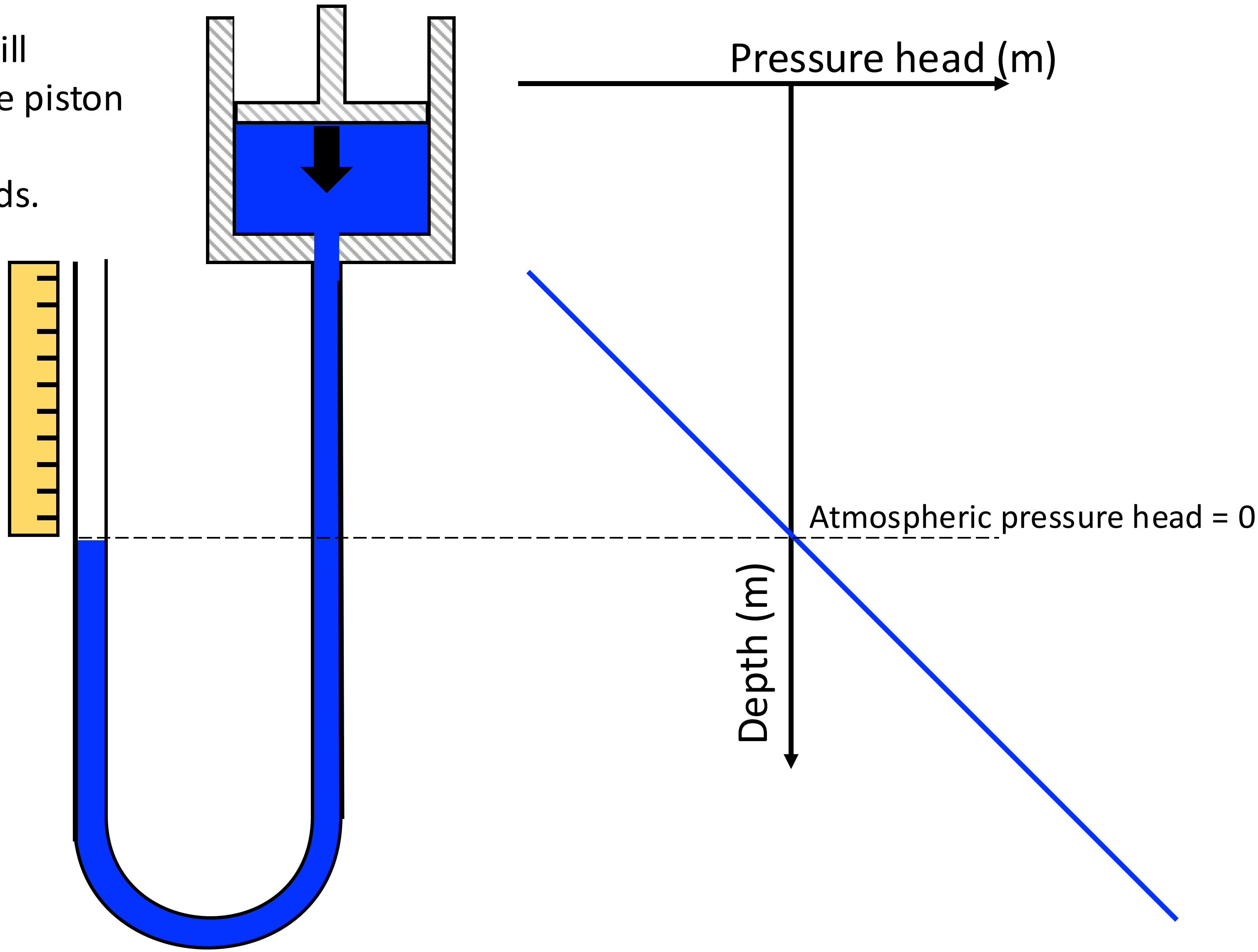
Drop the left-hand side. The tube is sealed, so water cannot move in/out. The depth of the water level from the top of the tube on the left is fixed.



Negative pressures

Instead of your thumb, use a simple piston:

Suction will induce the piston to move downwards.



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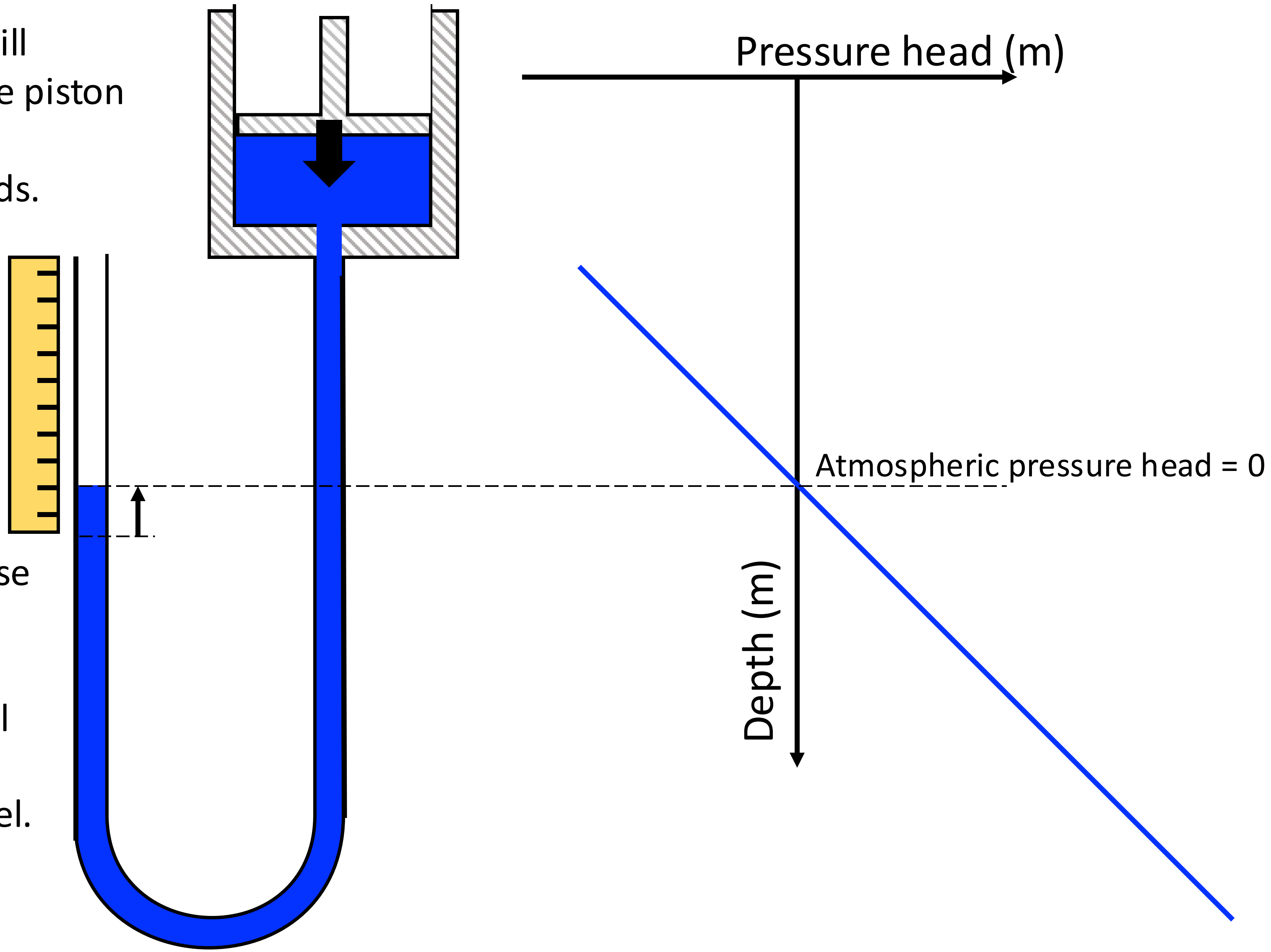


The release of water from the piston will raise the water level.

Forces in a capillary tube

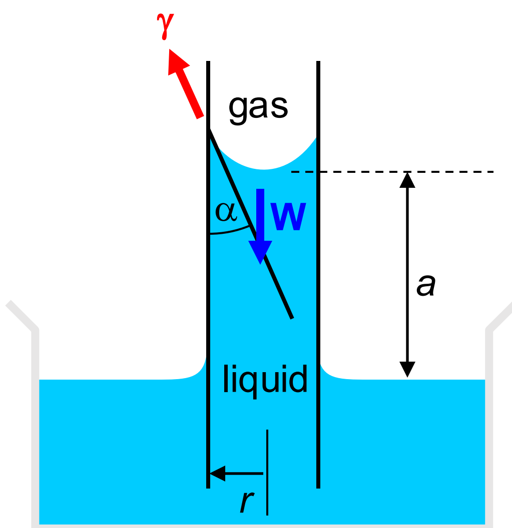
What happens when you insert a glass capillary tube in a water reservoir?

This is caused by surface tension – that is forces of attraction between the liquid and the solid.

