

Symbols

Sets

Name	Domains	Description
i, il	*	index
j	i	a duplicate of i
m	*	QC index
n	m	A duplicate of j
L	m, i	
D	m, i	Unloading Containers. U is a subset of index i
C	m, i	All Containers
li	*	AGV index
a, a1, a2, a2_1, a1_1	*	AGV actions
XR, XR1	*	Vertical Operational Area
YR	*	Horizontal Operational Area
YS	YR	Horizontal Seaside Operation Area
YL	YR	Horizontal Path
o	m, i, XR	
A.L.set	XR	
A.R.set	XR	
WT	m, i, a	set of total actions without virtual node. More precisely, just WHWV
WV	m, i, a	Vertical Actions
WH	m, i, a	Horizontal Actions
psi_1	m, i, m, i	sequence of Container jobs for QC
psi_2	m, i, m, i	sequence of Container jobs for ASC
h	m	
k	i	
x_t	XR	
SameAs	*, *	Set Element Comparison Without Checking

Parameters

Name	Domains	Description
S_Q		switch time for qc between two containers
v		AGV speed
Mnum		a very large number
o1	m, i	merely a copy of the o(m,i,XR), with XR treated as a number
G_Q	m, i	
G_Y	m, i	

Variables

Name	Domains	Description
z	m, i, m, i, li	used mainly for handling QC double cycling, it consists of 0 virtual point!
U_AGV	m, i, a, m, i, a	U_AGV(j_1,j_2) conducted before
U_QC	m, i, m, i, a	U_QC(j,WT) conducted before
P_X	m, i, a, XR	P_X(WV,x) finish V loc, These are defined on actions, NOT ON CONTAINERS!
P_Y	m, i, a, YR	P_Y(WH,y) finish H loc, These are defined on actions, NOT ON CONTAINERS!
T_Q	m, i	start time of QC
T_Y	m, i	Start time of agv putting cont on ASC
T_start	m, i, a	Start of agv for action (m,i,a)
t_AGV	m, i, a, m, i, a	t_AGV(WT_1,WT_2
X_position	m, i, a	X_position(WT)
Y_position	m, i, a	Y_position(WT)
obj		objective function

Equations

Name	Domains	Description
ADRP1		AGV Dispatching and Routing Problem
ADRP2		AGV Dispatching and Routing Problem
cnstr_2	m, i	
cnstr_3	m, i, li	C,B
cnstr_4	li	B with 0 virtual node
cnstr_5	li	"B with 0 virutal node"
cnstr_6	m, i	L
cnstr_7	m, i	D
cnstr_8	m, i, n, j, XR	C,C,XR
cnstr_9	m, i, n, j, YR	C,C,YR
cnstr_10	m, i, a	C,a
cnstr_11	m, i, a	C,a
cnstr_12	m, i	L
cnstr_13	m, i	D
cnstr_14	m, i, XR	D
cnstr_15	m, i	L
cnstr_16	m, i	D
cnstr_17	m, i	L
cnstr_18	m, i, XR	L
cnstr_19	m, i	D
cnstr_20	m, i, a, a, YR	WH,YR
cnstr_21	m, i, a, a, XR	WV,XR
cnstr_22	m, i, n, j	C,C
cnstr_23	m, i, a, n, j, a, YR, XR, a, a	WH,WH,YR,XR
cnstr_24	m, i, n, j, a	C, WH
cnstr_25	m, i, n, j, a, a	C, WH

Name	Domains	Description
cnstr_26_1	n, j, a, YS, m, i, a, a	WH,YS,D
cnstr_26_2	n, j, a, YS, m, i, a, a	WH,YS,D
cnstr_27	m, i, a, n, j, a, XR	WV, XR
cnstr_28	m, i, a, a	C,a
cnstr_29	m, i, i	C,C
cnstr_30	m, i, m, i	psi_1
cnstr_31	m, i, m, i	psi_2
cnstr_32	m, i	D
cnstr_33	m, i	L
cnstr_34	m, i, m, i	D,L
cnstr_35	m, i, m, i	L,D
cnstr_36	m, i, a	D,a
cnstr_37	m, i, a	D,a
cnstr_38	m, i, a, a, n, j, a	WT,WT
cnstr_39	m, i, a, XR	C,a,XR
cnstr_40	m, i, a, YR	C,a,YR
cnstr_41_1	m, i, a, m, i, a	
cnstr_41_2	m, i, a, m, i, a	
cnstr_41_3	m, i, a, m, i, a	
cnstr_41_4	m, i, a, m, i, a	

