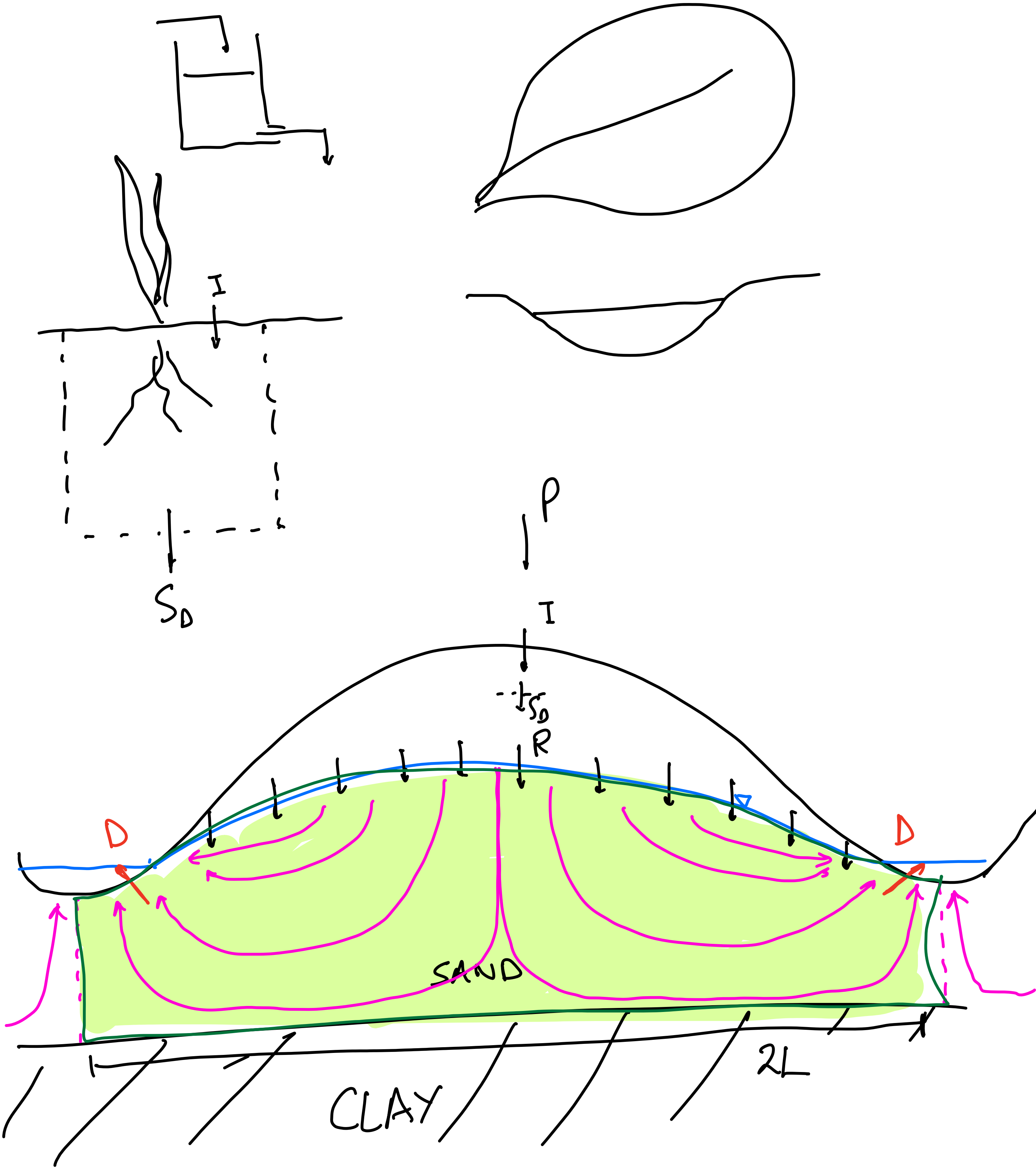


INPUTS : Groundwater recharge, R, (m/d)

