Faculty of Sciences
Faculty of Mathematics, Statistics, and Computer Science

Stochastic Process Recitation Session 8

Problem 1. Consider a Poisson process $\{Q(t), t \geq 0\}$ with rate θ, and suppose that each time the event occurs, it is classified as either a type I or a type II event, which occurs with probability p or $1-p$ respectively, independently of all other events. Let $N(t)$ and $M(t)$ denote respectively the number of type I and type II events occurring in $[0, t]$. Note that $Q(t) = N(t) + M(t)$. Prove that $\{N(t), t \geq 0\}$ and $\{M(t), t \geq 0\}$ are both Poisson processes having respective rates θp and $\theta(1-p)$. Furthermore, the two processes are independent.

Problem 2. Let $X_1, X_2, \ldots$ be independent positive continuous random variables with a common density function f, and suppose this sequence is independent of N, a Poisson random variable with mean λ. Define

$$N(t) := \text{number of } i \leq N : X_i \leq t$$

Show that $\{N(t), t \geq 0\}$ is a nonhomogeneous Poisson process with intensity function $\lambda(t) = \lambda f(t)$.

Problem 3. Customers arrive at a single-server queue in accordance with a Poisson process having rate λ. However, an arrival that finds n customers already in the system will only join the system with probability $1/(n+1)$. That is, with probability $n/(n+1)$ such an arrival will not join the system. Show that the limiting distribution of the number of customers in the system is Poisson with mean λ/μ. Assume that the service distribution is exponential with rate μ.

Problem 4. Customers arrive at a two-server service station according to a Poisson process with rate λ. Whenever a new customer arrives, any customer that is in the system immediately departs. A new arrival enters service, first with server 1 and then with server 2. If the service times at the servers are independent exponentials with respective rates μ_1 and μ_2, what proportion of entering customers completes their service with server 2? (i.e., what proportion of entering customers completes both services without being interrupted.)