Equation Definitions

ADRP1

$$\text{obj} \geq \text{T_Q}_{\text{m3,i3}} + \text{G_Q}_{\text{m3,i3}}$$

ADRP2

$$\text{obj} \geq \text{T_Y}_{\text{m3,i3}} + \text{G_Y}_{\text{m3,i3}}$$

cnstr_2_{m,i}

$$\sum_{li,n,j | (C_{n,j} \vee (n=\text{m0}) \wedge (j=\text{i0}))} z_{m,i,n,j,li} = 1 \quad \forall m, i \mid C_{m,i}$$

cnstr_3_{m,i,li}

$$\sum_{n,j | (C_{n,j} \vee (n=\text{m0}) \wedge (j=\text{i0}))} z_{n,j,m,i,li} = \sum_{h,k | (C_{h,k} \vee (h=\text{m0}) \wedge (k=\text{i0}))} z_{m,i,h,k,li} \quad \forall m, i, li \mid C_{m,i}$$

cnstr_4_{li}

$$\sum_{m,i | C_{m,i}} z_{\text{m0,i0},m,i,li} = 1 \quad \forall li$$

cnstr_5_{*li*}

$$\sum_{m,i|C_{m,i}} z_{m,i,m0,i0,li} = 1 \quad \forall li$$

cnstr_6_{*m,i*}

$$\sum_{li,n,j|(D_{n,j} \vee (n=m0) \wedge (j=i0))} z_{m,i,n,j,li} = 1 \quad \forall m,i \mid L_{m,i}$$

cnstr_7_{*m,i*}

$$\sum_{li,n,j|(L_{n,j} \vee (n=m0) \wedge (j=i0))} z_{m,i,n,j,li} = 1 \quad \forall m,i \mid D_{m,i}$$

cnstr_8_{*m,i,n,j,XR*}

$$P_X_{m,i,a4,XR} = P_X_{n,j,a0,XR}[(\sum_{li} z.L_{m,i,n,j,li} = 1)] \quad \forall m,i,n,j,XR \mid (WT_{m,i,a4} \wedge WH_{n,j,a0})$$

cnstr_9_{*m,i,n,j,YR*}

$$P_Y_{m,i,a4,YR} = P_Y_{n,j,a0,YR}[(\sum_{li} z.L_{m,i,n,j,li} = 1)] \quad \forall m,i,n,j,YR \mid (WT_{m,i,a4} \wedge WT_{n,j,a0})$$

cnstr_10_{*m,i,a*}

$$\sum_{XR} P_X_{m,i,a,XR} = 1 \quad \forall m,i,a \mid WT_{m,i,a}$$

cnstr_11_{*m,i,a*}

$$\sum_{YR} P_Y_{m,i,a,YR} = 1 \quad \forall m,i,a \mid WT_{m,i,a}$$

cnstr_12_{*m,i*}

$$\sum_{YL} P_Y_{m,i,a0,YL} = 1 \quad \forall m,i \mid (L_{m,i} \wedge WT_{m,i,a0})$$

cnstr_13_{*m,i*}

$$\sum_{YS} P_Y_{m,i,a0,YS} = 1 \quad \forall m,i \mid (D_{m,i} \wedge WT_{m,i,a0})$$

cnstr_14_{*m,i,XR*}

$$P_X_{m,i,a0,XR} = 1 \quad \forall m,i,XR \mid (D_{m,i} \wedge o_{m,i,XR} \wedge WT_{m,i,a0})$$

cnstr_15_{*m,i*}

$$\sum_{A_L_set, A_R_set} \sum_{x_t | ((x_t.val \geq A_L_set.val) \wedge (x_t.val \leq A_R_set.val))} (P_X_{m,i,a0,x_t} = 1 \quad \forall m, i \mid (L_{m,i} \wedge WT_{m,i,a0}))$$

cnstr_19_{*m,i*}

$$\sum_{A_L_set, A_R_set} \sum_{x_t | ((x_t.val \geq A_L_set.val) \wedge (x_t.val \leq A_R_set.val))} (P_X_{m,i,a3,x_t} = 1 \quad \forall m, i \mid (D_{m,i} \wedge WT_{m,i,a3}))$$

