Training Neural Networks Practical Issues

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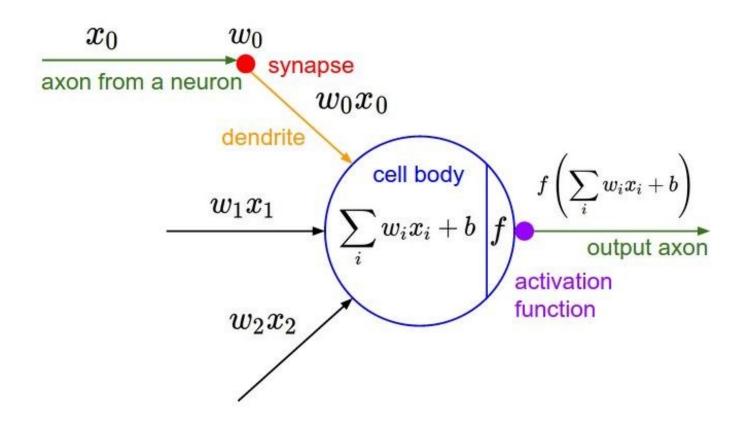
Most slides have been adapted from Fei Fei Li and colleagues lectures, cs231n

Outline

- Activation Functions
- Data Preprocessing
- Weight Initialization
- Choosing hyperparameters

Activation Functions

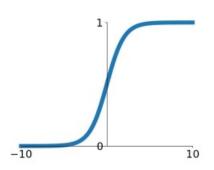
Neuron



Activation Functions

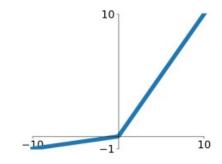
Sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



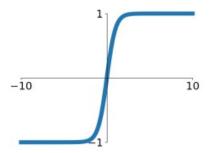
Leaky ReLU

 $\max(0.1x, x)$



tanh

tanh(x)

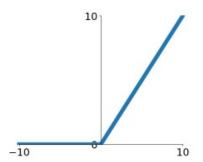


Maxout

 $\max(w_1^T x + b_1, w_2^T x + b_2)$

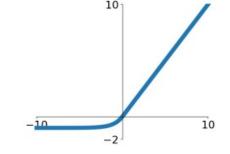
ReLU

 $\max(0, x)$

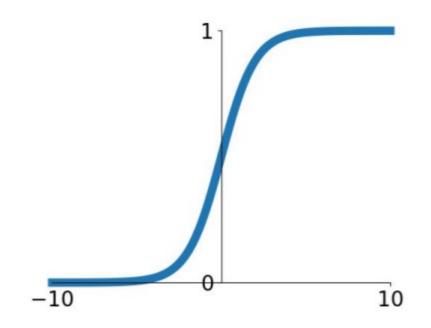


ELU

$$\begin{cases} x & x \ge 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

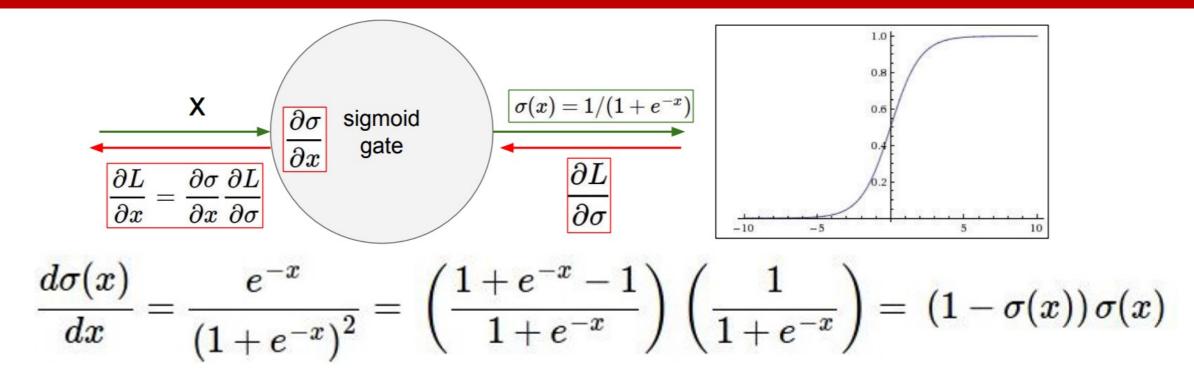


$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

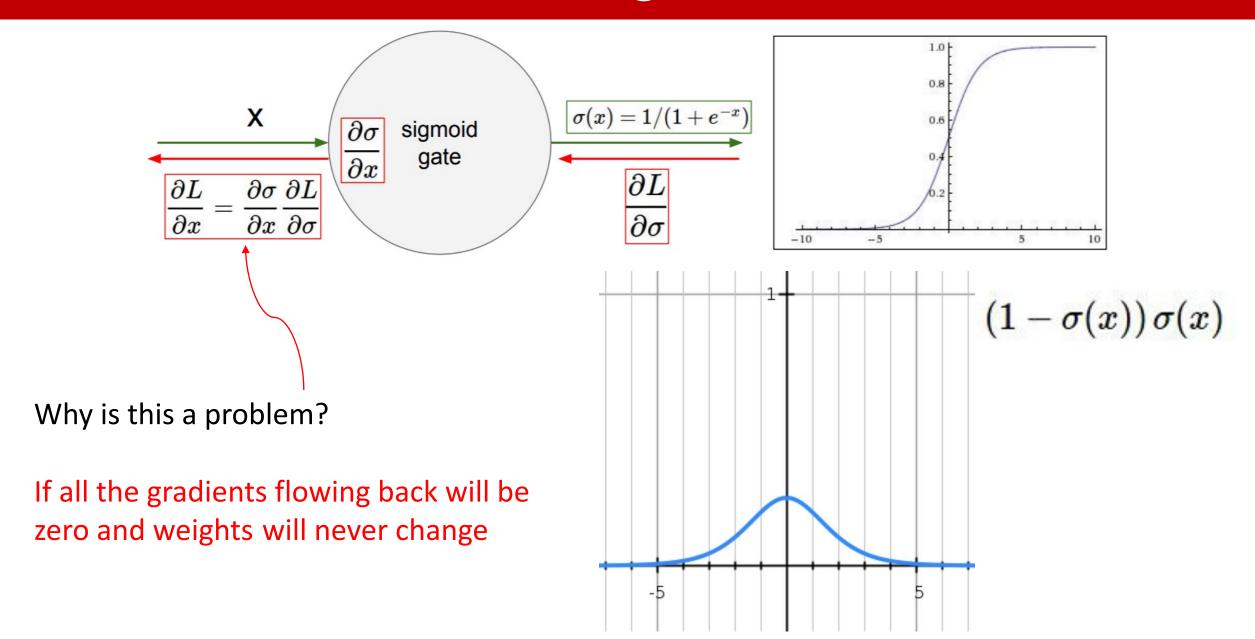
Problems:

