Faculty of Sciences

Faculty of Mathematics, Statistics, and Computer Science

Stochastic Process Assignment 8

Due Date: Tuesday, 13 May

Problem 1. Let $X(t)$ be a Poisson process with rate λ. Consider the following:

- Find the conditional probability that there are m events in the first s units of time, given that there are n events in the first t units of time, where $0 \leq m \leq n$ and $0 \leq s \leq t$.
- Let T_m denote the time of the mth event. Find the distribution function of T_m.
- Find the density of the random variable T_m in the previous part.

Problem 2. Let T be a random variable that is independent of the times when events occur. Suppose that T has an exponential density with parameter ν:

$$f_T(t) = \begin{cases} \nu e^{-\nu t}, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

- Find the distribution of $X(T)$, which is the number of events occurring by time T, where $X(t)$ is a Poisson process with rate λ.

Problem 3. Suppose that customers arrive at a facility according to a Poisson process having rate $\lambda = 2$. Let $X(t)$ be the number of customers that have arrived up to time t. Determine the following probabilities and conditional probabilities:

(a) $\Pr\{X(1) = 2\}$

(b) $\Pr\{X(1) = 2 \text{ and } X(3) = 6\}$

(c) $\Pr\{X(1) = 2 \mid X(3) = 6\}$

(d) $\Pr\{X(3) = 6 \mid X(1) = 2\}$

Problem 4. Shocks occur to a system according to a Poisson process of intensity λ. Each shock causes some damage to the system, and these damages accumulate. Let $N(t)$ be the number of shocks up to time t, and let Y_i be the damage caused by the ith shock. Then

$$X(t) = Y_1 + \cdots + Y_{N(t)}$$

is the total damage up to time t. Suppose that the system continues to operate as long as the total damage is strictly less than some critical value a, and fails otherwise.

- Determine the mean time to system failure when the individual damages Y_i have a geometric distribution with

$$\Pr\{Y = k\} = p(1-p)^k, \quad k = 0, 1, 2, \ldots$$

Problem 5. Customers arrive at a store according to a non-homogeneous Poisson process with rate function $\lambda(t) = 3 + 2t$ (customers per hour), for $0 \leq t \leq 4$, where t is time in hours after opening.

(a) Find the expected number of customers that arrive during the first 4 hours of business.

(b) Find the probability that exactly 10 customers arrive in the first 2 hours.

(c) Given that 5 customers arrived in the first hour, what is the probability that 2 arrived in the first 30 minutes?

(d) Let T_1 be the time of the first customer arrival. Find the probability density function of T_1.