cnstr_16_{*m,i*}

$$\sum_{Y_L} P_Y_{m,i,a3,Y_L} = 1 \quad \forall m, i \mid (D_{m,i} \wedge WT_{m,i,a3})$$

cnstr_17_{*m,i*}

$$\sum_{Y_S} P_Y_{m,i,a3,Y_S} = 1 \quad \forall m, i \mid (L_{m,i} \wedge WT_{m,i,a3})$$

cnstr_18_{*m,i,XR*}

$$P_X_{m,i,a3,XR} = 1 \quad \forall m, i, XR \mid (L_{m,i} \wedge o_{m,i,XR} \wedge WT_{m,i,a3})$$

cnstr_20_{*m,i,a1,a1_1,YR*}

$$P_Y_{m,i,a1,YR} = P_Y_{m,i,a1_1,YR} \quad \forall m, i, a1, a1_1, YR \mid (WH_{m,i,a1} \wedge WT_{m,i,a1_1} \wedge (\text{ord}(a1_1) = (\text{ord}(a1) - 1)))$$

cnstr_21_{*m,i,a1,a1_1,XR*}

$$P_X_{m,i,a1,XR} = P_X_{m,i,a1_1,XR} \quad \forall m, i, a1, a1_1, XR \mid (WH_{m,i,a1} \wedge WT_{m,i,a1_1} \wedge (\text{ord}(a1_1) = (\text{ord}(a1) - 1)))$$

cnstr_22_{*m,i,n,j*}

$$U_AGV_{m,i,a4,n,j,a1} \geq \sum_{li} z_{m,i,n,j,li} \quad \forall m, i, n, j \mid (WT_{m,i,a4} \wedge WT_{n,j,a1})$$

cnstr_23_{*m,i,a1,n,j,a2,YR,XR,a1_1,a2_1*}

$$U_AGV_{m,i,a1,n,j,a2} + U_AGV_{n,j,a2,m,i,a1} + 3 - P_Y_{m,i,a1,YR} - P_Y_{n,j,a2,YR} - \sum_{XR1 | (XR1.val \leq XR.val)} (P_X_{m,i,a1_1,XR1} + P_X_{n,j,a2,XR1} - P_X_{m,i,a1,XR1} - P_X_{n,j,a2_1,XR1}) \geq 0 \quad \forall m, i, a1, n, j, a2, YR, XR, a1_1, a2_1 \mid ((\text{ord}(a1_1) = (\text{ord}(a1) - 1)) \wedge (\text{ord}(a2_1) = (\text{ord}(a2) - 1)) \wedge WH_{m,i,a1} \wedge WH_{n,j,a2})$$

cnstr_24 _{m,i,n,j,a}

$$T_Q_{m,i} + G_Q_{m,i} + Mnum \cdot (1 - U_QC_{m,i,n,j,a}) \geq T_start_{n,j,a} \quad \forall m, i, n, j, a \mid (C_{m,i} \wedge WH_{n,j,a})$$

cnstr_25 _{$m,i,n,j,a1,a1_1$}

$$T_start_{n,j,a1} + t_AGV_{n,j,a1_1,n,j,a1} + Mnum \cdot (1 - U_QC_{m,i,n,j,a1}) \geq T_Q_{m,i} \quad \forall m, i, n, j, a1, a1_1 \mid (C_{m,i} \wedge WH_{n,j,a1} \wedge (\text{ord}(a1_1) = (\text{ord}(a1) - 1)))$$

cnstr_26_1 _{$n,j,a2,YS,m,i,a1,a2_1$}

$$(3 - U_QC_{m,i,n,j,a2} - P_Y_{m,i,a1,YS} - P_Y_{n,j,a2,YS} + (\sum_{XR \mid (XR.val \leq o1_{m,i})} P_X_{n,j,a2,XR} - \sum_{XR \mid (XR.val > o1_{m,i})} P_X_{n,j,a2_1,XR} [(\sum_{XR \mid (XR.val > o1_{m,i})} P_X.L_{n,j,a2_1,XR}])) \cdot Mnum + T_start_{n,j,a2} + t_AGV_{n,j,a2_1,m,i,a1} \geq T_Q_{m,i} + G_Q_{m,i} \quad \forall n, j, a2, YS, m, i, a1, a2_1 \mid (((a1 = a0) \wedge D_{m,i} \vee (a1 = a3) \wedge L_{m,i}) \wedge WH_{n,j,a2} \wedge (\text{ord}(a2_1) = (\text{ord}(a2) - 1)))$$