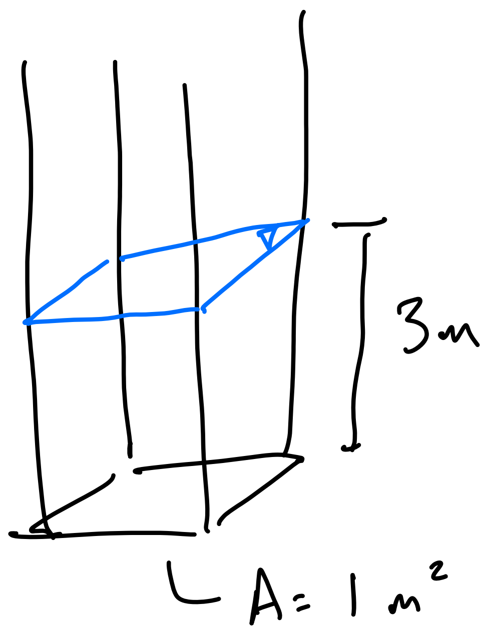
OUTPUTS : Groundwater discharge, D, (m^3/d)

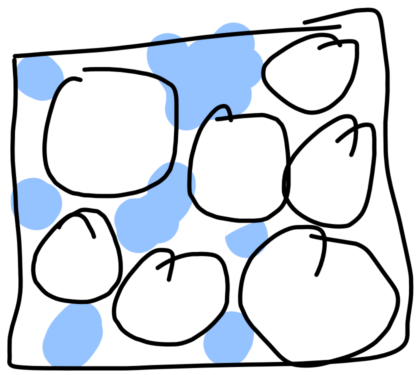
$$\Delta M = M_{IN} - M_{OUT}$$

$$= R.2L.\rho.\Delta t - 2D.\rho.\Delta t$$

$$\Delta(V_w \cancel{\rho}) = 2RL\cancel{\rho}\Delta t - 2D\cancel{\rho}\Delta t$$

$$\frac{\Delta V_w}{\Delta t} = 2RL - 2D$$





S_y = specific yield

$= 0.001 \rightarrow 0.15$

$\Delta V_w = 1 m^3 \quad \Delta h = 10 m$

$$\Delta V_w = \Delta h . A . S_y$$

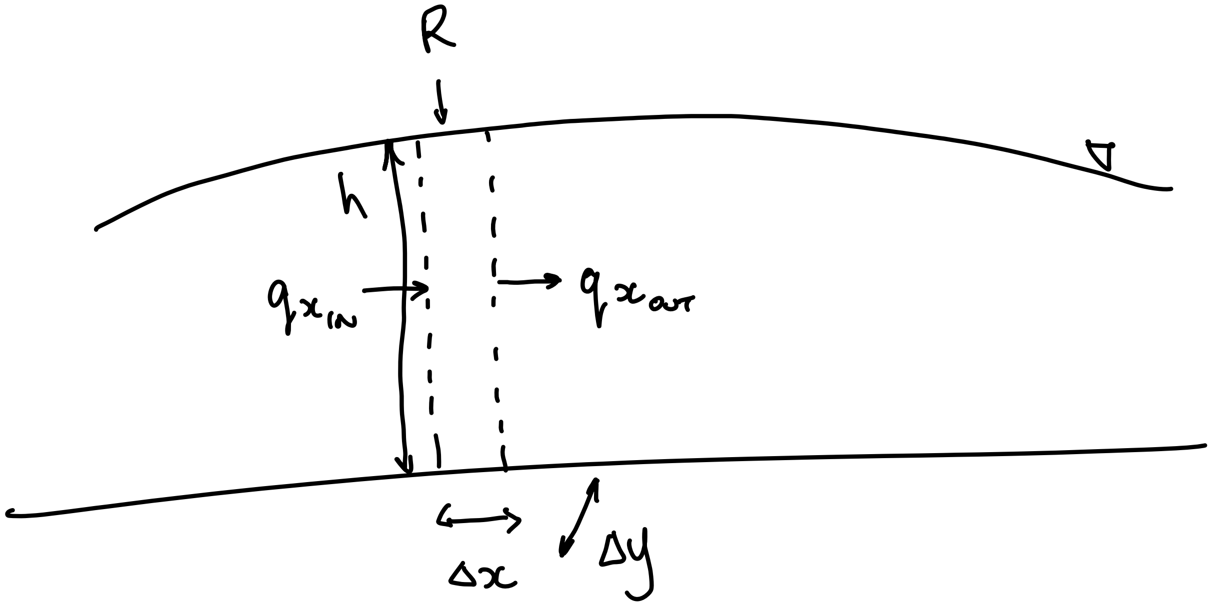
$$\Delta h = \frac{\Delta V_w}{A.S_y} = \frac{1}{1.0.1} = 10$$

$$\frac{\Delta h}{\Delta t} . A . S_y = 2.R.L - 2D$$

$$\frac{\Delta h}{\Delta t} \cdot 2L S_y = 2RL - 2D$$

$$S_y \cdot \frac{\Delta h}{\Delta t} = R - \frac{D}{L}$$

$$S_y \cdot \frac{dh}{dt} = R - \frac{D}{L}$$ ← Lumped water balance of Aq.