Resolve forces

γ = surface tension force

W = weight of column



Weight of column

$2\pi r\gamma \cos \alpha = \pi r^2 h \rho_w g$

Vertical component of surface tension

Hence:

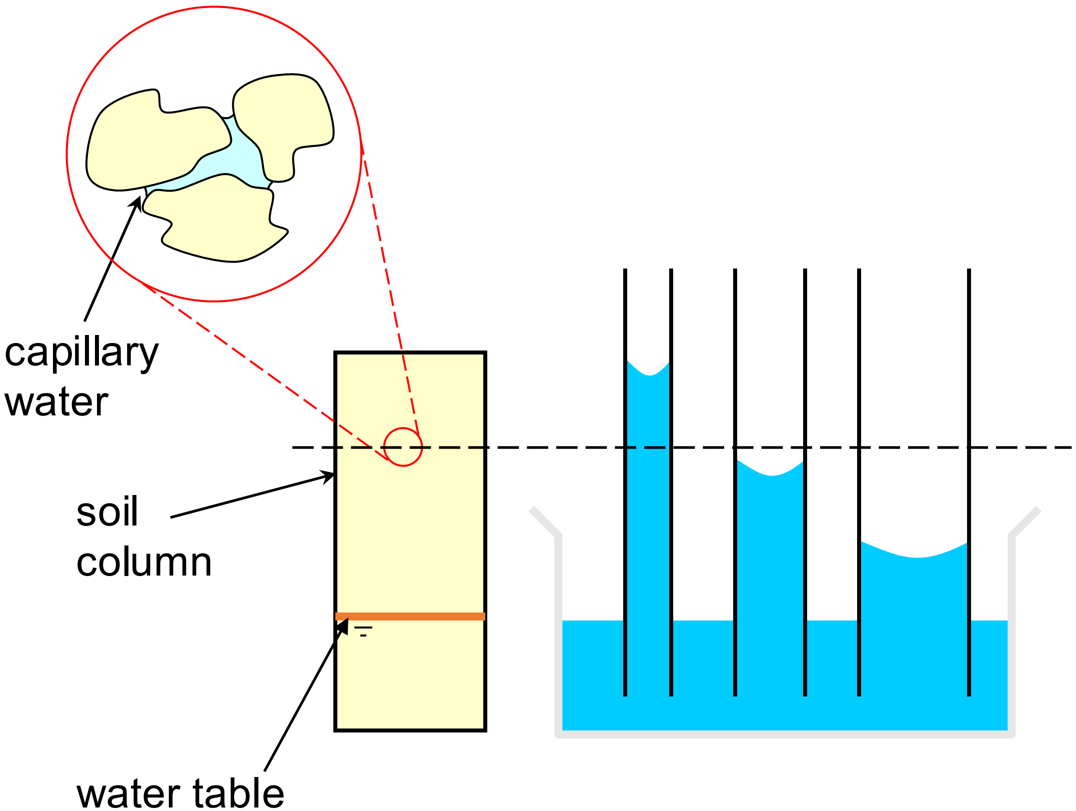
$$\boxed{a = \frac{2\gamma \cos\alpha}{r\rho_w g} \propto \frac{1}{r}}$$

Important point I: the amount of capillary rise is inversely proportional to the tube radius

Important point II: since the water in the capillary tube is raised above the free water surface, the pressure in the capillary tube are negative.

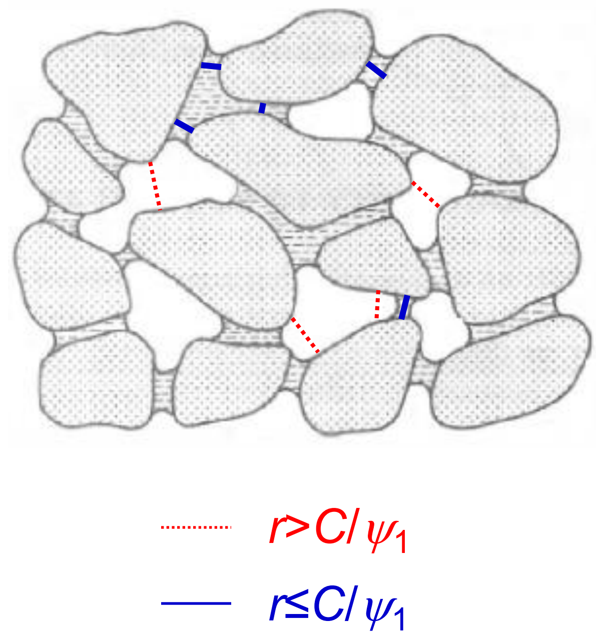
Soils compared with capillary tubes

Soil pores are understood to behave like capillary tubes. That is, (electro-static) forces of attraction between the water and soil, and self-attraction between water molecules result in the phenomenon of capillarity.



At a given elevation, only pores with a sufficiently small radius will be filled

So in reality, this looks something like this:



Soil moisture characteristic

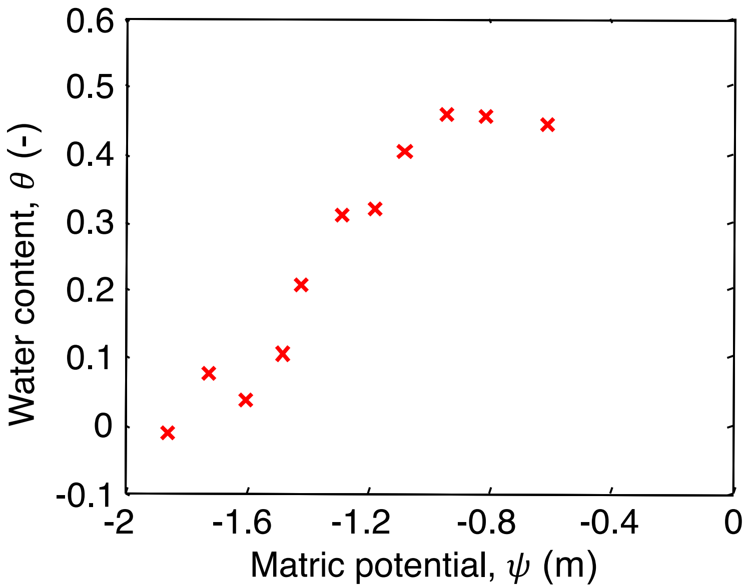
This is known as a **hanging-column experiment**.

For a given suction we calculate the matric potential, ψ (as shown)

We obtain a corresponding volume of water released from the sample, V_R

A corresponding water content is given by $\theta = n - \frac{V_R}{V_T}$ (where n is the porosity, and V_T is the sample volume).

From this we obtain pairs of θ, ψ values, which are plotted thus:



This plot is known as the **soil moisture characteristic** curve

Soil moisture characteristic

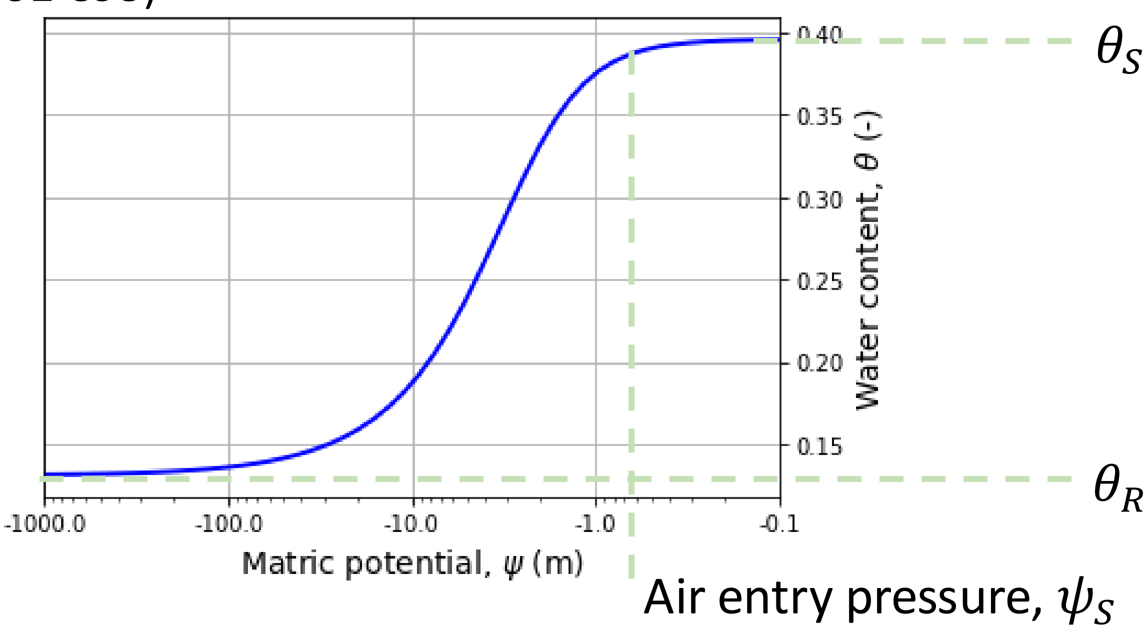
The SMC is different for different soil textures/structures.

It determines how much water the soil can retain at different suctions.

Atmospheric pressure corresponds to zero matric potential, which indicates that the soil is completely saturated.

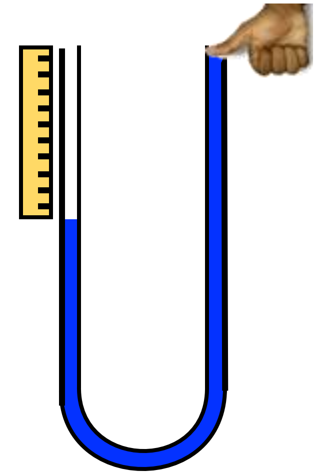
As ψ drops below zero, initially no pores empty (the capillary fringe), then at the entry pressure the largest pores start emptying, and the smallest pores are the last to empty, at high suctions (i.e. low ψ) values.