cnstr_26_2 _{$n,j,a2,YS,m,i,a1,a2_1$}

$$(3 - U_QC_{m,i,n,j,a2} - P_Y_{m,i,a1,YS} - P_Y_{n,j,a2,YS} + (- \sum_{XR \mid (XR.val \leq o1_{m,i})} P_X_{n,j,a2,XR} + \sum_{XR \mid (XR.val > o1_{m,i})} P_X_{n,j,a2_1,XR} [(\sum_{XR \mid (XR.val > o1_{m,i})} P_X.L_{n,j,a2_1,XR}])) \cdot Mnum + T_start_{n,j,a2} + t_AGV_{n,j,a2_1,m,i,a1} \geq T_Q_{m,i} + G_Q_{m,i} \quad \forall n, j, a2, YS, m, i, a1, a2_1 \mid (((a1 = a0) \wedge D_{m,i} \vee (a1 = a3) \wedge L_{m,i}) \wedge WH_{n,j,a2} \wedge (\text{ord}(a2_1) = (\text{ord}(a2) - 1)))$$

cnstr_27 _{$m,i,a1,n,j,a2,XR$}

$$U_AGV_{m,i,a1,n,j,a2} + U_AGV_{n,j,a2,m,i,a1} \geq P_X_{m,i,a1,XR} + P_X_{n,j,a2,XR} - 1 \quad \forall m, i, a1, n, j, a2, XR \mid (WV_{m,i,a1} \wedge WV_{n,j,a2})$$

cnstr_28 _{$m,i,a1,a1_1$}

$$U_AGV_{m,i,a1_1,m,i,a1} = 1 \quad \forall m, i, a1, a1_1 \mid (C_{m,i} \wedge ((a1 = a2) \vee (a1 = a3) \vee (a1 = a4)) \wedge (\text{ord}(a1_1) = (\text{ord}(a1) - 1)))$$

cnstr_29 _{$m,i,i1$}

$$T_Q_{m,i1} \geq T_Q_{m,i} + G_Q_{m,i} + S_Q \quad \forall m, i, i1 \mid (C_{m,i} \wedge (\text{ord}(i1) = (\text{ord}(i) + 1)) \wedge C_{m,i1})$$

cnstr_30 _{m,i,n,j}

$$T_Q_{n,j} \geq T_Q_{m,i} + G_Q_{m,i} \quad \forall m, i, n, j \mid \text{psi_1}_{m,i,n,j}$$

cnstr_31 _{m,i,n,j}

$$T_Y_{n,j} \geq T_Y_{m,i} + G_Y_{m,i} \quad \forall m, i, n, j \mid \text{psi_2}_{m,i,n,j}$$

cnstr_32 _{m,i}

$$T_Y_{m,i} \geq T_start_{m,i,a3} + t_AGV_{m,i,a2,m,i,a3} \quad \forall m, i \mid D_{m,i}$$

cnstr_33 _{m,i}

$$T_Q_{m,i} \geq T_start_{m,i,a3} + t_AGV_{m,i,a2,m,i,a3} \quad \forall m, i \mid L_{m,i}$$

cnstr_34 _{m,i,n,j}

$$T_Y_{n,j} + Mnum \cdot (1 - \sum_{li} z_{m,i,n,j,li}) \geq T_start_{m,i,a4} + t_AGV_{m,i,a3,m,i,a4} \quad \forall m, i, n, j \mid (D_{m,i} \wedge L_{n,j})$$

cnstr_35 _{m,i,n,j}

$$T_Q_{n,j} + Mnum \cdot (1 - \sum_{li} z_{m,i,n,j,li}) \geq T_start_{m,i,a4} + t_AGV_{m,i,a3,m,i,a4} \quad \forall m, i, n, j \mid (L_{m,i} \wedge D_{n,j})$$

cnstr_36 _{m,i,a}

$$T_start_{m,i,a} \geq T_Y_{m,i} + G_Y_{m,i} \quad \forall m, i, a \mid (D_{m,i} \wedge (a = a4) \vee L_{m,i} \wedge (a = a1))$$

cnstr_37 _{m,i,a}

$$T_start_{m,i,a} \geq T_Q_{m,i} + G_Q_{m,i} \quad \forall m, i, a \mid (D_{m,i} \wedge (a = a1) \vee L_{m,i} \wedge (a = a4))$$