$$\Delta M = M_{IN} - M_{OUT}$$

$$\rho \cdot \Delta h \, \Delta x \Delta y \, S_y = R \Delta x \Delta y \rho \cdot \Delta t + q_{x_{IN}} \cdot h \cdot \Delta y \cdot \rho \Delta t - q_{x_{OUT}} h \Delta y \rho \Delta t$$

$$S_y \cdot \frac{\Delta h}{\Delta t} = R + \frac{q_{x_{IN}} \cdot h}{\Delta x} - \frac{q_{x_{out}} \cdot h}{\Delta x} = R + \frac{h}{\Delta x}(q_{x_{IN}} - q_{x_{out}})$$

Let $q_{x_{out}} = q_{x_{IN}} + \Delta q$

$$\Delta q = q_{x\,out} - q_{x\,in}$$

$$S_y \frac{\Delta h}{\Delta t} = R - \frac{\Delta q . h}{\Delta x}$$

$$S_y \frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}(q.h) + R$$

Darcy's Law: $Q \propto \frac{\partial h}{\partial x}, A$

$$Q = -k.A.\frac{dh}{dx}$$

$$q = -k.\frac{dh}{dx}$$

$$S_y.\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\left(-k.\frac{\partial h}{\partial x}.h\right) + R$$

$$S_y \frac{\partial h}{\partial t} = k.\frac{\partial}{\partial x}\left(h.\frac{\partial h}{\partial x}\right) + R$$

$$\frac{\partial uv}{\partial x} = u.\frac{\partial v}{dx} + v.\frac{du}{dx}$$

$u = h$, $v = h$

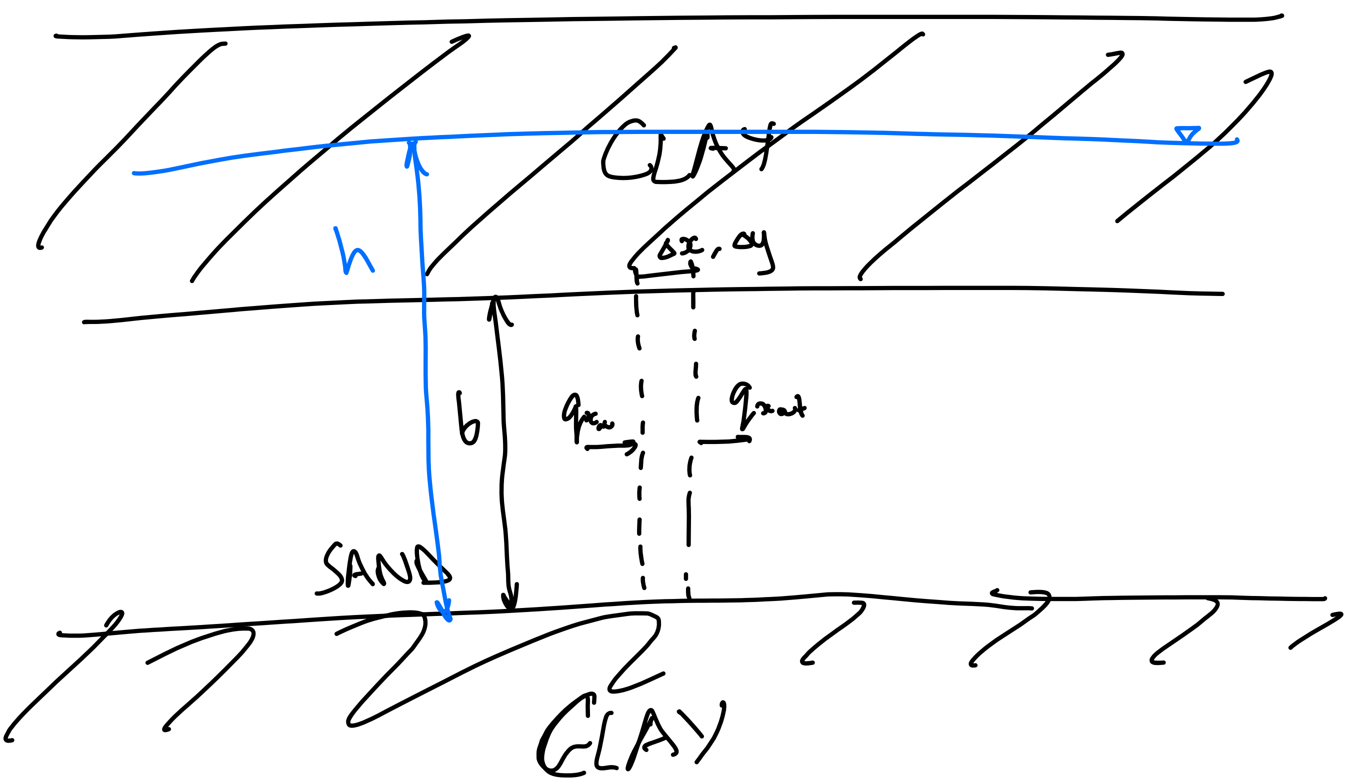
$$\frac{\partial h.h}{\partial x} = h.\frac{dh}{dx} + h.\frac{dh}{dx} = \frac{\partial h^2}{\partial x} = 2.h.\frac{\partial h}{\partial x}$$