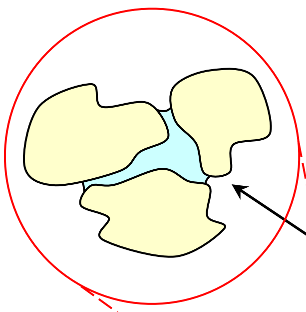
Below is the SMC for a silt loam textured soil, after van Genuchten (1980, Soil Sci. Soc. America J., 44:892-898)



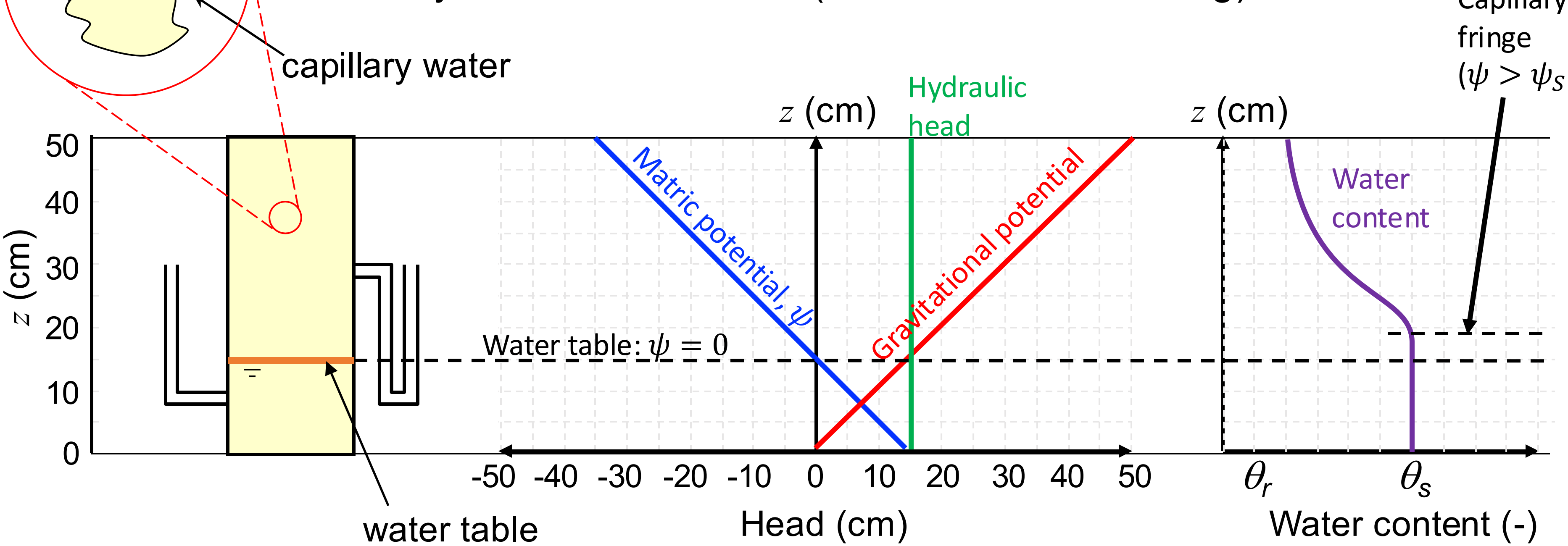
The hydrostatic profile

A hydrostatic profile is one where there is no-flow – where the hydraulic head is constant. The pressure head distribution is the same as in the u-tube from earlier:





The homogeneous soil column below contains water which is in hydrostatic conditions (i.e. no flow is occurring).



Water retention in different soil textures

Clay soils

High porosity

Low air-entry pressure (i.e. high suction)

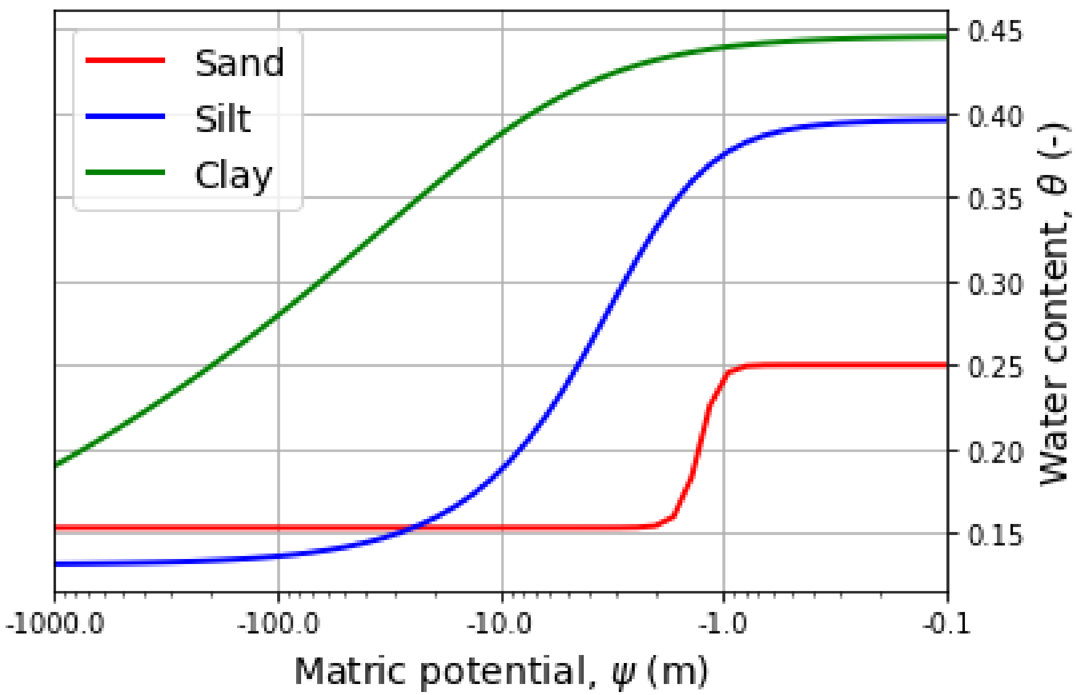
Drains slowly as ψ reduces

Sandy soils

Low porosity

High air-entry pressure

Drains rapidly as ψ reduces

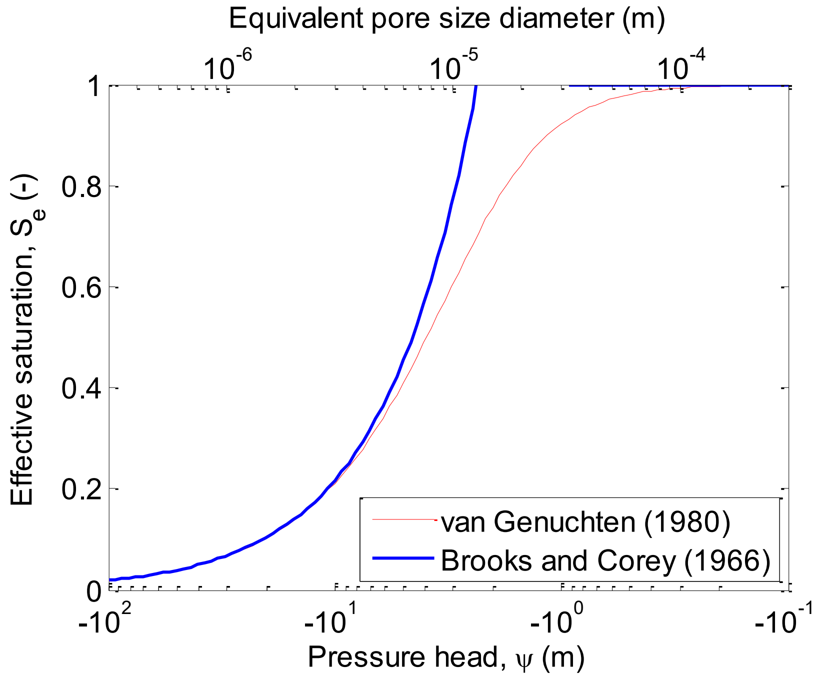


(after van Genuchten, 1980, Soil Sci. Soc. America J., 44:892-898)

SMC parametric relationships

Parametric relationships are used to describe the soil moisture characteristic.

