

# Stochastic Processes I

## Recitation Session 7

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**1.** Consider a two-server system where customers must go through server 1 and then server 2, and where the service time at server  $i$  is exponential with parameter  $\mu_i$ . Customer A arrives and finds server 1 free and two customers at server 2, one in service and one waiting. Find the expected value of the time  $T$  customer A spends in the system.

**2.** There are  $n$  different types of coupons and each time someone collects a coupon, it is independently of type  $i$  with probability  $p_i$ . We want to compute the expected value of the number  $N$  of coupons one needs to collect to have at least one coupon of each type. Assuming that coupons are collected at rate one and letting  $T$  be the time it takes to have the complete collection in this continuous-time setting, prove that:

$$E(N) = E(T) = \int_0^\infty \left( 1 - \prod_{i=1}^n (1 - e^{-p_i t}) \right) dt.$$

**3.** Suppose that passengers arrive at a train station as a Poisson process with rate  $\lambda$ . The only train departs after a deterministic time  $T$ . Let  $W$  be the combined waiting time for all passengers. Compute  $E(W)$ .

**4.** Let  $\{X(t) : t \geq 0\}$  be a Poisson process with rate  $\lambda$  :

**a)** If  $T_1, T_2, \dots, T_n$  are arrival times for the  $n$  events, show that  $\mathbb{P}(T_1 \leq t_1, \dots, T_n \leq t_n \mid X(t) = n)$  is free of  $\lambda$ .

**b)** Let  $T$  be a random variable that is independent of the times when events occur. Suppose that  $T$  has an exponential density with parameter  $\nu$ . Find the distribution of  $X(T)$ , which is the number of events occurring by time  $T$ .