$Y^{position}_{(m,i,\alpha)}$ The horizontal path where the finishing location of action (m, i, α) is located. Note that variables $X^{position}_{(m,i,\alpha)}$ and $Y^{position}_{(m,i,\alpha)}$ are used to link variables $P^X_{(m,i,\alpha),x}$ and $P^Y_{(m,i,\alpha),y}$ with $t^{AGV}_{(m,i,\alpha_1)(n,j,\alpha_2)}$, as shown in constraints (37) to (39), and $(m, i, 0)$ in those two variables indicates the starting location of container (m, i).

4.3. Mathematical model

Def of makespan

The object of the model is to minimize the makespan of all containers, i.e., the time when the last container has been completed, as shown in objective function (1). The finishing time is $T^Q_{(m,i_d)} + G^Q_{(m,i_d)}$ if the last container (m, i_d) of QC m is a loading job; otherwise, the makespan is $T^Y_{(m,i_d)} + G^Y_{(m,i_d)}$.

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$$[\textbf{ADRP model}]\ \text{Min} : \max_m \left\{ T^Q_{(m,i_d)} + G^Q_{(m,i_d)}, T^Y_{(m,i_d)} + G^Y_{(m,i_d)} \right\} \tag{1}$$

The constraints of the model can be divided into five groups as follows: the job assignment constraints, the location constraints of AGV actions, the conflict-free constraints of AGV actions, the time constraints, and the constraints on the range of variables. The explanations of the five groups and the constraints of this model are shown as follows.

(i) **Job assignment constraints.** The job assignment constraints determine which container the AGV transports and the sequence in which the AGV transports the containers, and also ensure that the multiple QC double cycling model is adopted. Constraint (2) ensures that each container is handled by one AGV, where 0 is a virtual node. Constraint (3) ensures the operation continuity of the AGVs, which means that each container has either one predecessor or one successor for AGV operation. Constraint (4) ensures that each AGV must start its operation from the virtual node. Constraint (5) ensures that each AGV must finish its operation at the virtual node. Constraints (6) and (7) ensure that the multiple QC double cycling model is used for the AGV operation.

$$\sum_{l\in B} \sum_{(n,j)\in C\cup\{0\}} Z_{(m,i)(n,j),l} = 1, \forall (m,i) \in C \tag{2}$$

$$\sum_{(n,j)\in C\cup\{0\}} Z_{(n,j)(m,i),l} = \sum_{(h,k)\in C\cup\{0\}} Z_{(m,i)(h,k),l}, \forall (m,i) \in C, \forall l \in B \tag{3}$$

$$\sum_{(m,i)\in C} Z_{0(m,i),l} = 1, \forall l \in B \tag{4}$$

$$\sum_{(m,i)\in C} Z_{(m,i)0,l} = 1, \forall l \in B \tag{5}$$

$$\sum_{l\in B} \sum_{(n,j)\in D\cup\{0\}} Z_{(m,i)(n,j),l} = 1, \forall (m,i) \in L \tag{6}$$

$$\sum_{l\in B} \sum_{(n,j)\in L\cup\{0\}} Z_{(m,i)(n,j),l} = 1, \forall (m,i) \in D \tag{7}$$

(ii) **Location constraints of AGV actions.** The location constraints of AGV actions determine the location of the AGV during its transportation. The container storage locations are known in this model, therefore the vertical path where the QC operation location of each container is located is given, and the vertical path range of the yard side operation location of each container is also known. The starting location of the unloading container for AGV transportation is on the quayside and the finishing location is on the yard side, and the loading container is the reverse of the unloading container, which means that the location constraints for the unloading and loading containers are also different. The location constraints of AGV actions can be explained as follows.

Constraints (8) and (9) indicate that if container (m, i) is the preceding job of (n, j) transported by the same AGV, the finishing location of (m, i) is the same as the starting location of (n, j). Constraint (8) is for vertical path and constraint (9) is for horizontal path. Constraints (10) and (11) ensure that there can only be one location for AGV action, including the starting location of action $(m, i, 1)$ and the finishing locations of actions $(m, i, 1)$ to $(m, i, 4)$. Constraint (12) indicates that the starting horizontal path of the loading container is in the AGV landside operation area. Constraint (13) indicates that the starting horizontal path of the unloading container is in the AGV seaside operation area. Constraint (14) indicates that the starting vertical path of the unloading container is the vertical path of the QC operation location. Constraint (15) indicates that the starting vertical path of the loading container is the vertical path corresponding to the block that stored container (m, i). Constraint (16) indicates that the finishing horizontal path of the unloading container is in the AGV landside operation area. Constraint (17) indicates that the finishing horizontal path of the loading container is in the AGV seaside operation area. Constraint (18) indicates that the finishing vertical path of the loading container is the vertical path of the QC operation location. Constraint (19) indicates that the finishing vertical path of the unloading container is the vertical path corresponding to the block that stored container (m, i). Constraints (20) and (21) ensure the locational continuity of the AGV transportation. Constraint (20) ensures that the starting location and finishing location of a horizontal action are on the same horizontal path. Constraint (21) ensures that the starting location and finishing location of a vertical action are on the same vertical path.

$$P^X_{(m,i,4),x} = P^X_{(n,j,0),x}, \sum_{l\in B} Z_{(m,i)(n,j),l} = 1, \forall (m,i),(n,j) \in C, \forall x \in X^R \tag{8}$$

$$P^Y_{(m,i,4),y} = P^Y_{(n,j,0),y}, \sum_{l\in B} Z_{(m,i)(n,j),l} = 1, \forall (m,i),(n,j) \in C, \forall y \in Y^R \tag{9}$$

$$\sum_{x\in X^R} P^X_{(m,i,\alpha),x} = 1, \forall (m,i) \in C, \forall \alpha \in \{0,1,2,3,4\} \tag{10}$$

$$\sum_{y\in Y^R} P^Y_{(m,i,\alpha),y} = 1, \forall (m,i) \in C, \forall \alpha \in \{0,1,2,3,4\} \tag{11}$$

$$\sum_{y\in Y^L} P^Y_{(m,i,0),y} = 1, \forall (m,i) \in L \tag{12}$$

$$\sum_{y\in Y^S} P^Y_{(m,i,0),y} = 1, \forall (m,i) \in D \tag{13}$$

$$P^X_{(m,i,0),O_{(m,i)}} = 1, \quad \forall (m,i) \in D \tag{14}$$

$$\sum_{x=A^L_{(m,i)}}^{A^R_{(m,i)}} P^X_{(m,i,0),x} = 1, \forall (m,i) \in L \tag{15}$$

$$\sum_{y\in Y^L} P^Y_{(m,i,3),y} = 1, \forall (m,i) \in D \tag{16}$$

$$\sum_{y\in Y^S} P^Y_{(m,i,3),y} = 1, \forall (m,i) \in L \tag{17}$$

$$P^X_{(m,i,3),O_{(m,i)}} = 1, \forall (m,i) \in L \tag{18}$$

$$\sum_{x=A^L_{(m,i)}}^{A^R_{(m,i)}} P^X_{(m,i,3),x} = 1, \forall (m,i) \in D \tag{19}$$