These tend to be simple, curve fitting equations for effective saturation, S_e, as a function of ψ:



Effective saturation

$$S_e(\psi) = \frac{\theta(\psi) - \theta_r}{\theta_s - \theta_r}$$

θ_r - residial moisture content

θ_s - saturated moisture content

Brooks and Corey (1966)

$$S_e(\psi) = \begin{cases} (\psi_s / \psi)^{\lambda}, & \psi < \psi_s \\ 1 & \psi \geq \psi_s \end{cases}$$

ψ_s - air entry pressure of largest pore with significant presence [L]

λ - empirical exponent

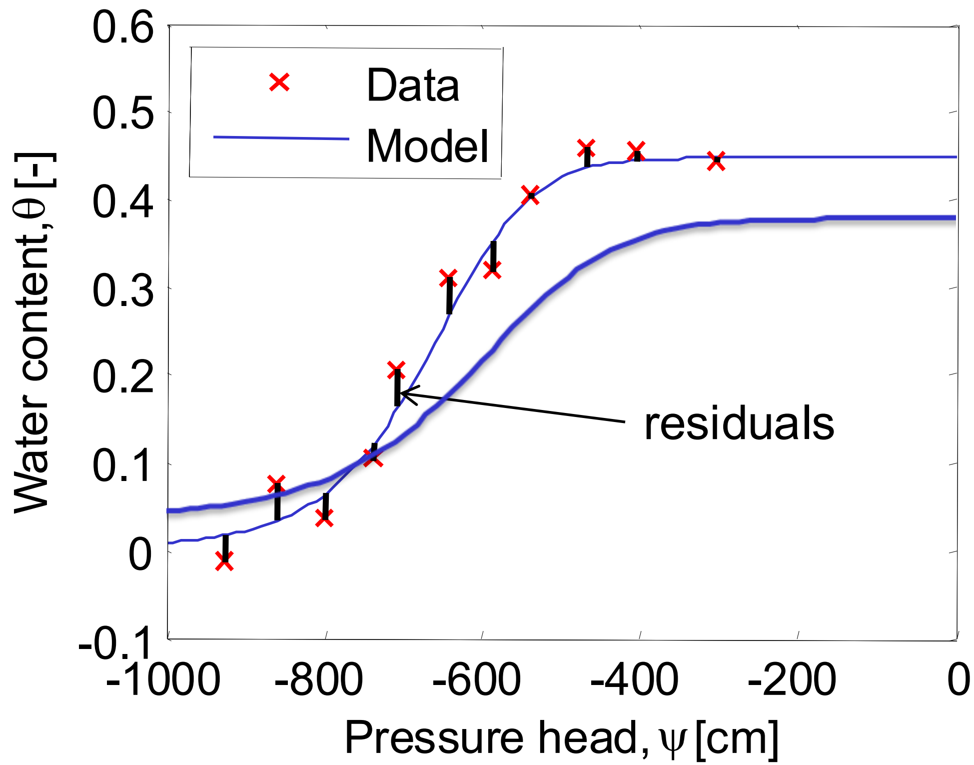
van Genuchten (1980)

$$S_e(\psi) = \left(\frac{1}{1 + |\alpha \psi|^n} \right)^m, m = 1 - 1/n$$

α - empirical parameter [L^{-1}]

n - empirical exponent

Fitting parametric relationships to SMCs

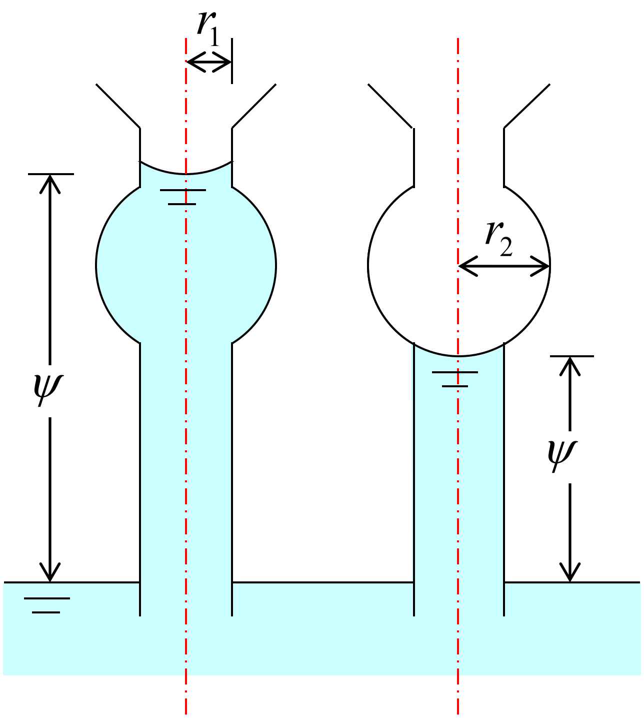


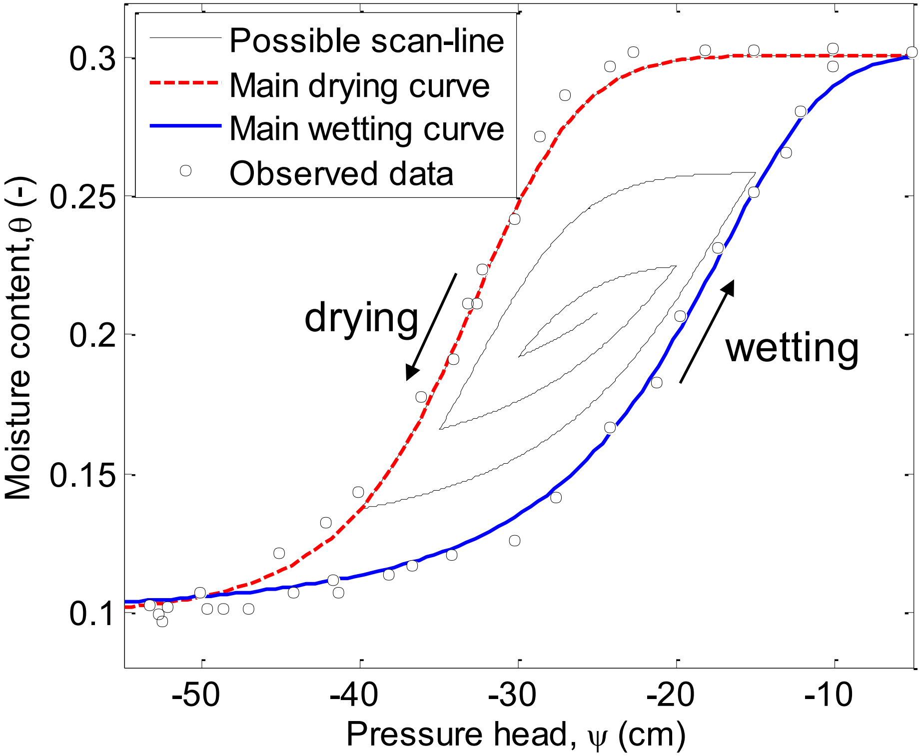
- Data can be obtained from field observations;
- Select a suitable parametric model, such as Brooks and Corey or van Genuchten;
- Modify parameter values to minimise the residuals.
- We can do this manually, trying to find the best visual fit
- It is sometimes more effective to use some numerical optimisation algorithm (e.g. *solver* in Excel, *fminsearch* in MATLAB, *fmin* in python/scipy)

Hysteresis

For some soils, θ may have a non-unique relationship with ψ, which depends on the previous wetting and drying history – this is **hysteresis**.

The ink bottle effect





(after Kool and Parker, 1987, *Water Resour. Res.*, 23:105-114)

REVIEW OF THE BASIC PHYSICS OF WATER MOVEMENT IN SOILS

Total potential in soils: hydraulic head

We saw that in the u-tube (and you will have learned that in saturated groundwater):

$$h = z + p$$

Flow in any fluid is driven by gradients in $\boldsymbol{h}$

In soils, we replace p (the pressure head) with ψ (the matric potential), and we note that ψ depends on how wet the soils are. Hence, the hydraulic head in soils (and in the unsaturated zone more generally) is

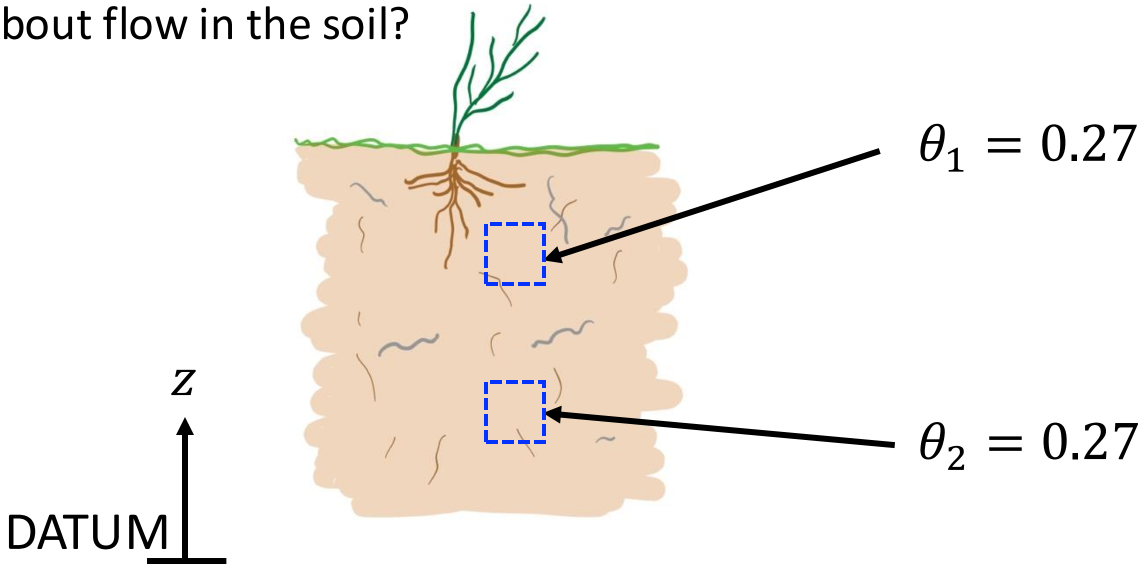
$$h = z + \psi$$

Again, flow is driven by gradients (that is, spatial differences) in h.

Repeat 10 times: ***"flow moves from high head to low head***".