1. Saturated neurons "kill" the gradients

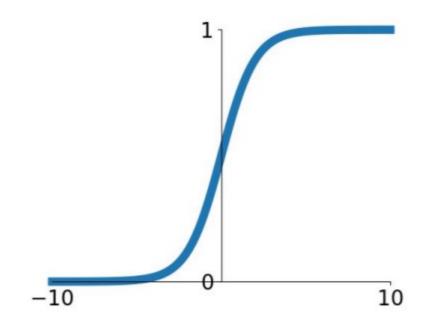
Sigmoid



- What happens when x = -10?
- What happens when x = 0?
- What happens when x = 10?



- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron



$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Problems:

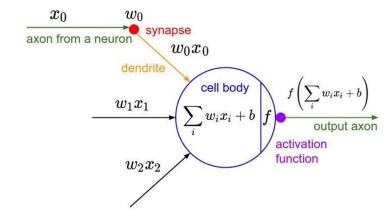
- 1. Saturated neurons "kill" the gradients
- 2. Sigmoid outputs are not zero-centered

Sigmoid

• Consider what happens when the input to a neuron (x) is always positive: $x_0 = x_0 = x_0$

 $f\left(\sum_{i}w_{i}\,x_{i}+b\right)$

• What can we say about the gradients on w?



 Consider what happens when the input to a neuron (x) is always positive:

$$f\left(\sum_{i}w_{i}\,x_{i}+b\right)$$

What can we say about the gradients on w?

We know that local gradient of sigmoid is always positive

$$\frac{\partial L}{\partial w} = \sigma \left(\sum_{i} w_{i} x_{i} + b \right) \left(1 - \sigma \left(\sum_{i} w_{i} x_{i} + b \right) \right) x \times upstream_gradient$$
We are assuming x is always positive

So!! Sign of gradient for all w_i is the same as the sign of upstream scalar gradient!

 Consider what happens when the input to a neuron is always positive...

 What can we say about the gradients on w? Always all positive or all negative 😊

$$f\left(\sum_{i} w_{i} x_{i} + b\right)$$

(For a single element! Minibatches help)

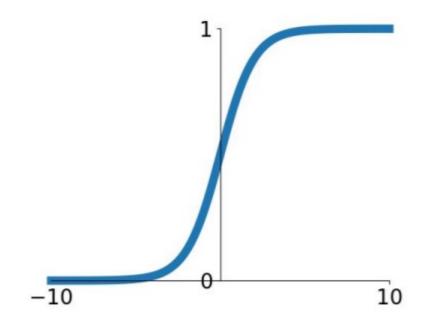
allowed gradient update directions zig zag path directions hypothetical optimal w vector

allowed

gradient

update

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron



$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

3 problems:

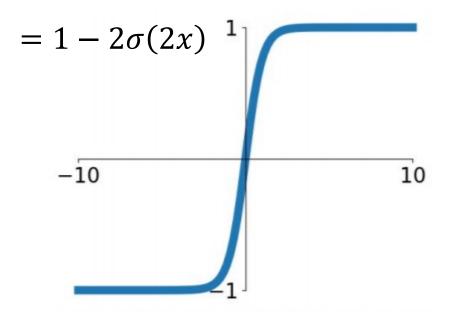
- 1. Saturated neurons "kill" the gradients
- 2. Sigmoid outputs are not zero-centered

Sigmoid

Activation functions: tanh

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

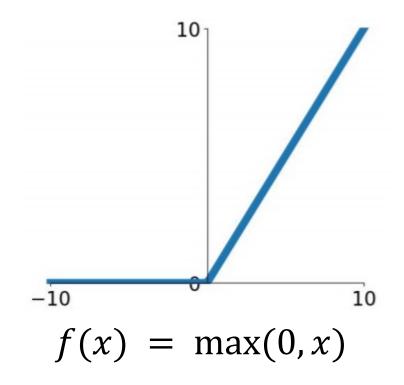
$$\tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$
 [LeCun et al., 1991]



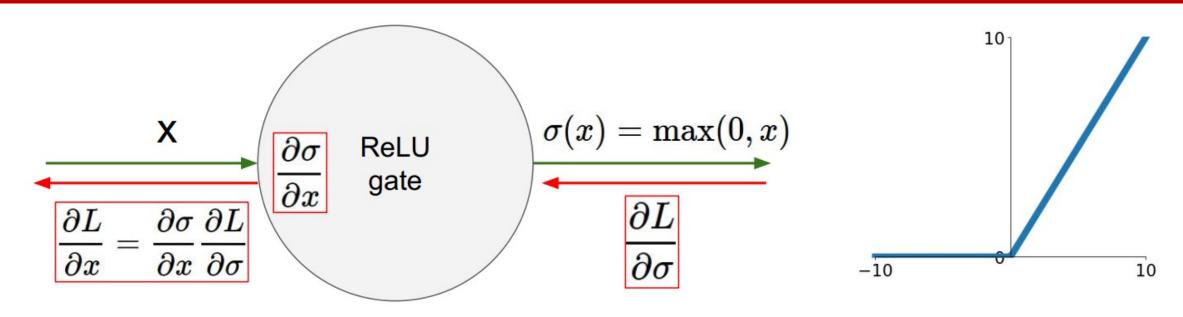
Activation functions: ReLU (Rectified Linear Unit)

[Krizhevsky et al., 2012]

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid



Activation functions: ReLU

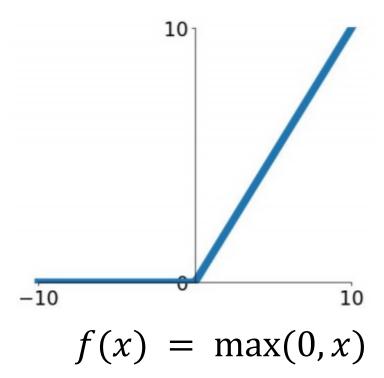


- What happens when x = -10?
- What happens when x = 0?
- What happens when x = 10?