$$h \cdot \frac{dh}{dx} = \frac{1}{2} \cdot \frac{\partial h^2}{\partial x}$$

$$S_y \frac{\partial h}{\partial t} = k \cdot \frac{\partial}{\partial x}\left(\frac{1}{2} \cdot \frac{\partial h^2}{\partial x}\right) + R$$

$$S_y \frac{\partial h}{\partial t} = \frac{k}{2} \cdot \frac{\partial^2 h^2}{\partial x^2} + R$$

Governing Equation for 1D, transient, GW flow in an unconfined aquifer, subject to uniform recharge.



$$\Delta V_w = \Delta h . A . b . S_s$$

$$\Delta M_1 = M_{in} - M_{out}$$

$$\Delta h . \Delta x \Delta y b . S_s . \rho = q_{x_{in}} . b . . \Delta y . \rho . \Delta t - q_{x_{out}} b . \Delta y . \rho . \Delta t$$

$$= - \Delta q . b . \Delta y . \rho . \Delta t$$

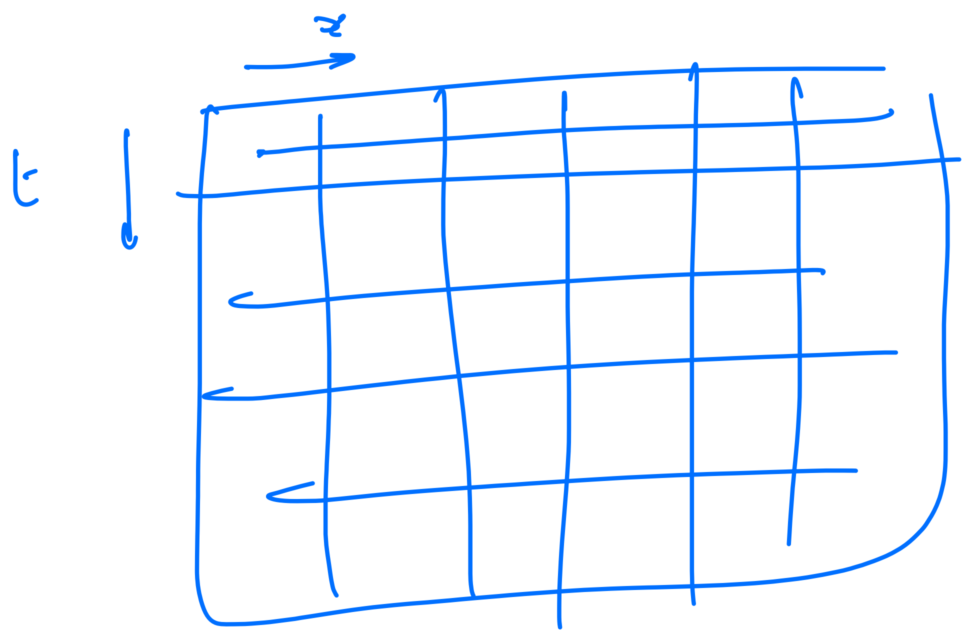
$$S_s . \frac{\Delta h}{\Delta t} = - \frac{\Delta q}{\Delta x}$$

