Stochastic Processess I
Recitation Session 7

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1. Consider a two-server system where customers must go through server 1 and then server 2, and where the service time at server i is exponential with parameter μ_i. Customer A arrives and finds server 1 free and two customers at server 2, one in service and one waiting. Find the expected value of the time T customer A spends in the system.

2. There are n different types of coupons and each time someone collects a coupon, it is independently of type i with probability p_i. We want to compute the expected value of the number N of coupons one needs to collect to have at least one coupon of each type. Assuming that coupons are collected at rate one and letting T be the time it takes to have the complete collection in this continuous-time setting, prove that:

$$E(N) = E(T) = \int_0^\infty \left(1 - \prod_{i=1}^{n}(1 - e^{-p_i t})\right) dt.$$

3. Suppose that passengers arrive at a train station as a Poisson process with rate λ. The only train departs after a deterministic time T. Let W be the combined waiting time for all passengers. Compute $E(W)$.

4. Let $\{X(t) : t \geq 0\}$ be a Poisson process with rate λ :

a) If $T_1, T_2, \cdots, T_n$ are arrival times for the n events, show that $\mathbb{P}(T_1 \leq t_1, \cdots, T_n \leq t_n \mid X(t) = n)$ is free of λ.

b) Let T be a random variable that is independent of the times when events occur. Suppose that T has an exponential density with parameter ν. Find the distribution of $X(T)$, which is the number of events occurring by time T.