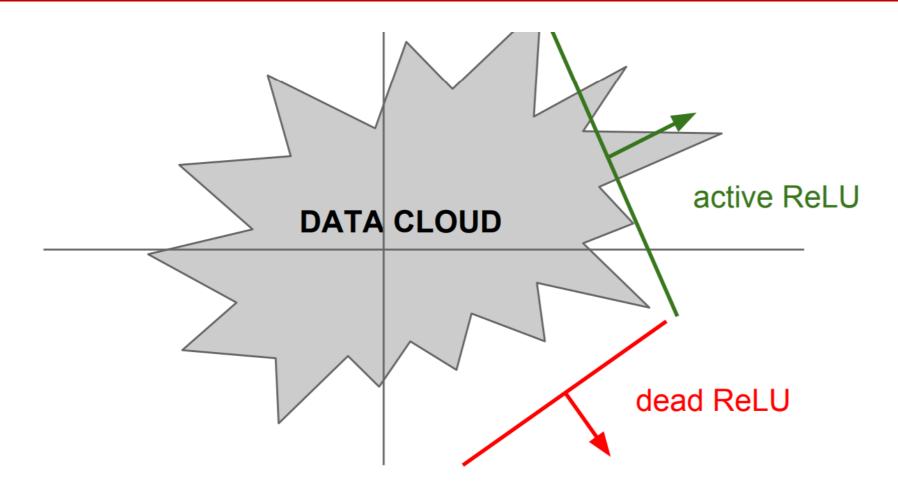
Activation functions: ReLU

[Krizhevsky et al., 2012]

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid
- Problems:
 - Not zero-centered output
 - An annoyance:
 - hint: what is the gradient when x < 0?



Activation functions: ReLU

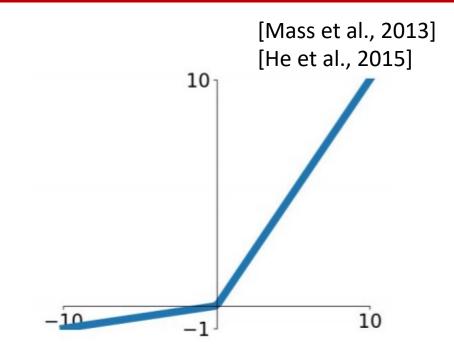


=> people like to initialize ReLU neurons with slightly positive biases (e.g. 0.01)

It can be due initialization or high learning rate

Activation functions: Leaky ReLU

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".



Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

 α is determined during backpropagation

Leaky ReLU

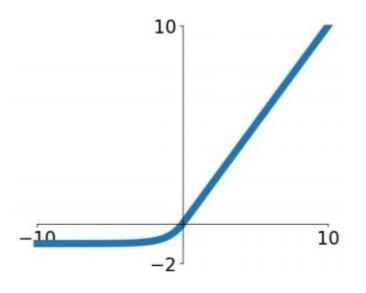
$$f(x) = \max(0.01x, x)$$

Activation Functions: Exponential Linear Units (ELU)

- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise

Computation requires exp()

[Clevert et al., 2015]



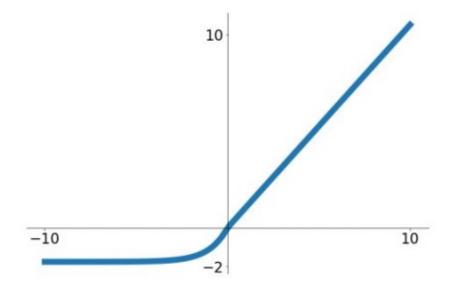
$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

(Alpha default = 1)

Activation Functions: Scaled Exponential Linear Units (SELU)

- Scaled version of ELU that works better for deep networks
- "Self-normalizing" property;
- Can train deep SELU networks without BatchNorm

[Klambauer et al. ICLR 2017]



$$f(x) = egin{cases} \lambda x & ext{if } x > 0 \ \lambda lpha(e^x - 1) & ext{otherwise} \end{cases}$$

 $\alpha = 1.6732632423543772848170429916717$ $\lambda = 1.0507009873554804934193349852946$

Maxout "Neuron"

[Goodfellow et al., 2013]

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

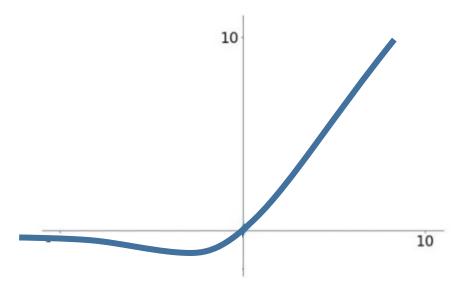
- Does not have the basic form of "dot product -> nonlinearity"
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

Problem: doubles the number of parameters/neuron :(

GELU

$$GELU(x) = x\phi(\alpha x) = x \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

- α controls the smoothness of the GELU function (generally $\alpha=1$)
- Used in BERT, ViT, and GPT

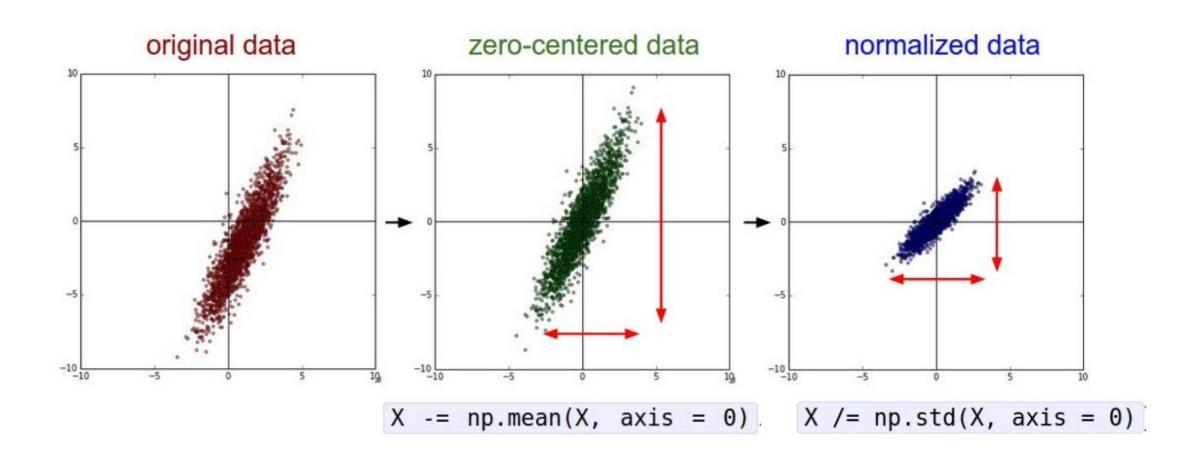


Activation functions

- In practice:
 - Use ReLU. Be careful with your learning rates
 - Try out Leaky ReLU / Maxout / ELU / SELU / GELU
 - To squeeze out some marginal gains
 - Use sigmoid and tanh only for gating purposes