Uniform water content profiles

Consider a homogeneous soil profile, with a completely uniform water content distribution – i.e. at any two depths, the water content is the same. What might this imply about flow in the soil?



Answer: We can say that if the soil is homogeneous and θ is constant, then ψ must also be constant. The hydraulic head at each point is therefore

$$h_1 = z_1 + \psi_1$$
$$h_2 = z_2 + \psi_2 = z_2 + \psi_1$$

The head difference is $\Delta h_{2,1} = h_2 - h_1 = z_2 - z_1 < 0$

Therefore, $h_2 < h_1$ so flow is occurring in the downward direction!

Unsaturated flow: Darcy’s law

Water flows from high hydraulic head to low hydraulic head. Darcy’s law quantifies this, stating that the flow per unit area, i.e. the flux, q, is proportional to the head gradient:

$$q \propto \frac{\Delta h}{l}$$

And the flow, Q, is the flux times the cross-sectional area, hence

$$Q \propto A\frac{\Delta h}{l}$$

And in both cases, we decided to call the constant of proportionality the *hydraulic conductivity*, K (m/d). Hence we have **Darcy’s law**

$$q = K\frac{\Delta h}{l} \quad \text{and} \quad Q = KA\frac{\Delta h}{l}$$

In continuous notation, we can write this as a differential equation

$$q = -k\frac{dh}{dz} \quad \text{and} \quad Q = -K.A\frac{dh}{dx}$$

Unsaturated hydraulic conductivity

- In saturated conditions, or strictly, when $\psi > \psi_s$, all pores are water filled. Hydraulic conductivity is constant and depends on the pore structure.
- In unsaturated conditions, as ψ decreases, so pores progressively empty, starting with the larger pores;
- In general, the larger pores tend also to be the most conductive pores;
- In these conditions two modes of flow may occur simultaneously:
    - As film creep along the walls of larger, partially water filled pores;
    - As tube flow, through smaller, completely water filled pores.
- Therefore in **unsaturated conditions** K is a non-linear function of ψ (or θ):

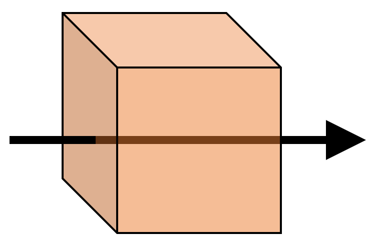
$$\lim_{\psi \to -\infty} K(\psi) = 0$$

$$\lim_{\theta \to 0} K(\theta) = 0$$

$$K(\psi \geq \psi_s) = K_s$$

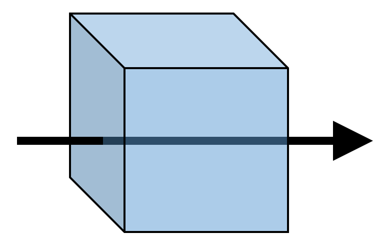
$$K(\theta = \theta_s) = K_s$$

Saturated zone:



Q=f (head gradient)

Unsaturated zone:



Q=f (head gradient, pressure head)

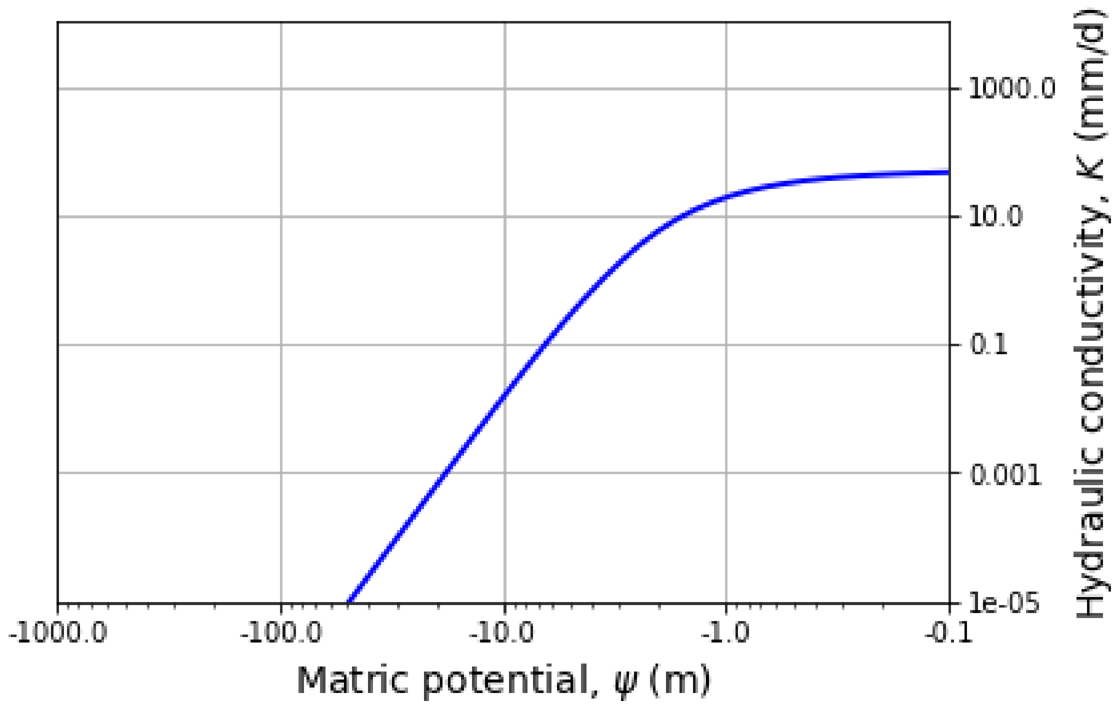
The $K(\psi)$ curve

The $K(\ \psi)$ curve, exactly like the SMC, is different for different soil textures/structures.

It determines how much water the soil can ~~retain~~ **transmit** at different suctions.

At saturation (i.e. atmospheric pressure, $\psi = 0$) K corresponds to the saturated hydraulic conductivity, which is the same as that used in saturated groundwater studies.

As ψ drops below zero, initially no pores empty (the capillary fringe), then at the entry pressure the largest pores, which are most conductive start emptying, leading to very rapid drop in the K (hence the log-scale below).



(after van Genuchten, 1980, Soil Sci. Soc. America J., 44:892-898)

$K(\psi)$ in different soil textures

Clay soils

Lower saturated K

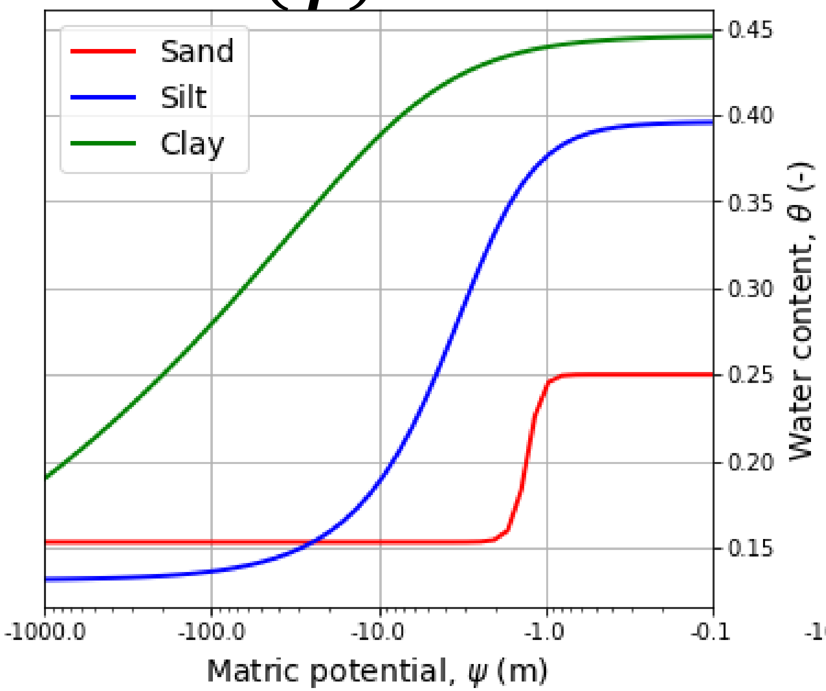
K drops more slowly, so higher K in drier conditions.