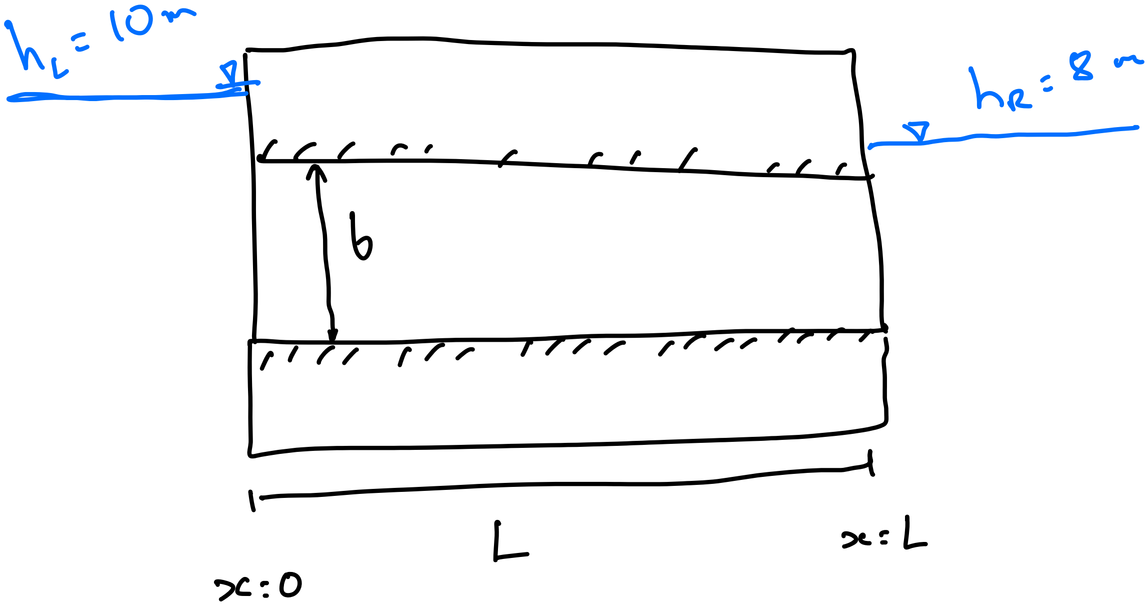
$$S_s \frac{\partial h}{\partial t} = - \frac{\partial q}{\partial x} \qquad q = - k . \frac{dh}{dx}$$

$$S_s . \frac{\partial h}{\partial t} = k . \frac{\partial}{\partial x} \left(\frac{\partial h}{\partial x} \right)$$

$$S_s . \frac{\partial h}{\partial t} = k . \frac{\partial^2 h}{\partial x^2}$$

Governing equation for 1D, transient, GW flow in a confined aquifer.





$$S_s \cancel{\frac{\partial h}{\partial t}} = K \frac{\partial^2 h}{\partial x^2} = 0$$

0 at Steady state

$$\frac{d^2 h}{dx^2} = 0$$

$$\frac{dh}{dx} = C_1$$

$$h = C_1 x + C_2$$

General Soln.

$$h(x = 0) = h_L$$

$$h(x = L) = h_R$$

$$x = 0 \rightarrow \quad h_L = \cancel{C_1 x}^{\,0} + C_2$$

$$h = C_1 x + h_L$$

$$x = L \rightarrow \quad h_R = C_1 . L + h_L$$

$$C_1 = \frac{h_R - h_L}{L}$$

$$h = \left(\frac{h_R - h_L}{L}\right)x + h_L$$

Particular Soln

S.S. solution for 1D unconfined Aq w/ Recharge

$$S_y \cancel{\frac{\partial h}{\partial t}}^{0} = \frac{K}{2} \cdot \frac{\partial^2 h^2}{\partial x^2} + R$$

$$\frac{\partial^2 h^2}{\partial x^2} = -\frac{2R}{K}$$

$$\frac{dh^2}{dx} = -\frac{2R}{K} \cdot x + C_1$$

$$h^2 = -\frac{\cancel{2}R}{K} \cdot \frac{x^2}{\cancel{2}} + C_1 x + C_2$$

General Soln

$$h^2 = -\frac{R}{K} \cdot x^2 + C_1 x + C_2$$

$h(x=0) = h_L$ $\quad$ $h(x=L) = h_R$

$x = 0 \rightarrow h_L^2 = C_2$

$$x = L \rightarrow \quad h_R^2 = -\frac{RL^2}{k} + C_1 . L + h_L^2$$

$$\frac{h_R^2 - h_L^2 + \frac{RL^2}{k}}{L} = C_1$$

$$\frac{h_R^2 - h_L^2}{L} + \frac{RL}{k} = C_1$$

$$h^2 = -\frac{R}{k} . x^2 + \left(\frac{h_R^2 - h_L^2}{L}\right) x + \frac{RLx}{k} + h_L^2$$