Data Preprocessing

Data Preprocessing



(Assume X [NxD] is data matrix, each example in a row)

Normalizing the input

On the training set compute mean of each input (feature)

$$-\mu_{i} = \frac{\sum_{n=1}^{N} x_{i}^{(n)}}{N}$$
$$-\sigma_{i}^{2} = \frac{\sum_{n=1}^{N} (x_{i}^{(n)} - \mu_{i})^{2}}{N}$$

• Remove mean: from each of the input mean of the corresponding input

$$-x_i \leftarrow x_i - \mu_i$$

Normalize variance:

$$-x_i \leftarrow \frac{x_i}{\sigma_i}$$

• If we normalize the training set we use the same μ and σ^2 to normalize test data too

Why zero-mean input?

Reminder: sigmoid

• Consider what happens when the input to a neuron is always positive...

 What can we say about the gradients on w? Always all positive or all negative <a>©

- this is also why you want zero-mean data!

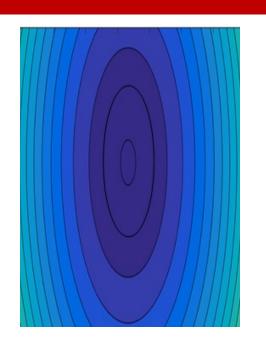
allowed gradient update directions allowed gradient update directions

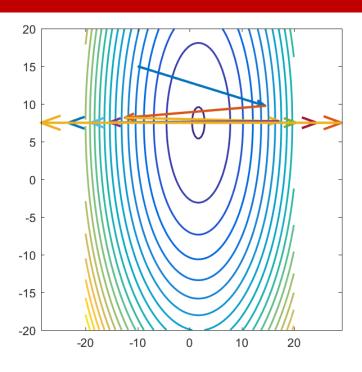
hypothetical optimal w vector

zig zag path

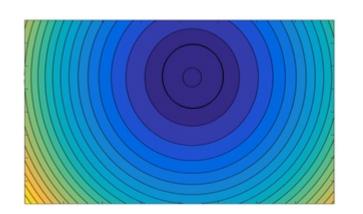
Why normalize inputs?

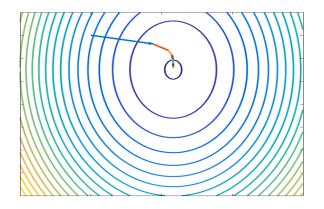
Very different range of input features





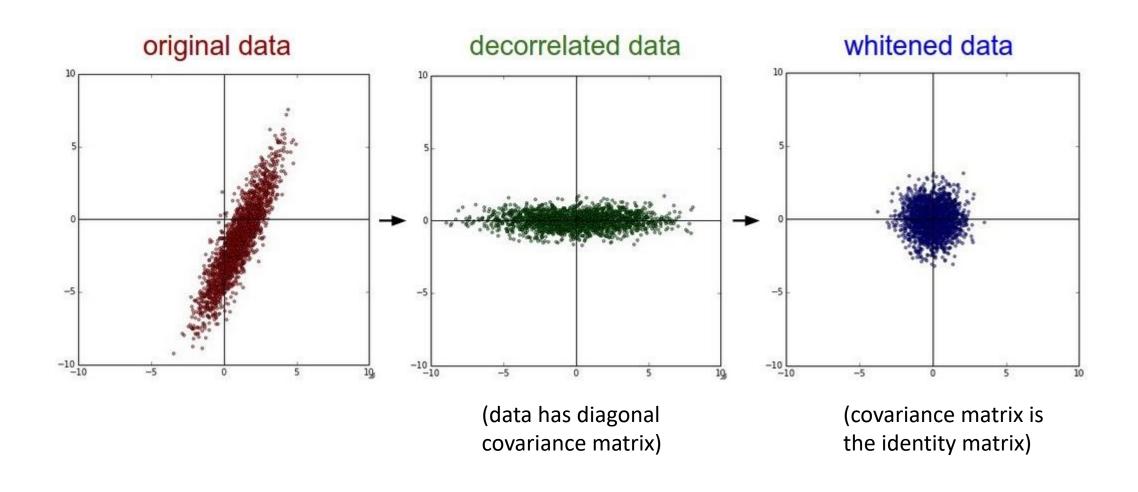
Normalized input: Speedup training





Data Preprocessing

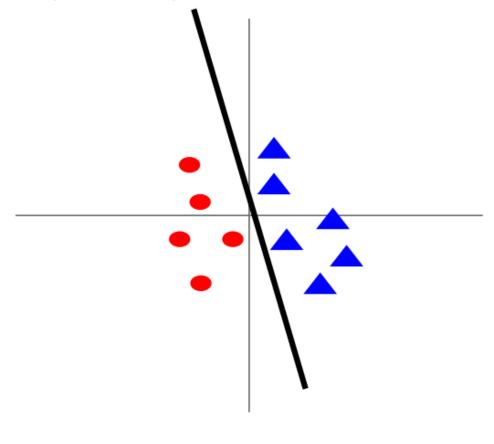
Not common to normalize variance, to do PCA or whitening



Data Preprocessing

Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize

After normalization: less sensitive to small changes in weights; easier to optimize



TLDR: In practice for Images: center only

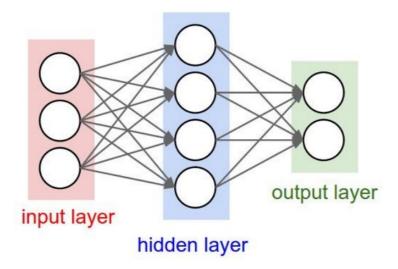
- Example: consider CIFAR-10 example with [32,32,3] images
- Subtract the mean image (e.g. AlexNet)
 - (mean image = [32,32,3] array)

- Subtract per-channel mean (e.g. VGGNet)
 - (mean along each channel = 3 numbers)
- Subtract per-channel mean and divide by per-channel std (e.g. ResNet)
 - (mean along each channel = 3 numbers)

Weight Initialization

Weight initialization

- How to choose the starting point for the iterative process of optimization
- Q: what happens when W=constant init is used?
- Key point: neurons with identical connections that are identically initialized will never diverge
 - All neurons of a layer do the same thing



Weight initialization

First idea: Small random numbers
 (gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

Works ~okay for small networks, but problems with deeper networks.

