## Faculty of Sciences
## Faculty of Mathematics, Statistics, and Computer Science

## Stochastic Process Assignment 2

Due Date: Tuesday, 18th March

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Problem 1. simplified model for the spread of a rumor goes this way: There are $N = 5$ people in a group of friends, of which some have heard the rumor and the others have not. During any single period of time, two people are selected at random from the group and assumed to interact. The selection is such that an encounter between any pair of friends is just as likely as between any other pair. If one of these persons has heard the rumor and the other has not, then with probability $\alpha = 0.1$ the rumor is transmitted. Let $X_n$ denote the number of friends who have heard the rumor at the end of the nth period.

Assuming that the process begins at time 0 with a single person knowing the rumor, what is the mean time that it takes for everyone to hear it?

Problem 2. Consider the Markov chain with state space $\{0, 1, 2\}$ and transition matrix:

$$P = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

If $A = \{1, 2\}$ and $X_0 = 0$:

a) Find the distribution of $T_A$. (Reminder: $T_A = \min\{n \geq 1, X_n \in A\}$ is the first hitting time)

b) Compute $\mathbb{E}[T_A]$.

Problem 3. Consider the random walk on the circle. More precisely, there are n points labeled $0, 1, 2, ..., n-1$ orderly and equidistantly placed on a circle. A particle moves from one point to an adjacent point in the manner of a random walk on $\mathbb{Z}$. This gives rise to an *hmc* with the transition probabilities $p_{i,i+1} = p \in (0, 1), p_{i,i-1} = 1 - p$, where by convention state $-1$ is $n - 1$ and state $n$ is 0. Compute the average time it takes to go back to 0 when initially in 0.

Problem 4. Charlie has been arrested and has 2 dollars; he can be free if he has 7 dollars. A police agrees to make a series of bets with him. If Charlie bets $X$ dollars, he wins $X$ dollars with probability 0.3 and loses $X$ dollars with probability 0.7. Find the probability that he wins 7 dollars before losing all of his money if :

a) He bets 1 dollar each time.

b) He bets, each time ,as much as possible but not more than necessary to bring his fortune up to 7 dollars.

c) Which strategy(a or b) gives Charlie the better chance to be free?

Problem 5. In a sequence of independent flips of a coin that comes up heads with probability 0.7, what is the probability that there is a run of three consecutive heads within the first 10 flips?