cnstr_38 _{$m,i,a1,a1_1,n,j,a2$}

$$T_start_{n,j,a2} + Mnum \cdot (1 - U_AGV_{m,i,a1,n,j,a2}) \geq T_start_{m,i,a1} + t_AGV_{m,i,a1_1,m,i,a1} \quad \forall m, i, a1, a1_1, n, j, a2 \mid (WT_{m,i}, WT_{n,j,a2} \wedge (\text{ord}(a1_1) = (\text{ord}(a1) - 1)))$$

cnstr_39 _{m,i,a,XR}

$$X_position_{m,i,a} = XR.val[(P_X.L_{m,i,a,XR} = 1)] \quad \forall m, i, a, XR \mid C_{m,i}$$

cnstr_40 _{m,i,a,YR}

$$Y_position_{m,i,a} = YR.val[(P_Y.L_{m,i,a,YR} = 1)] \quad \forall m, i, a, YR \mid C_{m,i}$$

cnstr_41_1 _{$m,i,a1,n,j,a2$}

$$t_AGV_{m,i,a1,n,j,a2} = \frac{X_position_{m,i,a1} - X_position_{n,j,a2} + Y_position_{m,i,a1} - Y_position_{n,j,a2}}{v} \quad \forall m, i, a1, n, j, a2 \mid (WT_{m,i,a1} \wedge WT_{n,j,a2} \wedge ((X_position.L_{m,i,a1} \geq X_position.L_{n,j,a2}) \wedge (Y_position.L_{m,i,a1} \geq Y_position.L_{n,j,a2})))$$

cnstr_41_2 _{$m,i,a1,n,j,a2$}

$$t_AGV_{m,i,a1,n,j,a2} = \frac{-X_position_{m,i,a1} + X_position_{n,j,a2} - Y_position_{m,i,a1} + Y_position_{n,j,a2}}{v} \forall m, i, a1, n, j, a2 \mid (WT_{m,i,a1} \wedge WT_{n,j,a2} \wedge ((X_position.L_{m,i,a1} \leq X_position.L_{n,j,a2}) \wedge (Y_position.L_{m,i,a1} \leq Y_position.L_{n,j,a2})))$$

cnstr_41_3 _{$m,i,a1,n,j,a2$}

$$t_AGV_{m,i,a1,n,j,a2} = \frac{X_position_{m,i,a1} - X_position_{n,j,a2} - Y_position_{m,i,a1} + Y_position_{n,j,a2}}{v} \forall m, i, a1, n, j, a2 \mid (WT_{m,i,a1} \wedge WT_{n,j,a2} \wedge ((X_position.L_{m,i,a1} \geq X_position.L_{n,j,a2}) \wedge (Y_position.L_{m,i,a1} \leq Y_position.L_{n,j,a2})))$$

cnstr_41_4 _{$m,i,a1,n,j,a2$}

$$t_AGV_{m,i,a1,n,j,a2} = \frac{-X_position_{m,i,a1} + X_position_{n,j,a2} + Y_position_{m,i,a1} - Y_position_{n,j,a2}}{v} \forall m, i, a1, n, j, a2 \mid (WT_{m,i,a1} \wedge WT_{n,j,a2} \wedge ((X_position.L_{m,i,a1} \leq X_position.L_{n,j,a2}) \wedge (Y_position.L_{m,i,a1} \geq Y_position.L_{n,j,a2})))$$

$$T_Q_{m,i} \geq 0 \forall m, i$$

$$T_Y_{m,i} \geq 0 \forall m, i$$

$$z_{m,i,m,i,li} \in \{0, 1\} \forall m, i, m, i, li$$

$$P_X_{m,i,a,XR} \in \{0, 1\} \forall m, i, a, XR$$

$$P_Y_{m,i,a,YR} \in \{0, 1\} \forall m, i, a, YR$$

$$U_AGV_{m,i,a,m,i,a} \in \{0, 1\} \forall m, i, a, m, i, a$$

$$U_QC_{m,i,m,i,a} \in \{0, 1\} \forall m, i, m, i, a$$

$$T_start_{m,i,a} \geq 0 \forall m, i, a$$

$$t_AGV_{m,i,a,m,i,a} \geq 0 \forall m, i, a, m, i, a$$

$$X_position_{m,i,a} \geq 0 \forall m, i, a$$

$$Y_position_{m,i,a} \geq 0 \forall m, i, a$$