Weight Initialization: Activation statistics

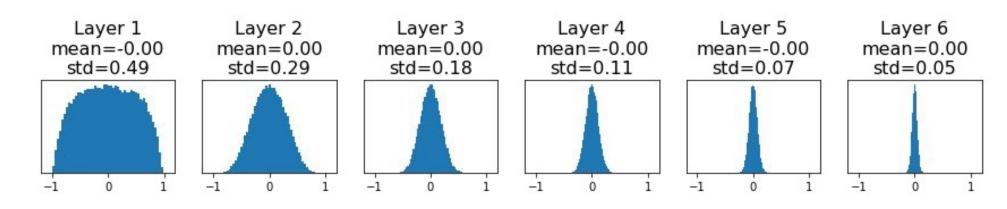
```
dims = [4096] * 7 Forward pass for a 6-layer
hs = [] net with hidden size 4096
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
hs.append(x)
```

What will happen to the activations for the last layer?

All activations tend to zero for deeper network layers

What do the gradients dL/dW look like?

All zero, no learning =(



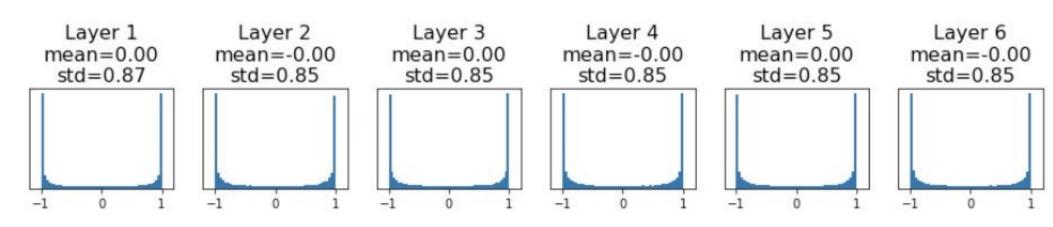
Weight Initialization: Activation statistics

What will happen to the activations for the last layer?

All activations saturate

What do the gradients look like?

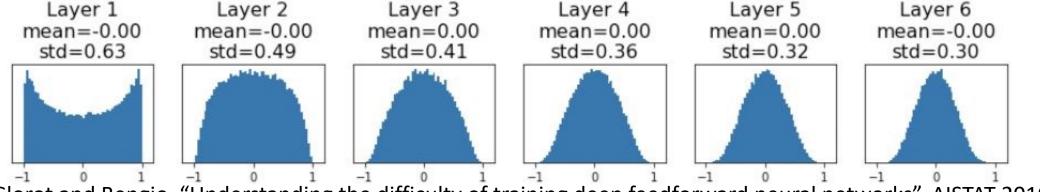
Local gradients all zero, no learning =(



Initialization: Gaussian with zero mean and $1/\sqrt{\text{fan_in}}$ standard deviation fan_in for fully connected layers = number of neurons in the previous layer

[Glorot et al., 2010]

"Just right": Activations are nicely scaled for all layers!



Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

"Just right": Activations are nicely scaled for all layers!
For conv layers, we will see Din is filter_size² * input_channels

```
Let: y = x_1 w_1 + x_2 w_2 + ... + x_{Din} w_{Din}

Assume: Var(x_1) = Var(x_2) = ... = Var(x_{Din})

We want: Var(y) = Var(x_1)
```

 $Var(y) = Var(x_1w_1+x_2w_2+...+x_{Din}w_{Din})$ [substituting value of y]

"Just right": Activations are nicely scaled for all layers!

```
Let: y = x_1 w_1 + x_2 w_2 + ... + x_{Din} w_{Din}

Assume: Var(x_1) = Var(x_2) = ... = Var(x_{Din})

We want: Var(y) = Var(x_i)
```

```
Var(y) = Var(x_1w_1 + x_2w_2 + ... + x_{Din}w_{Din})
= Din Var(x_iw_i)
[Assume all x<sub>i</sub>, w<sub>i</sub> are iid]
```

"Just right": Activations are nicely scaled for all layers!

```
Let: y = x_1 w_1 + x_2 w_2 + ... + x_{Din} w_{Din}

Assume: Var(x_1) = Var(x_2) = ... = Var(x_{Din})

We want: Var(y) = Var(x_i)
```

```
Var(y) = Var(x_1w_1 + x_2w_2 + ... + x_{Din}w_{Din})
= Din Var(x_iw_i)
= Din Var(x_i) Var(w_i)
[Assume all x<sub>i</sub>, w<sub>i</sub> are zero mean]
```

"Just right": Activations are nicely scaled for all layers!
For conv layers, Din is filter_size² * input_channels

```
Let: y = x_1 w_1 + x_2 w_2 + ... + x_{Din} w_{Din}

Assume: Var(x_1) = Var(x_2) = ... = Var(x_{Din})

We want: Var(y) = Var(x_i)
```

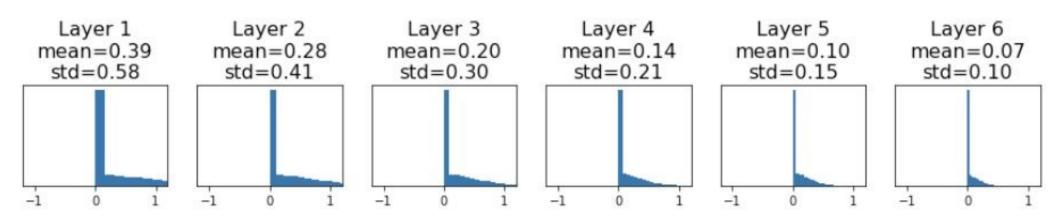
```
\begin{aligned} \text{Var}(\textbf{y}) &= \text{Var}(\textbf{x}_1 \textbf{w}_1 + \textbf{x}_2 \textbf{w}_2 + ... + \textbf{x}_{\text{Din}} \textbf{w}_{\text{Din}}) \\ &= \text{Din Var}(\textbf{x}_i \textbf{w}_i) \\ &= \text{Din Var}(\textbf{x}_i) \text{ Var}(\textbf{w}_i) \\ [\text{Assume all } \textbf{x}_i, \ \textbf{w}_i \text{ are iid}] \end{aligned}
```

So, $Var(y) = Var(x_i)$ only when $Var(w_i) = 1/Din$

Weight Initialization: What about ReLU?