Sandy soils

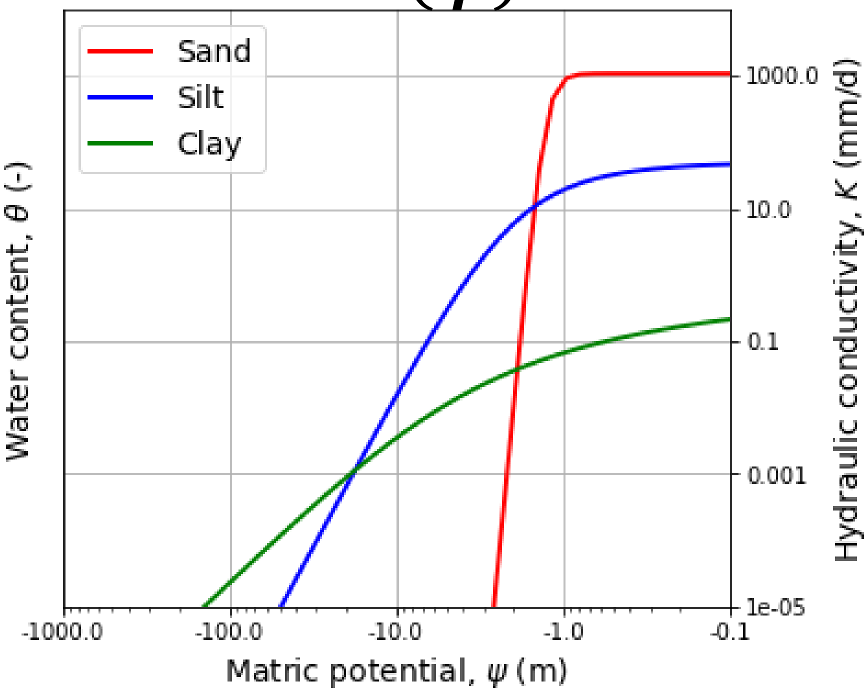
Higher saturated K

K drops very quickly, so hard to transmit water in drier conditions

$\boldsymbol{\theta(\psi)}$

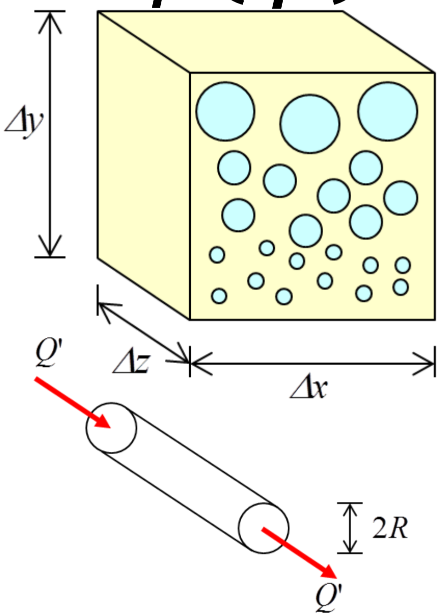


$\boldsymbol{K(\psi)}$



(after van Genuchten, 1980, Soil Sci. Soc. America J., 44:892-898)

$K_r(\psi)$: A physically based model

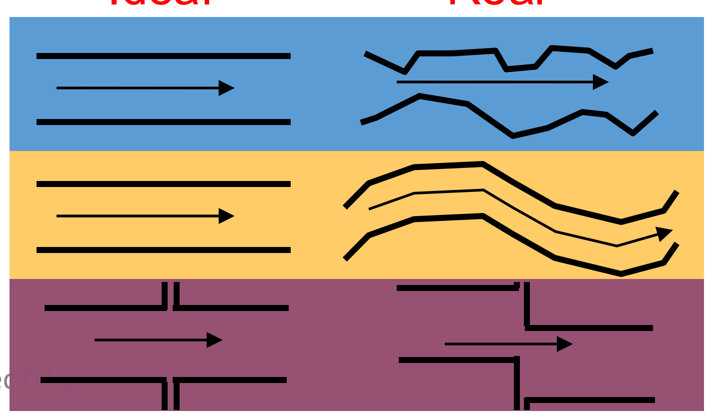


Physically based models have been proposed based on:

- The idea that at a given pressure only pores below a certain radius would conduct water (i.e. capillary theory)
- Flow and hence K is proportional to r^2 (Poiseuille's Law)
- r is proportional to $-1/\psi$ (capillary theory again)
- The distribution of pore radii can be described as a statistical distribution.

There are three reasons why the physically based relationships that have been proposed for $K(\psi)$ are problematic:

Ideal Real



- Pore surfaces are rough
- Flow pathways are tortuous
- Pore to pore connectivity is not perfect

$K(\psi)$ Parametric relationships

Parametric relationships for $K(\psi)$ have been derived based on physical reasoning and assuming ideal pores.

The effects of roughness, tortuosity and connectivity are then corrected for by using empirical correction factors.

A variety of parametric models have been proposed:

Brutsaert (1967)

$$K(\psi) = K_s S_e^{\eta}$$

Burdine (1953)

$$K(\psi) = K_s S_e^{\eta} \frac{\int_0^{S_e} \psi^{-2} dS_e}{\int_0^1 \psi^{-2} dS_e}$$

Mualem (1976)

$$K(\psi) = K_s S_e^{\eta} \left[\frac{\int_0^{S_e} \psi^{-1} dS_e}{\int_0^1 \psi^{-1} dS_e} \right]^2$$

η - correction factor [-]

Brutsaert (1967) *Trans. ASAE*, 10:400-404.
Burdine (1953) *Trans. AIME*, 198:71-78.
Mualem (1976) *Water Resour. Res.*, 12:513-522.

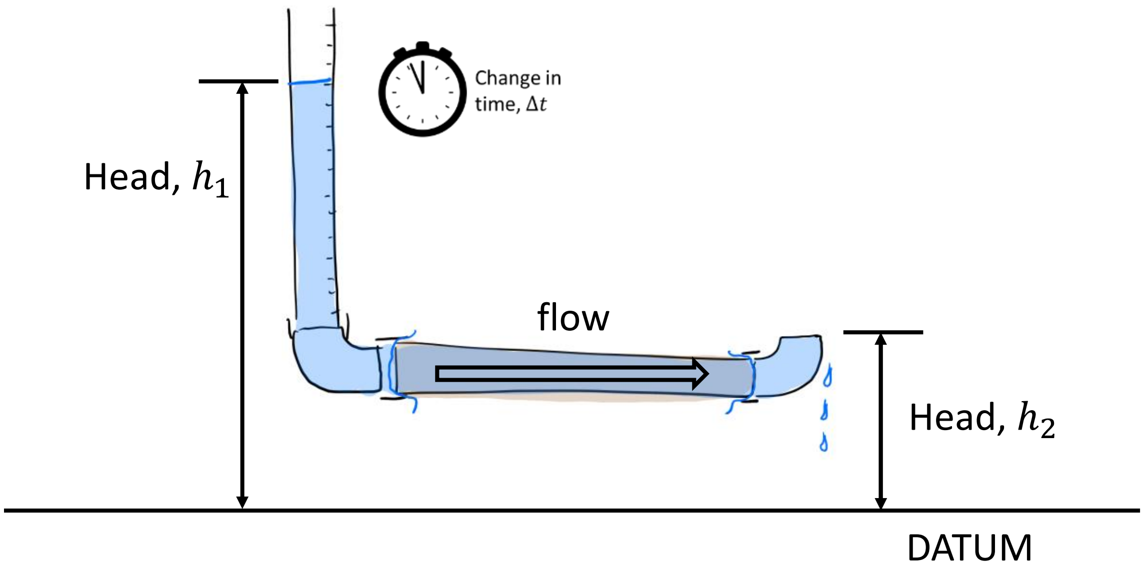
To solve the integral in the Burdine and Mualem models requires the parametric relationship for $\theta(\psi)$ to be given.

For common $\theta(\psi)$ relationships (e.g. van Genuchten, Brooks and Corey) these solutions are available in the literature.

Measuring K_S

The saturated hydraulic conductivity, K_S (m/d), can be measured in a variety of ways – the simplest/most classic method is to obtain a soil sample, and measure K_S using a permeameter.

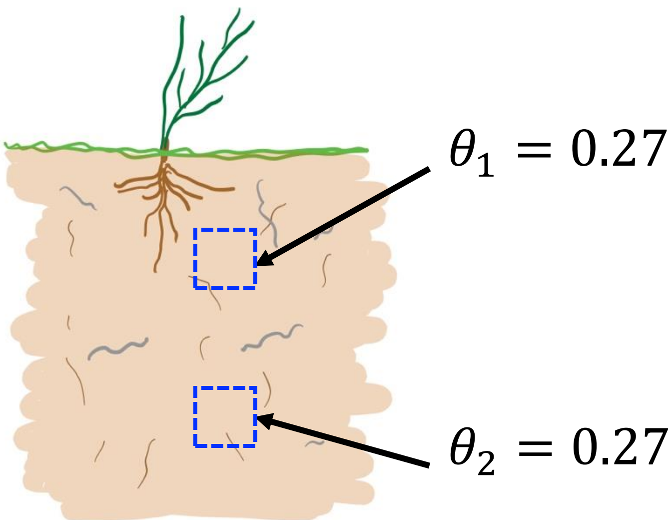
Shown here is a falling head permeameter, which is particularly simple to set up:



Measuring unsaturated $K(\psi)$

Measuring $K(\psi)$ when $\psi < 0$ is much harder to do. One way to do this, is to use a steady-state infiltration experiment. Consider again the uniform water content profile. We showed that there is downward flow. But what specific conditions are required to establish a uniform water content/matric potential profile?

It turns out that this condition is achieved under steady-state infiltration.



We can show this from Darcy's law:

$$q = -K(\psi)\frac{dh}{dz}$$

Replacing $h = \psi + z$ we have

$$q = -K(\psi)\left(\frac{d\psi}{dz} + \frac{dz}{dz}\right)$$

Now $\frac{dz}{dz} = 1$ and if we have uniform ψ with depth then $\frac{d\psi}{dz} = 0$. Hence

$$q = -K(\psi)$$

Since the soil is homogeneous $K(\psi)$ must be uniform, hence q is also uniform and hence steady.