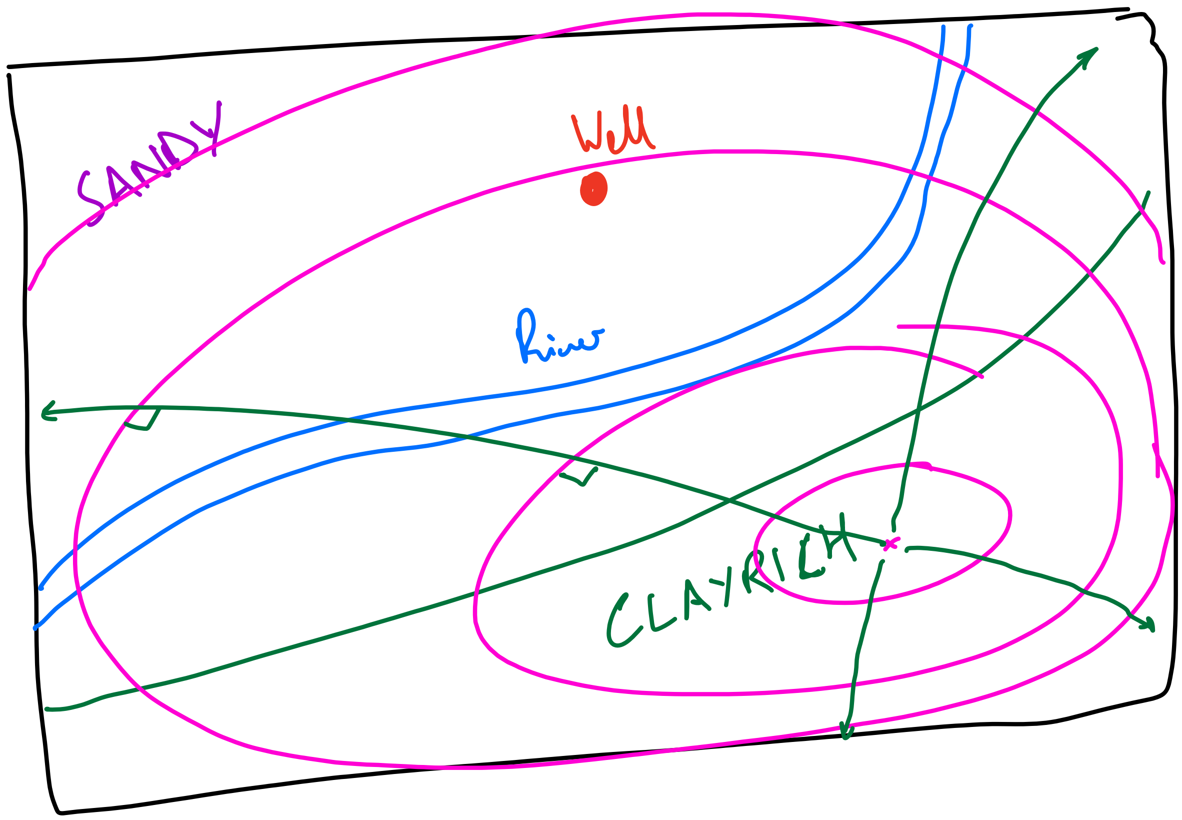
$$h^2 = \left(\frac{h_R^2 - h_L^2}{L}\right) x + \frac{Rx}{k}\left(L - x\right) + h_L^2$$

GW Divide:

$$\frac{dh^2}{dx} = -\frac{2R}{k} . x + \left(\frac{h_R^2 - h_L^2}{L}\right) + \frac{RL}{k} = 0$$

$$\frac{2R}{k} . x = \frac{h_R^2 - h_L^2}{L} + \frac{RL}{k}$$

$$x = \frac{k}{2R}\left(\frac{h_R^2 - h_L^2}{L}\right) + \frac{L}{2}$$


SANDY
Well
River
CLAYRICH