Xavier assumes zero centered activation function

Activations collapse to zero again, no learning =(



Weight Initialization: Kaiming / MSRA Initialization

Initialization: gaussian with zero mean and 1/√fan_{in}/2 standard deviation

dims = [4096] * 7
hs = []

x = np.random.randn(16, dims[0])

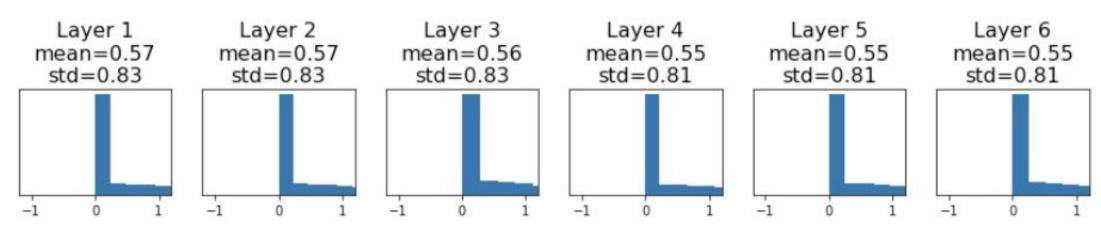
for Din, Dout in zip(dims[:-1], dims[1:]):

W = np.random.randn(Din, Dout) * np.sqrt(2/Din)

x = np.maximum(0, x.dot(W))

"Just right": Activations are nicely scaled for all layers!

hs.append(x)



He et al, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", ICCV 2015

Proper initialization is an active area of research...

- Understanding the difficulty of training deep feedforward neural networks by Glorot and Bengio, 2010
- Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013
- Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014
- Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015
- Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015
- All you need is a good init, Mishkin and Matas, 2015
- Fixup Initialization: Residual Learning Without Normalization, Zhang et al, 2019
- The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks, Frankle and Carbin, 2019

Choosing Hyperparameters (without tons of GPUs)

• Step 1: Check initial loss

Turn off weight decay, sanity check loss at initialization e.g. log(C) for softmax with C classes

- Step 1: Check initial loss
- Step 2: Overfit a small sample

Try to train to 100% training accuracy on a small sample of training data (~5-10 mini-batches); fiddle with architecture, learning rate, weight initialization

```
Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
finished optimization, best validation accuracy: 1.000000
```

Loss not going down? LR too low, bad initialization Loss explodes to Inf or NaN? LR too high, bad initialization

- Step 1: Check initial loss
- Step 2: Overfit a small sample
- Step 3: Find LR that makes loss go down

Use the architecture from the previous step, use all training data, turn on small weight decay, find a learning rate that makes the loss drop significantly within ~100 iterations

Good learning rates to try: 1e-1, 1e-2, 1e-3, 1e-4

Hyperparameter search

- Larger Neural Networks typically require a long time to train
 - so performing hyperparameter search can take many days/weeks

 A single validation set of respectable size substantially simplifies the code base, without the need for cross-validation with multiple folds

Validation strategy

- coarse -> fine validation in stages
 - Zoom in to smaller region of hyperparameters and sample very densely in them

• First stage: only a few epochs to get rough idea of what params work

• Second stage: longer running time, finer search ... (repeat as necessary)

- Step 1: Check initial loss
- Step 2: Overfit a small sample
- Step 3: Find LR that makes loss go down
- Step 4: Coarse grid, train for ~1-5 epochs

Choose a few values of learning rate and weight decay around what worked from Step 3, train a few models for ~1-5 epochs.

Good weight decay to try: 1e-4, 1e-5, 0

- Step 1: Check initial loss
- Step 2: Overfit a small sample
- Step 3: Find LR that makes loss go down
- Step 4: Coarse grid, train for ~1-5 epochs
- Step 5: Refine grid, train longer

Pick best models from Step 4, train them for longer (~10-20 epochs) without learning rate decay

For example: run coarse search for 5 epochs

```
val_acc: 0.412000, lr: 1.405206e-04, reg: 4.793564e-01, (1 / 100)
val_acc: 0.214000, lr: 7.231888e-06, reg: 2.321281e-04, (2 / 100)
val_acc: 0.208000, lr: 2.119571e-06, reg: 8.011857e+01, (3 / 100)
val_acc: 0.196000, lr: 1.551131e-05, reg: 4.374936e-05, (4 / 100)
val_acc: 0.079000, lr: 1.753300e-05, reg: 1.200424e+03, (5 / 100)
val_acc: 0.223000, lr: 4.215128e-05, reg: 4.196174e+01, (6 / 100)
val_acc: 0.441000, lr: 1.750259e-04, reg: 2.110807e-04, (7 / 100)
val_acc: 0.241000, lr: 6.749231e-05, reg: 4.226413e+01, (8 / 100)
val_acc: 0.482000, lr: 4.296863e-04, reg: 6.642555e-01, (9 / 100)
val_acc: 0.079000, lr: 5.401602e-06, reg: 1.599828e+04, (10 / 100)
val_acc: 0.154000, lr: 1.618508e-06, reg: 4.925252e-01, (11 / 100)
```