Steady-state infiltration experiments

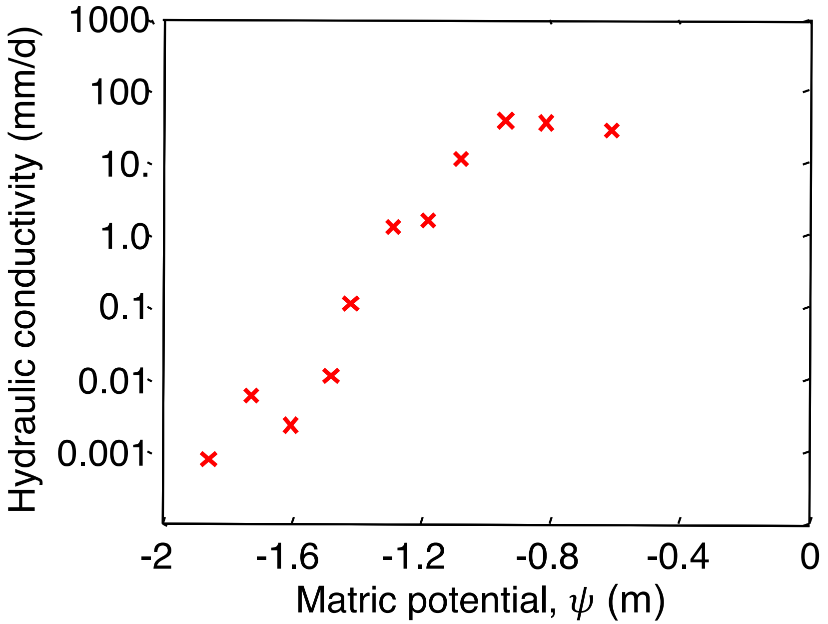
Steady-infiltration results in uniform θ, ψ and $K(\psi)$.

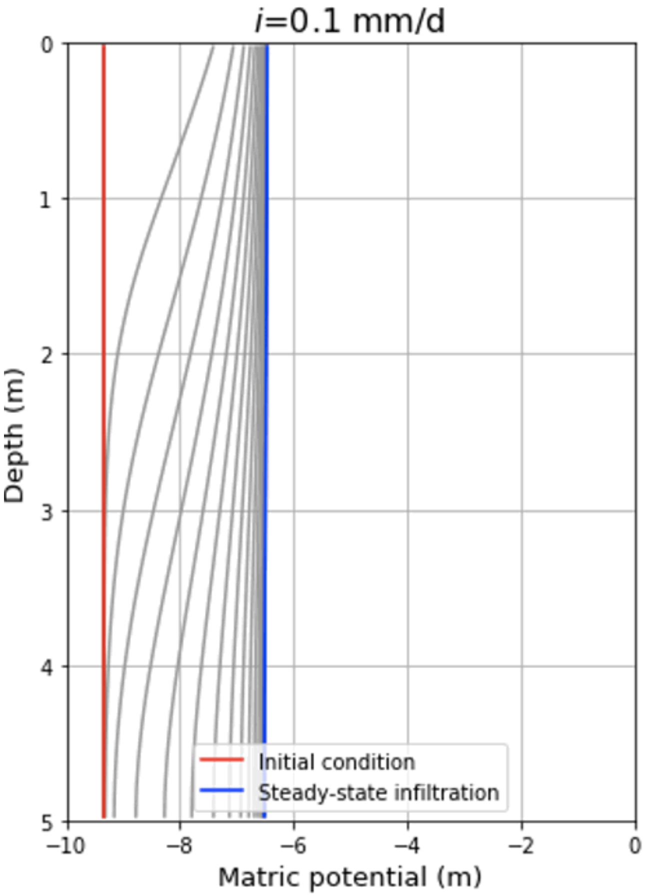
We just showed that during s.s. infiltration

$$i = -K(\psi)$$

Where i is the infiltration rate (m/d).

Hence, by irrigating a profile with different values of i and measuring the steady value of ψ we can determine pairs of $K(\psi)$ values:





$\theta(\psi)$ and $K(\psi)$ relationships

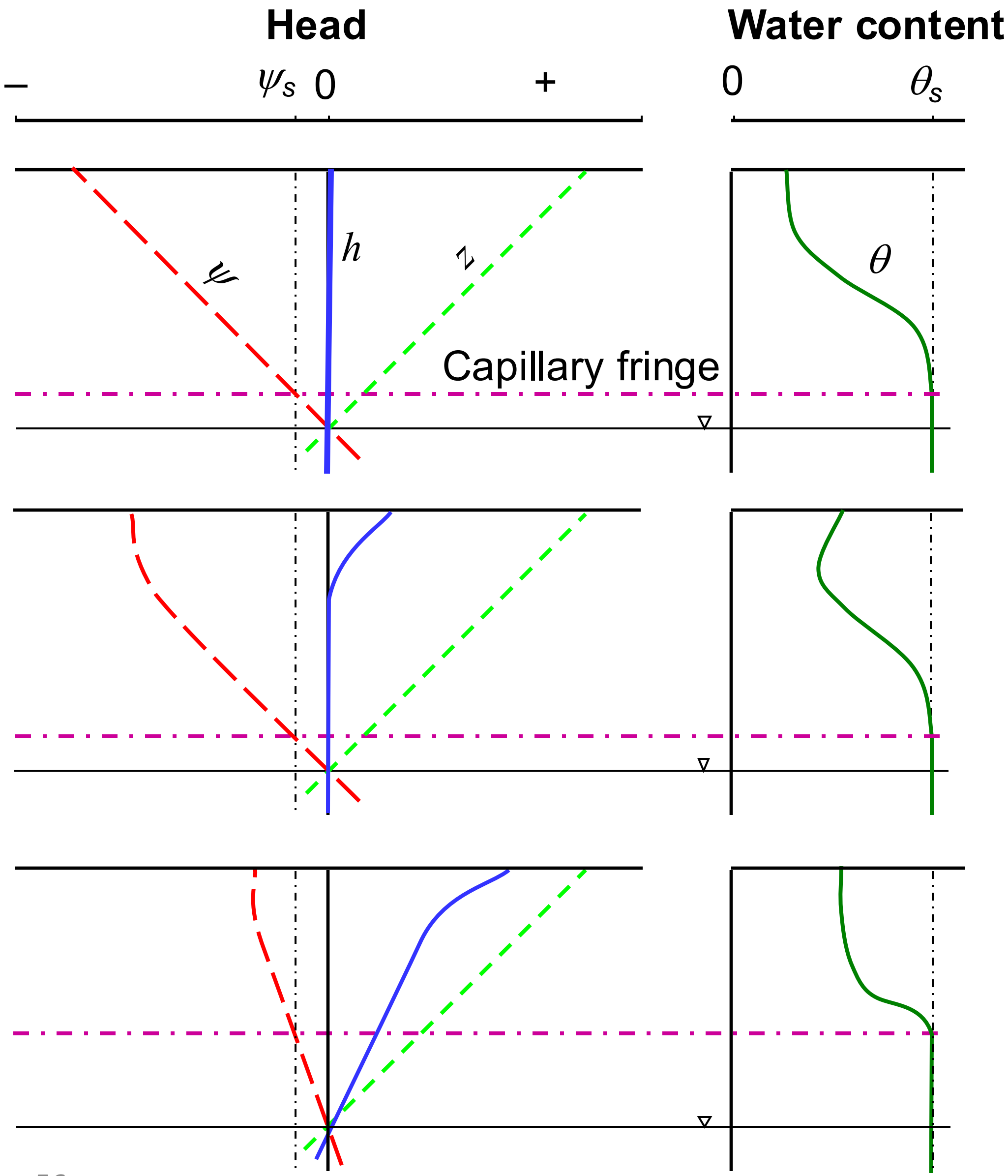
	Water content: $\theta = S_e(\psi).(\theta_s - \theta_r) + \theta_r$	
	Brooks and Corey $S_e(\psi) = \begin{cases} (\psi_s/\psi)^\lambda, & \psi < \psi_s \\ 1 & \psi \geq \psi_s \end{cases}$	**van Genuchten** $S_e(\psi) = \left(\dfrac{1}{1+\lvert\alpha\psi\rvert^n}\right)^m, m = 1 - 1/n$
Brutsaert	$K(\psi) = K_s K_r = K_s S_e^\eta$	$K(\psi) = K_s K_r = K_s S_e^\eta$
Burdine	$K(\psi) = K_s K_r = K_s S_e^{\eta+1+2/\lambda}$	Not tractable
Mualem	$K(\psi) = K_s K_r = K_s S_e^{\eta+2+2/\lambda}$	$K(\psi) = K_s K_r = K_s S_e^\eta \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2$

The van Genuchten-Mualem model combination is probably the most popular.

Traditionally for this combination, η was assumed to be 0.5. However, recent studies have shown much better results can be obtained if this is treated as a free parameter (Schaap & Leij, 2000, SSSA 64).