nice

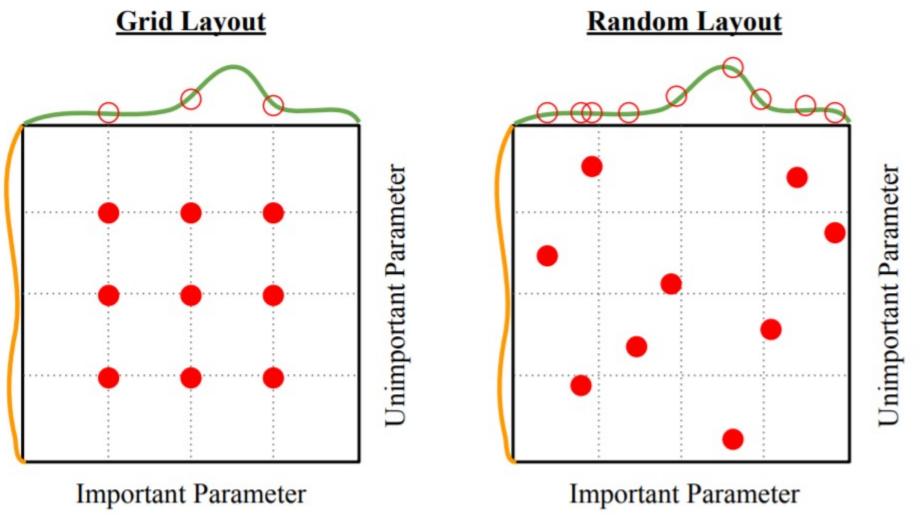
Now run finer search

```
max count = 100
                                               adjust range
                                                                               max count = 100
for count in xrange(max count):
                                                                               for count in xrange(max count):
      reg = 10**uniform(-5, 5)
                                                                                      reg = 10**uniform(-4, 0)
      lr = 10**uniform(-3, -6)
                                                                                     lr = 10**uniform(-3, -4)
                    val acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
                    val acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
                    val acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
                    val acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
                    val acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
                    val acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
                    val acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
                    val acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
                    val acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
                    val acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
                    val acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
                    val acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
                                                                                               But this best
                    val acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
                                                                                               cross-validation result is
                    val acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
                    val acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
                                                                                               worrying. Why?
                    val acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
                    val acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
                    val acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
                                                                                              Careful with best values on
                    val acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
                    val acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
                                                                                              border
                    val acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
```

val acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)

Random search vs. grid search

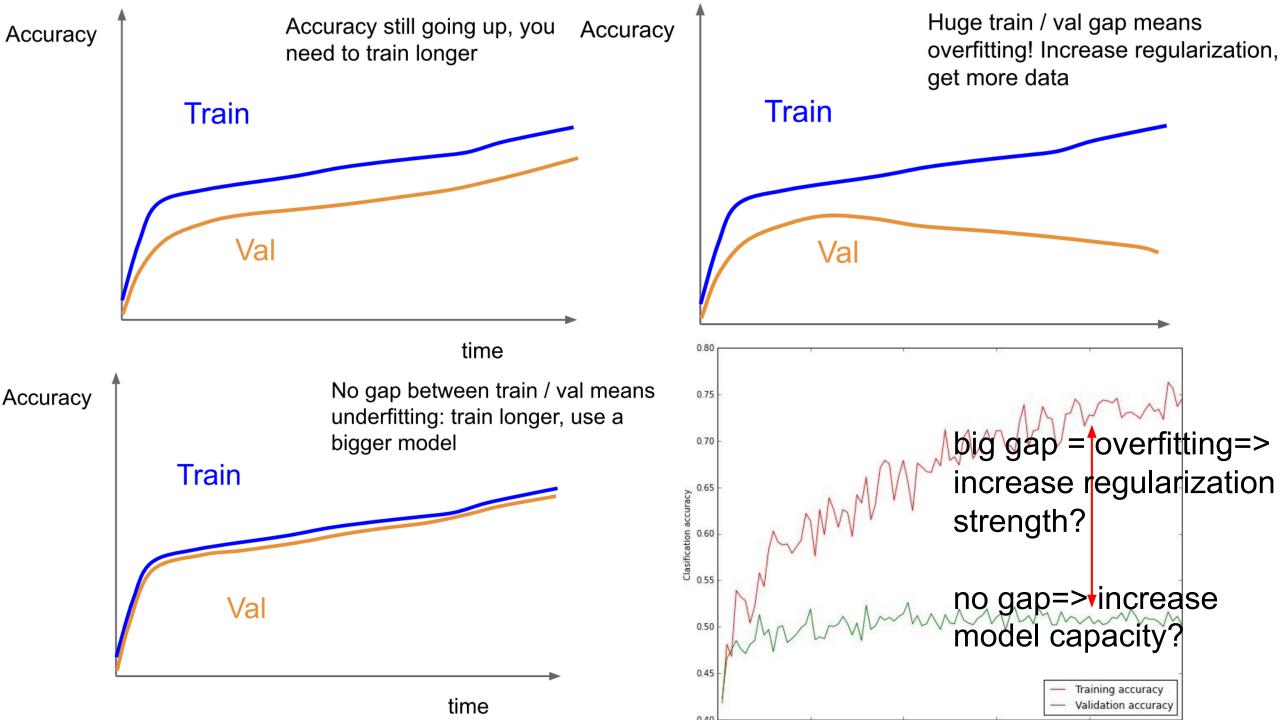
Random Search for Hyper-Parameter Optimization Bergstra and Bengio, 2012



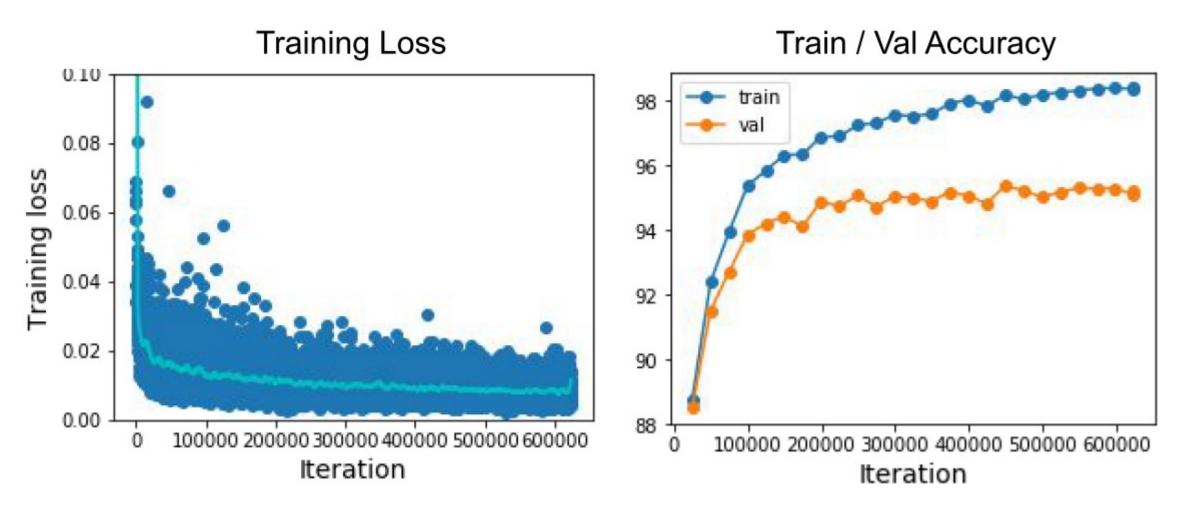
Random search

- Random search is more effective and richly exploring when:
 - The performance may not be such sensitive to the value of all hyperparameters (some parameters are actually much more important)
 - We don't know it in advance
 - More distinct values of the more important hyperparameter are tried.
 - Especially when the number of hyperparameters becomes larger

- Step 1: Check initial loss
- Step 2: Overfit a small sample
- Step 3: Find LR that makes loss go down
- Step 4: Coarse grid, train for ~1-5 epochs
- Step 5: Refine grid, train longer
- Step 6: Look at loss and accuracy curves

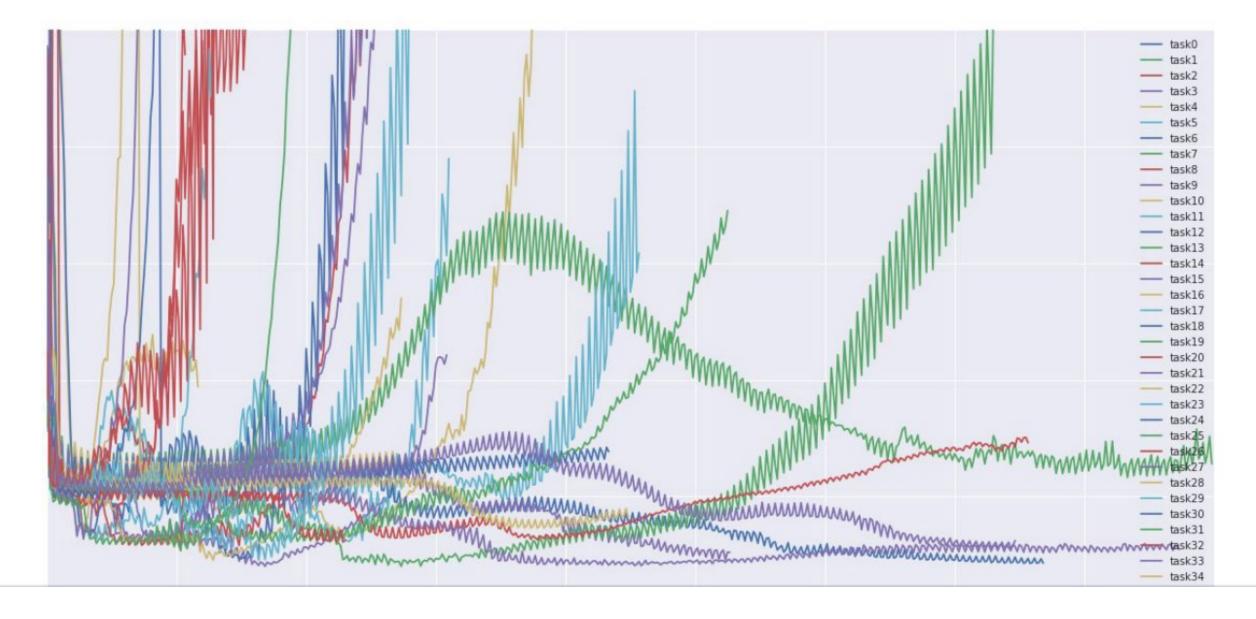


Look at learning curves



Losses may be noisy, use a scatter plot and also plot moving average to see trends better

You can plot all your loss curves for different hyperparameters on a single plot



- Step 1: Check initial loss
- Step 2: Overfit a small sample
- Step 3: Find LR that makes loss go down
- Step 4: Coarse grid, train for ~1-5 epochs
- Step 5: Refine grid, train longer
- Step 6: Look at loss and accuracy curves
- Step 7: Goto Step 5

Hyperparameter selection

- Finding a good set of hyperparameters to provide better convergence
 - initial learning rate α
 - regularization strength (L2 penalty, dropout strength)
 - # of hidden units and # of layers
 - mini-batch size
 - decay schedule (such as the decay constant), update type
 - parameters of optimization algorithms (momentum, adam, ...)
 - These are usually fixed to $\beta_1=0.9$, $\beta_2=0.999$, and $\epsilon=10^{-8}$ or 10^{-7}

Babysitting one model vs. training models in parallel

- When we do not have sufficient computational resources to train a lot of models
 - Watching performance of one model during the time and tune its parameters by nudging them up and down
- Train many different models in parallel (with different hyperparameetrs) and just pick the one that works best

 Babysitting one model called Panda while training many models in parallel called Cavier approach for selecting hyperparameters

Summary

- Activation Functions (use ReLU)
- Data Preprocessing (subtract mean and some times scale according to standard deviation)
- Weight Initialization (use Xavier init)
- Hyper-parameter Optimization (random sample hyperparams, in log space when appropriate)
- Babysitting the learning process

Setting up a problem

- Obtain training data
 - Use appropriate representation for inputs and outputs
- Choose the appropriate loss function
 - Choose regularization
- Choose network architecture
 - More neurons need more data
 - Deep is better, but harder to train
- Choose heuristics (batch norm, dropout, etc.)
- Choose optimization algorithm
 - E.g. Adam
- Perform a grid search for hyper parameters (learning rate, regularization parameter, ...) on held-out data
- Train
 - Evaluate periodically on validation data, for early stopping if required

Resource

- Please see the following notes:
 - http://cs231n.github.io/neural-networks-2/
 - http://cs231n.github.io/neural-networks-3/