TRANSIENT FLOW IN SOILS

Transient vertical flow profiles



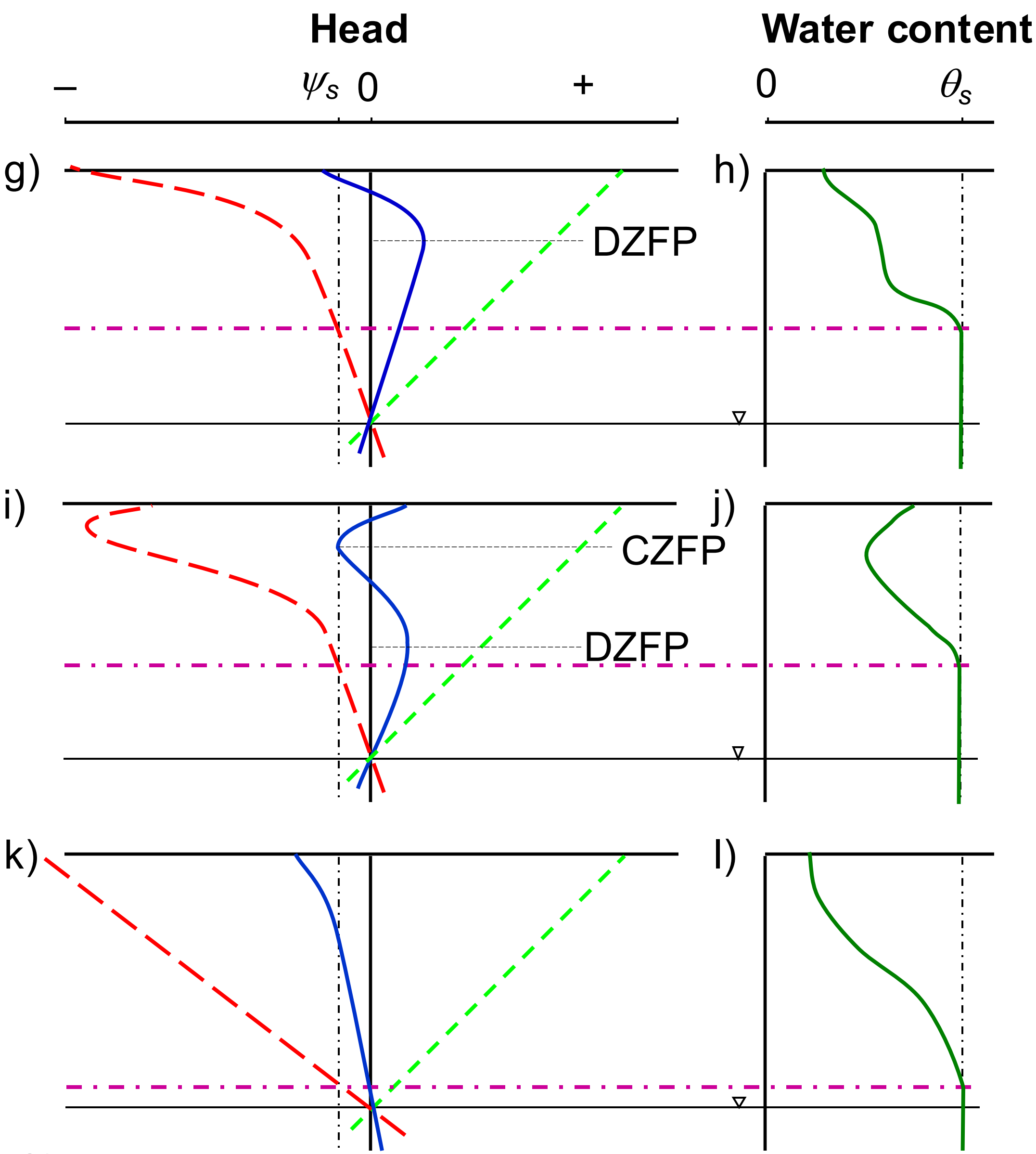
1. **The hydrostatic profile** (no flow). This (unrealistic) situation occurs after a sustained absence of rainfall, evaporation and water table movement.

2. **It rains!** Water infiltrating into the soil causes an increase in θ and ψ near the surface and hence an increase in h creates a negative downward head gradient – i.e. downward flow.

3. **It continues to rain!** As water moves down through the profile, the capillary fringe moves up and downward flow persists throughout the unsaturated zone.

(adapted from Price et al., 1993, Chapter in the "Hydrogeology of the Chalk of North West Europe")

Transient vertical flow profiles

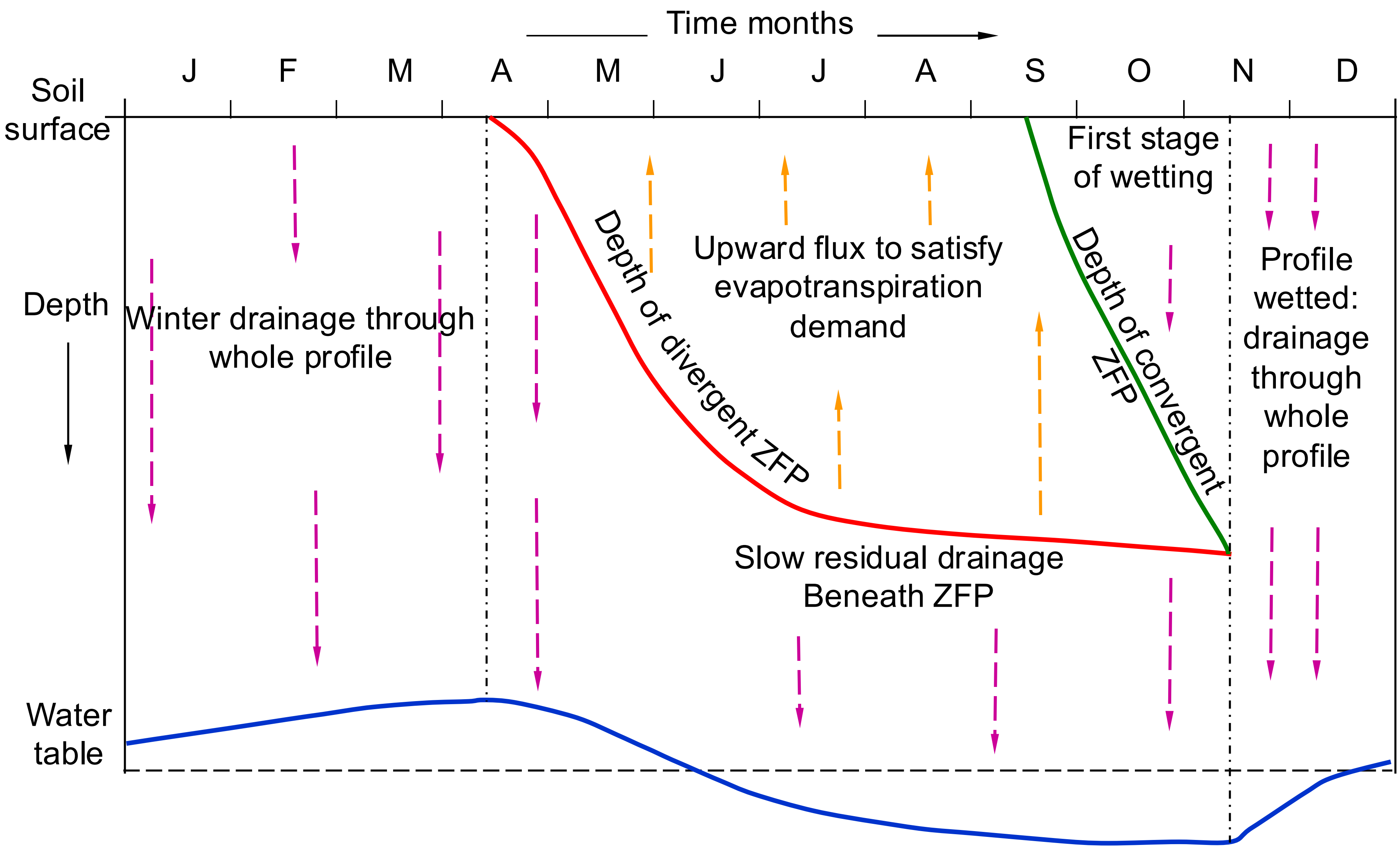


ZFP = zero flux plane

4. **It stops raining!** Water at the top is pulled upwards to satisfy evapotranspiration. Water at the bottom is still infiltrating downwards. Hence a divergent ZFP develops.
5. **It rains again!** As rainfall again exceeds evapotranspiration, a convergent ZFP develops (this may move down and cancel out the DZFP).
6. **Prolonged absence of rain**! The soil dries out. Any ZFPs are cancelled out. Flow is upward, from the water table to the atmosphere. The capillary fringe moves down.

(adapted from Price et al., 1993, Chapter in the "Hydrogeology of the Chalk of North West Europe")

Vertical flow patterns over a year



(after Wellings and Bell, 1980, Journal of Hydrology. They suggested this as conceptual model for recharge in the Chalk, and though arguably it has been subsequently overlooked, recent studies at Imperial suggest that this is an excellent conceptualisation)

A model for transient unsaturated flow

Unsaturated transient flow is governed by Richards' Equation:

$$C(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left(K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right)\right)$$

Where $C(\psi) = \frac{d\theta}{d\psi}$ which is obtained by differentiating the $\theta(\psi)$ relationship.

This partial differential equation must be solved numerically, a somewhat complicated problem. This is what Hydrus does, as well as various other software (see https://github.com/amireson/RichardsEquation_improved for my Richards' Equation solver).

You can see how the $\theta(\psi)$ and $K(\psi)$ relationships are required to solve this.

Solutions to Richards' Equation allow us to calculate the transient rates of infiltration, drainage and (with a few extra bits added) evapotranspiration. They also allow us to calculate the water content as a function of depth and time and the total profile storage as a function of time.

In the following slides information about the derivation and solution of Richards' Equation are provided, but we won't go through these in class.

Transient GW flow between two ponds